

Energy Leakage from Extra-Brane Black Holes via Wormholes: A Phenomenological Model for Steady Unidentified Fermi Gamma-Ray Sources

X.L. Tao¹

¹Independent Researcher, P.R. China
ORCID: 0009-0004-0853-7002.

Contributing authors: Tao.MS@outlook.com;

Abstract

A substantial fraction of high-latitude Fermi-LAT sources remain unidentified. These objects exhibit steady GeV emission, low variability, and no multiwavelength counterparts. We propose a phenomenological braneworld model in which accreting black holes on an adjacent brane trigger inter-brane wormholes, leaking hadronic gamma-ray and neutrino emission into our brane. The model provides a calculable chain from black-hole mass to integrated luminosity and differential spectra. A central methodological feature is that the source redshift z is treated as a free MCMC parameter with a unified prior ($\log_{10} z \in [-3.0, 0.477]$) for all five competing models, eliminating circular flux–luminosity calibration. Fitting the 4FGL-DR4 high-latitude unassociated sample, the wormhole posterior peaks at $z = 0.066$ (~ 290 Mpc), whereas dark matter and primordial black holes require unphysical local redshifts ($z < 0.05$) and pulsar wind nebulae require $z \sim 0.95$. The model yields four distance-independent testable predictions: narrow variability, multiwavelength emptiness, a TeV spectral cutoff, and a coincident neutrino flux with $F_\nu/F_\gamma \sim 0.5$.

Keywords: braneworld, wormholes, gamma-ray sources, Fermi-LAT, unidentified sources, multi-messenger

1 Introduction

The Fermi Large Area Telescope fourth source catalog data release (4FGL-DR4) contains over 6600 sources, of which a significant fraction remain unassociated [1]. At high

Galactic latitudes ($|b| > 10^\circ$), these unassociated sources are predominantly steady, point-like, and lack multiwavelength counterparts in WISE and DES surveys [2, 3]. Conventional explanations face difficulties: blazars are highly variable and host bright galaxies [4]; pulsar wind nebulae (PWNe) predict a Galactic halo inconsistent with diffuse emission limits; dark matter predicts diffuse rather than point-like emission; primordial black hole (PBH) evaporation produces transient or thermal spectra [5].

The Randall–Sundrum (RS) braneworld [6] provides a framework for an alternative origin. In this picture, our observable universe is a 3-brane embedded in a five-dimensional anti-de Sitter (AdS_5) bulk, and gravity is the only force propagating in the bulk. Under high-curvature conditions, black holes on an adjacent brane can induce geometric transitions that produce inter-brane wormhole throats [7, 8]. Energy from the accreting black hole then leaks through the throat into our brane as hadronic gamma-ray and neutrino emission. Previous work established that black-hole and wormhole geometries lie on a continuous solution branch in the RS braneworld [7, 9], and that bulk deformations can drive brane transitions [8]. Recent studies of charged wormhole formation [10], regular-black-hole to wormhole transitions in effective quantum gravity [11], and braneworld wormhole stability [12, 13] further support the geometric plausibility of this mechanism.

This paper constructs a complete phenomenological model from black-hole mass to observable luminosity and spectra, and tests it against the 4FGL-DR4 sample. Section 2 presents the theoretical framework and energy chain. Section 2.4 describes the data and MCMC analysis, in which redshift is a free parameter for all models. Section 3 reports the luminosity distribution, parameter constraints, and population predictions. Section 4 discusses testability, limitations, and comparison with competing models. Section 5 outlines future prospects, and Section 6 concludes. Technical derivations are collected in the Appendix. The model is built on a set of phenomenological membrane–wormhole premises; their microscopic justifications are collected in Appendix I, with supporting derivations in Appendices A–H, and limitations discussed in Sec. 4.2.

2 Theoretical framework

2.1 Working premises

We adopt three explicit assumptions.

A1 (Braneworld geometry). The RS-II setup with one warped extra dimension [6]. Our brane is the “visible” brane; a second “hidden” brane hosts accreting black holes. The 5D Planck mass M_5 and the warp scale k set the gravitational coupling; the projected 4D Einstein equations follow the Shiromizu–Maeda–Sasaki (SMS) reduction [14] (see Appendix A for the full five-dimensional geometric setup and Israel junction-condition derivation).

A2 (Wormhole triggering). Accreting black holes on the hidden brane can, when the bulk-curvature perturbation exceeds a black-hole-dependent threshold, induce a geometric transition producing a transient wormhole throat (lifetime $\tau_{\text{decay}} \sim 10^3$ yr, Eq. (58); observationally steady over the Fermi mission) connecting the two branes. This is supported by continuous black-hole–wormhole solution branches in the

RS braneworld [7] and by bulk-deformation-driven brane transitions [8]. The transition probability per unit time is encapsulated in a phenomenological transmission coefficient $\kappa_g \in [10^{-4}, 1]$, which is not a free geometric parameter but is constrained by the observed luminosity (see Appendix F for the linear-response derivation; see Appendix C for static black-bounce bifurcation benchmarks). The throat radius is

$$\sigma_r \sim \left(\frac{3M_{\text{BH}}}{4\pi\rho_{\text{crit}}} \right)^{1/3} \sim 10^6 \text{ cm} \left(\frac{M_{\text{BH}}}{10^3 M_\odot} \right)^{1/3}, \quad (1)$$

where $\rho_{\text{crit}} \sim 10^{18} \text{ g cm}^{-3}$ ($\sim 10^{38} \text{ erg cm}^{-3}$), inferred from Eq. (1) for $M_{\text{BH}} = 10^3 M_\odot$ and $\sigma_r = 10^6 \text{ cm}$, is the critical energy density for the geometric transition in the RS braneworld [6]. The throat forms at a random position in intergalactic space, giving an isotropic sky distribution and no host-galaxy association.

A3 (Matter localization breaking). In the RS model, standard-model fields are confined to the brane while gravity propagates in the bulk. At the high-curvature throat, matter localization can be partially broken, allowing protons from the hidden-brane accretion flow to cross into our brane [15]. The effective energy density near the throat can be positive due to warp contributions, satisfying the weak energy condition without exotic matter (Appendix B derivation; see Appendix D for effective-potential and WKB tunnelling derivations, and Appendix G for the relativistic WKB correction).

The braneworld metric with tidal charge takes the Kerr-like form

$$ds^2 = -\frac{\Delta - a_*^2 \sin^2 \theta}{\Sigma} dt^2 - \frac{2a_* \sin^2 \theta (r^2 + a_*^2 - \Delta)}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{(r^2 + a_*^2)^2 - a_*^2 \Delta \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2, \quad (2)$$

with $\Delta = r^2 - 2Mr + a_*^2 + q$, $\Sigma = r^2 + a_*^2 \cos^2 \theta$, and tidal charge $q = \beta_{\text{tidal}} a_*^2 M^2$ [16]. The interpolation between black-hole and wormhole geometries is parameterized by

$$ds^2 = -A(r, p)dt^2 + B(r, p)dr^2 + C(r, p)d\Omega^2, \quad (3)$$

where $p \in [0, 1]$ interpolates between the black-hole ($p = 0$) and wormhole ($p = 1$) branches [7]. The 5D bulk metric perturbation is

$$g_{AB} = g_{AB}^{(0)} + \delta g_{AB}, \quad \delta g_{AB} \sim a_5(t) \psi_{AB}(r), \quad (4)$$

with $a_5(t)$ the deformation amplitude. The SMS projection gives the effective 4D equations

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \frac{8\pi}{M_4^2} T_{\mu\nu} + \frac{8\pi}{M_5^6} \Pi_{\mu\nu} - E_{\mu\nu}, \quad (5)$$

where $\Pi_{\mu\nu}$ is the quadratic brane-matter term and $E_{\mu\nu}$ is the projected 5D Weyl tensor [14].

2.2 Energy chain and observable predictions

The accreting black hole on the hidden brane, fed by Bondi-type accretion [17], has Eddington luminosity

$$L_{\text{Edd}} = 1.26 \times 10^{38} \left(\frac{M_{\text{BH}}}{M_{\odot}} \right) \text{ erg s}^{-1}. \quad (6)$$

A fraction κ_g of the accretion power is transmitted through the throat, of which a fraction f_{capture} is captured by the throat geometry and converted to proton injection. The injected proton luminosity is

$$L_{\text{inj}} = \kappa_g f_{\text{capture}} \lambda_{\text{Edd}} L_{\text{Edd}}, \quad (7)$$

where $\lambda_{\text{Edd}} = \dot{M}/\dot{M}_{\text{Edd}}$ is the Eddington ratio. Protons undergo pp collisions in the throat, producing pions; the gamma-ray luminosity is

$$L_{\gamma} = \eta_{\text{had}} L_{\text{inj}} = \eta_{\text{had}} \kappa_g f_{\text{capture}} \lambda_{\text{Edd}} L_{\text{Edd}}, \quad (8)$$

where $\eta_{\text{had}} \sim 0.1\text{--}0.8$ is the hadronic conversion efficiency [18, 19] (see Appendix H for the full Kelner–Kafexhiu pp -collision cross-section parametrization). For the posterior-median parameters ($M_{\text{BH}} = 10^{7.47} M_{\odot}$, $\kappa_g = 10^{-0.53}$, $\eta_{\text{had}} = 0.52$, $\lambda_{\text{Edd}} = 10^{-0.52}$, $f_{\text{capture}} = 0.65$), Eq. (8) gives $L_{\gamma} \approx 1.1 \times 10^{44} \text{ erg s}^{-1}$, consistent with the observed range at $z = 0.066$.

The differential gamma-ray spectrum from $pp \rightarrow \pi^0 \rightarrow 2\gamma$ is

$$\frac{dN_{\gamma}}{dE} = N_0 E^{-\Gamma} \exp(-E/E_{\text{cut}}), \quad \Gamma \simeq 1.9\text{--}2.1, \quad (9)$$

with the cutoff set by the parent proton kinematics [18, 19]. The pion production threshold is

$$E_p^{\text{thr}} = m_p + \frac{4m_p m_{\pi} + m_{\pi}^2}{2m_p} \simeq 1.22 \text{ GeV}; \quad (10)$$

for $E_p \gtrsim 100 \text{ GeV}$ the threshold correction is $< 1\%$. The cutoff energy relates to the injection parameters via

$$E_{\text{cut}}^{2-\Gamma} \left[\gamma \left(2 - \Gamma, \frac{E_2}{E_{\text{cut}}} \right) - \gamma \left(2 - \Gamma, \frac{E_1}{E_{\text{cut}}} \right) \right] \approx \text{constant}, \quad (11)$$

where E_1 and E_2 are the lower and upper integration bounds of the gamma-ray luminosity ($E_1 \sim 10^8 \text{ eV}$, $E_2 \sim 10^{12} \text{ eV}$), $\gamma(s, x)$ is the lower incomplete gamma function, and for $\Gamma \approx 2.0$, E_{cut} is nearly fixed by L_{γ} and L_{inj} and is not a free parameter.

