

Metrarchic Field Theory:

A Geometric-Probability Research Architecture for Quantum Measure, Relativistic Geometry, Gauge Structure, and Matter Spectra within the Metrarchic Geometry Framework

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Abstract

Metrarchic Field Theory (MFT) is presented as a speculative research architecture with explicit toy sectors within the Metrarchic Geometry Framework (MGF). Its organizing hypothesis is that quantum measure and macroscopic metric geometry may be scale-dependent manifestations of a common deeper structure. A positive coframe metric and a prescribed conserved reference measure give the conditional identity $\rho_G = \rho_0/J$, where J is the metric volume ratio. Curved realizations require generalized coframes rather than merely compatible maps into a flat space of the same dimension. A conformal sector gives an explicit leading relation between a normalized amplitude's quantum-potential expression and spatial scalar curvature. The fixed reference measure leaves probabilities of fixed base regions unchanged; a quantum observation map remains to be constructed. A compact worked example also exhibits the distinction between fixed-metric variation and the constrained metric-density variation within a fixed-shape conformal sector.

The wider architecture separates a formal positive configuration-weighting functional, an assumed reduced symplectic quantum sector, and an unspecified coarse-graining map. Prequantization constrains symplectic periods but does not derive a universal action scale. Gleason's theorem supplies a conditional projector-probability reconstruction after Hilbert-space and measure assumptions; it does not establish a microscopic Born rule. Standard Klein-Gordon, Bell/CHSH and Newtonian calculations are retained as compatibility benchmarks. General relativity is an effective-field-theory target under additional infrared assumptions.

An internal finite-dimensional construction uses a nonzero simple two-form in $\mathbb{C}^5$ to select a rank-two plane and a postulated 3+2 splitting operator. The plane projector has determinant-one stabilizer $S(U(3) \times U(2))$, whereas a fixed two-form has the smaller stabilizer $SU(3) \times SU(2)$. Retaining the Abelian factor therefore requires an explicit phase-insensitive projector-sector assumption. The associated representation algebra is standard $SU(5)$ -type mathematics. Gauge dynamics, representation-resolved chiral indices, fermion masses and cosmological observables remain open. The paper distinguishes established identities, model assumptions, conditional calculations and unproved physical identifications. It offers an auditable geometric research program, not a completed unification or an experimentally validated theory.

Keywords: Metrarchic Geometry Framework; Metrarchic Field Theory; geometric measure; metric-measure geometry; deformation geometry; quantum foundations; geometric quantization; emergent

spacetime; gauge symmetry; spectral geometry; general relativity; effective field theory; toy model.

Status statement. The Metrarchic Geometry Framework (MGF) is the umbrella conceptual framework; Metrarchic Field Theory (MFT) is the speculative toy physical theory developed within it. Equations labeled as MFT postulates or toy assumptions are not established laws of nature. The aim of this manuscript is to specify a coherent architecture, recover selected known-physics limits conditionally, and identify the precise mathematical steps that remain unproved.

Conventions and distinct measures

The positive geometric sector uses real smooth metrics and orientation-preserving, invertible coframes. The Laplacian is $\Delta_h = \text{div}_h \nabla$, so on a flat Fourier mode its eigenvalue is $-|k|^2$. Relativistic signature is $(-+++)$; when c is explicit, $x^0 = ct$. Natural units $c = \hbar = 1$ are used only where stated. The local measure μ_G on a geometric base, the ensemble measure $\mathbb{P}_H$ on configurations, and a normalized quantum outcome measure are distinct objects. No equality between them is implied without a specified projection and normalization. The Hilbert space is denoted $\mathcal{H}_Q$, distinct from the stability Hessian $\mathcal{H}_G$. All statements involving $\ln \rho_G$ mean the logarithm of a dimensionless density ratio. Regularity, positivity and boundary assumptions accompany the calculations below.

Contents

1	Introduction	4
2	Related Work and Conceptual Boundaries	5
3	Foundational Postulates and Claim Discipline	8
4	Deep Deformation and the Geometric-Measure Identity	10
5	Quantum-Measure Sector	11
6	Deep Compatibility Functional and HPSS	13
7	Reduced State Geometry, Phase, and the Quantum of Action	15
8	Born-Rule Reconstruction and Entanglement	16
9	Relativistic Scalar Compatibility Check	17
10	Time and the Emergent Lorentzian Sector	18
11	Dimensionality as an Illustrative Selection Model	19
12	Gravity and the General-Relativistic Limit	20
13	Internal Spectral Geometry and Gauge Structure	21

