

# A Falsifiable Stochastic-Subgrid Framework for Intermittency-Driven Planetesimal Formation

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Planetesimal formation must cross two barriers at once: a *physical* barrier, whereby aerodynamic radial drift removes pebble-sized solids on  $10^2$ – $10^3$  yr timescales, and a *computational* barrier, whereby resolving the three-dimensional turbulence that concentrates those solids imposes Courant-limited time steps that make Myr integrations intractable. We present a stochastic-subgrid framework that advances the mean flow and grain growth deterministically on macroscopic ( $\Delta t \sim 10$ – $100$  yr) steps while injecting the unresolved, intermittent turbulent concentration as samples from a variance-gamma (VG) Lévy process embedded in an Ornstein–Uhlenbeck relaxation. A single intermittency parameter  $\nu$  – the excess kurtosis of the turbulent fluctuations, tending to the Gaussian limit as  $\nu \rightarrow 0$  – controls the rare, fat-tailed bursts that trigger streaming-instability-mediated gravitational collapse. The closure is deliberately falsifiable:  $\nu$  inferred from magnetohydrodynamic (MHD) turbulence should increase with magnetic activity and vanish in dead zones, and the excess kurtosis of coarse-grained increments should decay as  $\kappa_{\text{exc}} \propto \Delta t^{-1}$ , the signature of Lévy aggregation. We develop the model through an analytic collapse rate with a continuous mass function, a one-dimensional radial open system with grain growth, drift, diffusion and pebble supply, imposed and then *self-consistently generated* pressure traps, and an explicit, spatially-resolved convergence field. The mass-conserving model yields sharp, testable results: in a realistic drifting disk the concentration breach occurs in the far tail where the Gaussian probability is  $< 10^{-10}$ , so *intermittency is a prerequisite* for planetesimal formation (the yield rises  $\sim 30\times$  from  $\nu = 0$  to  $\nu = 2$ ); a self-consistent dead-zone edge – derived, together with the turbulence strength and intermittency, from a single magnetic-activity profile – produces a surface-density bump that localizes formation and makes it  $\nu$ -independent, while the adjacent smooth region is  $\nu$ -controlled, so intermittency and pressure traps are complementary and govern distinct regions. We compile seven predictions and, because MHD data are not yet in hand, present the verification methodology rather than its outcome: a pipeline that extracts the intermittency map, the aggregation exponent, and the convergence-to-velocity kurtosis ratio from any velocity field, validated on realistic synthetic surrogates, together with decision tables that fix in advance which measured outcome would refute the framework and which would robustify it. We position the framework honestly with respect to the turbulent-concentration and streaming-instability literature: the concentrating *mechanism* is inherited, while the Lévy closure, the macro-step integrator, the falsifiability design, and the self-consistent trap–intermittency dichotomy are new.

**Keywords:** protoplanetary disks — planetesimals — turbulence — hydrodynamics — methods: numerical — methods: statistical

## I. INTRODUCTION

### A. Two barriers

Growth from dust to planetesimals is obstructed first by a *physical* barrier. Gas in a protoplanetary disk orbits at slightly sub-Keplerian speed because it is partially pressure-supported; solids feel a headwind, lose angular momentum, and drift inward, with the fastest drift near Stokes number  $St \sim 1$  removing bodies at  $\sim 1$  AU on  $10^2$ – $10^3$  yr timescales [21]. The leading route across this “meter barrier” is the streaming instability (SI), an aerodynamic instability that concentrates solids until a clump exceeds the Roche density and collapses gravitationally into a  $\sim 10$ – $100$  km planetesimal [13, 23]. SI clumping requires the local solid-to-gas ratio to exceed a threshold that depends on Stokes number, pressure gradient, and turbulence [3, 16, 22].

There is also a *computational* barrier. The turbulence that regulates concentration – of magnetorotational [1] or hydrodynamic origin – is intrinsically three-dimensional and multi-scale; resolving it imposes Courant time steps orders of magnitude shorter than the  $10^5$ – $10^6$  yr required for planetesimals to appear, so direct simulation of the full problem over its natural duration is infeasible.

## B. Turbulent concentration and its statistics

That rare, intermittent turbulent events concentrate inertial particles far above the mean is established. The turbulent-concentration (TC) cascade program models the scale-to-scale partitioning of particle concentration with multiplier distributions calibrated from direct numerical simulation, integrates the resulting fat-tailed probability density function above a stability threshold, and predicts a planetesimal initial mass function (IMF) [4, 5, 9–11]; an analytic variant was given by Hopkins [12]. Independently, turbulent velocity increments are known to be non-Gaussian and heavy-tailed, and multifractal in their scale dependence [7, 19]. Our framework inherits the TC *mechanism* – rare bursts crossing a threshold – but adopts a different mathematical vehicle, a different computational role, and a falsifiability design absent from that program.

## C. Contributions

None of our contributions is a new physical mechanism; they are a new vehicle, a new computational strategy, a falsifiability design, and the predictions these enable.

1. A **variance–gamma Lévy–OU closure** for the turbulent driving field [2, 17, 18]: an analytically tractable subordinated Brownian motion whose single parameter  $\nu$  tunes intermittency continuously to the Gaussian limit, additive on the velocity/convergence field rather than multiplicative on the concentration as in the TC cascade.
2. A **macro-step stochastic–subgrid integrator**: the unresolved turbulence is not solved but sampled from its stationary fat-tailed distribution each macro step, freeing the integration from the Courant constraint – a stochastic-parametrization strategy imported from geophysical modeling.
3. A **falsifiability design** tying  $\nu$  to magnetic activity, with two sharp refutation tests on MHD data (Section III).
4. **Seven predictions** (Section V), of which the self-consistent trap–intermittency dichotomy (P4, P6) and the concentration-versus-velocity intermittency relation (P7) are, to our knowledge, new, together with a **verification methodology** (Section VI) that pre-registers which measured outcome refutes or robustifies the framework.

