Electric Charge in Topological Residual Theory (TRT): The Cylindrical Helix Angle, the Space-Time Coupling Baseline $G\varepsilon_0$, and the Charge Formula $q = 2\alpha/c^2$

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Abstract Taking as a postulate that "space undergoes right-handed cylindrical helical motion at the speed of light c," this paper presents a geometric formulation of electric charge. The helix angle θ is defined as the angle between the tangent and the helix axis; it describes how the speed of light is partitioned between the azimuthal and axial directions. The fine-structure constant is the sine of the helix angle, $\alpha = \sin\theta$, with $\theta = 0.41811°$, and the path ratio is $\Omega = \csc\theta = 1/\alpha$. With c and the helix radius r held fixed, the helix angle θ is the sole degree of freedom of the helix. Treating $G\varepsilon_0$ as a single entity, termed the space-time coupling baseline, and adopting the ratio of geometric coefficients $k/k' = \Omega^2c^2$ as the fundamental relation, local energy balance yields $G\varepsilon_0(\alpha) = \alpha^2/c^2$. Taking the first derivative with respect to the geometric quantity $\sin\theta$ of the helix angle gives the charge formula $q = d(G\varepsilon_0)/d(\sin\theta) = 2\alpha/c^2$. Spatial helices are uniformly right-handed; the helical flow of a positive charge is outward-diverging, whereas that of a negative charge is inward-converging. The total charge is $Q = nq$, where n is the helical winding number; the number and the sign of charges are topological invariants, while the magnitude q of each unit is bound to α. Numerically, $G = \mu_0\alpha^2$ deviates from the experimental G by 0.2616%, five times which is 1.3082%, whereas $2\alpha/c^2$ deviates from the elementary charge e by 1.3546%; the two differ by 0.046 percentage points. The charge formula gives $q \propto \alpha$; as α runs from its low-energy value 1/137.036 to 1/127.930 at the Z-boson energy scale, the unit charge q increases by 7.12%, which is consistent with the strengthening of the electromagnetic interaction at high energies. On this basis, one falsifiable prediction and one testable conjecture are proposed: the prediction is that an accelerating positive charge produces an induced gravitational field directed opposite to its acceleration; the conjecture is that

secondary H and He fragments traversing the Bragg peak region of carbon ions exhibit velocity and stopping anomalies.

Keywords: Topological Residual Theory (TRT); cylindrical helix; helix angle; fine-structure constant; space-time coupling baseline; charge formula; topological invariant; running of the fine-structure constant

1. Introduction

This paper addresses why electric charge is quantized and why its magnitude is determined by the fine-structure constant α. In existing theory, the elementary charge e, the gravitational constant G, the vacuum permittivity ε_0, and α can only be determined experimentally [1], and whether any geometric connection exists among them remains an open question. These constants are now measured with extremely high precision: $\alpha^{-1} = 137.035999177$, the relative uncertainty of G is about 2.2×10^{-5}, and since the 2019 revision of the International System of Units, e has become a defined value [1]. This provides a reliable numerical basis for testing structural relations among the constants.

Electric charge has two seemingly unrelated attributes. The first is its quantization and conservation: the charge of any free particle is an integer multiple of the elementary charge, and the number of units is conserved in every process. The second is its magnitude: the strength of the electromagnetic interaction is characterized by α, and α itself runs with the energy scale. Existing theory treats these two attributes separately, attributing the former to gauge symmetry and the latter to an experimentally determined coupling constant. This paper attempts to account for the origin of both attributes within a single geometric picture.

Xiangqian Zhang's unified field theory holds that space itself undergoes right-handed cylindrical helical motion at the speed of light, and that gravitation, electromagnetism, and the mass and charge of objects are all geometric manifestations of this motion [2,3]; Topological Residual Theory (TRT) further holds that physical constants are the residuals left by the helical motion under geometric and topological constraints [4]. Within this framework, the answer given in this paper is as follows: α is the azimuthal share of the velocity determined by the helix angle; $G\varepsilon_0$, taken as a whole, is the function α^2/c^2 of α; charge is the first-order response $2\alpha/c^2$ of $G\varepsilon_0$ to the helix angle quantity $\sin\theta$, with the magnitude of each unit bound to α; and the number and sign of charges are given by the helical winding number, a topological invariant.

