

The Floating Sun God of Konark

Earnshaw's Theorem, the Stability of a Boat, and a Magnetic Arrangement
that Could Hold a Statue in Mid-Air

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Abstract

Oral tradition about the thirteenth-century Sun Temple at Konark, Odisha, says that its image of the Sun God once floated in the air, held up by a great lodestone set at the top of the tower. The same tradition says this lodestone deflected the compasses of passing ships until sailors removed it. We first review what the historical record does and does not say. The levitation story is a nineteenth-century tradition. It belongs to a long family of similar tales, from Pliny's unfinished iron statue of Arsinoe to al-Qazwini's suspended idol of Somnath.

We then ask the physical question posed by the first author: *what arrangement of magnets could make such a statue float stably?* Earnshaw's theorem, extended by Braunbek, forbids stable levitation by any static arrangement of permanent magnets, soft iron and gravity. We prove the relevant forms of the theorem, including the sum rule $k_x + k_y + k_z = 0$ for the stiffnesses, and the extension to orientation and to induced magnetism.

Following the hint "what gives stability to a boat?", we analyse hydrostatic stability: buoyancy, heave stiffness $\rho g A_w$ and the metacentre. We show that buoyancy evades Earnshaw because it is a contact force exerted by matter, not a field in free space. The magnetic analogue is a *magnetic boat*. Lift comes from a magnet overhead, acting at the statue's crown, above a low centre of gravity; this plays the role of the metacentre. A pair of diamagnetic plates (graphite or bismuth) plays the role of the water and supplies the missing vertical stiffness.

We derive the complete set of stability conditions, evaluate them for lodestone, ferrite, samarium-cobalt and neodymium-iron-boron magnets, and give a worked design. A neodymium arrangement holds a 10 kg statue 15 cm below a 20 cm magnet, with graphite plates at gaps of about 12 mm. Lodestones fall short of the required forces by two to six orders of magnitude. A 52-tonne lodestone would also have deflected a compass by less than about 10^{-3} degree at one kilometre, even if fully magnetised, so the sailors' story cannot be literally true. Finally we survey the dynamic loopholes to Earnshaw: spin stabilisation, feedback, superconductivity and oscillating fields.

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1 Introduction

Few problems in elementary physics have so clean a negative answer as the levitation of one magnet by others. In 1842 Samuel Earnshaw showed that a collection of particles interacting through inverse-square forces cannot rest in stable equilibrium [11]. The argument carries over directly to permanent magnets in a static magnetic field. In 1939 Werner Braunbek extended it to every body with positive magnetic susceptibility, such as soft iron or a lodestone, placed in any static arrangement of magnets [12]. Gravity, being itself an inverse-square field, does not help.

The consequence is easily stated. Hang a magnet below another and adjust the distance until the pull balances the weight. The balance is unstable: if the lower magnet rises slightly it is pulled up harder and jumps to the upper magnet; if it sinks slightly the pull weakens and it falls.

Yet tradition around the world is full of objects said to hang in the air by magnetism. Iron statues are said to have hung in temples at Alexandria and Magnesia, and an idol at Somnath. The coffin of the Prophet was said to hang at Medina, and the image of the Sun God at Konark. Lowe [1] has collected and analysed the Greco-Roman and medieval examples. The Konark version adds a nautical detail. The lodestone that held the image was said to be so strong that it disturbed the compasses of ships passing along the coast of Odisha, until sailors climbed the tower and carried it away [13, 14].

This paper takes the first author's question seriously as a problem in physics: *given Earnshaw's theorem, what arrangement of magnets would make the statue float stably?* The question came with a hint: *what gives stability to a boat?* We will see that the hint is exactly right. A floating boat is a body in stable equilibrium under two forces, its weight and the buoyancy of the water. It is stable in the vertical direction (heave) and against tilting (roll and pitch), and neutral horizontally and in yaw. It escapes Earnshaw's theorem because buoyancy is not the gradient of a harmonic potential. It is a pressure force exerted by matter, and it weakens as the boat rises out of the water. A "magnetic boat" needs the same ingredient: a material that pushes back harder the closer the magnet comes. Diamagnetic materials do exactly this [9, 12, 15, 16].

The plan of the paper is as follows. Section 2 reviews the historical record of the Konark tradition and its relatives. Section 3 tests the sailors' story quantitatively: how far from a 52-tonne lodestone would a compass notice it? Section 4 proves Earnshaw's theorem in the forms we need. Section 5 analyses the stability of a floating body, including the metacentre. Section 6 builds the magnetic boat and derives its stability conditions. Section 7 puts in numbers for lodestones and for modern magnets. Section 8 surveys the dynamic ways around the theorem. Section 9 returns to Konark, and Section 10 concludes. Appendices collect the longer derivations.

2 The Konark tradition and the historical record

2.1 The temple

The Sun Temple at Konark stands on the coast of Odisha, about 35 km north-east of Puri. It was built around 1250 CE under Narasimhadeva I of the Eastern Ganga dynasty (r. 1238–1264) [17–19]. The whole temple is conceived as the chariot of the Sun God Surya. Twenty-four carved stone wheels line the platform, and seven horses draw it. The original sanctum tower (*rekha deula*) is estimated to have risen to about 70 m. It collapsed in the nineteenth century; the standard modern account dates the main fall to 1837, with a further collapse in 1848 [17]. The surviving assembly hall (*jagamohana*) was filled with sand and sealed in 1903 to prevent its collapse. The Archaeological Survey of India began removing the sand in 2022 [17]. The temple was inscribed as a UNESCO World Heritage Site in 1984.

The temple was well known to Mughal administrators. Abu'l Fazl's *Ain-i-Akbari* (1590s) describes it as a site that astonished its visitors. He records the tradition that the revenue of twelve years was spent on its construction [13, 20]. Early British surveyors described it in detail, notably Andrew Stirling in his 1825 account of Orissa [21], James Fergusson in his architectural illustrations [22], and Rajendralala Mitra in *The Antiquities of Orissa* [23]. Later came M. M. Ganguly [24] and the detailed modern studies of Boner [19] and Behera [18].

Iron was used extensively in the structure. Wrought-iron beams up to about 35 ft (10.7 m) long were forged from short segments and used to support the roofing, and iron clamps tie the stone blocks [13]. We shall see in Section 3 that this structural iron is magnetically comparable to the legendary lodestone.

2.2 The floating image and the lodestone

In its usual modern form, the tradition has three parts [13, 14]:

1. The image of the Sun God in the sanctum floated in the air without visible support. It contained iron and was held by magnets.
2. The chief magnet was a great lodestone set at or near the top of the tower. Popular retellings give its mass as 52 tonnes. Iron plates or further magnets in the walls kept the image in place.
3. The lodestone was so powerful that it disturbed the compasses of ships passing along the coast. Sailors therefore removed it. In different versions they are Portuguese, or "Mughal" (Muslim) sailors whose ships had been troubled. The removal led to the collapse of the tower, and the priests carried the deity to Puri.

Bahinipati, Biswal and Suar [13] trace the sailors' version to Stirling's 1825 account [21]. In that version discontented Muslim sailors removed the stone, and the priests then migrated to Puri. They also note the alternative version, that the image was "suspended in the air through the use of magnets", which they regard as lacking affirmative evidence. Lowe [1] classifies the Konark story as a nineteenth-century oral tradition, in which the capstone magnet is said to have been destroyed by Portuguese or by British agents. We are not aware of any medieval Odia inscription, entry in the *Madala Panji* (the temple chronicle of Puri), or European navigational record that describes either a floating image or a compass disturbed at Konark, and the 52-tonne figure appears to come only from modern retellings. We therefore treat the story as a tradition worth testing physically, not as a historical fact.

2.3 Sailors, the "Black Pagoda", and compasses

Konark certainly mattered to sailors, though not for the reason the tradition gives. Its dark tower was a conspicuous landmark from the Bay of Bengal. European accounts called it the *Black Pagoda* as early as 1676, in contrast to the whitewashed Jagannath temple at Puri, the *White Pagoda* [17, 18]. Both appear on European coastal charts. Ships coasting between the Hooghly and the Coromandel coast used them to fix their positions.

The magnetic compass reached the Bay of Bengal long before the Portuguese. A clear description of the magnetised needle, including magnetic declination, appears in Shen Kuo's *Dream Pool Essays* (1088). Chinese texts record its use at sea by the early twelfth century [25]. In Europe it is described by Alexander Neckam (c. 1190) and analysed by Petrus Peregrinus (1269) [26]. Arab and Indian Ocean pilots used it by the later Middle Ages. Vasco da Gama

reached Calicut in 1498, and the Portuguese were active on the coast of Bengal and Odisha in the sixteenth century. So the chronology of the sailors' story is at least possible: ships passing Konark could have carried compasses.

The physics is another matter (Section 3). It is worth noting that iron ships and iron cargoes *do* disturb compasses. The problem was severe enough that Airy devised his famous corrections for iron ships in 1839 [27], and navigators still swing their compasses to tabulate deviation [28]. A navigator who found his compass misbehaving near a coast would have had many local causes to blame before a temple ashore.