## II. MODEL

### A. Local domain and stochastic closure

We work in a local shearing patch at radius  $R_0$  with orbital frequency  $\Omega$  and Cartesian coordinates  $(x, y, z)$ , nondimensionalizing time by  $\Omega^{-1}$ , velocity by the sound speed  $c_s$ , and length by the scale height  $H = c_s/\Omega$ . The gas velocity is Reynolds-decomposed,  $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$ , with the mean flow  $\bar{\mathbf{u}}$  (Keplerian shear plus a sub-Keplerian drift set by the dimensionless pressure gradient  $\Pi = \eta v_K/c_s$ ) advanced deterministically, and the fluctuation  $\mathbf{u}'$  modeled stochastically. The quantity that drives concentration is the convergence  $s(t) = -\nabla \cdot \mathbf{v}_p$  of the particle velocity, which we drive with the variance–gamma (VG) process

$$L^{\text{VG}}(t) = \theta \mathcal{G}(t; \nu) + \sigma W(\mathcal{G}(t; \nu)), \quad \mathbb{E}[\mathcal{G}(t)] = t, \quad \text{Var}[\mathcal{G}(t)] = \nu t, \quad (1)$$

a Brownian motion with drift  $\theta$  and volatility  $\sigma$  subordinated to a gamma clock  $\mathcal{G}$  of variance rate  $\nu$  [18]. Its increment over an interval  $\Delta t$  has variance  $(\sigma^2 + \theta^2 \nu) \Delta t$  and, for  $\theta = 0$ , excess kurtosis

$$\kappa_{\text{exc}}(\Delta t) = \frac{3\nu}{\Delta t}, \quad (2)$$

so  $\nu$  is (up to normalization) the excess kurtosis and  $\nu \rightarrow 0$  recovers Gaussian statistics. The fluctuation obeys the Lévy-driven Ornstein–Uhlenbeck equation  $du'_i = -u'_i/\tau_e dt + dL_i^{\text{VG}}$  with eddy turnover time  $\tau_e$  [2]; its stationary variance,  $\frac{\tau_e}{2}(\sigma^2 + \theta^2 \nu)$ , is required to match the turbulent velocity dispersion  $\alpha c_s^2$ , which fixes a self-consistent diffusivity  $D = \alpha c_s H$  that closes the diffusion, collapse, and concentration terms below with a single parameter.

### B. Concentration and collapse

Because the macro step satisfies  $\Delta t \gg \tau_e$ , the OU memory decays within a step and the concentration is drawn from its stationary distribution. Writing the enhancement as  $\varepsilon = \rho_p/\rho_g$  and its settling equilibrium as  $\varepsilon_0 = Z\sqrt{(\alpha + \text{St})/\alpha}$  for metallicity  $Z$ ,

$$\ln \varepsilon = \ln \varepsilon_0 + \chi, \quad \chi \sim \text{VG}(\nu, \sigma_\chi, \theta_\chi), \quad \langle \chi \rangle = 0, \quad (3)$$

with standard deviation  $\sigma_\chi$ , excess kurtosis  $3\nu$ , and skewness set by  $\theta_\chi$ . Collapse is triggered when  $\varepsilon \geq \varepsilon_{\text{crit}}(\text{St}; \Pi, \alpha)$ , and, following the diffusion-regulated picture [14], the clump forms at the diffusion-gravity balance length obtained from  $t_{\text{ff}} \simeq l_c^2/D$  with  $t_{\text{ff}} = (G\rho_p)^{-1/2}$ :

$$l_c = \left( \frac{D^2}{G\rho_p} \right)^{1/4}, \quad M_p = \rho_p l_c^3, \quad (4)$$

evaluated at the actual  $\rho_p = \varepsilon\rho_g$  reached in the burst. The breach probability is the one-dimensional quadrature over the subordinator,

$$\mathbb{P}[\chi \geq c] = \int_0^\infty \text{Gamma}(g; \frac{1}{\nu}, \nu) \bar{\Phi}\left(\frac{c' - \theta_\chi(g-1)}{\sqrt{g}}\right) dg, \quad c' = \frac{c\sqrt{1+\theta_\chi^2\nu}}{\sigma_\chi}, \quad c = \ln \frac{\varepsilon_{\text{crit}}}{\varepsilon_0}, \quad (5)$$

which is analytic in  $\nu$  and free of Monte-Carlo noise. Each collapsing region samples a continuous mass function

$$\frac{dN}{d \ln M_p} \propto \left( \frac{M_p}{M_c} \right)^{1-\gamma} \exp\left[-\left( \frac{M_p}{M_{\text{max}}} \right)^\beta\right], \quad M_c = \rho_p l_c^3, \quad (6)$$

with slope  $\gamma \approx 1.6$  consistent with SI simulations [20] and a gravitational-length cutoff  $M_{\text{max}}$ ; a fixed characteristic mass is the  $\gamma \rightarrow \infty$  limit. Because  $M_c \propto \rho_p^{1/4}$ , deeper bursts yield smaller, more numerous bodies.

### C. Radial open system

We embed the closure on a radial grid on which the dust surface density evolves by advection, diffusion, and the collapse sink,

$$\frac{\partial \Sigma_d}{\partial t} = -\frac{1}{R} \frac{\partial}{\partial R} [R(\Sigma_d v_{\text{drift}} + F_{\text{diff}})] - \dot{\Sigma}_{\text{pl}}, \quad (7)$$

with drift  $v_{\text{drift}} = -2\text{St}/(1+\text{St}^2)\eta v_K$ , concentration diffusion  $F_{\text{diff}} = -D_d \Sigma_g \partial_R(\Sigma_d/\Sigma_g)$  and  $D_d = D/(1+\text{St}^2)$  [24], grain growth  $d\text{St}/dt = \text{St} \Omega Z_{\text{eff}}$  ( $Z_{\text{eff}} = \Sigma_d/\Sigma_g$ ) toward the fragmentation/drift barrier, and an outer boundary pebble flux  $\dot{M}_{\text{peb}}$  [15]. Because a breaching region must be resupplied by drift before it can re-form, the sink is supply-limited,

$$\dot{\Sigma}_{\text{pl}} = \varepsilon_{\text{eff}} \Sigma_d \frac{\mathbb{P}[\chi \geq c]}{\tau_{\text{drift}}}, \quad \tau_{\text{drift}} = R/|v_{\text{drift}}|, \quad (8)$$

so a fraction  $\varepsilon_{\text{eff}}\mathbb{P}$  of the drifting pebble column is captured per drift time, and  $\varepsilon_0 = Z_{\text{eff}}\sqrt{(\alpha + \text{St})/\alpha}$  becomes time- and radius-dependent. Radial transport is sub-cycled to satisfy its (mild) Courant condition within each macro step.