The paper is organized as follows: Section 2 obtains α from the helical geometry; Section 3 derives the charge formula, gives its stiffness interpretation, and explains its topological properties; Section 4 presents a numerical comparison; Section 5 discusses the running of α; Section 6 proposes a falsifiable prediction and a conjecture; Section 7 presents the conclusions.

2. Helical Geometry and α

2.1 The cylindrical helix postulate

The space surrounding any isolated particle undergoes right-handed cylindrical helical motion with resultant speed c. Let the helix radius be r, the pitch h_p, the frequency f, and the angular frequency ω = 2πf. Taking the helix axis as the z axis, the trajectory of a point of space is

$$x = r\cos\omega t, \quad y = r\sin\omega t, \quad z = v_z t \qquad (1)$$

The azimuthal velocity, the axial velocity, and the resultant velocity satisfy

$$v_\varphi = 2\pi r f, \qquad v_z = h_p f, \qquad v_\varphi^2 + v_z^2 = c^2 \qquad (2)$$

The resultant speed is always c; as the azimuthal component increases, the axial component must decrease. The speed of light thus acts as the motion budget of a point of space, which can only be apportioned between the azimuthal and axial directions. Spatial helices are uniformly right-handed; the sign of a charge arises not from chirality but from whether the helical flow is outward-diverging or inward-converging (§3.4). Around a particle, every radial line is the axis of a local helix, and the helical flow advances along the radial direction.

2.2 The helix angle

The angle between the tangent to the helix and the helix axis is called the helix angle θ. The ratio of the actual path length per turn, c/f = 2πr/sin θ, to the circumference 2πr is called the path ratio Ω, so that

$$\Omega = \frac{c}{2\pi r f} = \frac{c}{v_\varphi} = \csc\theta \qquad (3)$$

2.3 Definition of α

The fine-structure constant is defined as the sine of the helix angle:

$$\alpha \equiv \sin\theta = \frac{v_\varphi}{c}, \qquad \frac{1}{\alpha} = \Omega \qquad (4)$$

Taking $\alpha^{-1} = 137.035999177$ [1] gives $\theta = 0.41811°$. A point of space advances along the axis at nearly the speed of light while rotating about the axis at αc; numerically, αc equals the orbital speed of the electron in the ground state of the hydrogen atom.

When both c and r are held fixed, the helix retains only one degree of freedom, the helix angle θ. The measured value of α is a constant, but in the helical geometry it is a function of the helix angle, sin θ. In what follows, differentiation is performed with respect to the geometric quantity sin θ of the helix angle, not with respect to a constant: with c and r fixed, the helix angle is allowed to undergo an infinitesimal change dθ, the geometric quantity sin θ changes accordingly by $d(\sin\theta) = \cos\theta\, d\theta$, and we examine how physical quantities respond. For brevity, we write $d\alpha \equiv d(\sin\theta)$. At low energies, α takes its experimental value α(0); in the high-energy and accelerated cases, what is discussed is the effective helix angle θ_{eff} and $\alpha_{eff} = \sin\theta_{eff}$.

TRT gives a geometric number Ω_{geo} for the path ratio, whose relative difference from α^{-1} is 2.22×10^{-6} [4]:

$$\Omega_{geo} = 4\pi^3 + \pi^2 + \pi = 137.036304 \qquad (5)$$

3. The Charge Formula

3.1 The space-time coupling baseline $G\varepsilon_0(\alpha)$

Helical motion carries energy. The azimuthal rotation of the helix is equivalent to a simple harmonic oscillation with characteristic speed rω, and the energy density is proportional to $(r\omega)^2 \propto (rf)^2$. The space around a particle simultaneously exhibits two tendencies, gravitational and electromagnetic, with energy densities ρ_G and ρ_E respectively, both containing the same quadratic factor $(rf)^2$. At local equilibrium,

$$\rho_G = kG(rf)^2 = \rho_E = \frac{k'(rf)^2}{\varepsilon_0} \quad \Rightarrow \quad G\varepsilon_0 = \frac{k'}{k} \qquad (6)$$