2.4 Magnetic suspension stories elsewhere

The Konark tradition is one of a large family. Table 1 lists the principal members, following Lowe's survey [1]. Two features recur. First, an iron image is held by a lodestone in the ceiling, sometimes with a second one in the floor. Augustine describes a statue suspended between a magnet above and one below [29]; that arrangement is exactly unstable, for the reasons proved in Section 4. Second, sailors and ships appear: Ptolemy's Maniolae islands, whose magnetic rocks were said to hold back ships fastened with iron nails [30], and the magnetic mountain of the *Thousand and One Nights*, which pulls the nails out of a ship [31]. Sceptics appear too. Thomas Browne in 1646 rejected the tale of the Prophet's suspended coffin [32], and William Gilbert's *De Magnete* (1600), the founding text of experimental magnetism, set out how weak lodestones actually are [33].

The Somnath and Konark stories are both Indian, and the Somnath story is roughly contemporary with the building of Konark. It is notable that al-Qazwini's Somnath idol, like Augustine's iron statue, is described as held by magnets on several sides. That is precisely the intuitive design, and it is precisely what Earnshaw forbids.

2.5 What the record establishes

Three conclusions can be drawn from the historical record.

1. The temple was a major navigational landmark, the "Black Pagoda" of European sailors.
2. The floating image and the compass-disturbing lodestone are attested only in later tradition, first recorded in the nineteenth century.
3. The story belongs to an ancient and widespread family of magnetic-suspension legends, several of which describe arrangements that are physically unstable.

None of this settles whether the builders of Konark attempted something remarkable with magnets. Physics, however, can say what such an attempt would have required. To that we now turn.

3 Could a lodestone have deflected the ships' compasses?

The sailors' story makes a quantitative claim: a magnet in the temple disturbed compasses at sea. This claim can be tested with nothing more than the field of a magnetic dipole and the strength of the Earth's field.

Table 1: Accounts of objects said to be suspended by magnets. Classical and medieval entries follow Lowe [1].

Source	Date	Account
Pliny, <i>NH</i> 34.148 [34]	1st c. CE	The architect Timochares began to roof a temple of Arsinoe at Alexandria with lodestone, so that her iron statue would hang in the air. The project was abandoned at his death.
Ampelius, <i>Liber memorialis</i> 8	2nd–4th c.	An iron Victory at Magnesia hangs without support and does not move in wind or rain.
Ausonius, <i>Mosella</i> 314–17	4th c.	Reports the Arsinoe statue as completed.
Augustine, <i>City of God</i> 21.6 [29]	5th c.	An iron statue hangs between magnets above and below, giving an appearance of a miracle.
Rufinus, <i>Hist. Eccl.</i> 2.23	c. 400	An iron Sun image (Helios) hangs from a magnet in the ceiling of the Serapeum.
Cassiodorus, <i>Variae</i> 1.45	6th c.	An iron Cupid hangs without attachment in a temple of Diana.
Ptolemy, <i>Geography</i> 7.2 [30]	2nd c.	The Maniolae islands hold back ships with iron nails by their magnetic stone.
<i>Thousand and One Nights</i> [31]	medieval	A magnetic mountain draws the nails from a ship, which falls apart.
al-Qazwini on Somnath [1]	c. 1263	The idol of Somnath hangs between magnets in the dome. Removing stones makes it descend.
Embrico of Mainz, <i>Vita Mahumeti</i> [1]	11th–12th c.	The Prophet’s coffin hangs from a magnet in an arched shrine (rejected by Browne [32]).
Konark tradition [1, 13, 21]	19th c. (oral)	The Sun image floats, held by a lodestone at the top of the tower. The lodestone disturbs ships’ compasses and is removed by sailors.

3.1 The Earth’s field at Konark

A compass needle aligns with the horizontal component B_H of the local field. Konark lies at about 19.9° N, 86.1° E. The International Geomagnetic Reference Field [35] gives a total intensity there of roughly $44 \mu\text{T}$, with an inclination of roughly 28° . The horizontal component is therefore

$$B_H \approx 39 \mu\text{T}. \quad (1)$$

(The geomagnetic field drifts slowly, so in the thirteenth century the values differed by perhaps ten to twenty per cent. Such a change does not affect any conclusion below.)

3.2 The magnetic moment of a 52-tonne lodestone

Lodestone is magnetite, Fe_3O_4 , with density $\rho_{\text{mt}} \approx 5200 \text{ kg m}^{-3}$. Its saturation magnetisation is $\sigma_s \approx 92 \text{ A m}^2 \text{ kg}^{-1}$, so

$$M_s = \rho_{\text{mt}} \sigma_s \approx 4.8 \times 10^5 \text{ A m}^{-1}, \quad \mu_0 M_s \approx 0.60 \text{ T} \quad (2)$$

[2, 4]. Natural lodestones are not saturated. Their remanence is generally believed to be acquired from the intense, brief fields of lightning strikes [5], and it is a small fraction of M_s . In this paper

we bracket the remanent magnetisation of a good lodestone by

$$10^3 \text{ A m}^{-1} \lesssim M_{\text{lode}} \lesssim 5 \times 10^4 \text{ A m}^{-1}, \quad (3)$$

and use M_s as an absolute upper bound. A 52-tonne block has volume

$$V = \frac{5.2 \times 10^4 \text{ kg}}{5200 \text{ kg m}^{-3}} = 10 \text{ m}^3, \quad (4)$$

roughly a cube 2.15 m on a side. If the whole block were uniformly magnetised, its moment would be

$$m = MV \in [10^4, 5 \times 10^5] \text{ A m}^2, \quad m_{\text{max}} = M_s V = 4.8 \times 10^6 \text{ A m}^2. \quad (5)$$

Real lodestones are polycrystalline and not uniformly magnetised, so the true moment would be smaller still.

3.3 Dipole field and compass deviation

Far from the block (at distances $r \gg 2 \text{ m}$) its field is that of a dipole [36, 37]:

$$\mathbf{B}_d(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} [3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}], \quad |\mathbf{B}_d| \leq \frac{\mu_0 m}{2\pi r^3}. \quad (6)$$

The maximum, twice the equatorial value, occurs on the axis of the dipole. A compass needle points along $\mathbf{B}_H + \mathbf{B}_{d,H}$. If $B_{d,\perp}$ is the component of the dipole field perpendicular to the local meridian, the deviation δ satisfies

$$\tan \delta = \frac{B_{d,\perp}}{B_H + B_{d,\parallel}} \implies \delta \lesssim \frac{\mu_0 m}{2\pi r^3 B_H} \quad (\delta \ll 1). \quad (7)$$

Inverting this, a deviation δ can be produced only within the distance

$$r_\delta = \left(\frac{\mu_0 m}{2\pi B_H \tan \delta} \right)^{1/3}. \quad (8)$$

For $\delta = 1^\circ$ (about the smallest deviation a helmsman would notice):

$$M = 10^4 \text{ A m}^{-1} : \quad r_{1^\circ} \approx 31 \text{ m}, \quad (9)$$

$$M = M_s \text{ (saturated)} : \quad r_{1^\circ} \approx 112 \text{ m}. \quad (10)$$

At one kilometre, a typical distance for a ship coasting past a shore landmark, the deviation is

$$\delta(1 \text{ km}) \approx \frac{2 \times 10^{-7} m}{(10^3)^3 \times 3.9 \times 10^{-5}} \text{ rad} = 5.1 \times 10^{-12} m \text{ rad}, \quad (11)$$

that is, 3×10^{-5} degree for $m = 10^5 \text{ A m}^2$, and 1.4×10^{-3} degree even for a fully saturated block. Figure 1 shows the full curves.

The tower was about 60 m tall. Even a compass carried to its foot, about 60 m below a capstone lodestone with $M = 10^4 \text{ A m}^{-1}$, would deviate by only

$$\delta(60 \text{ m}) \approx \frac{2 \times 10^{-7} \times 10^5}{(60)^3 \times 3.9 \times 10^{-5}} \approx 2.4 \times 10^{-3} \text{ rad} \approx 0.14^\circ. \quad (12)$$

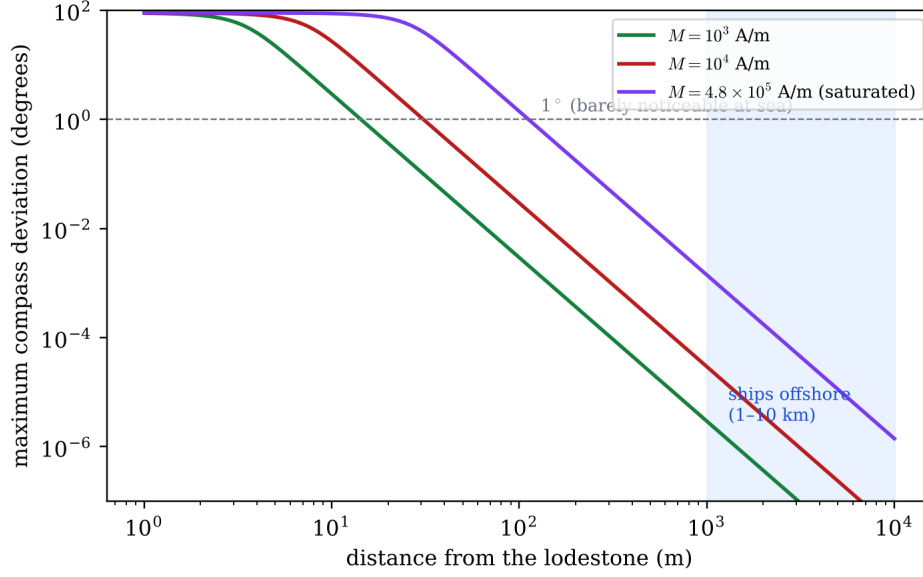


Figure 1: Maximum compass deviation produced by a 52-tonne (10 m^3) lodestone, from eq. (7) with $B_H = 39 \mu\text{T}$, for three values of the magnetisation. The deviation exceeds 1° only within about 30 m, or about 110 m for a fully saturated block. At the distances of passing ships it is at most 10^{-3} degree.