### D. Self-consistent dead-zone traps

Rather than impose a pressure bump, we generate one from a single magnetic-activity profile  $\Lambda(R)$ . Following layered-accretion and dead-zone models [6, 8], we set the turbulence, the intermittency, and the viscous surface density from the same  $\Lambda$ :

$$\alpha(R) = \alpha_{\text{dead}} + (\alpha_{\text{MRI}} - \alpha_{\text{dead}})g(\Lambda), \quad \nu(R) = \nu_{\text{max}} \frac{\Lambda}{\Lambda + \Lambda_0}, \quad \Sigma_g(R) = \frac{\dot{M} \Omega}{3\pi \alpha c_s^2}, \quad (9)$$

with  $g(\Lambda) = \Lambda/(\Lambda + \Lambda_0)$ , and recompute  $\eta(R)$  numerically from the pressure profile. The viscous steady state  $\Sigma_g \propto 1/\alpha$  turns an annular dead zone (an  $\alpha$  minimum) into a surface-density maximum, hence a pressure bump and a drift trap, at the dead-zone edge; the same  $\Lambda(R)$  sets  $\nu(R)$  low in the dead zone and high outside.

### E. Spatially-resolved concentration field

Equation (3) samples  $\chi$  independently per radius, an effectively zero-dimensional treatment. We test it by generating a spatially-correlated VG velocity field  $u'_R(R, t)$  – a common gamma subordinator shared across radius through a Gaussian copula, preserving the local  $\nu(R)$ , with correlation length  $\ell_{\text{corr}} \sim H$  – computing the convergence  $s = -\frac{1}{R}\partial_R(Ru'_R)$  explicitly, and integrating  $d \ln \varepsilon = [s - \lambda(\ln \varepsilon - \ln \varepsilon_0)]dt$  with  $\lambda = D/\ell_{\text{corr}}^2$ .

## III. FALSIFIABILITY FRAMEWORK

The framework's central physical hypothesis – that the intermittency is sourced by MHD turbulence – is made refutable on MHD data. *Calibration*: fitting VG parameters to velocity increments across a suite of runs indexed by a magnetic parameter  $\Lambda$  (Elsasser number or plasma  $\beta$ ) should give

$$\nu = \nu(\Lambda), \quad \frac{d\nu}{d\Lambda} > 0, \quad \lim_{\Lambda \rightarrow 0} \nu(\Lambda) = 0, \quad (10)$$

i.e. stronger magnetic activity yields fatter tails and dead zones return Gaussian residuals. *Refutation A*: a pure-hydrodynamic run must satisfy  $\nu_{\text{hydro}} \leq \nu_{\text{tol}}$ ; comparable fat tails without a field falsify the magnetic origin. *Refutation B*: the macro-step injection is justified only if  $\kappa_{\text{exc}}(\Delta t) \propto \Delta t^{-p}$  with  $p \approx 1$  (Eq. 2); a steeper decay means the jumps wash out by the central limit theorem before reaching the macro scale. All three tests share the moment estimator  $\hat{\nu} = \kappa_{\text{exc}} \Delta t/3$ , so calibration and refutation live in one statistical frame.

## IV. NUMERICAL METHOD AND RESULTS

### A. Mechanism and self-consistency checks

We integrate an ensemble of independent local regions on macro steps; each draws  $\varepsilon$  from Eq. (3), tests the gate, and forms bodies from Eqs. (4)–(6). Figure 1 establishes the mechanism for a minimum-mass-solar-nebula fiducial: the conversion efficiency rises with  $\nu$  and the size distribution shifts to smaller, more numerous bodies, while the built-in self-tests recover  $\kappa_{\text{exc}} = 3\nu$ , an aggregation exponent  $p = 1.00$ , and  $\hat{\nu} \approx \nu$ , confirming that the falsifiability statistics behave as designed.

### B. The open system: intermittency as a prerequisite

In the radial open system the result sharpens (Fig. 2). Radial drift depletes  $\Sigma_d$  so that  $\varepsilon_0 \ll \varepsilon_{\text{crit}}$ ; the breach then moves into the far tail, where the Gaussian ( $\nu = 0$ ) probability falls below  $10^{-10}$  and formation nearly ceases, whereas fat tails restore it. The planetesimal yield rises by a factor of  $\sim 30$  from  $\nu = 0$  to  $\nu = 2$ . In the regime real drifting disks occupy, intermittency is therefore not an enhancement but a prerequisite. The scheme conserves solid mass to machine precision ( $\sim 10^{-3} M_{\oplus}$  of  $\sim 130 M_{\oplus}$  processed); absolute efficiency in a smooth disk is low and formation is front-loaded, the signature that motivates traps.

### C. Traps and intermittency are complementary

An imposed pressure bump halts drift at its maximum; solids pile up, formation is sustained, and the efficiency jumps from  $\sim 0.2\%$  to  $\sim 76\%$  above a threshold amplitude (Fig. 3). Decisively, with a trap the