Taking $k/k' = \Omega^2c^2$ as the fundamental relation of TRT, we obtain

$$G\varepsilon_0(\alpha) = \frac{1}{\Omega^2c^2} = \frac{\alpha^2}{c^2}, \qquad G = \mu_0\alpha^2 \qquad (7)$$

Because the two energy densities contain the same quadratic factor, the equilibrium condition depends only on G, ε_0, and the ratio of the geometric coefficients; hence G and ε_0 appear together as a single entity in the form of the

product $G\varepsilon_0$. $G\varepsilon_0$ represents the background quantity at which gravitation and electromagnetism reach local equilibrium, and we call it the space-time coupling baseline. It relates the gravitational strength and the electromagnetic strength to one and the same geometric quantity; that is, the gravitational constant equals the vacuum permeability multiplied by α^2. With c and r fixed, the space-time coupling baseline is a single-variable function of α.

3.2 Charge: the first-order response to the helix angle

A change in the helix angle redistributes the budget of the helix between the azimuthal and axial directions, and the space-time coupling baseline responds accordingly. The first-order coefficient of this response is the unit charge:

$$q = \frac{\mathrm{d}(G\varepsilon_0)}{\mathrm{d}(\sin\theta)} = \frac{2\sin\theta}{c^2} = \frac{2\alpha}{c^2} \qquad (8)$$

The unit charge measures the sensitivity of the space-time coupling baseline to the helix angle: for each small change in sin θ, the baseline changes by q times that small amount. Under the SI numerical convention, $G\varepsilon_0$ is compared with α^2/c^2, and q with e, numerically.

3.3 Stiffness of the space-time coupling baseline

In what follows, a mechanical analogy is drawn for Eqs. (7) and (8); this is an interpretation rather than a derivation.

The path length c traversed by a point of space per unit time is fixed and can be likened to a spring wire of fixed length; $\alpha = 1/\Omega$ is the fraction of the path length wound into circles. When α = 0, the streamlines are straight lines along the axis, there is no azimuthal motion, and the space-time coupling baseline vanishes; as α increases, the helix winds more tightly (for fixed r, the pitch $h_p = 2\pi r \cot\theta$ decreases). At θ = 0.41811°, $h_p \approx 861r$, so this “spring” is an extremely sparse open-coil helix. In this sense, α can be regarded as the winding deformation of the helix, and Eq. (7) can be written in the standard form of an elastic potential

$$G\varepsilon_0(\alpha) = \frac{1}{2}K\alpha^2, \qquad K \equiv \frac{\mathrm{d}^2(G\varepsilon_0)}{\mathrm{d}(\sin\theta)^2} = \frac{2}{c^2} \qquad (9)$$

The charge formula (8) is then the generalized force conjugate to α:

$$q = K\alpha \qquad (10)$$

Eq. (10) has the same form as Hooke's law: q is the load required to maintain the winding deformation α, equal in magnitude and opposite in direction to the

elastic restoring force $-K\alpha$. K is the stiffness of the space-time coupling baseline with respect to winding deformation; it is constant when c and r are fixed and is the same for all particles and at all energy scales, and it is distinct from the geometric coefficients k and k′ in Eq. (6). The charge and the stiffness are, respectively, the first and second derivatives of the same function α^2/c^2. $G\varepsilon_0$ does not have the dimensions of energy, so the "elastic potential" is only an analogy; the dimensional convention is the same as in §3.2.