3.4 The temple's own iron

The temple contains a great deal of wrought iron: beams up to 10.7 m long and many clamps [13]. Soft iron in the Earth's field becomes magnetised. A long bar of soft iron with susceptibility $\chi \gg 1$ and demagnetising factor N has effective susceptibility [2]

$$\chi_{\text{eff}} = \frac{\chi}{1 + N\chi}, \quad N \simeq \frac{\ln(2p) - 1}{p^2} \quad (\text{aspect ratio } p \gg 1). \quad (13)$$

For a beam with $p \approx 30$, $N \approx 3.4 \times 10^{-3}$. With $\chi \approx 200$ (wrought iron at low field) this gives $\chi_{\text{eff}} \approx 120$. In the Earth's field $H_E = B_E/\mu_0 \approx 35 \text{ A m}^{-1}$, the beam acquires $M \approx 4 \times 10^3 \text{ A m}^{-1}$.

Twenty tonnes of such iron (2.5 m^3) would carry an induced moment of about 10^4 A m^2 . That is comparable to the lower end of the lodestone estimate in eq. (5). If any magnetic disturbance at Konark could be noticed, it would come as much from the iron of the building as from a lodestone. In either case it would be confined to a few tens of metres.

3.5 Could the lodestone pull a ship?

The other half of the legend, ships drawn towards the shore, is weaker still. A body of volume V_s and effective susceptibility χ_{eff} in a field B experiences the force (derived in Section 4)

$$\mathbf{F} = \frac{\chi_{\text{eff}} V_s}{2\mu_0} \nabla(B^2). \quad (14)$$

On the axis of a dipole, $B = \mu_0 m / (2\pi r^3)$ and $|\nabla B^2| = 6B^2/r$. Take one cubic metre of iron fittings aboard ship, with $\chi_{\text{eff}} \sim 100$, at $r = 1 \text{ km}$ from a lodestone with $m = 10^5 \text{ A m}^2$. Then $B = 2 \times 10^{-11} \text{ T}$ and

$$F \approx \frac{100 \times 1}{2 \times 1.26 \times 10^{-6}} \times \frac{6 \times (2 \times 10^{-11})^2}{10^3} \approx 10^{-16} \text{ N}. \quad (15)$$

This is less than the weight of a single bacterium. The Earth's own field gradient exerts far larger forces on the same iron.

3.6 Verdict on the sailors' story

Equations (7)–(8) are robust. They depend only on the cube of the distance and linearly on the moment, and no plausible lodestone can supply the many orders of magnitude needed for a measurable effect at sea. The nautical part of the Konark legend therefore cannot be literally true. What *is* true is that the temple was a crucial visual landmark, the Black Pagoda, and that compasses aboard ship were notoriously affected by the ship's *own* iron [27, 28]. Both facts may have fed the legend. A navigator who found his compass in error while taking bearings on the Black Pagoda could easily have attributed the error to the pagoda.

4 Earnshaw's theorem and its extensions

4.1 The original theorem

Consider a test charge q in the electrostatic potential $\phi(\mathbf{r})$ of fixed charges. In a charge-free region, ϕ satisfies Laplace's equation, $\nabla^2\phi = 0$. The potential energy $U = q\phi$ is therefore *harmonic*. A harmonic function has no strict local minimum in the interior of its domain: by the mean-value property, its value at a point equals its average over any small sphere centred there. In terms of the Hessian $K_{ij} = \partial_i\partial_j U$ at an equilibrium point,

$$\text{tr } K = k_x + k_y + k_z = \nabla^2 U = 0, \quad (16)$$

where k_x, k_y, k_z are the eigenvalues of K , i.e. the spring constants along the principal directions. If any one of them is positive, at least one other must be negative. This is the content of Earnshaw's theorem [11]: no stable static equilibrium exists for a body acted on only by inverse-square forces.

4.2 Permanent magnets

Let a rigid body carry a fixed magnetic moment $\mathbf{m}$ (a permanent magnet) in a static field $\mathbf{B}(\mathbf{r})$ produced by fixed magnets and steady currents outside the region of interest. There $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{B} = 0$, so

$$\nabla^2 \mathbf{B} = \nabla(\nabla \cdot \mathbf{B}) - \nabla \times (\nabla \times \mathbf{B}) = 0. \quad (17)$$

Each Cartesian component B_j is harmonic. For a fixed orientation of the body the energy is

$$U(\mathbf{r}) = -\mathbf{m} \cdot \mathbf{B}(\mathbf{r}) + Mgz, \quad (18)$$

where we include the weight of the body (mass M) and take z upward. This is a linear combination of harmonic functions, so $\nabla^2 U = 0$, and the sum rule (16) holds. For an extended magnet, $U = -\int \mathbf{M} \cdot \mathbf{B} dV$ is a superposition of such terms and is again harmonic in the translation coordinates. Gravity adds the term Mgz , which is also harmonic (indeed linear), so gravity can never supply the missing stiffness.

4.3 Allowing the body to rotate

One might hope that letting the magnet turn as well as translate would help. It does not.

Proposition 1. *A rigid body carrying permanent magnetisation, in a static external field and uniform gravity, has no stable equilibrium in its six rigid-body coordinates.*

Proof. Let $q = (\mathbf{r}, \Omega)$ be position and orientation, and let $U(q)$ be the energy. At a strict local minimum of U the full 6×6 Hessian is positive definite. Then so is every principal sub-matrix, in particular the 3×3 block of second derivatives with respect to $\mathbf{r}$ at fixed Ω . But at fixed Ω the energy $U(\cdot, \Omega)$ is harmonic in $\mathbf{r}$, by (17)–(18). That block therefore has trace zero, and a trace-zero matrix cannot be positive definite. This is a contradiction. $\square$

The proof assumes that the body’s magnetisation is fixed relative to the body. It fails if the moment can reorient *relative* to the body. It also fails if the motion is not static: a spinning top is governed by dynamics, not by the minimum of U . Both loopholes are exploited in Section 8.

4.4 Induced magnetism: soft iron and lodestone

A soft-iron statue, or one carrying unmagnetised magnetite, acquires a moment proportional to the applied field. For a small body of volume V and effective susceptibility χ_{eff} (including demagnetisation), the induced moment is $\mathbf{m} = \chi_{\text{eff}} V \mathbf{B} / \mu_0$. The energy, counting the work done in magnetising it, is [16, 36]

$$U = -\frac{\chi_{\text{eff}} V}{2\mu_0} B^2 + Mgz. \quad (19)$$

Its Laplacian follows from the identity (Appendix A)

$$\nabla^2(B^2) = 2 \sum_{i,j} \left(\partial_i B_j \right)^2 + 2 \mathbf{B} \cdot \nabla^2 \mathbf{B} = 2 \sum_{i,j} \left(\partial_i B_j \right)^2 \geq 0. \quad (20)$$

So B^2 is *subharmonic* and has no local maximum in free space. For $\chi_{\text{eff}} > 0$ the energy (19) is superharmonic, $\nabla^2 U \leq 0$, and has no local minimum. Soft iron and magnetite are drawn towards stronger fields, and there is no isolated point of maximum field strength to rest at. This is Braunbek’s extension [12]. The intuitive designs in the old stories therefore cannot work: an iron image held by lodestones above, below, or all around. Augustine’s statue between two magnets [29] and al-Qazwini’s idol at Somnath [1] both fail.

4.5 The diamagnetic loophole

Now suppose instead $\chi < 0$. Diamagnetic materials, such as bismuth, graphite, water and most organic matter, are weakly repelled by magnetic fields. Then

$$U = +\frac{|\chi|V}{2\mu_0} B^2 + Mgz, \quad \nabla^2 U = \frac{|\chi|V}{\mu_0} \sum_{i,j} (\partial_i B_j)^2 \geq 0. \quad (21)$$

A subharmonic function *can* have a local minimum, so stable levitation becomes possible in principle. William Thomson (Lord Kelvin) noted as early as 1847 that diamagnetic bodies seek field minima [15]. Braunbek showed in 1939 that stable static levitation requires materials with $\mu_r < 1$, i.e. diamagnets or superconductors [12]. The modern demonstrations, a living frog floating in a 16 T solenoid and a small magnet floating between bismuth plates or near a fingertip, are described by Berry and Geim [16], Geim *et al.* [38] and Simon and Geim [9].