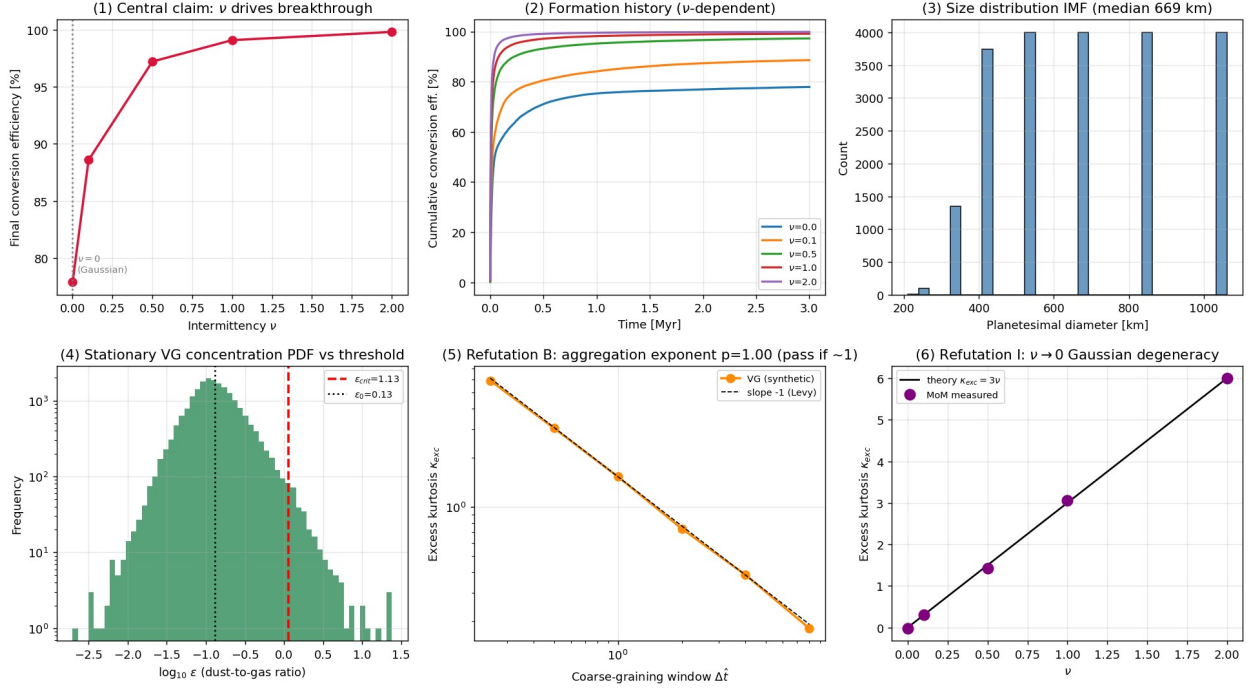


FIG. 1. Mechanism and self-consistency (closed-box). (1) conversion efficiency rises with  $\nu$ ; (2) cumulative efficiency by  $\nu$ ; (3) size distribution; (4) stationary VG concentration PDF and the breakthrough threshold; (5) aggregation self-test,  $p = 1.00$ ; (6)  $\nu \rightarrow 0$  Gaussian degeneracy,  $\kappa_{exc} = 3\nu$ .

efficiency is *independent* of  $\nu$ , because the raised  $\varepsilon_0$  makes the threshold easy to breach, whereas without a trap it depends steeply on  $\nu$ . Intermittency and traps thus operate in complementary regimes.

#### D. A self-consistent dead-zone factory

When the trap is generated from  $\Lambda(R)$  through Eq. (9), the two regimes appear together and self-consistently (Fig. 4). The viscous surface density bumps by  $\sim 60\times$  at the dead-zone edge, a drift trap forms there, and planetesimal formation localizes at the trap. Decomposing by region: at the trap, formation is  $\nu$ -independent ( $149$  vs  $149 M_\oplus$  with the full  $\nu(R)$  and with  $\nu \rightarrow 0$ ), while the smooth active region is  $\nu$ -driven ( $0.083$  vs  $0.001 M_\oplus$ , an  $\sim 80\times$  dependence). A single magnetic profile thus produces a  $\nu$ -independent trap factory adjacent to a  $\nu$ -controlled smooth zone – the dead-zone edge is a self-consistent planetesimal factory. The characteristic diameter at the trap ( $\sim 200$  km) approaches the canonical scale.

#### E. The spatial field justifies the zero-dimensional closure

Resolving the convergence field explicitly (Fig. 5) yields two findings. First, the turbulent coherence length is at or below the disk-scale radial grid ( $\ell_{corr} \lesssim H \lesssim \Delta R$  for  $n_R \lesssim 200$ ), so the field is effectively uncorrelated between cells and the formation results are unchanged ( $M_{pl} = 149$  vs  $151 M_\oplus$ ): the efficient per-radius closure is vindicated at disk resolution, with the explicit field reserved for sub- $H$  zooms. Second,

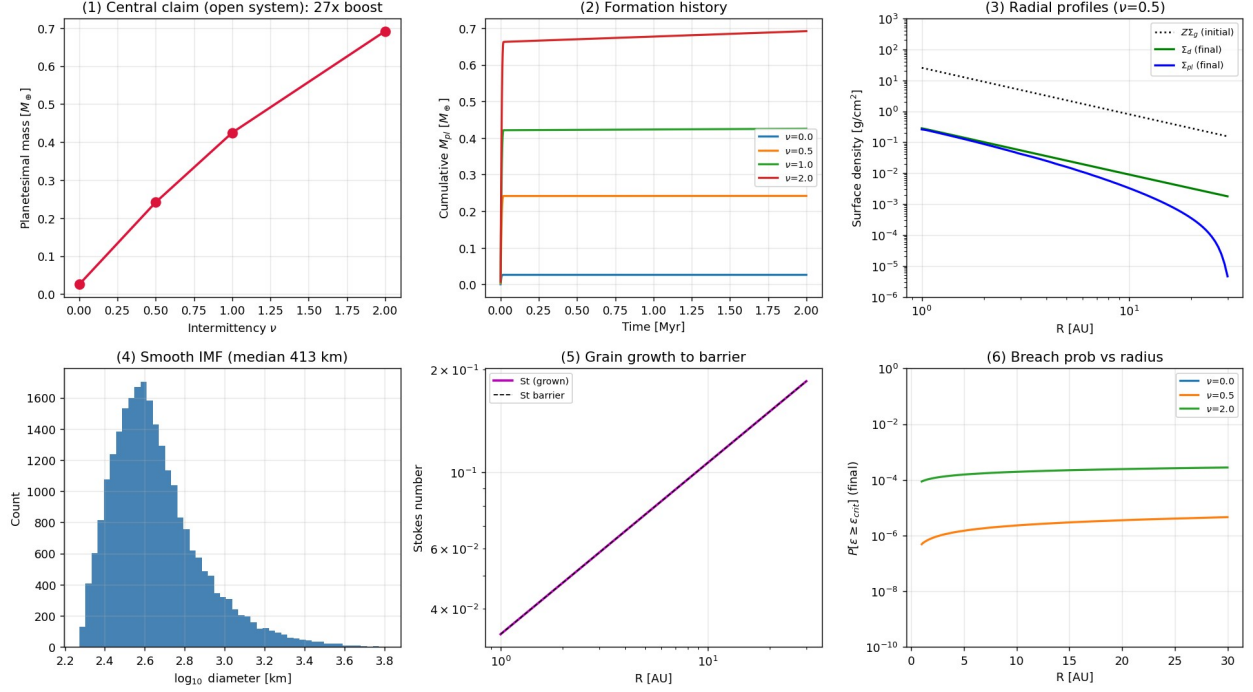


FIG. 2. Radial open system. (1)  $\sim 30\times$  yield boost with  $\nu$ ; (2) formation history; (3) radial profiles of  $Z\Sigma_g$  (initial),  $\Sigma_d$ ,  $\Sigma_{\text{pl}}$ ; (4) smooth IMF; (5) grain growth to the barrier; (6) breach probability versus radius (the  $\nu = 0$  curve lies below  $10^{-10}$ ).

the divergence operation lowers the intermittency – the velocity field has  $\kappa_{\text{exc}} \approx 3\nu$ , but its divergence, the actual concentration driver, has  $\kappa_{\text{exc}} \approx 2.95$  – so the velocity intermittency  $\nu$  does not translate one-to-one to the concentration intermittency.