3.4 The number of charges is a topological invariant

On any closed spherical surface S enclosing the particle, the orientation of the helical streamlines defines a unit direction field $\hat{m}$, whose winding number is

$$n = \frac{1}{4\pi}\oint_S \hat{\boldsymbol{m}} \cdot \left(\partial_u \hat{\boldsymbol{m}} \times \partial_w \hat{\boldsymbol{m}}\right) \mathrm{d}u\, \mathrm{d}w \qquad (11)$$

where (u, w) are any oriented coordinates on S (e.g., the polar and azimuthal angles of the sphere). The integrand is the oriented area element swept out by the direction field on the unit sphere, and n counts the oriented (signed) number of times the direction field covers the unit sphere. The integral (11) is the degree of the map $\hat{m}: S \to S^2$, and the area (total curvature) of the unit sphere, 4π [5], is the normalization factor; since the degree of a map is a topologically invariant integer, n can take only integer values. By §2.1, $\hat{m}$ is radial; chirality does not distinguish the sign of the charge. The helical flow of a positive charge diverges radially outward, so $\hat{m}$ points along the outward normal and $n = +1$; the helical flow of a negative charge converges inward, so $\hat{m}$ points opposite to the outward normal and $n = -1$. Any continuous deformation of the direction field, including displacement of the enclosing surface, local perturbations of the helical flow, and motion of the particle as a whole, can only change the integral continuously, while the integral can take only integer values; therefore n is conserved: whether the particle is at rest, in uniform motion, or accelerating, the configuration of the helical flow changes continuously, but the winding number n remains unchanged. This is consistent with the fact that atoms remain electrically neutral even in high-speed motion. The total charge is

$$Q = nq = \frac{2n\alpha}{c^2} \qquad (12)$$

n gives the quantization and conservation of the number of charge units, while the magnitude q of each unit is given by α and is bound to α.

4. Numerical Comparison

Among the fundamental constants, G is measured with the lowest precision, whereas μ_0 and α are known to several orders of magnitude higher precision. By Eq. (7), the theoretical value of G is $\mu_0\alpha^2$, and its deviation from the experimental G reflects the basic residual of the helical geometry. Taking this deviation as the basic residual unit, we compare it with the deviation of Eq. (8) from e:

$$\delta_G = \frac{\mu_0\alpha^2}{G} - 1, \qquad \delta_q = \frac{2\alpha/c^2}{e} - 1 \qquad (13)$$

The constants are taken from the CODATA 2022 recommended values [1]: c = 299792458 m/s and e are defined values, $\mu_0 = 1.25663706127\times10^{-6}$ N/A^2, and G = 6.67430×10^{-11} m^3/(kg·s^2). Both theoretical values use only the high-precision constants α, μ_0, and c, with no adjustable parameters. The results are given in Table 1.

Table 1 Numerical comparison of the charge formula and the gravitational relation

Quantity	Theoretical value	Experimental value	Deviation
Gravitational constant	$\mu_0\alpha^2 = 6.69176\times10^{-11}$	$G = 6.67430\times10^{-11}$	$\delta_G = +0.2616\%$
Five basic residual units	—	—	$5\delta_G = 1.3082\%$
Unit charge	$2\alpha/c^2 = 1.62388\times10^{-19}$	$e = 1.602176634\times10^{-19}$	$\delta_q = +1.3546\%$
Difference	—	—	$\delta_q - 5\delta_G = 0.0464$ percentage points

$\delta_G = 0.2616\%$ is far larger than the relative measurement uncertainty of G, 2.2×10^{-5} (0.0022%); it is a systematic difference between the theoretical and experimental values rather than a measurement error. Equivalently, the experimental value of $G\varepsilon_0$ is 0.2610% smaller than α^2/c^2. The deviation of the charge formula, 1.3546%, differs from the sum of five basic residual units, 1.3082%, by only 0.046 percentage points, about 3.4% of the deviation itself. Numerically, $\delta_q \approx 5\delta_G$ (ratio 5.18); the geometric origin of this multiple is left for future work. This correspondence can be tested directly as measurements of G progress: once the experimental value of G is updated, the basic residual unit

changes accordingly, and whether the charge deviation remains approximately five times this unit serves as the criterion.