4.6 Geometry of the instability

Figure 2(a) shows the energy landscape near the equilibrium point of a magnet hanging below another, using the numbers of the design in Section 7. The contours form a saddle: stable horizontally, with $k_x = k_y = 718 \text{ N m}^{-1}$, and unstable vertically, with $k_z = -1437 \text{ N m}^{-1} = -2k_x$, exactly as the sum rule demands. Panel (b) shows the same landscape after the diamagnetic “keel” of Section 6 has been added. The saddle has become a bowl.

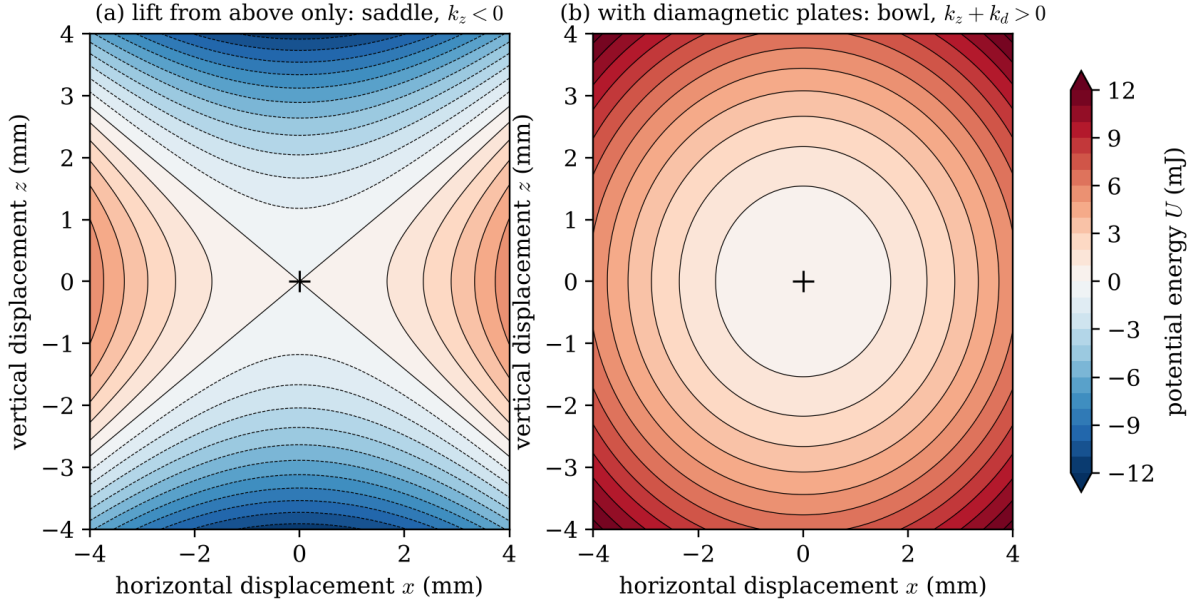


Figure 2: Potential energy near equilibrium for the 10 kg design of Section 7. (a) Lift from an overhead magnet alone gives a saddle, with $k_x > 0$ and $k_z = -2k_x < 0$, as required by Earnshaw's sum rule (16). (b) Adding diamagnetic plates at a gap of $d = 12.5$ mm contributes $k_d \approx 2.3 \times 10^3 \text{ N m}^{-1}$ and turns the saddle into a bowl.

5 Why a boat is stable

The hint in the problem, “what gives stability to a boat?”, points to the oldest stable levitation known: flotation. Archimedes analysed both the equilibrium and the stability of floating bodies [39]. The theory of the metacentre was completed by Bouguer [40] and Euler [41] in the eighteenth century and is now standard in naval architecture [42, 43].

5.1 Heave: vertical stability

A body of mass M floats in water of density ρ when the displaced volume V satisfies Archimedes' principle,

$$\rho g V = Mg. \quad (22)$$

If the body rises by a small distance ζ , the immersed volume falls by $A_w \zeta$, where A_w is the area of the waterplane, and the buoyancy falls by $\rho g A_w \zeta$. The net restoring force is $-\rho g A_w \zeta$, so the heave stiffness is

$$k_z^{\text{boat}} = \rho g A_w > 0. \quad (23)$$

In energy terms,

$$U(\zeta) = Mg\zeta + \rho g \int_0^\zeta (A_w \zeta') d\zeta' + \text{const} = U_0 + \frac{1}{2} \rho g A_w \zeta^2, \quad (24)$$

a parabola with a minimum. Horizontal translations leave the immersed volume unchanged, so

$$k_x^{\text{boat}} = k_y^{\text{boat}} = 0. \quad (25)$$

The stiffnesses sum to $\rho g A_w > 0$, violating the sum rule (16). This is no contradiction: buoyancy is the resultant of the pressure of a fluid in contact with the hull. It is not the gradient of a harmonic potential in empty space. The boat is stable vertically, neutral horizontally (it drifts and must be moored), and stable in tilt, as we now show.

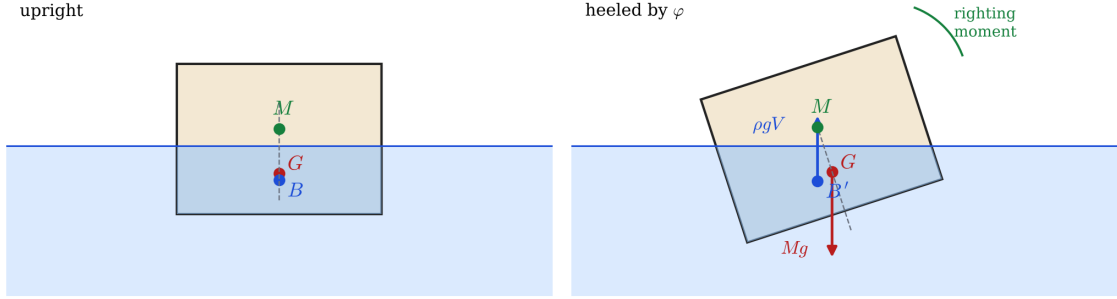


Figure 3: Hydrostatic stability of a box-shaped hull, computed exactly (beam 3, draft 1, $KG = 0.6$). When the hull heels, the centre of buoyancy moves from B to B' , towards the immersed side. The buoyancy then acts along a line through the metacentre M , which lies above G , and the couple formed with the weight rights the boat.

5.2 Roll and pitch: the metacentre

Let the boat heel through a small angle φ . The weight acts at the centre of gravity G , which is fixed in the body. The buoyancy acts at the centre of buoyancy, the centroid of the immersed volume. This point moves from B to B' , because a wedge of hull emerges on one side and an equal wedge is immersed on the other (Figure 3). The vertical through B' meets the centreline at the *metacentre* M . Taking moments of the two wedges gives the classical result [42, 43]

$$BM = \frac{I_w}{V}, \quad I_w = \int_{\text{waterplane}} y^2 dA, \quad (26)$$

where I_w is the second moment of the waterplane area about the axis of heel. The righting moment is

$$\tau = -Mg GM \sin \varphi, \quad GM = KB + BM - KG, \quad (27)$$

where K is the keel, and the angular stiffness in roll is $\kappa = Mg GM$. The boat is stable if the metacentre lies above the centre of gravity, $GM > 0$.

For a box-shaped hull of beam b and draft T , $I_w = Lb^3/12$ and $V = LbT$, so

$$BM = \frac{b^2}{12T}, \quad KB = \frac{T}{2}, \quad GM = \frac{T}{2} + \frac{b^2}{12T} - KG. \quad (28)$$

A wide, shallow hull with ballast low down is very stable. Figure 3 is drawn to scale for $b = 3$, $T = 1$ and $KG = 0.6$ (arbitrary units), which give $GM = 0.65$.

5.3 What the boat teaches

The boat combines three distinct mechanisms:

1. **A vertical restoring force that weakens as the body rises**, eq. (23). It comes from matter (the water) that the hull displaces and cannot penetrate.
2. **An effective point of support above the centre of gravity**, the metacentre, which gives stability in tilt, eq. (27).

3. **Neutrality in the remaining directions** (surge, sway and yaw), handled by mooring or by steering.

A magnetic suspension can copy the second mechanism directly: lift a statue by its crown. It can do better than the boat on the third: an overhead magnet gives *positive* horizontal stiffness. Only the first mechanism, the water, is missing, and by the sum rule it is missing exactly where the overhead magnet gives a negative stiffness. The ingredient that supplies it must be a material medium that the magnet approaches and is pushed back by. A diamagnet is exactly such a medium. Table 2 sets out the correspondence.

Table 2: Correspondence between the stability of a floating boat and of the proposed magnetic suspension (Section 6).