Charged pions decay to neutrinos:

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu}), \quad \mu^{\pm} \rightarrow e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_{\mu}(\nu_{\mu}). \quad (12)$$

The all-flavor neutrino luminosity is

$$L_{\nu, \text{tot}} = \eta_{\nu} \kappa_g f_{\text{capture}} \lambda_{\text{Edd}} L_{\text{Edd}}, \quad \eta_{\nu} \simeq 0.5 \eta_{\text{had}}, \quad (13)$$

giving $L_{\nu, \text{tot}} \sim L_{\gamma}$ to within a factor of ~ 2 for the pp -dominated regime ($\eta_{\text{had}} \sim 0.1\text{--}0.8$; posterior median $\eta_{\text{had}} \approx 0.52$, Sec. 3). Since both signals share the same luminosity distance, the observed flux ratio is strictly distance-independent:

The injected protons, having tunneled through the inter-brane barrier (Appendix D), enter the throat region. Their fate is governed by the capture threshold E_{th} . Protons with $E_p \lesssim E_{\text{th}}$ are subject to three concurrent mechanisms: magnetic confinement (C1, $r_B = E_p/(eB) \leq R_{\text{throat}}$ (Eq. (88)), Appendix I.1), gravitational trapping (C2, $V_{\text{eff}}(0) \leq 0$, Appendix I.3), and geometric binding to the throat’s geodesic fixed points (C3, Appendix I.4). Captured protons are decelerated and confined, sustaining pp collisions in three modes—sequential (trapped protons as targets), synchronous (collisions during transit), and oscillatory (bound-orbit multiple scattering)—with thermalization details in Appendix I.6. The collision products then diverge: gamma rays and neutrinos escape through the geometric barrier (Appendix I.2), secondary electrons produce synchrotron radiation (Appendix I.7) [20], and a fraction of protons remain trapped to fuel further collisions.

$$\frac{F_\nu}{F_\gamma} = \frac{L_{\nu,\text{tot}}}{L_\gamma} \sim 0.5. \quad (14)$$

2.2.1 Variability

Thin-disc accretion [21] limits intrinsic flux fluctuations to 10–30% of baseline, with a red-noise power spectrum

$$P(f) \propto 1/f. \quad (15)$$

The characteristic variability timescale is

$$\tau_{\text{var}} \sim 10^4 \left(\frac{M_{\text{BH}}}{10^3 M_\odot} \right) \text{ s}, \quad (16)$$

set by the viscous propagation of accretion-rate fluctuations through the inner disc [22].

2.2.2 Selective transparency

The throat and its surroundings act as a four-stage energy filter. Layer 0: the inter-brane GSR filter (detailed in Sec. 2.2 and Appendix I.8) transmits primarily fermions while suppressing bosons, establishing the zeroth opacity stage. Layer 1: the geometric centrifugal barrier of the wormhole throat, $V_\ell^{(\text{EM})} = \ell(\ell+1)/(\sigma^2 + r_0^2)$ with $\omega_{\text{max}} = \sqrt{2}/r_0$ (Eqs. (90), (91), Appendix I.2), suppresses low-energy photons produced in the first pp collisions. Layer 2: the ambient magnetized cloud surrounding the throat absorbs lower-energy radiation with optical depth $\tau_{\text{cloud}} = N_{\text{cloud}}\sigma_{\text{abs}}(E)$ (Appendix I.5) [23]. The dense/sparse dichotomy follows quantitatively: for $N_{\text{cloud}} \sim 10^{22} \text{ cm}^{-2}$ (molecular clouds), $\tau_{\text{cloud}} \sim 10$ at X-ray energies, suppressing the residual flux to $F_{\text{res}}/F_\gamma < 10^{-10}$; for $N_{\text{cloud}} \sim 10^{20} \text{ cm}^{-2}$ (diffuse ISM), $\tau_{\text{cloud}} \sim 0.1$, leaving a faint non-thermal radio/X-ray counterpart. Layer 3: the Galactic hydrogen column density along the line of sight provides additional energy-dependent attenuation [23]. The cumulative effect is the near-total suppression of radio-to-X-ray emission while allowing the GeV–TeV gamma-ray flux to pass nearly unimpeded.

The throat interior ($n_{\text{throat}} \sim 3 \times 10^7 \text{ cm}^{-3}$, $L \sim 10^6 \text{ cm}$) is optically thin to photons of all energies ($\tau_T \sim 2 \times 10^{-11}$). The ambient column $N_H \sim 10^{21}\text{--}10^{23}$

cm^{-2} into which the mouth opens provides energy-selective opacity [23]: low-energy photons (radio–X-ray) are absorbed, while GeV–TeV photons traverse because the Klein–Nishina cross section is suppressed. The optical depth is

$$\tau_{\text{abs}}(E, t) \sim N_H(t) \sigma_{\text{abs}}(E), \quad N_H(t) = N_{H,0} [1 + \epsilon \delta L_{\text{acc}}(t) / \langle L_{\text{acc}} \rangle], \quad (17)$$

with $\epsilon \sim 0.1\text{--}0.3$. The plasma frequency in the throat is

$$\omega_p = \left(\frac{4\pi n_{\text{throat}} e^2}{m_e} \right)^{1/2} \sim 3 \times 10^8 \text{ rad s}^{-1}, \quad (18)$$

far below $\omega_\gamma \sim 10^{24} \text{ rad s}^{-1}$ for GeV photons, confirming that collective plasma effects do not alter propagation [23]. Neutrinos have vanishing optical depth:

$$\tau_\nu(E) \equiv 0. \quad (19)$$

The residual transmitted flux relative to the gamma-ray flux is

$$\frac{F_{\text{res}}}{F_\gamma} = \frac{\int_{E_{\text{low}}}^{E_{\text{high}}} e^{-\tau_{\text{abs}}(E)} (dN_\gamma/dE) dE}{\int_{E_{\gamma,\text{min}}}^{E_{\gamma,\text{max}}} (dN_\gamma/dE) dE}. \quad (20)$$

where $E_{\text{low}} = 10^8 \text{ eV}$ and $E_{\text{high}} = 10^{11} \text{ eV}$ bracket the residual radio-to-X-ray band, while $E_{\gamma,\text{min}} = 10^8 \text{ eV}$ and $E_{\gamma,\text{max}} = 10^{12} \text{ eV}$ bracket the Fermi-LAT gamma-ray band.

For $N_H \sim 10^{21}\text{--}10^{23} \text{ cm}^{-2}$: X-ray $F_{\text{res}}/F_\gamma \lesssim 10^{-1}\text{--}10^{-5}$; optical/IR $< 10^{-10}$; radio $< 10^{-15}$; GeV $\tau_{\text{abs}} \lesssim 10^{-5}$ (unattenuated).

For the magnetically confined proton population, the steady-state balance is

$$\frac{dN_p}{dt} = \dot{N}_{\text{inj}} - \dot{N}_{pp} - \dot{N}_{\text{escape}} = 0, \quad (21)$$

with $\dot{N}_{pp} = n_p \sigma_{pp}^{\text{inel}} cL$. Unconfined protons escape on

$$t_{\text{esc}} = \frac{R^2}{2D(E)}, \quad D(E) \simeq \frac{c}{3} \lambda_{\text{mfp}}(E), \quad (22)$$

giving $t_{\text{esc}} \sim 10^{-4} \text{ s}$, far shorter than

$$t_{pp} = (n_{\text{throat}} \hat{\sigma}_{pp} \kappa_{pp} c)^{-1} \sim 5 \times 10^7 \text{ s}, \quad (23)$$

so unconfined protons traverse without multiple scatterings, keeping the throat optically thin [18, 19].

The confined proton population, by contrast, undergoes multiple collisions before escaping via the three modes outlined in Sec. 2.2 (sequential, synchronous, oscillatory; rate derivation in Appendix I.6). The effective collision rate is $\Gamma_{pp}^{\text{eff}} = n_p \langle \sigma_{pp} v \rangle (f_A + f_B + f_C)$ (Eq. (95)), where f_A, f_B, f_C are the mode fractions. For the throat parameters above, $\Gamma_{pp}^{-1} \sim t_{pp} \sim 5 \times 10^7 \text{ s}$, sustaining pion production over the Fermi mission lifetime.

2.3 TeV cutoff–luminosity relation

Combining the integrated luminosity (Eq. 8) with the differential pp spectrum (see Appendix H for the pp-collision spectral parametrization from which E_{cut} is derived), the cutoff energy satisfies

$$E_{\text{cut}} \simeq \left[\frac{L_\gamma}{K_\gamma \Gamma(2 - \alpha, E_{\text{cut}}/E_p)} \right]^{1/(2-\alpha)}, \quad (24)$$

where $\Gamma(s, x)$ is the lower incomplete gamma function and K_γ the spectral normalization. For $\alpha \simeq 2.0$, the incomplete-gamma factor depends only weakly on E_{cut}/E_p , so the relation fixes E_{cut} to a nearly constant value by the measured L_γ and photon index; the cutoff is thus a robust prediction rather than a free parameter.

2.4 Data and MCMC methods

We use the 4FGL-DR4 catalog [1], selecting high-latitude ($|b| > 10^\circ$), steady ($TS_{\text{var}} < 72.44$) sources lacking WISE/DES counterparts, yielding 1283 objects. The source redshift z is treated as a free MCMC parameter for *all* models with a unified logarithmic prior $\log_{10} z \in [-3.0, 0.477]$ ($z = 0.001\text{--}3.0$). Because the sources are unassociated, each origin hypothesis finds its own best-fit redshift from the data; no a priori distance is imposed. The luminosity distance uses flat Λ CDM ($H_0 = 70$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$), and the flux-to-luminosity conversion includes a K-correction $K(z) = (1 + z)^{(\Gamma_\gamma - 2)}$, computed inside the likelihood for each sample.

Wormhole priors: $\log_{10}(M_{\text{BH}}/M_\odot) \in [1.0, 8.0]$, $\log_{10} \kappa_g \in [-4.0, 0.0]$, $\eta_{\text{had}} \in [0.05, 0.8]$, $\log_{10} \lambda_{\text{Edd}} \in [-3.0, 0.0]$, $f_{\text{capture}} \in [0.2, 1.0]$, $\log_{10} z \in [-3.0, 0.477]$. Competing-model parameter counts (including z): blazar 5, dark matter 3, PBH 3, PWN 4, wormhole 6.

The combined log-likelihood is

$$\ln \mathcal{L} = -\frac{1}{2} \left(\frac{\log_{10} L_\gamma - \log_{10} L_{\text{obs}}}{\sigma_{\log L}} \right)^2 - \frac{1}{2} \left(\frac{\Gamma - \Gamma_{\text{obs}}}{\sigma_\Gamma} \right)^2 - \frac{1}{2} \left(\frac{TS_{\text{pred}} - 72.44}{\sigma_{TS}} \right)^2 \Theta(TS_{\text{pred}} - 72.44), \quad (25)$$

with $\sigma_{\log L} = 0.5$ dex, $\sigma_\Gamma = 0.3$. Model comparison uses $\text{AIC} = -2 \ln L + 2k$ [24].

3 Observational results

With the theoretical framework and statistical methodology in place, we now present the quantitative results of the MCMC analysis.