## V. PREDICTIONS

Predictions tagged *[model]* are exercised above; those tagged *[data]* await MHD or observational input, for which the pipeline of Section VI is ready.

**P1 Magnetic-intermittency map** *[data]*.  $\nu$  inferred from velocity increments rises monotonically with magnetic activity and vanishes in dead zones.

**P2 Intermittency as a prerequisite** *[model]*. In the drift-depleted regime, Gaussian turbulence cannot form planetesimals ( $\mathbb{P} < 10^{-10}$ ); the yield rises  $\sim 30\times$  from  $\nu = 0$  to  $\nu = 2$ , and  $\sim 80\times$  in the self-consistent active zone.

**P3 Burst-density–size anticorrelation** *[model]*.  $M_c \propto \rho_p^{1/4}$ : deeper bursts give smaller, more numerous bodies.

**P4 Trap–intermittency complementarity** *[model]*. Where a trap raises  $\epsilon_0$ , formation is  $\nu$ -independent; elsewhere it is  $\nu$ -controlled – shown with imposed and self-consistent traps (149 vs 149  $M_\oplus$  at the trap).

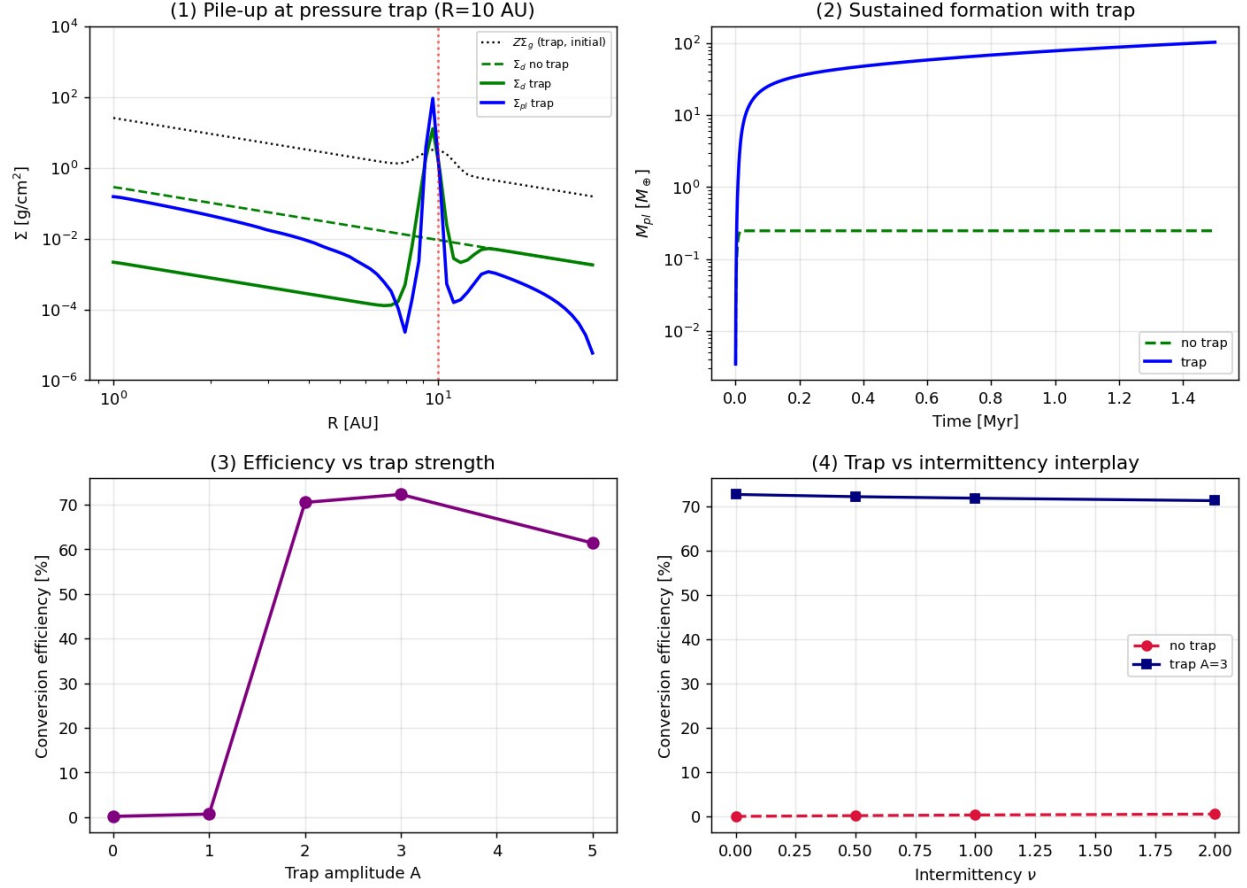


FIG. 3. Imposed pressure trap. (1) pile-up; (2) sustained formation versus a frozen no-trap value; (3) efficiency versus trap amplitude; (4) complementarity – efficiency versus  $\nu$  with (flat) and without (steep) a trap.

**P5 Lévy aggregation signature [data].**  $\kappa_{\text{exc}} \propto \Delta t^{-1}$ ; a steeper decay falsifies the additive macro-injection.

**P6 Dead-zone edges as self-consistent factories [model].** A single  $\Lambda(R)$  yields, at the dead-zone edge, a viscous  $\Sigma_g$  bump ( $\sim 60\times$ ) that localizes and dominates formation, with elevated  $\nu$  just outside; the size follows the local  $\rho_p$  through Eq. (4).

**P7 Concentration intermittency below velocity intermittency [model].** Because the concentration driver is  $-\nabla \cdot \mathbf{v}_p$ , its excess kurtosis ( $\approx 2.95$ ) is lower than the velocity field's ( $\approx 3\nu$ ); the pipeline measures the velocity  $\nu$ , so the effective concentration intermittency is somewhat weaker.

## VI. VERIFICATION METHODOLOGY

Predictions P1, P5, and P7 require turbulent velocity fields from non-ideal MHD simulations indexed by magnetic activity. Such data are not yet in hand; we therefore present the *method* by which they will be tested, validate it on realistic synthetic surrogates, and pre-register which measured outcome would *refute* the framework and which would *robustify* it.