Degree of freedom	Floating boat	Magnetic boat (statue)
Heave (z)	stable, $k_z = \rho g A_w$ (water displaced)	unstable from the magnet alone ($k_z = -2k_x$); stabilised by diamagnetic plates, $k_d > 2k_x$
Surge, sway (x, y)	neutral, $k = 0$ (moored)	stable, $k_x = k_y = \frac{1}{2}mB''$ from the overhead magnet
Roll, pitch	stable if $GM > 0$; $\kappa = Mg GM$	stable; $\kappa = Mg\ell + mB$ (lift at crown above G)
Yaw	neutral	neutral for an axisymmetric field; weakly stable if the field is made slightly non-axisymmetric
Source of restoring force	contact pressure of water	diamagnetic repulsion: induced magnetisation of the plates
Why Earnshaw does not apply	buoyancy is not a harmonic potential	$U \propto + \chi B^2$ is subharmonic, eq. (21)

6 A magnetic boat: the arrangement and its stability

We now build the arrangement suggested by the boat (Figure 4). It has four elements:

- (A) **An overhead lifting magnet** fixed in the ceiling of the sanctum, magnetised vertically.
- (B) **A crown magnet** fixed in the head-dress of the statue, magnetised so that it is *attracted* upward (its moment parallel to the overhead field).
- (C) **A low centre of gravity**: the statue is light above and weighted at the base, so that the lift acts a distance ℓ above G. This is the analogue of the metacentre.
- (D) **A diamagnetic keel**: two plates of pyrolytic graphite or bismuth, fixed to the structure (for instance inside a sun-disc or halo behind the head), just above and just below the crown magnet. This is the analogue of the water.

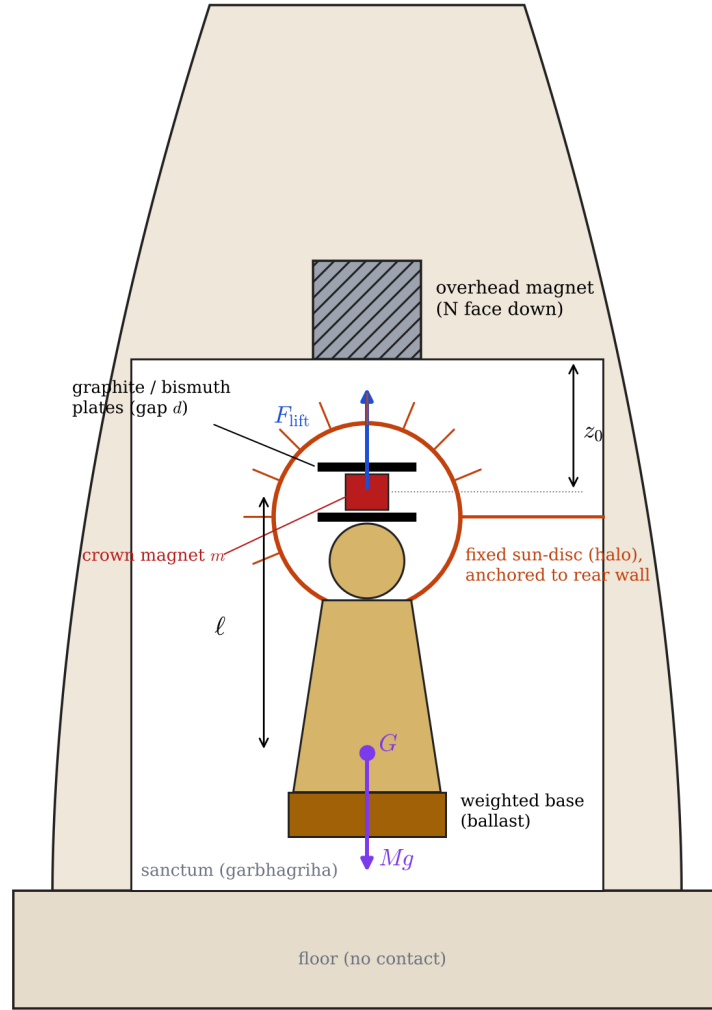


Figure 4: The proposed arrangement, schematic and not to scale. An overhead magnet in the ceiling of the sanctum lifts the statue by a magnet in its crown, a distance z_0 below. The centre of gravity G lies a distance ℓ below the point of lift, so the statue hangs upright like a boat whose metacentre is above G . Two diamagnetic plates (graphite or bismuth), held in a fixed sun-disc behind the head, sit at a gap d above and below the crown magnet and supply the vertical stiffness that Earnshaw's theorem denies to the magnets alone. Nothing touches the statue.

6.1 Lift from above

Model the overhead magnet as a uniformly magnetised cylinder of radius a , length L and remanence $B_r = \mu_0 M$, with its lower face in the plane $s = 0$. Here s measures distance *downward* from that face. On the axis its field is (Appendix C)

$$B(s) = \frac{B_r}{2} \left[\frac{s+L}{\sqrt{(s+L)^2 + a^2}} - \frac{s}{\sqrt{s^2 + a^2}} \right]. \quad (29)$$

Far away it reduces to the dipole field $B \simeq \mu_0 m_A / (2\pi s^3)$ with $m_A = M\pi a^2 L$. Treat the crown magnet as a point dipole m aligned with the field. With the statue's weight included, the energy as a function of depth is

$$U(s) = -m B(s) - Mgs, \quad (30)$$

since the gravitational energy falls as the statue descends. Equilibrium at $s = s_0$ requires

$$U'(s_0) = 0 \quad \Longleftrightarrow \quad m |B'(s_0)| = Mg, \quad (31)$$

since $B' < 0$: the field weakens with distance. The vertical stiffness is

$$k_s = U''(s_0) = -m B''(s_0) < 0, \quad (32)$$

because the field is a convex, decreasing function of s below the magnet ($B'' > 0$). The equilibrium is unstable vertically, as Earnshaw requires.

6.2 Horizontal stiffness from the paraxial expansion

Off the axis, at radius ρ , the vertical field component follows from $\nabla^2 B_z = 0$ in cylindrical coordinates (Appendix C):

$$B_z(\rho, s) = B(s) - \frac{\rho^2}{4} B''(s) + \frac{\rho^4}{64} B^{(4)}(s) - \dots \quad (33)$$

For a crown magnet that stays vertical, the energy is $U = -m B_z(\rho, s) - Mgs$, so

$$k_x = k_y = \left. \frac{\partial^2 U}{\partial x^2} \right|_0 = \frac{m B''(s_0)}{2} > 0. \quad (34)$$

The sum rule is satisfied identically:

$$k_x + k_y + k_s = \frac{1}{2} m B'' + \frac{1}{2} m B'' - m B'' = 0. \quad (35)$$

The statue is thus stable sideways: it is drawn back towards the axis of the overhead magnet. In this respect the magnetic boat is *better* than a real boat, which is only neutral horizontally (eq. 25). The price is paid in the vertical direction, where $k_s = -2k_x$.

6.3 Tilt: the effective metacentric height

Let the statue tilt through a small angle θ about a horizontal axis through the crown magnet. Two restoring torques act on it:

1. the weight, acting a distance ℓ below the point of lift, gives $\tau_g = -Mg\ell \sin \theta$, exactly as a pendulum or a boat;

2. the crown magnet, tilted away from the axial field, gives $\tau_m = -mB \sin \theta$, from $U = -mB \cos \theta$.

The total angular stiffness is

$$\kappa = Mg\ell + mB(s_0), \quad (36)$$

and by analogy with eq. (27) we may define an effective metacentric height

$$GM_{\text{eff}} = \frac{\kappa}{Mg} = \ell + \frac{mB(s_0)}{Mg}. \quad (37)$$

Both terms are positive. Tilt stability is therefore automatic provided the statue is lifted by its *top*. If the statue were instead pushed up from below by repelling magnets, the analogue of a boat with G above M , it would capsize. The tilt would grow until the repelling magnets turned to attract, which is why the old idea of floating a magnet on a repelling base always fails by flipping.

6.4 The diamagnetic keel

A point dipole $\mathbf{m}$ at distance d from the plane face of a half-space of relative permeability $\mu_r = 1 + \chi$ is acted on by an *image* dipole. The image sits at the mirror point, a distance $2d$ away (Appendix B), and has moment

$$\mathbf{m}' = \frac{\mu_r - 1}{\mu_r + 1} \mathbf{m} \simeq \frac{\chi}{2} \mathbf{m} \quad (|\chi| \ll 1), \quad (38)$$

pointing in the same direction as $\mathbf{m}$ when $\mathbf{m}$ is perpendicular to the face. Two coaxial parallel dipoles a distance r apart attract each other with force $3\mu_0 mm' / (2\pi r^4)$. For a diamagnet, $\chi < 0$, the image is reversed and the force is a *repulsion* of magnitude

$$F_d(d) = \frac{3\mu_0 m (|\chi| m / 2)}{2\pi (2d)^4} = \frac{3\mu_0 |\chi| m^2}{64\pi d^4}. \quad (39)$$

Now place two such plates symmetrically, one at gap d above the crown magnet and one at gap d below. If the statue sinks by ζ , the lower gap closes to $d - \zeta$ and the upper gap opens to $d + \zeta$. The net upward force is

$$F_{\text{net}} = F_d(d - \zeta) - F_d(d + \zeta) \simeq -2F'_d(d) \zeta = \frac{8F_d(d)}{d} \zeta, \quad (40)$$

so the plates add a vertical stiffness

$$k_d = \frac{8F_d(d)}{d} = \frac{3\mu_0 |\chi| m^2}{8\pi d^5}. \quad (41)$$

Infinite plates are invariant under horizontal translation, so to this order they add nothing to k_x , and the horizontal stability of eq. (34) is preserved. The plates are the water: they push back harder the closer the magnet comes, with the steep d^{-5} dependence that makes a narrow gap effective. Because the repulsion is induced in the plates, the energy is of the form (21) and the sum rule does not apply to it.