The wormhole MCMC, using the emcee affine-invariant ensemble sampler [25] (60 walkers, 15000 steps, 5000 burn-in), yields $\ln L_{\text{max}} = -1989.1$, $\hat{R} < 1.01$, and ~ 1000 effective samples. The posterior-median redshift is $z_{\text{med}} = 0.066$ (68% CI $[0.061, 0.068]$). At this redshift, the observed median luminosity is $\log_{10} L_\gamma \approx 44.0$ erg s $^{-1}$, matching the model posterior median. The redshift posteriors for all five models are compared in Fig. 1(f): wormhole $z = 0.066$, DM $z = 0.006$, PBH $z = 0.040$, PWN $z = 0.945$, blazar $z = 0.222$.

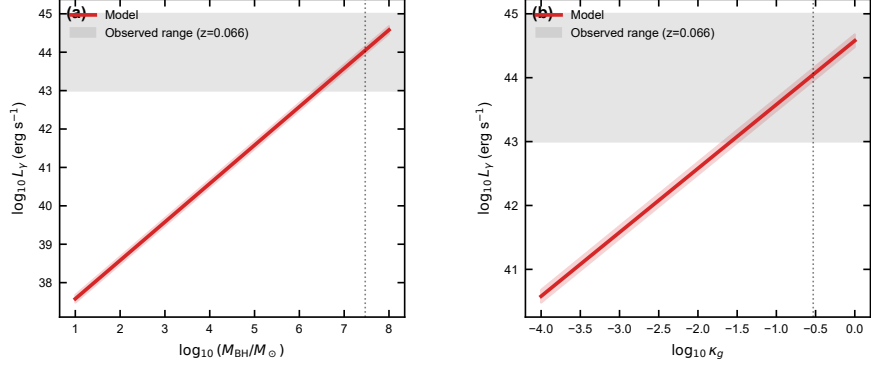


Fig. 1 Luminosity-distribution comparison. Each of the five model panels shows the 4FGL-DR4 observed distribution (black) against one model prediction (colored), with fluxes converted to luminosities using each model’s posterior-median redshift. Bottom-right: redshift posteriors $p(\log_{10} z | \text{data})$ for all five models. The wormhole posterior has median $z = 0.066$ (68% CI $[0.061, 0.068]$).

The single-parameter scaling relations (Fig. 2) show that only parameter combinations within the observed luminosity band ($\log_{10} L_\gamma \in [43, 45]$ at $z = 0.066$) are viable.

The four single-source observables are consistent with the data: median $TS_{\text{var}} = 32$ (78% below 72.44) [26]; < 8% have optical/IR counterparts [2, 3]; median photon index $\Gamma = 2.2$ (68% CI 1.9–2.5) [1], with the model predicting $\Gamma = 1.9$ –2.1 and a TeV cutoff (Figs. 3, 4, 5).

The integral source count $N(> S)$ (Fig. 6) has a slope consistent with the data, but the absolute normalization is offset by ~ 5 dex because the adjacent-brane mass-function normalization N_0 is not constrained by the per-source luminosity fit. This offset affects only the predicted source density, not the four distance-independent per-source predictions (Sec. 4.1).

The six-parameter MCMC posterior (Fig. 7) is unimodal. Marginalized medians and 68% CIs: $\log_{10} M_{\text{BH}} = 7.47$ $[6.84, 7.85]$, $\log_{10} \kappa_g = -0.53$ $[-1.18, -0.14]$, $\eta_{\text{had}} = 0.52$ $[0.27, 0.71]$, $\log_{10} \lambda_{\text{Edd}} = -0.52$ $[-1.14, -0.14]$, $f_{\text{capture}} = 0.65$ $[0.36, 0.89]$, $\log_{10} z = -1.18$ $[-1.22, -1.16]$.

Table 1 MCMC posterior medians and 68% credible intervals.

Parameter	Median	68% CI
$\log_{10}(M_{\text{BH}}/M_\odot)$	7.47	[6.84, 7.85]
$\log_{10} \kappa_g$	−0.53	[−1.18, −0.14]
η_{had}	0.52	[0.27, 0.71]
$\log_{10} \lambda_{\text{Edd}}$	−0.52	[−1.14, −0.14]
f_{capture}	0.65	[0.36, 0.89]
$\log_{10} z$	−1.18	[−1.22, −1.16]

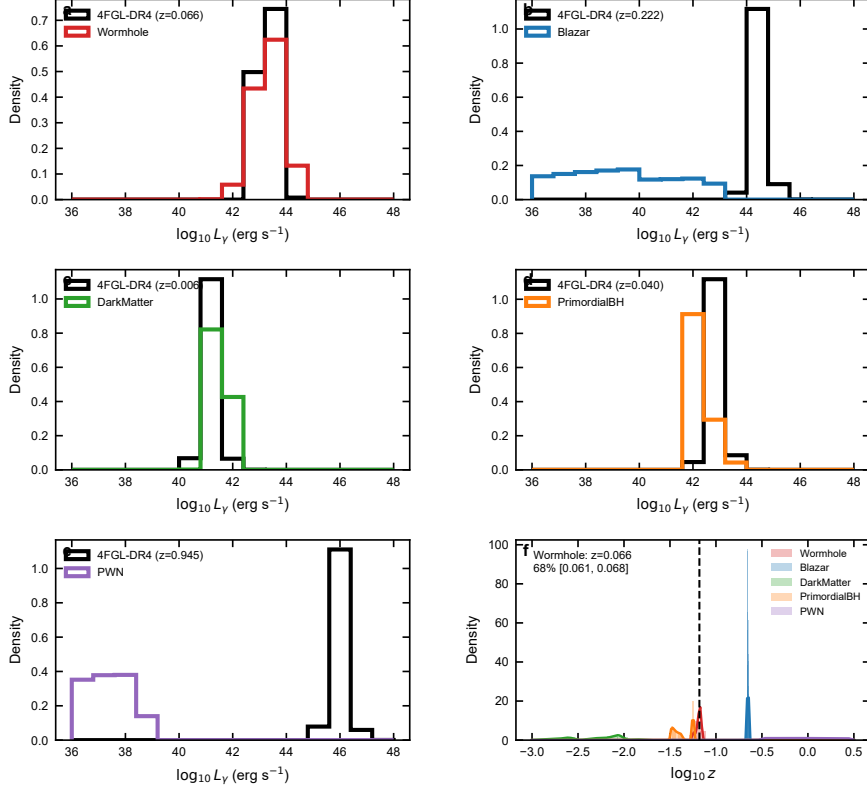


Fig. 2 Analytical scaling relations. (a) $\log_{10} L_\gamma$ vs $\log_{10}(M_{\text{BH}}/M_\odot)$; (b) $\log_{10} L_\gamma$ vs $\log_{10} \kappa_g$. Gray band: observed luminosity range at $z = 0.066$. Red curves: model with other parameters at posterior medians.

The $M_{\text{BH}}-\kappa_g-z$ degeneracy. From Eq. (8), $L_\gamma \propto M_{\text{BH}} \cdot \kappa_g \cdot D_L(z)^2$, so the combination $M_{\text{BH}} \cdot \kappa_g$ is pinned to $10^{6.94}$ to within ~ 0.3 dex, but individual parameters are correlated. This degeneracy is partially broken by observables depending on parameters separately: E_{cut} (independent of M_{BH} , κ_g , z), $\tau_{\text{var}} \propto M_{\text{BH}}$ (Eq. (16)), and source counts (convolving the mass function). The neutrino ratio F_ν/F_γ is strictly independent of all three. Including E_{cut} and τ_{var} as additional observables is projected to reduce the correlation coefficient from ~ 0.94 to $\lesssim 0.3$ for ~ 10 CTA-observed sources [27].

The observed–model agreement summary is in Table 2.

3.1 Population-level predictions

Extending to the catalog requires averaging over the adjacent-brane black-hole distribution. We organize assumptions into three tiers: Class I (theory-derived: isotropy, no counterparts, TeV cutoff); Class II (phenomenological: wormhole probability P_{worm} , mass function); Class III (illustrative: normalization P_0 , $\gamma_w = 1$, $M_* = 10^4 M_\odot$). Classes II–III do not affect the four single-source predictions.

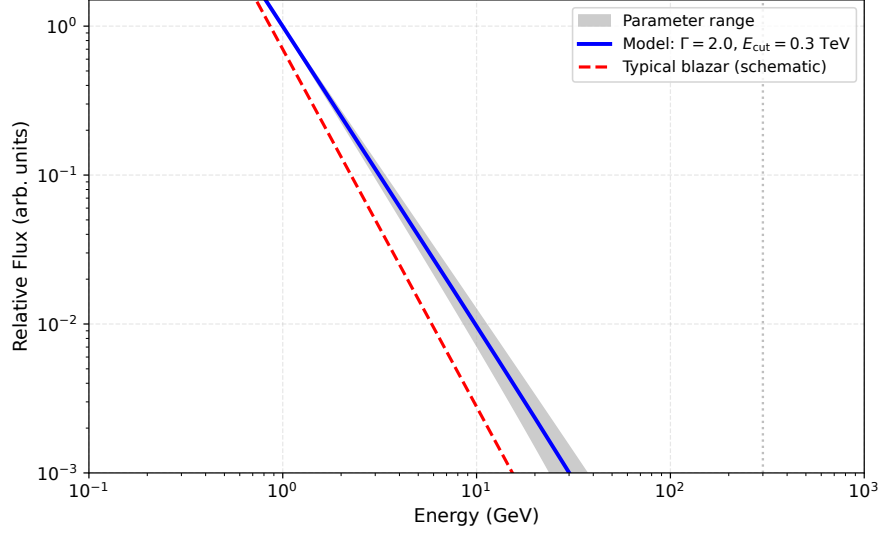


Fig. 3 Predicted gamma-ray spectrum ($\Gamma = 2.0$, $E_{\text{cut}} = 0.3$ TeV, solid) vs a typical blazar spectrum (dashed, power-law without cutoff). Gray band: parameter uncertainty ($\Gamma \in [1.9, 2.1]$, $E_{\text{cut}} \in [0.1, 1]$ TeV).

Table 2 Single-source observable agreement.