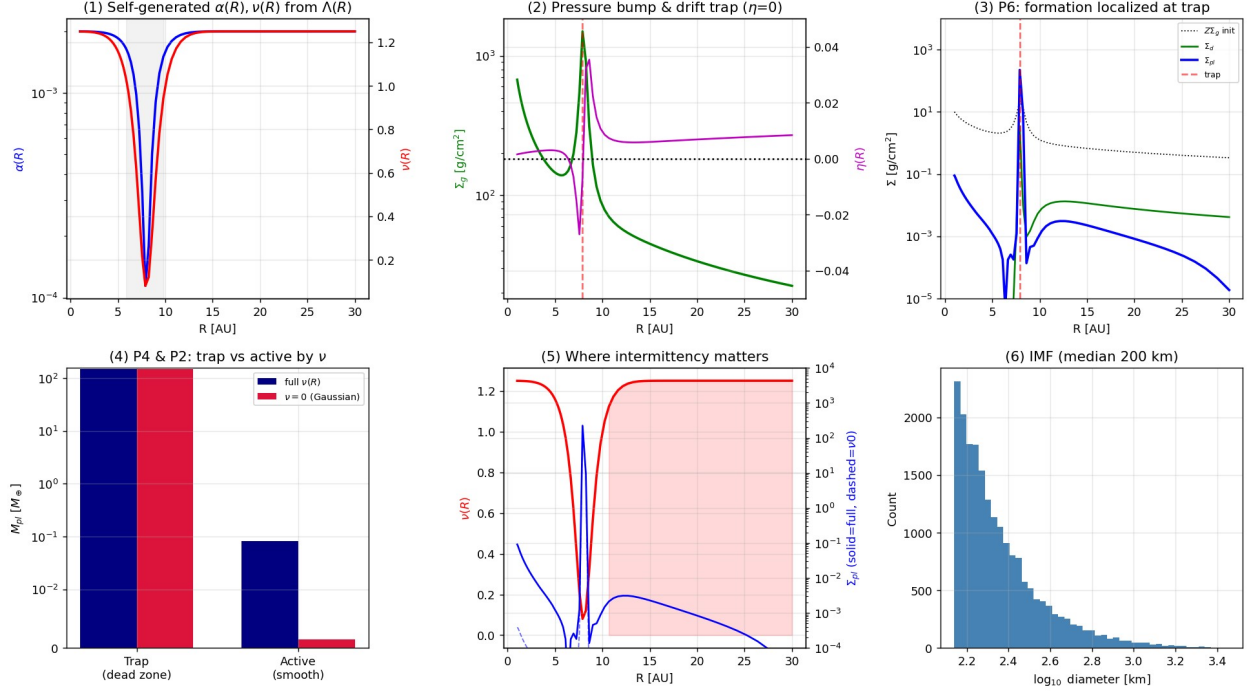


FIG. 4. Self-consistent dead-zone trap from  $\Lambda(R)$ . (1) self-generated  $\alpha(R)$ ,  $\nu(R)$ ; (2) viscous  $\Sigma_g$  bump and drift trap ( $\eta = 0$ ); (3) formation localized at the trap; (4) region decomposition – trap  $\nu$ -independent, active  $\nu$ -driven; (5) where intermittency matters; (6) IMF (median  $\sim 200$  km).

### A. Pipeline and surrogate

The pipeline reduces any velocity field  $u(x, t)$  to three diagnostics: the intermittency  $\hat{\nu} = \kappa_{\text{exc}}[\delta u]/3$  from the excess kurtosis of temporal increments (P1); the aggregation exponent  $p$  in  $\kappa_{\text{exc}}(\Delta t) \propto \Delta t^{-p}$  from coarse-grained increments (P5); and the ratio of the excess kurtosis of the convergence to that of the velocity (P7). We exercise it on a surrogate that reproduces the two features that matter – a turbulent spatial spectrum and intermittency clustering – by setting  $u'(x, t) = e^{h(x, t)/2} \xi(x, t)$ , where  $\xi$  has a Kolmogorov  $k^{-5/3}$  spatial spectrum and short temporal correlation and  $h$  is a space-time-correlated log-volatility field of standard deviation  $\sqrt{\ln(1 + \nu)}$  that injects fat-tailed, clustered increments of excess kurtosis  $\approx 3\nu$ ; the injected map is  $\nu(\Lambda) = \nu_{\text{max}}\Lambda/(\Lambda + \Lambda_0)$ . Figure 6 shows that the surrogate reproduces the  $k^{-5/3}$  spectrum and non-Gaussian increments, and that the pipeline recovers the injected  $\nu(\Lambda)$  and its dead-zone limit, discriminates the three aggregation regimes – Lévy ( $p \approx 1$ ), persistent/multifractal ( $p < 1$ ; cf. Frisch 7, She & Lévéque 19), and washout ( $p > 1$ ) – and discriminates the three intermittency mechanisms through the convergence-to-velocity kurtosis ratio. When MHD snapshots become available, a single loader returns the same  $(n_x, n_t)$  array and the identical routines apply.

### B. Outcome tables: refutation versus robustification

Tables I–III fix, in advance, the mapping from each measurable outcome to a model response: a *refutation* (a stated hypothesis fails and must be abandoned or amended), a *robustification* (the outcome confirms an

v4 Update 2: velocity  $\kappa_{\text{exc}}=3.19$  vs driver  $\kappa_{\text{exc}}=2.95$  ( $3\nu=3.75$ ) — differentiation lowers intermittency