The image formula is exact for a point dipole and a half-space. For a crown magnet whose size is comparable to d , and for plates of finite thickness and width, it gives the right order of magnitude. A precise design would use a finite-element field calculation; the estimates below should be read with an uncertainty of a factor of about two.

6.5 The complete stability conditions

Collecting the results, the statue floats stably if and only if all of the following hold.

$$(i) \text{ equilibrium: } m |B'(s_0)| = Mg, \quad (42)$$

$$(ii) \text{ vertical: } k_z \equiv k_d - mB''(s_0) = \frac{3\mu_0|\chi|m^2}{8\pi d^5} - mB''(s_0) > 0, \quad (43)$$

$$(iii) \text{ horizontal: } k_x = k_y = \frac{1}{2}mB''(s_0) > 0 \quad (\text{automatic}), \quad (44)$$

$$(iv) \text{ tilt: } \kappa = Mg\ell + mB(s_0) > 0 \quad (\text{automatic for } \ell > 0), \quad (45)$$

$$(v) \text{ clearance: } |\zeta|_{\max} \ll d. \quad (46)$$

Yaw, rotation about the vertical, is neutral for an axisymmetric field, exactly as for a boat. A slight asymmetry in the overhead magnet, for instance a rectangular rather than circular face, makes yaw weakly stable as well.

Condition (ii) can be solved for the largest permissible gap:

$$d < d_{\max} = \left(\frac{3\mu_0|\chi|m}{8\pi B''(s_0)} \right)^{1/5}. \quad (47)$$

The fifth root makes $d_{\max}$ remarkably insensitive to everything except the geometry. For the designs below, $d_{\max}$ is always about 11–15 mm.

Condition (v) sets the trimming tolerance. A mismatch ΔW between lift and weight displaces the equilibrium by $\zeta = \Delta W/k_z$. Requiring, say, $\zeta < d/4$ gives

$$\frac{\Delta W}{Mg} < \frac{k_z d}{4Mg}. \quad (48)$$

6.6 Temperature, damping and natural frequencies

The remanence of a permanent magnet falls with temperature at a rate $\alpha = -d \ln B_r / dT$. The lift is proportional to the product of the two remanences, so it falls at 2α , and the statue sinks by

$$\frac{d\zeta}{dT} = \frac{2\alpha Mg}{k_z}. \quad (49)$$

For NdFeB, $\alpha \approx 1.2 \times 10^{-3} \text{ K}^{-1}$; for SmCo, $\alpha \approx 0.35 \times 10^{-3} \text{ K}^{-1}$ [2, 3].

The small oscillations have natural frequencies

$$f_z = \frac{1}{2\pi} \sqrt{\frac{k_z}{M}}, \quad f_x = \frac{1}{2\pi} \sqrt{\frac{k_x}{M}}, \quad f_\theta = \frac{1}{2\pi} \sqrt{\frac{\kappa}{I_p}}, \quad (50)$$

where I_p is the moment of inertia about the crown. Damping comes from air and from eddy currents. A thin copper plate placed behind the graphite gives strong, contact-free damping of all motions without affecting the static balance.

7 Numbers: lodestone versus modern magnets

7.1 Materials

Table 3 collects the relevant properties. Two numbers matter most. The first is the remanence B_r , since lifting forces scale as B_r^2 when both magnets are of the same material. The second is the susceptibility $|\chi|$ of the diamagnet, which fixes the keel stiffness. For whole-body diamagnetic levitation, without any magnet in the statue, the requirement follows from eq. (21):

$$B \frac{\partial B}{\partial z} \geq \frac{\mu_0 \rho g}{|\chi|}. \quad (51)$$

Table 3: Properties of magnetic and diamagnetic materials relevant to levitation [2–10]. The lodestone values are the order-of-magnitude bracket adopted in this paper, eq. (3).

<i>(a) Permanent magnets</i>				
Material	B_r (T)	α (%/K)	density (kg m^{-3})	remarks
Lodestone (natural magnetite)	0.001–0.06	–	5200	remanence from lightning; non-uniform
Magnetite, saturated (bound)	0.60	–	5200	not achievable as remanence
Ferrite (Ba/Sr hexaferrite)	0.4	–0.2	4900	1950s [8]
SmCo ($\text{Sm}_2\text{Co}_{17}$)	1.05	–0.035	8400	1967 [7]; thermally stable
NdFeB	1.3	–0.12	7500	1984 [6]; strongest
<i>(b) Diamagnets</i>				
Material	χ (SI)		density	$\mu_0 \rho g / \chi $ ($\text{T}^2 \text{m}^{-1}$)
Water (frog)	-9.05×10^{-6}		1000	1360
Bismuth	-1.66×10^{-4}		9780	726
Pyrolytic graphite ($\perp$)	-4.5×10^{-4}		2200	60
Superconductor (Meissner)	–1		8000	0.1

7.2 Why lodestones fail

Figure 5(a) shows the mass that can be held, $m|B'(s)|/g$, for the geometry of our design. The overhead magnet is a cylinder 20 cm in diameter and 20 cm long; the crown magnet is a cylinder 5 cm in diameter and 5 cm long; both are of the same material. With lodestone ($M = 10^4 \text{ A m}^{-1}$), the lift is only about 6 g even at contact. Yet the crown lodestone itself weighs 0.5 kg. The arrangement cannot even lift its own crown magnet: it falls short by a factor of about 100. With the most generous bracket, $M = 5 \times 10^4 \text{ A m}^{-1}$, the lift at contact rises to about 0.14 kg, still a factor of four short. At a useful gap of about 15 cm, the lift available for a 10 kg statue falls short by two to six orders of magnitude across the whole bracket of eq. (3).

Panel (b) asks a different question. Could the legendary 52-tonne lodestone, modelled as a cylinder 2.4 m in diameter and 2.2 m long, hold an image of solid iron if the iron were fully saturated ($M_s^{\text{Fe}} = 1.71 \times 10^6 \text{ A m}^{-1}$)? The force per unit volume on saturated iron is $M_s^{\text{Fe}} |\partial_z B|$, and it must exceed $\rho_{\text{Fe}} g = 7.7 \times 10^4 \text{ N m}^{-3}$. For any realistic lodestone magnetisation the ratio stays below one at every distance. Only for a fictitious, fully saturated 52-tonne block does it exceed one, and then only within about 1.6 m. Even in that case the vertical instability of Section 4 would remain.

7.3 A worked design with NdFeB

Take an overhead NdFeB cylinder with $a = 10 \text{ cm}$ and $L = 20 \text{ cm}$ (mass 47 kg). The crown magnet is an NdFeB cylinder of radius 2.5 cm and length 5 cm, with

$$m = \frac{B_r}{\mu_0} V = \frac{1.3}{4\pi \times 10^{-7}} \times 9.82 \times 10^{-5} \approx 102 \text{ A m}^2, \quad (52)$$

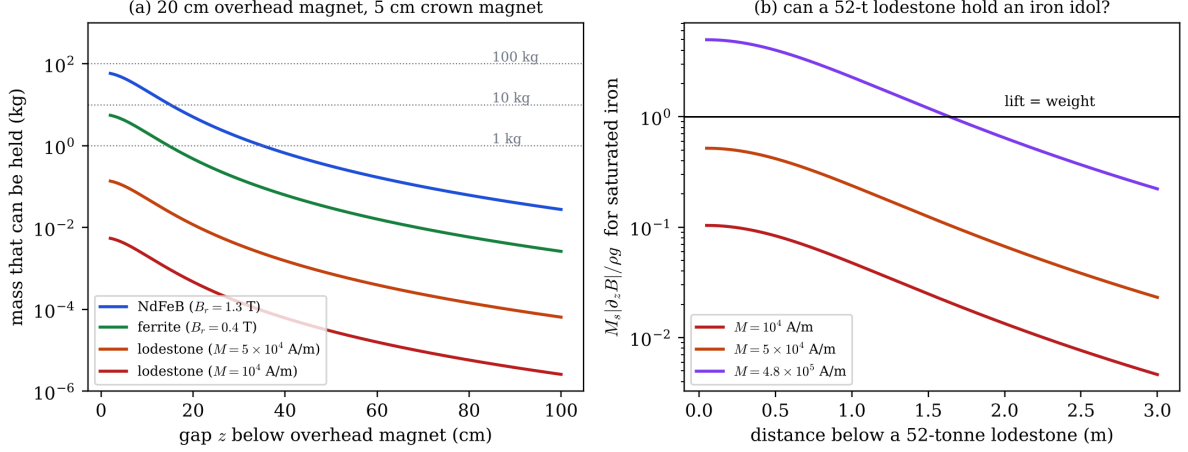


Figure 5: (a) Mass that can be held below a 20 cm cylindrical magnet by a 5 cm crown magnet of the same material, against the gap (point-dipole model for the crown magnet). (b) Ratio of the magnetic force to the weight for saturated soft iron below a 52-tonne lodestone, for three magnetisations. Lodestones fall short by orders of magnitude in both cases.