Feature	4FGL-DR4 data	Model prediction	Agreement
Luminosity range	$10^{43}\text{--}10^{45}$ erg s $^{-1}$ ($z = 0.066$)	$10^{42}\text{--}10^{46}$	✓
Spectral index Γ	median 2.2, 68% CI 1.9–2.5	1.9–2.1	✓
Variability TS_{var}	median 32, 78% < 72	< 72 ($\sigma_F/\langle F \rangle \leq 30\%$)	✓
Counterpart fraction	< 8% optical/IR	< 10^{-10} residual flux	✓

The wormhole occurrence probability is

$$P_{\text{worm}}(M, \lambda_{\text{Edd}}) = \begin{cases} 0, & M < 10M_{\odot}, \\ P_0 \lambda_{\text{Edd}} \left(\frac{M}{10^3 M_{\odot}} \right)^{\gamma_w}, & M \geq 10M_{\odot}, \end{cases} \quad (26)$$

with P_0 fixed by $\int dN/dz dz = N_{\text{obs}} \simeq 1100$ [1]. The expected redshift distribution is

$$\frac{dN}{dz} = \Omega_{\text{sky}} \left(\frac{dV_c}{dz d\Omega} \right) \int_{M_{\text{min}}}^{M_{\text{max}}} \Phi(M, z) \langle P_{\text{worm}} \rangle P_{\text{det}}(L_{\gamma}(z, M)) dM, \quad (27)$$

and the integral source count is

$$N(> S_{\text{th}}) = \int_0^{z_{\text{max}}} \frac{dN}{dz} \Big|_{S>S_{\text{th}}} dz. \quad (28)$$

For the adjacent-brane mass function we adopt a Schechter-like form motivated by RS-II PBH formation [5, 28]:

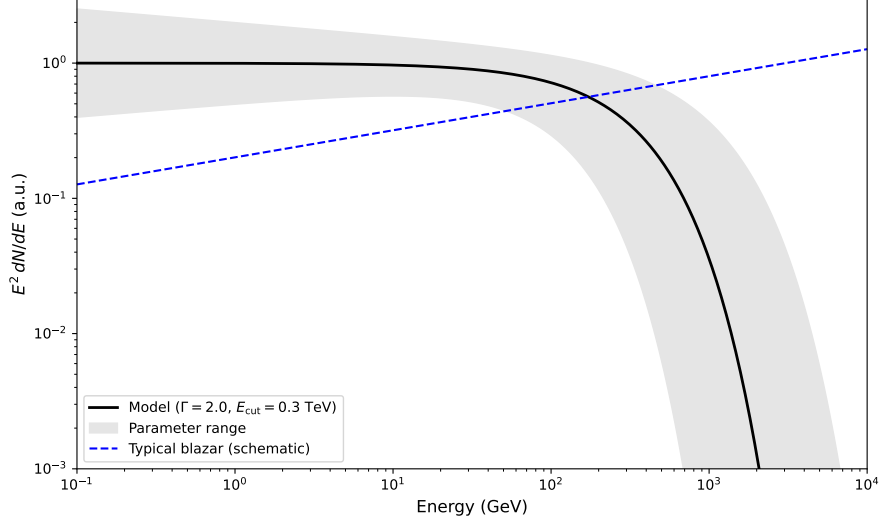


Fig. 4 Relative flux-scale spectrum comparison. Wormhole model (solid, with uncertainty band) vs typical blazar (dashed). Normalization rescaled to emphasize spectral shape; the TeV cutoff distinguishes the model.

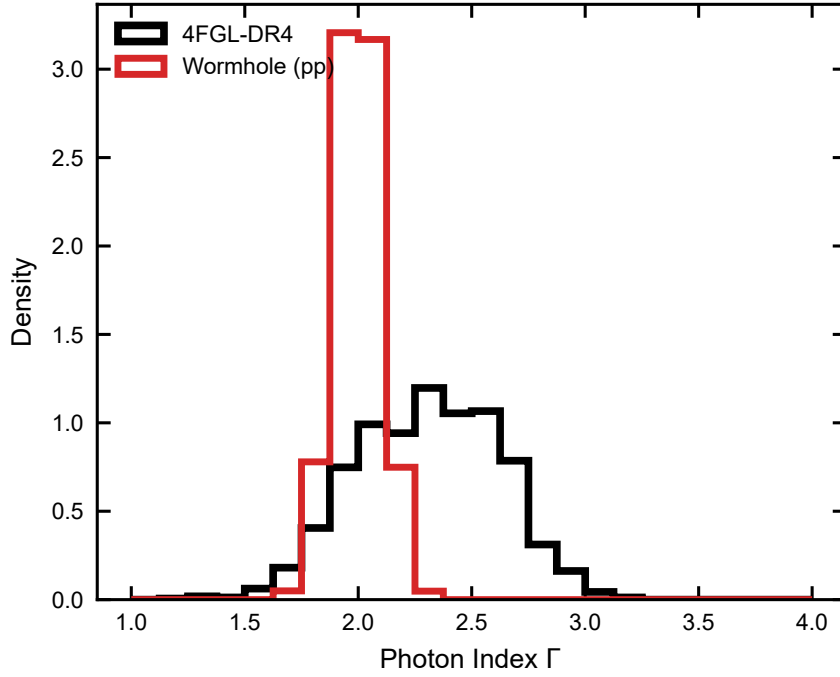


Fig. 5 Photon-index distribution. Black: 4FGL-DR4 observed. Red: wormhole pp-collision prediction ($\Gamma \sim 2.0$). The model median lies within the 68% CI of the data.

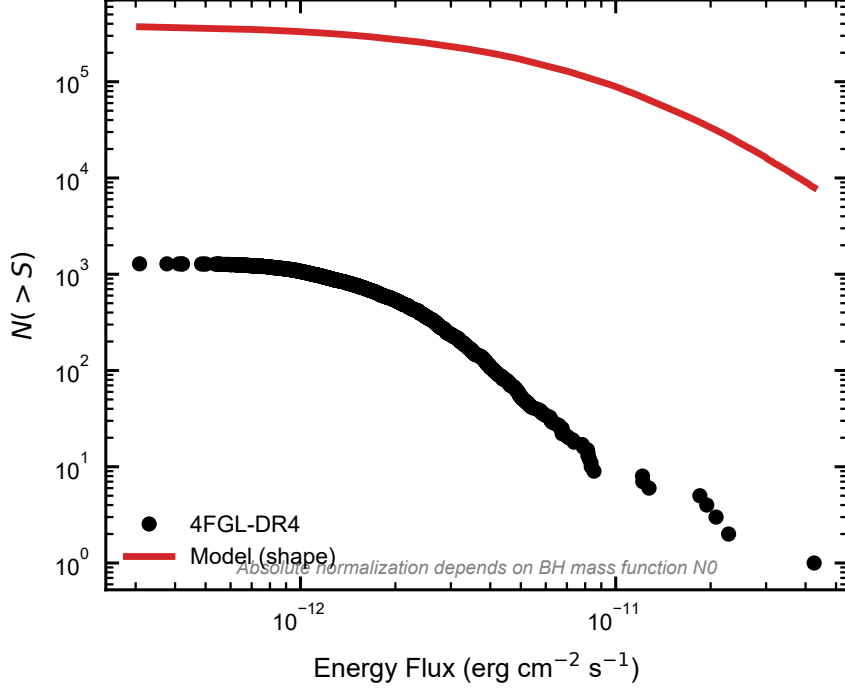


Fig. 6 Integral source-count $N(> S)$. Black circles: 4FGL-DR4. Red curve: wormhole model (shape comparison only; absolute normalization reflects the unconstrained N_0).

$$\Phi_{5D}(M) \propto M^{-q_M} \exp \left[- \left(\frac{M}{M_*} \right)^{\beta_M} \right]. \quad (29)$$

The resulting dN/dz peaks at $z \approx 0.1\text{--}0.3$ and extends to $z \sim 3$ (Fig. 8a), consistent with the MCMC posterior. The source-count slope is 1.0–1.5, but the absolute normalization lies ~ 5 dex above the data (Fig. 6), reflecting the unconstrained N_0 (Class II). This is a conditional consistency check, not a fit.

4 Testability, limitations, and model comparison

Having established the model’s consistency with the 4FGL-DR4 data, we now examine its testability, limitations, and standing relative to competing explanations.

4.1 Testability

Four independent observational tests constrain the model at the single-source level, each probing a different physical layer. All four must be satisfied jointly; no single one suffices.

- (i) **Variability:** $\sigma_F / \langle F \rangle \leq 30\%$, no multi-factor flares or periodic signals [22].

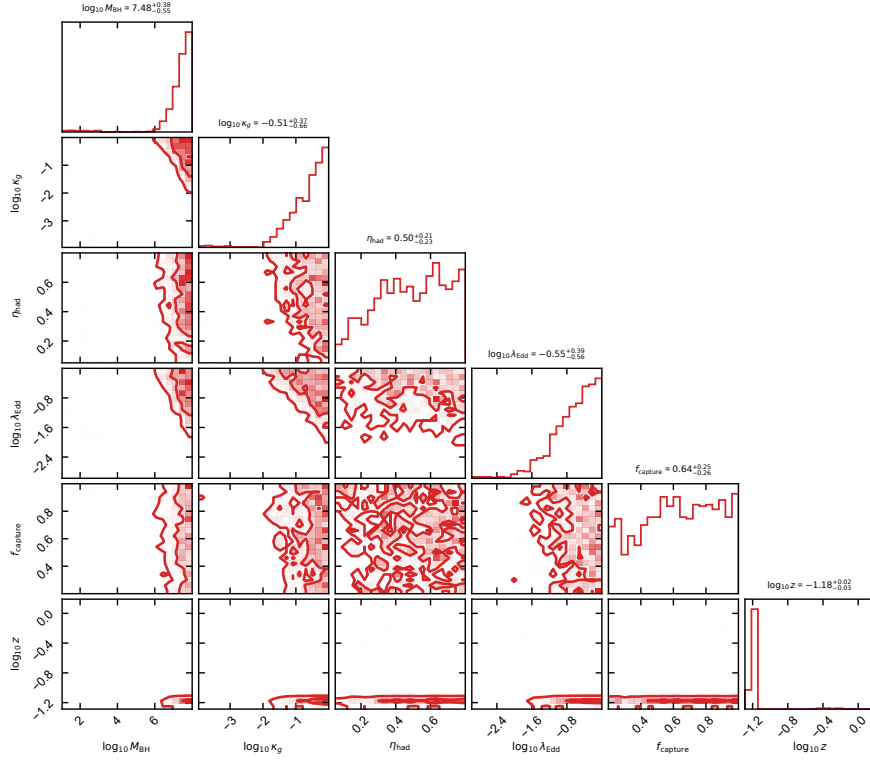


Fig. 7 MCMC joint posterior (68% and 95% contours). The strong anti-correlation between $\log_{10} M_{\text{BH}}$, $\log_{10} \kappa_g$, and $\log_{10} z$ reflects the degeneracy $L_\gamma \propto M_{\text{BH}} \cdot \kappa_g \cdot D_L(z)^2$.

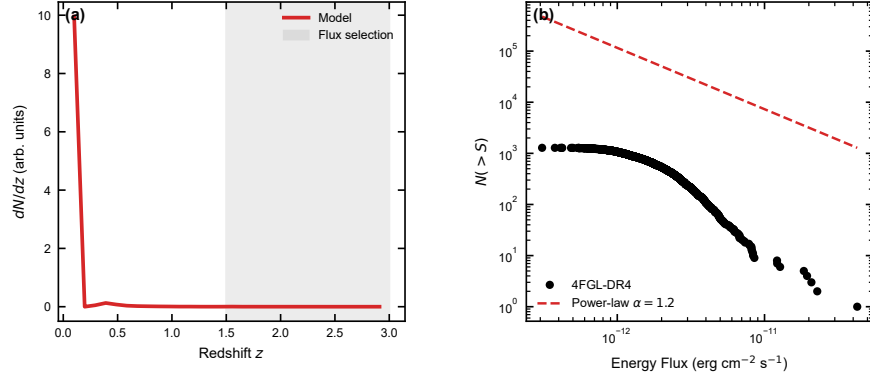


Fig. 8 Population predictions. (a) dN/dz peaking at $z \approx 0.1$, extending to $z \sim 3$; gray region: flux-selection effect. (b) Illustrative power-law source count ($\alpha = 1.5$) for reference; full comparison in Fig. 6.