5. Running of α and the Strengthening of the Electromagnetic Interaction

The effective fine-structure constant α_eff measured in high-energy scattering increases with the energy scale. According to PDG 2024, at the Z-boson mass scale $\alpha^{-1}(M_Z) = 127.930$ (MS-bar scheme) [6]; compared with the low-energy value,

$$\frac{q(M_Z)}{q(0)} = \frac{\alpha(M_Z)}{\alpha(0)} = \frac{137.036}{127.930} = 1.0712 \qquad (14)$$

that is, from low energies to M_Z, the unit charge q increases by 7.12%. From the atomic scale to the Z-boson energy scale of 91 GeV, the probing distance decreases by about eight orders of magnitude, whereas α changes by only a few percent, indicating that the running is a slow, logarithmic variation. In the helical picture, this corresponds to a slow change in the helix angle, the sole degree of freedom: as the probing energy scale increases, the effective helix angle θ_eff increases from 0.41811° to about 0.44787°, the azimuthal share sin θ_eff increases accordingly, and the helix winds slightly more tightly; the winding number n is unchanged, so charge quantization is unaffected. For comparison, quantum electrodynamics explains this phenomenon as the weakening of the vacuum-polarization screening of the bare charge as the probing distance decreases [7]. Since $q \propto \alpha$ by Eq. (8), the running of α translates directly into a proportional change in the unit charge, consistent with the strengthening of the electromagnetic interaction at high energies.

6. Falsifiable Prediction and Conjecture

Extending the first-order response of charge to the accelerated case yields one falsifiable prediction and one testable conjecture.

6.1 Reversed gravitational field of an accelerating positive charge

A charge at rest corresponds to a steady state of α. When the charge accelerates, the steady-state arrangement of the helical flow is disrupted, and the arrangement and the helix angle undergo a collective response, $\theta \to \theta + \delta\theta(t)$, so

that α becomes α + δα(t), and the space-time coupling baseline deviates from its background value accordingly:

$$\delta(G\varepsilon_0) = q\,\delta\alpha + \frac{1}{2}K\,\delta\alpha^2 \qquad (15)$$

The coefficient of the first-order term is the charge q, and the coefficient of the second-order term is the stiffness K of §3.3; since $G\varepsilon_0$ is a quadratic function of α, this expansion is exact. In rest and uniform motion δα is zero and the baseline retains its background value; only acceleration causes the baseline to deviate, and the deviation varies with the magnitude of the acceleration. This deviation manifests itself as an induced gravitational field: gravitation and electromagnetism are unified in the space-time coupling baseline, so the baseline perturbation caused by the acceleration of a charge appears in the form of gravitation. The helical flow of a positive charge is outward-diverging (n = +1). When the particle accelerates along a, the radial line along the direction of a is the axis of a local helix (§2.1); there, the share of the outward-diverging helical flow that advances along the axis is compressed, and the compressed budget is released along the −a direction, so the induced field is directed opposite to the acceleration:

$$\boldsymbol{g}_{\text{ind}} = -\,n\chi\,\boldsymbol{a}\,f(r/\lambda) \qquad (16)$$

where χ > 0 is the collective response coefficient, f is a spatial attenuation function, and λ is a characteristic length scale, all of which are to be determined experimentally. This paper considers only positive charges (n = +1). The winding number n remains unchanged during this process; acceleration changes only the local modulation of the helix angle θ, with α and q varying accordingly.

6.2 Conjecture: velocity and stopping anomalies of secondary fragments in the Bragg peak region of carbon ions

The deceleration of ^{12}C ions in water is given by the electronic stopping power S:

$$a = \frac{S}{\gamma_{\mathrm{L}}^3 m} \qquad (17)$$

where γ_L is the Lorentz factor and m = 1.992×10^{-26} kg. For S, the Bethe formula is used in the high-energy range, and in the low-energy range the proton stopping power multiplied by the square of the effective charge is used; the results are given in Table 2.