and mass 0.74 kg. The statue, including its crown magnet and ballast, has total mass $M = 10$ kg. Solving eq. (31) numerically with eq. (29):

$$s_0 = 15.1 \text{ cm}, \quad B(s_0) = 83.6 \text{ mT}, \quad |B'(s_0)| = 0.966 \text{ T m}^{-1}, \quad (53)$$

$$k_s = -mB'' = -1.44 \times 10^3 \text{ N m}^{-1}, \quad k_x = k_y = +718 \text{ N m}^{-1}. \quad (54)$$

From eq. (47), the maximum keel gap is 13.7 mm for pyrolytic graphite and 11.2 mm for bismuth. Choosing $d = 12.3$ mm (90% of the graphite limit) gives

$$k_d = 2.43 \times 10^3 \text{ N m}^{-1}, \quad k_z = k_d + k_s = +996 \text{ N m}^{-1}. \quad (55)$$

All three translational stiffnesses are now positive. For tilt, with the centre of gravity $\ell = 0.6$ m below the crown,

$$\kappa = Mg\ell + mB = 58.9 + 8.5 = 67.4 \text{ N m rad}^{-1}, \quad GM_{\text{eff}} = 0.69 \text{ m}. \quad (56)$$

The natural frequencies (eq. 50) are $f_z = 1.6$ Hz and $f_x = 1.35$ Hz. Taking $I_p \approx 4 \text{ kg m}^2$, the tilt frequency is $f_\theta \approx 0.65$ Hz. These are gentle motions, like a boat's slow roll.

The trim tolerance (eq. 48) is about 3% of the weight, roughly 300 g. The temperature drift (eq. 49) is 0.24 mm K^{-1} . A daily swing of 10 K would move the statue by 2.4 mm, a fifth of the gap. That is acceptable but not comfortable. Two remedies exist. SmCo magnets reduce the drift by a factor of about three (Table 4). Alternatively, the massive stone walls of a sanctum, which damp temperature swings to a small fraction of the outside range, help considerably.

7.4 Scaling to a larger statue

The stiffnesses scale simply. If every length in the design is multiplied by λ while the materials are kept fixed, then $m \propto \lambda^3$ and B is unchanged at corresponding points, so $B' \propto \lambda^{-1}$ and $B'' \propto \lambda^{-2}$. The lift therefore scales as $m|B'| \propto \lambda^2$, while the weight of a statue of fixed density scales as λ^3 . Larger statues are proportionally harder to lift, which is the usual square-cube law. The keel gap, from eq. (47), scales as $(\lambda^3/\lambda^{-2})^{1/5} = \lambda$, so it grows in proportion. A statue twice the size, with all magnets twice the size, must therefore have half the mean density, which hollow construction can provide.

Table 4: Worked designs. Overhead magnet: cylinder 20 cm $\times$ 20 cm. Crown magnet: cylinder 5 cm $\times$ 5 cm, of the same material. Keel: pyrolytic graphite at $d = 0.9 d_{\max}$.

	NdFeB	SmCo	Ferrite	Lodestone ($M = 10^4$)
Crown moment m (A m ²)	102	82	31	0.98
Maximum lift (kg, near contact)	61	40	5.8	0.006
Statue mass chosen (kg)	10	5	1	none ^a
Hanging gap s_0 (cm)	15.1	16.9	14.7	—
Field at crown $B(s_0)$ (mT)	83.6	54.8	26.8	—
k_x (N m ⁻¹)	718	344	72	—
k_s without keel (N m ⁻¹)	-1437	-688	-145	—
$d_{\max}$ graphite / bismuth (mm)	13.7 / 11.2	14.6 / 12.0	13.5 / 11.1	—
Net k_z with keel (N m ⁻¹)	996	477	100	—
f_z, f_x (Hz)	1.6, 1.35	1.55, 1.3	1.6, 1.35	—
Trim tolerance (% of weight)	3.1	3.2	3.1	—
Thermal drift (mm K ⁻¹)	0.24	0.07	0.39	—

^aThe lodestone arrangement cannot lift even its own 0.5 kg crown magnet.

7.5 Whole-body diamagnetic levitation

The alternative is a statue that is itself diamagnetic, with no magnet inside, floating over a magnet below. Table 3 shows that this is possible for pyrolytic graphite if $B \partial_z B \gtrsim 60 \text{ T}^2 \text{ m}^{-1}$. Arrays of NdFeB magnets with alternating poles achieve this, with $B \approx 0.5 \text{ T}$ and $\partial_z B \approx 250 \text{ T m}^{-1}$, but only within one or two millimetres of their surface. The resulting levitation height is about a millimetre. A thin graphite image of the Sun God could thus be made to hover just above a magnetic floor. It would be a delightful demonstration, but not the floating idol of the legend. With lodestone, $B \partial_z B$ is at most of order $10^{-2} \text{ T}^2 \text{ m}^{-1}$, three to four orders of magnitude too small.

8 Other ways around Earnshaw's theorem

The theorem forbids *static* stability with *fixed* magnetisation and $\mu_r \geq 1$. Every working levitation device violates at least one of these assumptions [16, 44]. The diamagnetic keel violates the third. We now summarise the others, since any of them could in principle float a statue.

8.1 Spin: the Levitron

The Levitron, patented by Harrigan in 1983 [45], is a magnetised top that hovers above a ring magnet while it spins. Its moment points *against* the field of the base, so that it is repelled. Without spin it would flip over and fall (the capsizing boat of Section 6). With spin, gyroscopic stiffness prevents the flip.

The subtle point, explained by Berry [46] and by Simon, Heflinger and Ridgway [47], is that the spinning moment precesses rapidly about the *local* field direction. It therefore follows that direction adiabatically as the top moves. Averaged over the precession, the energy becomes

$$U_{\text{eff}}(\mathbf{r}) = +m |\mathbf{B}(\mathbf{r})| + Mgz. \quad (57)$$

It depends only on the field strength, and $|\mathbf{B}|$ is subharmonic (Appendix A):

$$\nabla^2 |\mathbf{B}| = \frac{\sum_{ij} (\partial_i B_j)^2 - |\nabla |\mathbf{B}||^2}{|\mathbf{B}|} \geq 0, \quad (58)$$

so U_{eff} can have a minimum. Stability requires the spin to be fast enough for gyroscopic stability and for the adiabatic following of the field, yet slow enough to remain within the narrow window of stable heights [46–48]. The adiabatic condition is that the precession frequency $\omega_p \simeq mB/(I_3\omega_s)$ be much larger than the translational frequencies. The top must also be weighted to within a narrow tolerance.

Spin stabilisation is practical only for objects of a few tens of grams, and air drag stops the spin within minutes. A spinning sun-disc is a pleasing image for a Sun temple, but not a practical support for a statue.

8.2 Feedback: active magnetic suspension

An electromagnet whose current is controlled by a position sensor can hold a magnet or an iron body below it indefinitely. This is the principle of commercial floating displays and of electromagnetic maglev trains [49]. Linearising about equilibrium, with force $F_c = -K_p\zeta - K_d\dot{\zeta}$ supplied by the controller,

$$M\ddot{\zeta} = |k_s|\zeta - K_p\zeta - K_d\dot{\zeta}, \quad (59)$$

which is stable when $K_p > |k_s|$ and $K_d > 0$. The controller supplies exactly the positive stiffness that Earnshaw forbids to static fields. It needs a sensor, an amplifier and a continuous supply of power.

8.3 Superconductors

A superconductor expels magnetic flux (the Meissner effect [50]) and so behaves as a perfect diamagnet, with $\chi = -1$. The requirement (51) then falls to about $0.1 \text{ T}^2 \text{ m}^{-1}$, which is trivially met. Type-II superconductors also pin flux lines. They can then be held stably above *or below* a magnet in any orientation, and loads of hundreds of kilograms have been levitated [44, 51]. The cost is cryogenic cooling, to about 77 K with liquid nitrogen for the cuprate superconductors.

8.4 Oscillating fields and eddy currents

A conductor in an alternating field carries induced eddy currents that oppose the field. At frequencies where the skin depth is small it therefore acts as a diamagnet with $\chi_{\text{eff}} \rightarrow -1$. This is the basis of electrodynamic levitation of metals [52]. More generally, a rapidly oscillating force $\mathbf{F}_1 \cos \omega t$ superposed on a static potential U_0 produces, on average, the effective potential of Kapitza [53]:

$$U_{\text{eff}} = U_0 + \frac{\langle |\mathbf{F}_1|^2 \rangle}{4M\omega^2}, \quad (60)$$

which can turn a saddle into a bowl, as for the inverted pendulum on a vibrating pivot. Both mechanisms need a source of alternating power.

8.5 Summary

Of all the ways to beat Earnshaw's theorem, only diamagnetism needs *neither* power, cooling nor motion. It is therefore the only one consistent with a passive temple installation, and it is exactly the mechanism that the boat analogy points to.

9 Discussion: what might have been at Konark?

The physics yields three firm conclusions about the tradition.