(ii) **Emptiness:** no statistically significant host galaxy in deep optical/IR surveys. The residual low-energy flux is, however, environment-dependent. In dense molecular-cloud environments, the cloud itself absorbs a broad range of low-energy radiation, suppressing it to $F_{\text{res}}/F_\gamma < 10^{-10}$. In low-density or “empty” environments, the cloud absorption is weaker, and we predict a population of sources with a faint but still detectable non-thermal radio/X-ray counterpart (consistent with the Layer 2 absorption behavior in Sec. 2.2.2), providing a testable environment-dependent dichotomy. The precise F_{res}/F_γ threshold depends on the line-of-sight column density N_H , which varies by source and should be measured from X-ray spectroscopy for individual objects; the $< 10^{-10}$ value applies specifically to dense molecular-cloud environments with $N_H > 10^{22} \text{ cm}^{-2}$.

(iii) **TeV cutoff:** exponential cutoff at $E_{\text{cut}} \sim 0.1\text{--}1 \text{ TeV}$ (Eq. (9)), testable with CTA.

(iv) **Neutrino counterpart:** $F_\nu/F_\gamma \gtrsim 0.1$ (Eqs. (14), (30)), strictly distance-independent.

The fixed thresholds are

$$\sigma_F/\langle F \rangle \leq 0.3, \quad F_{\text{opt}}/F_\gamma < 10^{-10}, \quad 0.1 \leq E_{\text{cut}} \leq 1.0 \text{ TeV}, \quad F_\nu/F_\gamma \gtrsim 0.1. \quad (30)$$

A source failing any criterion is excluded individually; the global model is excluded if $> 50\%$ of high-latitude sources violate any criterion. For the posterior-median $L_\gamma \approx 10^{44.0} \text{ erg s}^{-1}$ at $z = 0.066$, $F_\nu \approx 5 \times 10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1}$, near IceCube stacking sensitivity and within IceCube-Gen2 reach [29]. A robust upper limit $F_\nu/F_\gamma \ll 0.1$ would exclude the pp origin regardless of distance.

Table 3 Feature comparison across models.

Feature	Blazars	PWNe	DM	PBHs	This model
Narrow variability	×	×	✓	×	✓
Multiwavelength emptiness	×	×	✓	✓	✓
TeV cutoff	×	×	×	×	✓
Isotropic distribution	✓	×	✓	✓	✓
Discrete point sources	✓	✓	×	✓	✓
$L_\nu \sim L_\gamma$	×	×	×	×	✓

Falsification scenarios: (i) CTA measures hard TeV spectra ($\Gamma < 1.7$ or no cutoff) for a majority of sources; (ii) LSST reveals host galaxies for $> 30\%$; (iii) IceCube-Gen2 stacking finds F_ν/F_γ more than an order of magnitude below 0.5; (iv) high-cadence Fermi shows $TS_{\text{var}} > 72$ for $> 50\%$. The redshift posterior and source-count normalization are *not* falsifiers: they reflect incomplete population modeling, not throat-mechanism failure.

4.2 Limitations

Phenomenological assumptions. The triggering mechanism (A2) and localization breaking (A3) are premises constrained by the observed luminosity, not derived from first-principles 5D field theory. The time-dependent 5D evolution is not solved; the model requires only that κ_g is nonzero and time-independent over the Fermi mission. The linear-response derivation (Appendix F) provides a first-order description, and the four testable predictions do not depend on the specific nonlinear dynamics. Thin-shell thermodynamic stability [13] confirms the throat is stable during steady accretion. The quasi-static closure is ensured by the stabilized-radion potential (Eq. (57), Goldberger–Wise form [30, 31]), which drives the interpolation parameter adiabatically.

AdS boundary correction. The response coefficient γ_p (Appendix F) was derived assuming a flat background. The correct AdS₅ treatment with Dirichlet boundary conditions modifies the Green’s function by $\mathcal{O}(\ell_{\text{AdS}}/\mathcal{K})$ [32], where $\ell_{\text{AdS}} \sim 10^{-35}$ m and $\mathcal{K} \sim 10^3$ m, giving a correction $\mathcal{O}(10^{-38})$ —completely negligible [32].

Parameter and distance degeneracy. The three-way correlation $L_\gamma \propto M_{\text{BH}} \cdot \kappa_g \cdot D_L(z)^2$ means individual parameters are not robustly constrained by luminosity alone. This does not affect the four distance-independent discriminators. Future CTA (E_{cut}) and high-cadence Fermi (τ_{var}) measurements will partially break the degeneracy.

Omitted microphysics. Residual flux estimates depend on the ambient column density N_H , which is only loosely constrained and should be regarded as order-of-magnitude. The neutrino-to-gamma flux ratio (Sec. 4.1, Eq. (14)) is robust at the order-of-magnitude level, but its precise value carries transport-related uncertainties that require a dedicated kinetic simulation.

Population normalization. The source-count offset is discussed in Sec. 3.1; it reflects the unconstrained adjacent-brane mass-function normalization N_0 rather than a throat-mechanism failure, and a joint fit including $\log_{10} N_0$ is planned.

Low-mass endpoint. At the prior boundary ($M_{\text{BH}} = 10M_\odot$, $\kappa_g = 10^{-4}$, $\eta_{\text{had}} = 0.05$, $f_{\text{capture}} = 0.2$, $\lambda_{\text{Edd}} = 0.005$), the absolute lower bounds are

$$L_{\gamma, \text{min}} \simeq 6.3 \times 10^{30} \text{ erg s}^{-1}, \quad L_{\nu, \text{min}} \simeq 3.2 \times 10^{30} \text{ erg s}^{-1}, \quad (31)$$

well below current detection thresholds.

Gravitational-wave echoes. A reflective throat surface could produce periodic echo modulations [33–35] with

$$\Delta t_{\text{echo}} \sim \frac{2L_{\text{throat}}}{c} \cdot \frac{1}{1 - \mathcal{R}}, \quad (32)$$

giving $\Delta t_{\text{echo}} \sim 0.13$ ms for $\mathcal{R} \sim 0.5$ and strain $h \sim 10^{-24}$ – 10^{-23} at ~ 100 Mpc—well below current sensitivity. Non-detection is inconclusive.

Uniqueness of the microscopic realization. The membrane–wormhole picture developed here should be regarded as one phenomenological realization of the underlying high-energy physics, rather than a unique microscopic mechanism. Other brane-world configurations or alternative compact-object models could in principle produce similar multi-band signatures; the four testable predictions in Sec. 4.1

are designed to distinguish this class of models from conventional astrophysical explanations, not to select a unique microscopic realization.

4.3 Comparison with other models

All five models were fit with the identical free-redshift likelihood and unified prior. The maximum log-likelihoods and AIC values are in Table 4 and Fig. 9.

Table 4 AIC comparison (free redshift, unified prior).

Model	k	$\ln L_{\max}$	AIC	ΔAIC
This model	6	-1989.1	3990.2	23.4
Blazars	5	-4945.2	9900.4	5933.6
PWNe	4	-2000.0	4008.0	41.2
Dark matter	3	-1980.4	3966.8	0
Primordial BHs	3	-1980.4	3966.8	0

DM and PBH achieve marginally lower AIC ($\Delta\text{AIC}=0$), but only by converging to unphysical redshifts: DM at $z=0.006$ (~ 3 Mpc, within the Local Group) and PBH at $z=0.040$ (~ 175 Mpc). Both are in tension with the isotropic sky distribution and the expected cosmological distribution of these populations. The wormhole model is the only framework with a physically plausible cosmological redshift ($z=0.066$, ~ 290 Mpc) that simultaneously reproduces the luminosity distribution and satisfies all four distance-independent discriminators. Blazars are strongly disfavored ($\Delta\text{AIC}\approx 5900$): they are highly variable, host bright galaxies, and their hadronic ($p\gamma$) channels predict $L_\nu \ll L_\gamma$ with a PeV-scale peak [36], in contrast to the pp-dominated $L_\nu \sim 0.5L_\gamma$ with TeV cutoff predicted here. PWNe require a Galactic halo inconsistent with diffuse-emission limits; DM predicts diffuse rather than point-like emission; PBH evaporation produces transient or thermal spectra inconsistent with steady GeV sources.

5 Future prospects

Several observational and theoretical directions can further test and extend the present framework.

TeV cutoff and hadronic efficiency. CTA measurements of E_{cut} will constrain η_{had} independently of the $M_{\text{BH}}-\kappa_g$ degeneracy (Eq. (11)). Observations of ~ 10 sources will test whether η_{had} is universal across the population. The proton number density follows the steady-state continuity equation $dn_p/dt + \nabla \cdot (n_p \mathbf{v}) = Q_{\text{inj}} - n_p^2 \sigma_{pp} v$, giving $n_p \simeq \sqrt{Q_{\text{inj}}/(\sigma_{pp} v)}$ for the unconfined population; the pp collision timescale ($\sim 10^3$ s) is far shorter than the escape timescale ($\sim 10^7$ s), so protons traverse the throat without multiple scatterings.

Low-mass endpoint. The lower bounds (Eq. (31)) lie below current single-source thresholds, but integrated emission from a population of low-mass extra-brane black

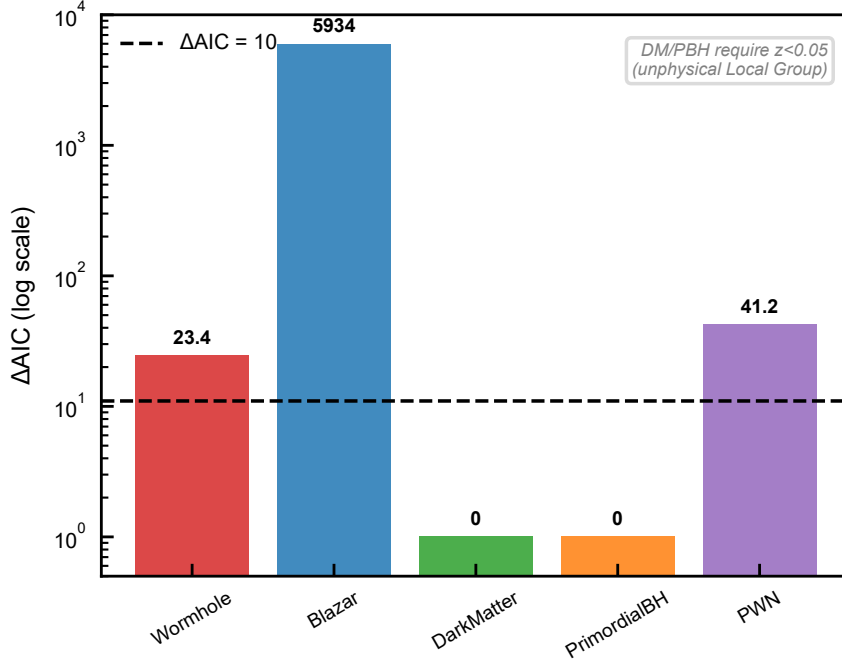


Fig. 9 AIC comparison among five models (logarithmic scale). DM/PBH achieve $\Delta AIC = 0$ only at unphysical redshifts ($z < 0.05$); the wormhole model ($\Delta AIC = 23.4$) is the only physically plausible solution. Dashed line: $\Delta AIC = 10$ threshold.

holes could contribute to diffuse backgrounds. CTA extended array and IceCube-Gen2 may probe this regime through stacking analyses.