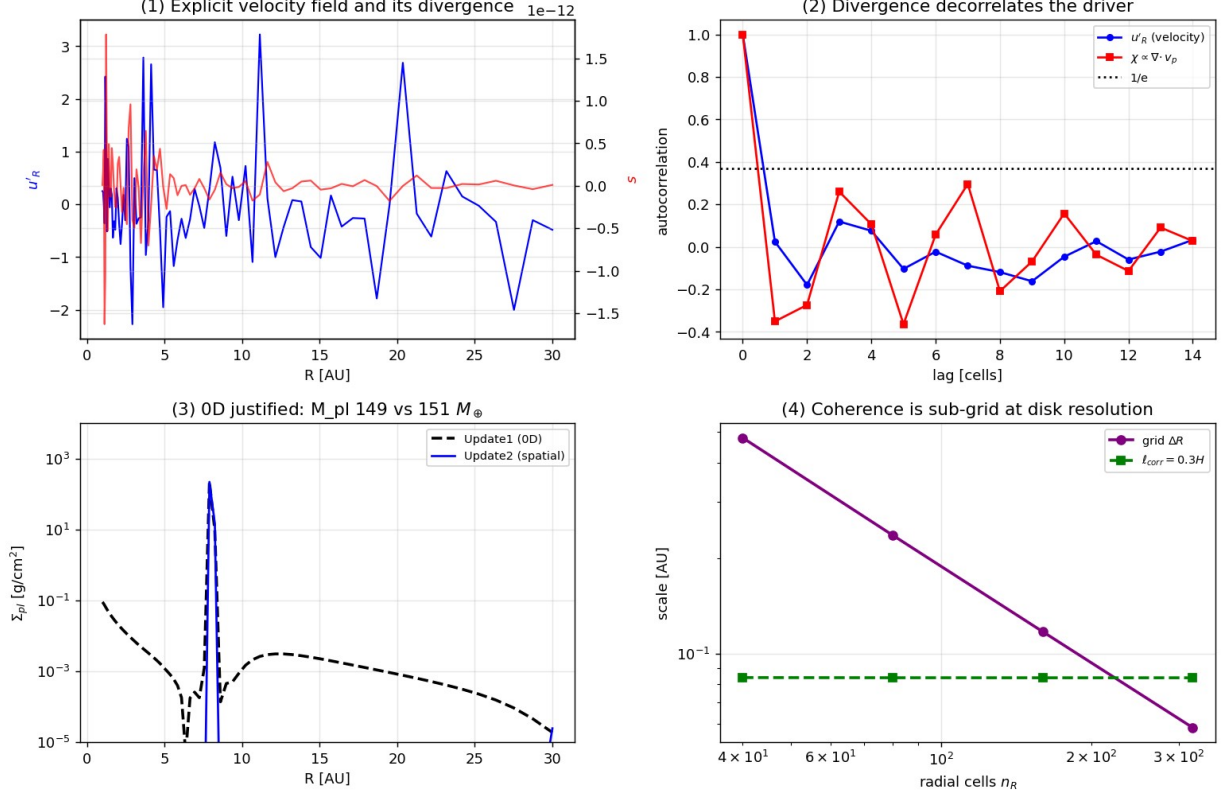


FIG. 5. Spatially-resolved concentration field. (1) explicit velocity field and its divergence; (2) the divergence decorrelates the driver; (3)  $\Sigma_{\text{pl}}$  from the spatial field versus the zero-dimensional model (nearly identical); (4) the coherence length is sub-grid at disk resolution.

assumption and tightens the model), or a *generalization* (the outcome requires replacing a component with a broader one while preserving the core mechanism). This pre-registration is what makes the tests genuine.

TABLE I. P1 – magnetic origin of intermittency. Measured:  $\hat{\nu}(\Lambda)$ .

| Measured outcome                                                            | Interpretation             | Model response                                                           |
|-----------------------------------------------------------------------------|----------------------------|--------------------------------------------------------------------------|
| $\hat{\nu}$ increases, $\hat{\nu} \rightarrow 0$ as $\Lambda \rightarrow 0$ | Magnetic origin confirmed  | <b>Robustify:</b> adopt $\nu(\Lambda)$ as a calibrated law               |
| $\hat{\nu}$ independent of $\Lambda$                                        | No magnetic control        | <b>Refute:</b> intermittency not magnetically sourced                    |
| $\hat{\nu} > \nu_{\text{tol}}$ at $\Lambda \rightarrow 0$                   | Fat tails in the dead zone | <b>Refute (A):</b> a non-magnetic source exists                          |
| $\hat{\nu}$ increases but floors at $\nu_{\text{floor}} > 0$                | Magnetic plus baseline     | <b>Amend:</b> $\nu(\Lambda) = \nu_{\text{floor}} + \text{magnetic term}$ |

Several outcomes redirect rather than kill the framework: a persistent  $p < 1$  or a compressive P7 would replace a component while preserving the core mechanism and the falsifiability design. Others are sharp failure modes: a  $\Lambda$ -independent  $\hat{\nu}$ , a dead-zone  $\hat{\nu} > \nu_{\text{tol}}$ , or a washout  $p > 1$  would each, on its own, refute a load-bearing assumption. The pipeline is exposed through a loader returning a velocity array of shape  $(n_x, n_t)$  for each run in a  $\Lambda$ -indexed suite; the downstream routines are format-agnostic. The minimal dataset is a handful of non-ideal MHD disk runs spanning active to dead-zone conditions, plus one pure-hydrodynamic run for Refutation A.