First, a lodestone at the top of a 60-metre tower can have had no effect on an image in the sanctum below. Equations (6) and (29) show that the field of even a 52-tonne block is a few millitesla at one or two metres. At tens of metres it is less than the Earth's field. Any magnetic suspension must have its lifting magnet within a few centimetres to tens of centimetres of the image, in the ceiling of the sanctum and not at the top of the tower.

Second, no arrangement of lodestones and iron alone could have been stable (Section 4). An iron image held between lodestones above and below, as described for the statues of Augustine and of Somnath, is unstable in every version. An image surrounded by lodestones in the walls is unstable too.

Third, even the stable arrangement of Section 6 could not have been built from lodestones, which are two to six orders of magnitude too weak (Section 7). Pyrolytic graphite, the best natural-material keel, is a modern product. Ordinary graphite has a susceptibility several times smaller, and bismuth was not isolated as a distinct metal until early modern times.

What, then, might lie behind the tradition? Several possibilities, none of which we can test, are consistent with the physics:

1. The image was supported in a way that was not obvious to the viewer. In the dim light of a sanctum a fine suspension, or a support hidden behind drapery or offerings, would be invisible. Magnets might even have been used to take part of the weight off such a support, a "near-levitation" that is perfectly possible.
2. The story is an explanation after the fact. Iron and lodestone were familiar, and the motif of the floating image was ancient and widespread (Table 1). The motif may have attached itself to the temple once its tower had fallen and its image had gone.
3. The nautical element grew from two true facts: the Black Pagoda was a vital landmark, and ships' compasses were often wrong. These were joined in a story that the landmark caused the error.

We emphasise that the tradition embodies one correct physical intuition: the image was held *from above*. Of all magnetic suspensions, lifting from above is the only one that is stable in tilt and stable sideways. It is the analogue of a boat's metacentre. What the tradition lacked, and what no lodestone could supply, was the water.

10 Conclusions

1. The floating Sun God of Konark is attested only in later tradition. It is first recorded in the nineteenth century and belongs to a widespread family of magnetic-suspension stories stretching back to Pliny [1, 13, 21].
2. A 52-tonne lodestone deflects a compass by 1° only within about 30 m, or about 110 m if fully saturated. At the distance of passing ships the deflection is below 10^{-3} degree. The temple's own structural iron is magnetically comparable. The sailors' story cannot be literally true.
3. Earnshaw's theorem, in the sum-rule form $k_x + k_y + k_z = 0$, forbids stable static levitation by permanent magnets, soft iron and gravity, even when the body is free to rotate. Braunschweig's extension shows that bodies with $\chi > 0$ are always drawn towards stronger fields, which have no isolated maxima.

4. A boat escapes the theorem because buoyancy is a contact force that weakens as the boat rises ($k_z = \rho g A_w$), and because the metacentre lies above the centre of gravity ($GM > 0$).
5. The magnetic boat copies both features. An overhead magnet lifts the statue by its crown, giving $k_x = \frac{1}{2}mB'' > 0$ and tilt stiffness $\kappa = Mg\ell + mB$. Diamagnetic plates at gap d supply $k_d = 3\mu_0|\chi|m^2/(8\pi d^5)$, which exceeds the vertical instability mB'' when $d < d_{\max}$ (eq. 47).
6. With NdFeB, a 10 kg statue floats 15 cm below a 47 kg magnet, with graphite plates at 12 mm and natural frequencies near 1.5 Hz. With SmCo the design is thermally steadier. With ferrite it carries about 1 kg. With lodestones it cannot even lift its own crown magnet.
7. Spin, feedback, superconductivity and oscillating fields offer other escapes from Earnshaw's theorem, but all need motion, power or cooling. Diamagnetism alone gives a passive, permanent levitation, a true magnetic boat.

A Subharmonicity of B^2 and $|\mathbf{B}|$

In a source-free region each component satisfies $\nabla^2 B_j = 0$. Then

$$\nabla^2(B^2) = \sum_j \nabla^2(B_j^2) = \sum_j [2B_j \nabla^2 B_j + 2|\nabla B_j|^2] = 2 \sum_{i,j} (\partial_i B_j)^2 \geq 0. \quad (61)$$

For $|\mathbf{B}| = \sqrt{B^2}$, differentiate $B^2 = |\mathbf{B}|^2$ twice:

$$\nabla^2(B^2) = 2|\mathbf{B}|\nabla^2|\mathbf{B}| + 2|\nabla|\mathbf{B}||^2, \quad (62)$$

so

$$\nabla^2|\mathbf{B}| = \frac{\sum_{i,j} (\partial_i B_j)^2 - |\nabla|\mathbf{B}||^2}{|\mathbf{B}|}. \quad (63)$$

Now $\partial_i|\mathbf{B}| = \sum_j B_j \partial_i B_j / |\mathbf{B}|$, and by the Cauchy–Schwarz inequality $(\partial_i|\mathbf{B}|)^2 \leq \sum_j (\partial_i B_j)^2$. Summing over i shows that the numerator is non-negative, so $\nabla^2|\mathbf{B}| \geq 0$. Neither B^2 nor $|\mathbf{B}|$ can therefore have a local maximum in free space, although both can have minima. This is why paramagnets and ferromagnets ($\chi > 0$, seeking maxima) cannot be held stably, while diamagnets and anti-aligned spinning tops (seeking minima) can.

B The image of a dipole in a permeable half-space

In a current-free region, write $\mathbf{H} = -\nabla\phi_M$, with sources represented by magnetic poles of strength q_m . At the plane interface $z = 0$ between a region of permeability μ_1 (containing the pole, at height d) and a half-space of permeability μ_2 , the boundary conditions are continuity of $H_{\parallel}$ and of $B_z = \mu H_z$. They are satisfied, exactly as in the electrostatic problem of a charge near a dielectric [36], by an image pole

$$q'_m = -\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} q_m \quad (64)$$

at $z = -d$ for the field in region 1. A dipole perpendicular to the interface is a pair of poles $\pm q_m$ at heights d and $d + \delta$. Its image is the pair $\mp k q_m$ at $-d$ and $-(d + \delta)$, with $k = (\mu_2 - \mu_1)/(\mu_2 + \mu_1)$. This is a dipole of moment $k m$ parallel to the original. With $\mu_1 = \mu_0$ and $\mu_2 = \mu_0(1 + \chi)$,

$$k = \frac{\chi}{2 + \chi} \simeq \frac{\chi}{2}, \quad (65)$$

which is eq. (38). The force between two coaxial dipoles m_1, m_2 a distance r apart follows from $U = -m_2 B_1(r)$ with $B_1 = \mu_0 m_1 / (2\pi r^3)$:

$$F = -\frac{\partial U}{\partial r} = -\frac{3\mu_0 m_1 m_2}{2\pi r^4} \quad (\text{attractive for } m_1 m_2 > 0). \quad (66)$$

Setting $m_1 = m$, $m_2 = km$ and $r = 2d$ gives eq. (39), which is repulsive when $\chi < 0$.

C Field of a cylindrical magnet and the paraxial expansion

A uniformly magnetised cylinder (magnetisation $M\hat{z}$, radius a , length L) is equivalent to two discs of magnetic surface charge $\pm M$ on its end faces. A disc of charge density σ produces on its axis, at distance s , the field

$$H(s) = \frac{\sigma}{2} \left[1 - \frac{s}{\sqrt{s^2 + a^2}} \right]. \quad (67)$$

Superposing the near face ($-M$, facing the statue below a north-down magnet) and the far face ($+M$ at distance $s + L$) and multiplying by μ_0 gives eq. (29). Its derivatives, needed for eqs. (31)–(34), are

$$B'(s) = \frac{B_r}{2} \left[\frac{a^2}{((s+L)^2 + a^2)^{3/2}} - \frac{a^2}{(s^2 + a^2)^{3/2}} \right], \quad (68)$$

$$B''(s) = \frac{3B_r a^2}{2} \left[\frac{s}{(s^2 + a^2)^{5/2}} - \frac{s+L}{((s+L)^2 + a^2)^{5/2}} \right]. \quad (69)$$

Off the axis, B_z is harmonic and axisymmetric. Write $B_z(\rho, s) = \sum_{n \geq 0} c_n(s) \rho^{2n}$ and substitute into

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial B_z}{\partial \rho} \right) + \frac{\partial^2 B_z}{\partial s^2} = 0. \quad (70)$$

This gives $4(n+1)^2 c_{n+1} = -c_n''$, hence

$$B_z(\rho, s) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{\rho}{2} \right)^{2n} B^{(2n)}(s), \quad (71)$$

whose first terms are eq. (33).

D Numerical method

All numbers in Section 7 and in Figures 2–1 were computed from eqs. (29), (31), (32), (34) and (41). Derivatives were taken by central differences with steps of 10^{-6} – 10^{-5} m, and equilibria were found with Brent's method. The crown magnet is treated as a point dipole, and the keel plates as half-spaces. Both approximations are adequate at the order-of-magnitude level when the gaps are comparable to the magnet size, but a construction design would call for a full three-dimensional magnetostatic calculation. The figure of the floating box hull (Figure 3) was computed exactly by clipping the heeled hull polygon at the waterline and enforcing constant displacement.

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