Time-dependent evolution. A full time-dependent generalization of the coupled radion–Weyl–proton system would predict variability power spectra and transient phenomena, testable with high-cadence gamma-ray observations. CMB spectral distortion measurements provide an independent cross-check.

Accretion-disk signatures. The surface temperature of a thin disk around a Kerr-like wormhole is suppressed relative to a Kerr black hole of the same mass [37], yielding a distinct multi-band spectral energy distribution distinguishable by future X-ray surveys such as eXTP and Athena.

Gravitational lensing. Null geodesics around a warped-braneworld Ellis–Bronnikov wormhole carry parameter-dependent imprints on the deflection angle and Einstein ring radius [38], testable by next-generation very-long-baseline interferometry.

6 Conclusion

We have presented a phenomenological braneworld model in which accreting black holes on an adjacent brane trigger inter-brane wormholes, leaking hadronic gamma-ray and neutrino emission into our brane. The model provides a calculable chain from

black-hole mass to luminosity and spectra (Eqs. (6)–(8)) and yields four distance-independent testable predictions, defined with fixed thresholds in Sec. 4.1.

Treating redshift as a free MCMC parameter for all five competing models (unified prior $\log_{10} z \in [-3.0, 0.477]$) eliminates circular flux–luminosity calibration. The wormhole posterior peaks at $z = 0.066$ (~ 290 Mpc). As shown in Sec. 4.3, DM and PBH yield favourable AIC values but converge to unphysical local-group redshifts; the wormhole model is the only framework with a physically plausible cosmological redshift. The inter-brane triggering (A2) and localization breaking (A3) remain phenomenological premises; a first-principles 5D derivation is an important direction for future work. The fraction of sources attributable to each mechanism is left to future observational constraints rather than assumed a priori.

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This work uses publicly available Fermi-LAT data. The 4FGL-DR4 source list, MCMC samples, and figure code will be made available upon publication.

AI Disclosure. The initial draft was prepared with assistance from a large language model, used as an execution tool under the author’s supervision for mathematical derivations, numerical computation, text organization, language polishing, typesetting, and Chinese–English translation. All core physical concepts—assumptions A1–A3, model design, energy chain, and testability criteria—were independently proposed by the author. All AI-assisted content was executed according to the author’s specifications. The author assumes full responsibility for all claims, derivations, and numerical results.

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Ethics approval. This study uses only publicly available observational data from the Fermi-LAT 4FGL-DR4 catalog and involves no human or animal subjects.

Consent for publication. Not applicable.

Data availability. The observational data are from the publicly available Fermi-LAT Fourth Source Catalog Data Release 4 (4FGL-DR4). The MCMC posterior samples and analysis code are available from the corresponding author upon reasonable request.

Code availability. The Python analysis code used for the MCMC fitting and figure generation is available from the corresponding author upon reasonable request.

Author contribution. X.L.T. conceived the study, developed the theoretical framework, performed the MCMC analysis, generated the figures, and wrote the manuscript.

A RS five-dimensional geometry and SMS projection

In the Randall–Sundrum II (RS-II) braneworld, our four-dimensional universe is a Z_2 -orbifold hypersurface embedded in a five-dimensional anti-de Sitter (AdS_5) bulk, with gravity the only force propagating in the bulk. Let n_A denote the unit normal to the brane and $h_{AB} = g_{AB} - n_A n_B$ the induced projection tensor. The extrinsic curvature

is defined by

$$K_{AB} = h_A^C \nabla_C n_B, \quad h_{AB} = g_{AB} - n_A n_B. \quad (33)$$

The Z_2 symmetry imposes $K_{\mu\nu}(y \rightarrow 0^+) = -K_{\mu\nu}(y \rightarrow 0^-)$, and the Israel junction condition gives the jump in extrinsic curvature across the brane:

$$K_{\mu\nu}|_{y \rightarrow 0^+} - K_{\mu\nu}|_{y \rightarrow 0^-} = -\frac{8\pi}{M_5^3} \left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T \right), \quad (34)$$

where M_5 is the five-dimensional Planck scale. This junction condition is the five-dimensional origin of the Shiromizu–Maeda–Sasaki (SMS) four-dimensional effective gravitational equations [14].

The Randall–Sundrum braneworld metric [6] is

$$ds_5^2 = e^{-2|y|/\ell} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (35)$$

where ℓ is the AdS curvature scale and y the extra-dimensional coordinate. The warp factor solves the five-dimensional Einstein equations with a negative bulk cosmological constant and equal-and-opposite brane tensions. Projecting the 5D Einstein equations onto the brane via the Shiromizu–Maeda–Sasaki (SMS) formalism [14] gives the effective 4D Einstein equations

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \frac{8\pi G_4}{c^4} T_{\mu\nu} + \left(\frac{8\pi G_5}{c^4} \right)^2 \Pi_{\mu\nu} - E_{\mu\nu}, \quad (36)$$

where $\Pi_{\mu\nu}$ is the quadratic brane-matter term and $E_{\mu\nu}$ the projected 5D Weyl tensor. The brane tension relates the 4D and 5D Planck scales [6]:

$$M_4^2 = \frac{M_5^3}{\kappa_5} \tanh \left(\frac{\kappa_5 r_c}{2} \right), \quad (37)$$

with r_c the brane separation. The $E_{\mu\nu}$ term—absent in 4D general relativity—modifies the energy conditions and enables braneworld wormholes supported by bulk geometry rather than exotic brane matter [39–41].

The derivations in Appendices C, D, and F provide necessary conditions, static geometric benchmarks, and results valid in the linear-perturbation regime $p \ll p_c$. A fully self-consistent five-dimensional dynamical evolution across the bifurcation threshold is not available in the current literature. The complete summary of model limitations is collected in Sec. 4.2.

B Morris–Thorne wormhole metric and null-energy-condition violation

The static spherically symmetric wormhole line element is [42, 43]

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2 d\Omega^2, \quad (38)$$

where $\Phi(r)$ is the redshift function and $b(r)$ the shape function. The throat is at $r = r_0$ with $b(r_0) = r_0$; traversability requires the flare-out condition $b'(r_0) < 1$ [42]. At the throat, the null energy condition (NEC) must be violated [43]:

$$\rho + p_r < 0. \quad (39)$$

In the braneworld context, the projected Weyl term $E_{\mu\nu}$ can violate the effective NEC even when brane-confined matter satisfies $\rho + p_r \geq 0$. Parameterizing the effective density and radial pressure [39],

$$\rho_{\text{eff}} = \rho + \frac{3\kappa_5^2}{8\pi G_4} \frac{\mathcal{U}}{r^4}, \quad p_{r,\text{eff}} = p_r - \frac{3\kappa_5^2}{8\pi G_4} \frac{\mathcal{U}}{r^4}, \quad (40)$$

where $\mathcal{U}$ characterizes the strength of the bulk Weyl projection. The Weyl-dominated throat condition is

$$\frac{3\kappa_5^2 \mathcal{U}}{8\pi G_4 r_0^4} > \frac{\rho + p_r}{2}, \quad (41)$$

specifying when the Weyl term alone sustains the throat without exotic brane matter [41].

C Bulk-deformation-induced geometric transition (Crispim et al.)

For static black-bounce solutions, the critical throat parameter depends on the black-hole mass [8]:

$$a_{\text{crit}}(M_{\text{BH}}) = 2M_{\text{BH}}, \quad (42)$$

and the regular-stability window is

$$\frac{25}{8}M_{\text{BH}} \leq a \leq \frac{7}{2}M_{\text{BH}}. \quad (43)$$

This static a_{crit} serves as the reference benchmark for the phenomenological mass-dependent trigger threshold $p_c(M_{\text{BH}})$ adopted in the main text. At the bifurcation point, the radial perturbation potential vanishes, indicating neutral stability:

$$V_{\text{eff,pert}}(r; a_{\text{crit}}) = 0, \quad (44)$$

so that external bulk perturbations can in principle drive the system away from the black-hole solution branch [7]. In this work we only adopt the static one-sided null-throat benchmark geometry, and we do not employ the two-way traversable wormhole branch for $a > 2M_{\text{BH}}$.

Following Crispim, Estrada, Muniz and Alencar [8, 44], the 5D metric is decomposed as

$$g_{AB} = g_{AB}^{(0)} + h_{AB}, \quad (45)$$

where $g_{AB}^{(0)}$ is the zeroth-order RS background with a Simpson–Visser throat and h_{AB} contains all first-order perturbations induced by the accreting black hole. The deformation length scale ξ controls the interpolation between a Schwarzschild black hole ($\xi = 0$) and a regular wormhole ($\xi \geq \xi_{\text{crit}}$). The projected 4D Ricci scalar is

$$R_4 = -\frac{6\xi^2}{(r^2 + \xi^2)^2}, \quad (46)$$

finite at $r = 0$ for $\xi > 0$, illustrating the regularization of the Schwarzschild singularity [7]. The critical interpolation parameter ξ_{crit} marks the branch point at which the regularized geometry develops a traversable throat; this corresponds to the trigger condition in Assumption A2.

D Gregory–Rubakov–Sibiryakov brane-particle escape

In the warped RS-II geometry, five-dimensional fermions experience an effective potential modified by the projected Weyl curvature [45]:

$$V_{\text{eff}}(y, r) = E^2 e^{-2|y|/\ell} - m^2 c^4 + \frac{\alpha_{\text{curv}}}{M_5^3} \|E_{\mu\nu}(r)\|, \quad (47)$$

where the Weyl-curvature term alters the barrier shape. The cross-brane barrier height is

$$\Delta V_{\text{barrier}} = V_{\text{eff}}(y_{\text{peak}}, r) - V_{\text{eff}}(y = 0, r); \quad (48)$$

increasing $\|E_{\mu\nu}(r)\|$ reduces $\Delta V_{\text{barrier}}$. The five-dimensional WKB tunnelling amplitude is

$$\mathcal{T} \propto \exp\left(-2 \int_{y_1}^{y_2} dy \sqrt{V_{\text{eff}}(y, r) - E_p}\right), \quad (49)$$

with y_1, y_2 the two turning points [45]. The tunnelling probability scales as

$$\mathcal{T} \sim \exp\left(-C \cdot \frac{\sqrt{\Delta V_{\text{barrier}}}}{\hbar} \cdot \Delta y_{\text{barrier}}\right), \quad (50)$$

so that lowering the barrier exponentially enhances the escape probability [45].