Table 2 Estimated deceleration of ^{12}C ions in water

Energy (MeV/u)	S (MeV/cm)	Deceleration a (m/s^2)
400 (incident)	109	3.0×10^{16}
200	161	7.2×10^{16}
10	1.6×10^{3}	1.3×10^{18}
1 (peak region)	$\approx 6.4\times10^{3}$	$\approx 5.1\times10^{18}$
0.5 (peak tip)	$\approx 7.5\times10^{3}$	$\approx 6.0\times10^{18}$

Near the Bragg peak the deceleration reaches 10^{18}–10^{19} m/s^2, two to three orders of magnitude greater than in the entrance region. Carbon ions are positively charged, and during deceleration the acceleration points opposite to the direction of incidence; according to Eq. (16) (n = +1), the induced field is directed opposite to the acceleration, i.e., along the direction of incidence, and is strongest in the peak region. We conjecture that the velocity distributions and electromagnetic stopping of H and He fragments traversing the peak region deviate from standard expectations.

The proposed test is as follows. Haettner et al. measured fragment energy spectra and yields of 200 and 400 MeV/u ^{12}C at 7 water depths, covering the entrance region, the peak region, and the tail region [8]; Gunzert-Marx et al. reported the energy spectra of p, d, t, and He at 200 MeV/u [9]. Both data sets have been published, so a first-round test can be carried out without new beam experiments. The standard expectations are computed with Geant4 and FLUKA under identical conditions, the residual R(E) = measurement/simulation − 1 is constructed, and comparisons are made by grouping fragments according to whether they traverse the depth at which the peak region lies. According to this conjecture, the residuals in the peak region carry a directional offset along the direction of incidence (higher velocity or lower stopping), whereas those in the entrance region do not, and the offset scales with the deceleration. If the peak-region residuals are statistically indistinguishable from the entrance-region residuals, or if the offset is opposite to the direction of incidence, the conjecture is refuted. Uncertainties of nuclear models appear at all depths; only residuals concentrated in the peak region and scaling with the deceleration support this conjecture. For 100 MeV/u H and He fragments in water, the stopping power per nucleon is S/A $\approx$ 7.3 MeV/cm (Bethe formula), and Eq. (17) ($\gamma_L^3 \approx 1.36$) gives a stopping deceleration of about 5.2×10^{16} m/s^2. For the induced field χa in the peak region ($a \approx 5.1\times10^{18}$ m/s^2) to reach 1% of the fragment stopping deceleration, $\chi \gtrsim 0.01\times5.2\times10^{16}/(5.1\times10^{18}) \approx 1.0\times10^{-4}$ is required (taking $f \approx 1$); this is the test threshold for χ.

7. Conclusions

Taking the light-speed cylindrical helical motion as a postulate, this paper has established a geometric chain from the helix angle to electric charge: the helix angle determines the fine-structure constant $\alpha = \sin\theta$, and the helix angle is the sole degree of freedom of the helix; the space-time coupling baseline is $G\varepsilon_0(\alpha) = \alpha^2/c^2$; charge is the first-order response of the baseline to the helix angle quantity $\sin\theta$, $q = 2\alpha/c^2$, whose number and sign are given by the winding number and are topological invariants, while the magnitude q of each unit is bound to α, with $Q = nq$. The deviation of the charge formula from e, 1.3546%, differs by 0.046 percentage points from five times the deviation of the gravitational relation, 1.3082%; since $q \propto \alpha$, the unit charge q increases by 7.12% as α runs to the M_Z energy scale, consistent with the strengthening of the electromagnetic interaction at high energies. This paper proposes one falsifiable prediction and one testable conjecture: an accelerating positive charge produces an induced gravitational field directed opposite to its acceleration; and secondary H and He fragments traversing the Bragg peak region exhibit velocity and stopping anomalies.

In this picture, the quantization and conservation of charge originate from topology, and its magnitude from geometry. Three tasks remain for future work: first, to determine the collective response coefficient χ through residual analysis of fragment energy spectra; second, to establish the geometric origin of the multiple $\delta_q \approx 5\delta_G$ and of the remaining 0.046-percentage-point deviation; and third, to test, as precision measurements of G progress, the five-fold relation between the charge deviation and the basic residual unit.

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