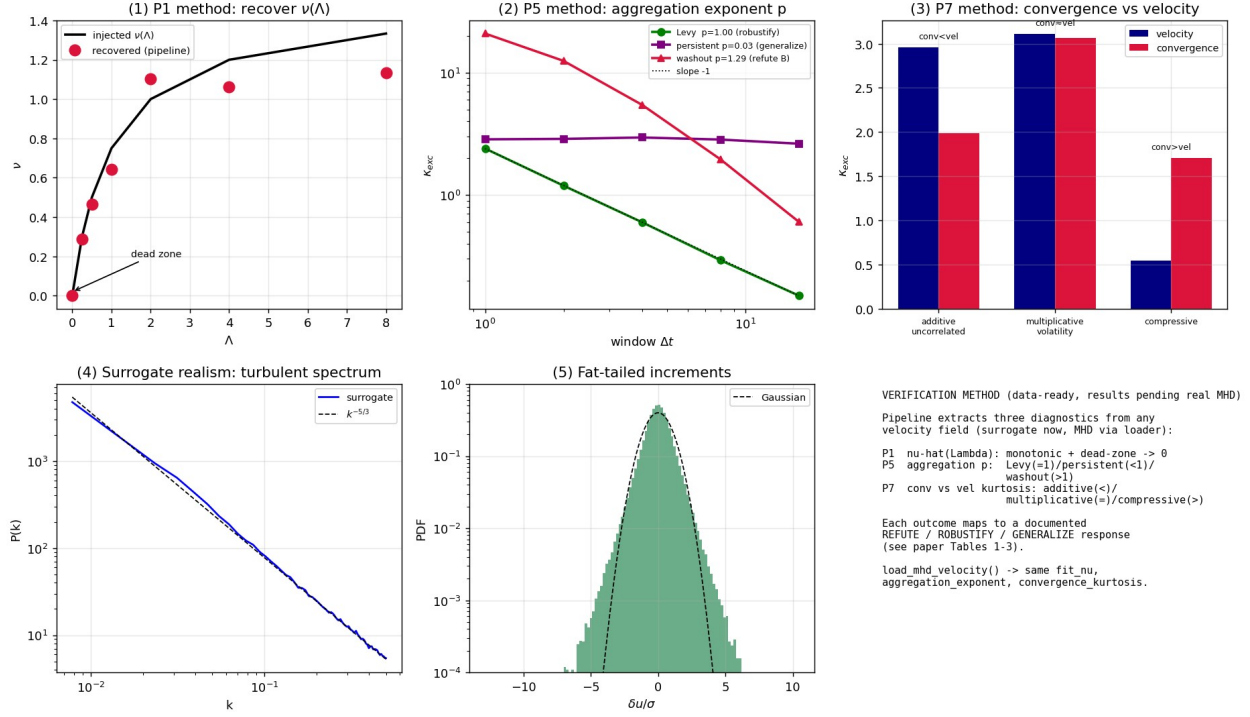


FIG. 6. Verification methodology on realistic synthetic surrogates. (1) P1: recovery of the injected  $\nu(\Lambda)$  with dead-zone limit. (2) P5: the aggregation exponent discriminates Lévy ( $p \approx 1$ ), persistent ( $p < 1$ ), and washout ( $p > 1$ ). (3) P7: the convergence-to-velocity kurtosis ratio discriminates additive ( $<$ ), multiplicative ( $\approx$ ), and compressive ( $>$ ) intermittency. (4)  $k^{-5/3}$  spectrum; (5) fat-tailed increments.

TABLE II. P5 – Lévy aggregation. Measured: exponent  $p$ .

| Measured outcome | Interpretation                     | Model response                                                                                                        |
|------------------|------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| $p \approx 1$    | Lévy aggregation                   | <b>Robustify:</b> the additive macro-injection is justified                                                           |
| $p > 1$          | Jumps cancel under coarse-graining | <b>Refute (B):</b> macro-injection invalid; require finer $\Delta t$ or a different temporal closure                  |
| $p < 1$          | Persistent / multifractal          | <b>Generalize:</b> replace VG with a multifractal or log-Poisson closure; the mechanism survives, the vehicle changes |

## VII. LIMITATIONS

We state these plainly. The absolute conversion efficiency in a smooth disk is low and formation is front-loaded there, resolved only where traps sustain the solid loading; the intermittency  $\nu$  is calibrated on surrogates and awaits real MHD data (P1); the concentration field is zero-dimensional at disk resolution, a choice tested and justified but exact only in that regime; the mass-function exponents  $\gamma, \beta$  and the pressure traps of the imposed variant are prescribed rather than derived (the self-consistent variant removes the latter); and fiducial diameters ( $\sim 200$  km at the trap) still exceed the canonical  $\sim 100$  km and depend on  $\alpha, R$ , and the achieved  $\rho_p$ . These are standard-physics bookkeeping items, separable from the novel core – the

TABLE III. P7 – concentration versus velocity intermittency. Measured:  $\kappa_{\text{conv}}/\kappa_{\text{vel}}$ .

| Measured outcome                                   | Interpretation               | Model response                                                                                              |
|----------------------------------------------------|------------------------------|-------------------------------------------------------------------------------------------------------------|
| $\kappa_{\text{conv}} < \kappa_{\text{vel}}$       | Additive / grid-uncorrelated | <b>Robustify &amp; correct:</b> apply the differentiation correction to the $\nu \rightarrow$ formation map |
| $\kappa_{\text{conv}} \approx \kappa_{\text{vel}}$ | Multiplicative / coherent    | <b>Simplify:</b> the velocity $\nu$ is usable directly                                                      |
| $\kappa_{\text{conv}} > \kappa_{\text{vel}}$       | Compressive enhancement      | <b>Revise:</b> compressive modes drive concentration and must be modeled                                    |

VG closure and the falsifiability design – which is exercised correctly throughout.

## VIII. DISCUSSION

The concentrating *mechanism* – rare intermittent turbulent concentration crossing a threshold – is the TC program of Cuzzi et al. [4], Hartlep & Cuzzi [10], and we do not claim it as new. What is new is the *vehicle* (an additive VG Lévy–OU closure on the driving field, rather than a multiplicative cascade on the concentration), the *computational role* (a macro-step stochastic-subgrid integrator that escapes the Courant constraint), the *falsifiability design* (the  $\nu(\Lambda)$  map with dead-zone degeneracy and the aggregation test), and the physical content the extensions expose: that intermittency is a prerequisite in the regime real disks occupy (P2), and that intermittency and pressure traps are complementary, with a single magnetic profile making the dead-zone edge a  $\nu$ -independent factory while the smooth zone is  $\nu$ -controlled (P4, P6) – a statement about *where* each mechanism governs the outcome that, to our knowledge, has not been framed this way. The spatially-resolved test both vindicates the efficient closure at disk resolution and reveals that the convergence field carries lower intermittency than the velocity field (P7). We therefore present this as a methods-and-prediction contribution, positioned relative to the turbulent-concentration, streaming-instability, pebble-flux, and dead-zone literatures, not as a claim to have resolved planetesimal formation.

## IX. CONCLUSIONS

A variance–gamma Lévy–OU closure injects intermittent turbulent concentration on macroscopic time steps, with a single, magnetically-tied parameter  $\nu$  controlling the bursts that trigger collapse. Developed through an analytic continuous-IMF rate, a radial open system, imposed and self-consistent dead-zone traps, and an explicit spatial concentration field, the mass-conserving model yields seven predictions. The sharpest results are that intermittency is a prerequisite for planetesimal formation in drifting disks; that a self-consistent dead-zone edge is a  $\nu$ -independent planetesimal factory adjacent to a  $\nu$ -controlled smooth zone; and that the concentration intermittency lies below the velocity intermittency the calibration pipeline measures. Predictions P1, P5, and P7 are directly actionable on existing MHD data, and Section VI provides the pipeline that will test them and pre-registers, in decision tables, which outcome would refute the framework and which would robustify it.

## DATA AVAILABILITY

The simulation and analysis code, including the stochastic-subgrid integrator, the self-consistent dead-zone module, the calibration pipeline, and the synthetic-surrogate verification suite, will be made available on reasonable request. No proprietary data were used.

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