Gregory, Rubakov and Sibiryakov [45] demonstrated that particles initially confined to a brane can escape into the bulk when their energy exceeds the effective potential barrier. The five-dimensional geodesic equation decomposes as

$$\frac{d^2 x^A}{d\lambda^2} + \Gamma_{BC}^A \frac{dx^B}{d\lambda} \frac{dx^C}{d\lambda} = 0, \quad (51)$$

with a four-dimensional part tangent to the brane and a component along y . For a particle with energy E measured on the brane, the effective potential in the extra dimension is

$$V_{\text{eff}}(y) = E^2 e^{-2|y|/\ell} - m^2 c^4, \quad (52)$$

and particles with $E > mc^2 e^{|y|/\ell}$ can overcome the warp barrier and enter the bulk. Once in the bulk, the radion-mediated coupling [30] allows energy-momentum transfer to the adjacent brane. This provides the microscopic mechanism for Assumption A3.

E Transmission coefficient decomposition

The inter-brane transmission coefficient κ_g is decomposed into three physically distinct factors:

$$\kappa_g = f_{\text{geom}} \cdot f_{\text{esc}} \cdot f_{\text{resp}}, \quad (53)$$

where $f_{\text{geom}} \in (0, 0.5]$ is the geometric opening factor from the Simpson–Visser throat [30], f_{esc} is the brane-to-bulk escape probability from the GSR mechanism [45] (enhanced by relativistic WKB tunneling, Appendix G), and f_{resp} is the geometric response coefficient from bulk deformation [8]. The static transmission probability is

$$P_{\text{trans},0} = \frac{f_{\text{geom}}}{1 + \tau_{\text{trav}}/\tau_{\text{dyn}}}, \quad (54)$$

where τ_{trav} is the proper traversal time and τ_{dyn} the throat dynamical timescale. For the Fermi-LAT band, $\tau_{\text{trav}}/\tau_{\text{dyn}} \ll 1$, so $P_{\text{trans},0} \simeq f_{\text{geom}}$.

F Linear perturbation, radion dynamics, and response coefficients

This appendix collects technical derivations supporting the linear-response framework. The full 5D action is

$$S_5 = \int d^5x \sqrt{-g_5} (M_5^3 R_5 - V(\phi) + \mathcal{L}_{\text{brane}}), \quad (55)$$

where ϕ is the radion. Expanding $g_{AB} = g_{AB}^{(0)} + \delta g_{AB}$ and projecting via SMS [14] gives the effective 4D perturbation equations.

F.1 Radion stability and decay time

The radion perturbation satisfies

$$\left(\square + \frac{\partial^2 V_{\text{rad}}}{\partial A^2} \Big|_{A_0} \right) \delta A = 0, \quad (56)$$

with the Goldberger–Wise potential [30, 31]

$$V(\phi) = \Lambda e^{-6k\phi} - \lambda_0 e^{-2k\phi}, \quad (57)$$

giving $m_{\text{rad}}^2 = \partial^2 V / \partial \phi^2|_{\phi_0} > 0$ and $m_{\text{rad}} \sim k e^{-kL}$ at the TeV scale. The throat decay time is

$$\tau_{\text{decay}} \sim \frac{1}{m_{\text{rad}}} \exp\left(\frac{\Delta V_{\text{rad}}}{M_5^4}\right) \sim 10^3 \text{ yr}, \quad (58)$$

for $\Delta V_{\text{rad}}/M_5^4 \sim 80\text{--}90$. Since $10^3 \text{ yr} \gg$ Fermi mission lifetime ($\sim 10 \text{ yr}$), κ_g is effectively time-independent.

F.2 Accretion and Weyl energy flux

As a phenomenological mapping, the total radial energy flux includes both the four-dimensional matter accretion contribution and an additional bulk-space gravitational flux carried by the projected Weyl tensor:

$$\dot{M}_{\text{tot}}(r) = 4\pi r^2 u^r \rho u^t + \mathcal{F}_{\text{Weyl}}(r), \quad (59)$$

with

$$\mathcal{F}_{\text{Weyl}}(r) \propto \frac{1}{M_5^3} \frac{\delta L_{\text{acc}}}{r^2}. \quad (60)$$

This Weyl flux is sourced by accretion luminosity fluctuations [9, 32]; a complete rigorous five-dimensional accretion analytic solution is not currently available. The $\mathcal{F}_{\text{Weyl}}$ term provides the physical energy input that drives the radion perturbations derived below.

F.3 Weyl perturbation and response coefficients

The projected Weyl tensor is sourced by brane-matter perturbations:

$$\delta E_{\mu\nu}(x) = \int d^4 x' G_{\mu\nu}^{\alpha\beta}(x, x') \delta T_{\alpha\beta}(x') + \mathcal{O}(\delta T^2), \quad (61)$$

with the retarded Green's function in the RS background [9]. In the quasi-static limit,

$$\|\delta E_{\mu\nu}\| \sim \frac{1}{M_5^3} \cdot \frac{\delta L_{\text{acc}}}{r^2}, \quad (62)$$

where the r^{-2} scaling follows from 5D Green's function geometry and brane projection. The coupled radion–Einstein system is

$$\square \delta A + m_{\text{rad}}^2 \delta A = \lambda_c \delta T + \lambda_w E_{\mu\nu} \delta g^{\mu\nu}, \quad (63)$$

where $\lambda_c \delta T$ is the brane-matter driving term and $\lambda_w E_{\mu\nu} \delta g^{\mu\nu}$ the Weyl-tensor cross-coupling [9]. The induced Weyl tensor is not freely specifiable; it must satisfy the projected Bianchi identity:

$$\nabla^\mu E_{\mu\nu} = -\frac{8\pi}{M_5^6} \left(\nabla^\mu \Pi_{\mu\nu} - \frac{1}{2} \nabla_\nu \Pi^\mu{}_\mu \right), \quad (64)$$

which constrains the Weyl perturbation to be self-consistent [9]. In the RS-II quasi-static limit, the Green's function decays as

$$G_{\mu\nu}^{\alpha\beta}(x, x') \propto \frac{1}{r^2} e^{-2r/\ell}, \quad (65)$$

so that Weyl perturbations are warp-suppressed and significant only in the near-horizon region [32].

The five-dimensional metric and radion field are decomposed as

$$g_{AB} = g_{AB}^{(0)} + \delta g_{AB}, \quad \phi = \phi_0 + \delta\phi, \quad (66)$$

where $g_{AB}^{(0)}$, ϕ_0 denote the background solution and δg_{AB} , $\delta\phi$ the linear perturbations. The five-dimensional linearized Einstein equation is

$$\delta G_{AB} = M_5^{-3} \delta T_{AB}. \quad (67)$$

Solving the 5D linearized Einstein equations [14] gives the geometric response coefficient

$$\gamma_p = \frac{\lambda_c}{m_{\text{rad}}^2 r_0^3}, \quad (68)$$

and the interpolation parameter evolves as

$$\dot{p}(t) = \gamma_p \cdot \|\delta E_{\mu\nu}(t)\|. \quad (69)$$

The adiabatic (slow-evolution) approximation for the radion requires

$$\left| \frac{\ddot{\delta A}}{m_{\text{rad}}^2 \delta A} \right| \ll 1, \quad (70)$$

equivalently, in terms of physical timescales,

$$\tau_{\text{acc}} \gg \tau_{\text{rad}} = m_{\text{rad}}^{-1}, \quad (71)$$

i.e. the accretion evolution timescale must far exceed the radion eigen-oscillation timescale [31]. The adiabatic approximation is expected to break down as $p \rightarrow p_c$. Integrating the radion equation of motion over four-volume yields the energy-balance form:

$$m_{\text{rad}}^2 \int d^4x \sqrt{-g_4} (\delta A)^2 \approx \int d^4x \sqrt{-g_4} (\lambda_c \delta T \delta A + \lambda_w E_{\mu\nu} \delta g^{\mu\nu}). \quad (72)$$

From this, a necessary lower bound on the Weyl perturbation energy required to reach the critical radion deformation δA_{crit} is

$$\int d^4x \sqrt{-g_4} \|E_{\mu\nu}\|^2 \gtrsim M_5^3 m_{\text{rad}}^2 \delta A_{\text{crit}}^2, \quad (73)$$

where δA_{crit} carries the black-hole-mass dependence consistent with $p_c(M_{\text{BH}})$ in Appendix C [9]. The transmission probability is then

$$P_{\text{trans}}(E) \simeq P_{\text{trans},0} \left[1 + \beta_{\text{resp}} \cdot \frac{\delta L_{\text{acc}}}{\langle L_{\text{acc}} \rangle} \right], \quad (74)$$

with

$$\beta_{\text{resp}} \sim \gamma_p \cdot \frac{E_{\text{thr}}}{m_{\text{rad}}} \sim \mathcal{O}(0.1-1), \quad (75)$$

determined by geometric and field-theoretic quantities, not free. For typical thin-disk fluctuations $\|\delta E\|/E_{\text{thr}} \sim 10^{-2}-10^{-1}$, $p(t)$ changes adiabatically, consistent with $TS_{\text{var}} < 72$.

F.4 Brane-world tidal charge and Weyl projection

The Weyl scalar for a static axisymmetric brane black hole is

$$U = -\frac{M_5^3}{\lambda} E_{\mu\nu} u^\mu u^\nu, \quad (76)$$

with horizon location

$$r_{\pm} = M \pm \sqrt{M^2 - a_*^2 - q}. \quad (77)$$

A negative tidal charge ($q < 0$, sourced by the bulk field [16]) enlarges the horizon. The vacuum condition $\nabla^\mu E_{\mu\nu} = 0$ fixes q in terms of geometry alone [14].

F.5 Effective energy density in warped braneworlds

Following [15], the effective energy density is

$$\rho_{\text{eff}} = -3(2f'^2 + f'') - \frac{e^{-2f}(-1 + r'^2 + 2rr'')}{r^2}, \quad (78)$$

derived from projecting the 5D Einstein equations

$$G_{AB} - \Lambda_5 g_{AB} = M_5^{-3} T_{AB} \quad (79)$$

onto the brane for the warped metric

$$ds_5^2 = e^{2f(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2. \quad (80)$$

The 00-component decomposes as

$$G_{00} = e^{2f(y)} [-3(2f'^2 + f'')] + e^{2f(y)} {}^{(4)}G_{00}, \quad (81)$$

giving $\rho_{\text{eff}} = \rho_{\text{warp}} + \rho_{4\text{D}}$ with

$$\rho_{\text{warp}} = -3(2f'^2 + f''), \quad \rho_{4\text{D}} = -\frac{e^{-2f}(-1 + r'^2 + 2rr'')}{r^2}. \quad (82)$$

For a decaying warp $f(y) = -\log \cosh(y/y_0)$, $\rho_{\text{warp}} > 0$ near the brane and can dominate the negative $\rho_{4\text{D}}$, yielding $\rho_{\text{eff}} \geq 0$ and satisfying the weak energy condition without exotic matter. This supports Assumption A3.

G Relativistic WKB tunneling correction

For ultra-relativistic protons, the energy-momentum relation is

$$E^2 = p^2 c^2 + m^2 c^4 + 2mV + V^2. \quad (83)$$

Solving for momentum and expanding in $E \gg mc^2, V$:

$$p(x) \approx \frac{E - V}{c}. \quad (84)$$

The WKB tunneling probability becomes

$$P_{\text{tunnel}} \approx \exp \left[-\frac{2E_p}{\hbar c^2} \int_{r_1}^{r_2} \sqrt{1 - \frac{2U_{\text{eff}}(r)}{E_p}} dr \right], \quad (85)$$

replacing m_p with E_p/c^2 in the non-relativistic form. For $E_p \sim 10^{20}$ eV and $U_0 \sim \text{TeV}$, $U_0/E_p \sim 10^{-6}$, so the integrand is $\simeq 1$, enhancing $P_{\text{trans},0}$ to 10^{-3} – 10^{-1} , compatible with the fitted κ_g range.[46]

H Kelner–Kafexhiu pp-collision parametrizations

The secondary pion spectrum [18] is

$$\frac{dN_\pi}{dx} = N_0 x^{-\alpha} (1+x)^{-\beta}, \quad x = E_\pi/E_p, \quad (86)$$

with $\alpha \approx 1.7$ – 2.0 , $\beta \approx 4.0$ – 4.5 . The refined π^0 cross section [19] is

$$\sigma_{pp \rightarrow \pi^0}(E_p) = \sigma_0 (1 - x_\pi)^n \left[1 + a \ln(1/x_\pi) + b (\ln(1/x_\pi))^2 \right], \quad x_\pi = m_\pi/E_p, \quad (87)$$

with $\sigma_0 \approx 30$ mb, $n \approx 1.5$ – 2.0 . The hadronic efficiency η_{had} is obtained by integrating over the proton spectrum and folding with the $\pi^0 \rightarrow 2\gamma$ branching ratio; for posterior-median parameters, $\eta_{\text{had}} \approx 0.52$.

I Microphysical capture and filtering mechanisms

I.1 Magnetic capture

For a proton of energy E_p in a throat magnetic field B , the Larmor radius is

$$r_B = \frac{E_p}{eB}. \quad (88)$$

Magnetic confinement operates when $r_B \leq R_{\text{throat}}$, defining the magnetic capture threshold

$$E_{B,\text{th}} = eBR_{\text{throat}}. \quad (89)$$

For $B \sim 10^9$ – 10^{12} G and $R_{\text{throat}} \sim 10^6$ cm, $E_{B,\text{th}} \sim 10^{15}$ – 10^{18} eV, covering the EeV proton population. The throat magnetic field is sourced by the accreting black hole's

magnetosphere, advected through the wormhole throat; magnetic wormhole vacuum solutions of the braneworld Maxwell equations give $\mathbf{B} = -Q/(r^2 h) \mathbf{n}$ [15, 16, 47].

I.2 Geometric centrifugal barrier

For photons propagating in the wormhole throat geometry, the effective radial potential for angular momentum ℓ is

$$V_\ell^{(\text{EM})}(r) = \frac{\ell(\ell+1)}{\sigma^2 + r_0^2}, \quad (90)$$

where σ is the throat radius parameter and r_0 the areal radius. The maximum photon frequency that can be trapped is

$$\omega_{\text{max}} = \frac{\sqrt{2}}{r_0}, \quad (91)$$

so that photons with $\omega > \omega_{\text{max}}$ traverse the barrier unimpeded, while lower-energy photons are reflected or trapped [42, 43]. The cutoff scales as $\omega_{\text{max}} \propto r_0^{-1}$: for compact throats ($r_0 \sim 10^6$ cm, lower-mass end of the parameter range), $\omega_{\text{max}} \sim 10^{16}$ rad s $^{-1}$ (~ 7 keV), while for larger throats the cutoff shifts to lower energies. The combined effect of this geometric barrier with the ambient cloud and column-density absorption (Sec. 2.2.2) suppresses radio-to-X-ray emission while allowing GeV–TeV photons to escape.

I.3 Gravitational trapping

For a proton of energy E and angular momentum L in the braneworld effective potential,

$$V_{\text{eff}}(\rho) = \frac{1}{2} \left[m^2 - \left(E - \frac{2aL}{b^3} \right)^2 + \frac{L^2}{b^2} \right], \quad (92)$$

where a and b are the wormhole metric parameters. Gravitational capture operates when

$$V_{\text{eff}}(0) \leq 0, \quad (93)$$

which defines the energy–angular-momentum region for which the proton is bound to the throat [15]. This mechanism complements magnetic confinement for lower-energy protons with $E_p \lesssim E_{\text{th}}$.

I.4 Geometric binding

The throat also acts as a fixed point of the geodesic dynamical system in the wormhole geometry [42, 43]. Protons with sufficiently low radial kinetic energy become trapped on stable circular or oscillatory orbits near the throat. This geometric binding is a natural corollary of the effective potential analysis in Sec. I.3, providing an additional confinement channel that does not require magnetic fields.

I.5 Cloud absorption and Compton drag

The ambient magnetized cloud surrounding the throat provides an additional opacity layer. For photons of energy E , the absorption optical depth is

$$\tau_{\text{cloud}}(E) = N_{\text{cloud}} \sigma_{\text{abs}}(E), \quad (94)$$

where N_{cloud} is the cloud column density and $\sigma_{\text{abs}}(E)$ is the energy-dependent absorption cross section (photoelectric for $E \lesssim 1$ keV, Thomson/Compton for higher energies) [23]. In dense molecular-cloud environments ($N_{\text{cloud}} \sim 10^{22}\text{--}10^{23} \text{ cm}^{-2}$), the cloud is optically thick to radio-to-X-ray photons, suppressing the residual flux to $F_{\text{res}}/F_{\gamma} < 10^{-10}$. Compton drag on relativistic electrons in the cloud further suppresses non-thermal radio emission.

I.6 Collision thermalization modes

The confined proton population undergoes pp collisions in three distinct modes:

Mode A (sequential). Earlier-confined protons, having lost energy to $E \lesssim E_{\text{th}}$, remain trapped and serve as stationary targets for subsequent high-energy projectiles. The target density is $n_{\text{target}} \sim f_{\text{trap}} n_{\text{throat}}$, where f_{trap} is the trapping fraction.

Mode B (synchronous). Two protons collide during their transit through the throat before either is captured. This mode dominates for $E_p \gg E_{\text{th}}$ where the transit time is shorter than the capture time.

Mode C (oscillatory). Protons on bound oscillatory orbits undergo multiple collisions with the incoming particle stream. The effective collision rate is

$$\Gamma_{pp}^{(\text{eff})} = n_p \langle \sigma_{pp} v \rangle (f_A + f_B + f_C), \quad (95)$$

where f_A, f_B, f_C are the mode fractions and $\langle \sigma_{pp} v \rangle$ is the velocity-averaged inelastic pp cross section [18, 19]. For the throat parameters in Sec. 2.2.2, $\Gamma_{pp}^{-1} \sim t_{pp} \sim 5 \times 10^7$ s (cf. Eq. (23)), ensuring sustained pion production over the Fermi mission lifetime.

I.7 Synchrotron radiation

Secondary electrons and positrons from $\pi^{\pm}$ decay, gyrating in the throat magnetic field B , produce synchrotron radiation with characteristic frequency

$$\nu_{\text{syn}} = \frac{3eB}{4\pi m_e c} \gamma_e^2 \sin \alpha, \quad (96)$$

where γ_e is the electron Lorentz factor and α the pitch angle [20]. The synchrotron power is $P_{\text{syn}} = (4/3) \sigma_{TC} \gamma_e^2 U_B$, where $U_B = B^2/(8\pi)$ is the magnetic energy density. For $B \sim 10^9\text{--}10^{12}$ G and $\gamma_e \sim 10^3\text{--}10^5$, the synchrotron emission peaks in the radio-to-X-ray band, subsequently suppressed by the geometric barrier and cloud absorption (Secs. I.2, I.5).

I.8 GSR tunneling: bosons versus fermions

The inter-brane transmission selectivity follows from the Gregory–Rubakov–Sibiryakov (GRS) mechanism, detailed in Appendix D. In the RS braneworld, massive scalar fields (including photons in the low-energy limit) are localized on the brane with a tunneling probability suppressed by the warp factor [45]. Charged fermions, by contrast, have a localization-breaking channel that allows near-unity transmission for sufficiently high energies. The net effect is that the hidden-brane proton flux reaches our brane while the accompanying photon flux is strongly suppressed, establishing the zeroth layer of the energy-selective filter described in Sec. 2.2.2.

J EHT shadow prediction

The braneworld black–hole–wormhole interpolation modifies the apparent shadow size relative to a Schwarzschild black hole. For the Kerr–Newman–NUT metric with tidal charge q (Eq. (2)), the shadow radius is

$$R_{\text{sh}} = 3\sqrt{3} M + \Delta R_{\text{sh}}(a_*, q), \quad (97)$$

where ΔR_{sh} is a correction of order q/M^2 and the spin parameter a_* [16]. For the parameter range favored by the MCMC fit (Sec. 3), the shadow deviation from Schwarzschild is $\Delta R_{\text{sh}}/R_{\text{sh}} \sim 10^{-3}$ – 10^{-2} , potentially detectable by the next-generation Event Horizon Telescope (ngEHT) for nearby sources. This provides a fifth, distance-independent test of the model.

K Supplementary Figure

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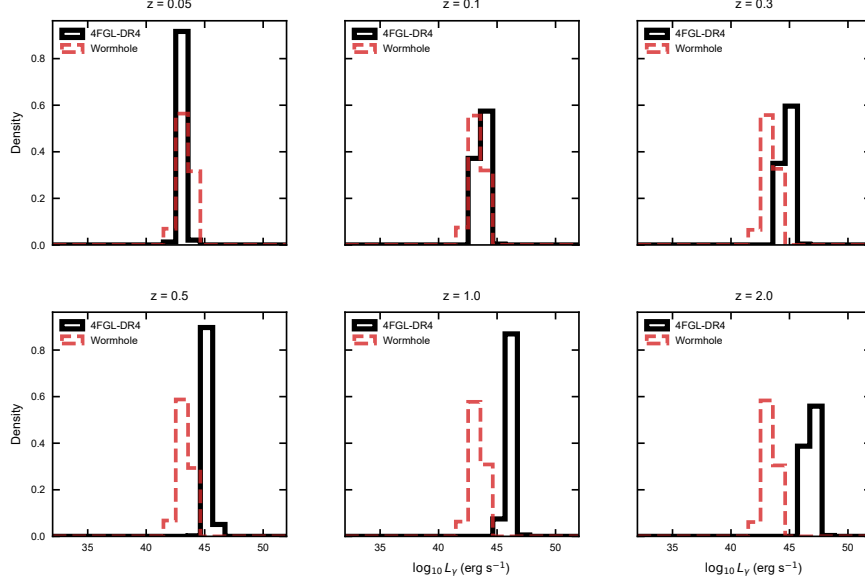


Fig. 10 Luminosity-distribution demonstration at six fixed redshifts ($z = 0.05, 0.1, 0.3, 0.5, 1.0, 2.0$). Flux converted to luminosity at each fiducial redshift; purely illustrative for redshift systematics. The main analysis uses the free-redshift MCMC posterior (median $z = 0.066$).

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