14 Chirality, Generations, and Mass	24
15 Constants and Couplings as Geometric Data	25
16 Cosmological Regimes	26
17 Falsifiability and Quantitative Research Targets	26
18 Limitations and Open Problems	27
19 Discussion	27
20 Conclusion	28
A Notation and Status Ledger	29
B Weak-Conformal Derivation Check	30
C Explicit Compact Coframe Example and Constrained Variation	30
D Rank-Two Internal Splitting Check	31
E Newtonian Point-Mass Check	32
F Bell/CHSH Benchmark Calculation	32
G Internal $U(1)$ Normalization Check	33

1 Introduction

Modern fundamental physics contains two extraordinarily successful but structurally different descriptions of nature. Quantum theory assigns complex amplitudes and probability measures to possible outcomes, while general relativity describes gravitation as the dynamics of spacetime geometry. Quantum field theory adds internal gauge connections, representation spaces, and particle spectra. The mathematical compatibility of these structures does not by itself identify a common primitive object from which they arise. The Metrarchic Geometry Framework (MGF) is introduced as an umbrella geometric research program built around the possibility that measure, metric, connection, and spectral structure share a common deeper origin. Metrarchic Field Theory (MFT) is the deliberately restricted toy theory developed here to make that umbrella idea mathematically explicit and auditable.

The term *metrarchic* is used here as a coined label for the organizing role of metric and measure geometry; it is not asserted to be an established term of art. Within the broader MGF program, the central MFT proposal can be stated compactly. Let $\mathfrak{G}$ denote a deep geometric state. Then the observed quantities usually treated as independent are represented schematically as

$$\mathfrak{G} \longrightarrow \{\rho_G, g_{\mu\nu}, \mathcal{A}_G, \mathcal{D}_G, \dots\}. \quad (1)$$

The intended common-origin constraint is stronger than merely adding a scalar to a metric because the density ρ_G is not intended to be an additional matter field living on a pre-existing metric. It is instead a density of geometric measure relative to an effective metric volume. In a restricted deformation model, the same coframe deformation, together with a prescribed reference measure, determines both the metric and the local measure density. This motivates the guiding principle of the paper:

quantum probability and relativistic geometry
 are scale-dependent observables of one deeper geometry.

(2)

This principle has antecedents but should not be confused with them. The Madelung formulation rewrites the Schrödinger equation in hydrodynamical variables [1]; Bohm’s causal formulation isolates a quantum-potential term built from the amplitude [2, 3]; Fisher-information approaches show how amplitude-gradient terms can reproduce Schrödinger dynamics under additional assumptions [4, 5]; weighted Riemannian geometry naturally combines measure density and curvature [11]; geometric quantization relates symplectic geometry to quantum phase and action units [9]; and general relativity is consistently organized as a low-energy effective field theory [13]. MFT does not claim ownership of any of these established constructions. Its proposed contribution is a specific, explicitly limited coframe/measure construction within a broader research architecture. The expanded comparison in Section 2 identifies close precedents; no priority claim for the general geometric program or demonstrated physical inequivalence is made.

A speculative unification framework is most vulnerable when it overstates derivations. For that reason, this manuscript uses a strict claim taxonomy. A statement is identified, where relevant, as one of: (i) established mathematics or physics; (ii) a MFT postulate or definition; (iii) a toy-model ansatz; (iv) a consequence derived within explicitly stated assumptions; or (v) a conjecture/open research target. In particular, the manuscript does not claim a derivation of the Standard Model, the numerical value of Planck’s constant, three spatial dimensions, three fermion generations, or the Einstein equation from a microscopic MFT action. Instead, it specifies mechanisms that make those questions sharper and

identifies where empirical or mathematical input remains.

The architecture is deliberately layered. A positive compatibility functional is used at the deep level rather than a Lorentzian spacetime action. A reduced stable state space is separately endowed with a symplectic structure in the quantum sector. Finally, a coarse-graining map connects the deep and reduced descriptions to effective spacetime physics. This separation makes clear where familiar effective equations are assumed as benchmarks rather than recovered from a microscopic model.

2 Related Work and Conceptual Boundaries

MFT belongs to an existing family of geometric approaches to quantum theory and gravitation. Probability densities, quantum potentials, conformal metrics, symplectic structures and internal spectral data have substantial prior histories. This section identifies close mathematical and physical precedents, distinguishes their mechanisms from the assumptions used here, and limits the contribution claimed for MFT. The comparison is selective, not a proof of priority or an exhaustive originality search. Similarity of motivation does not establish mathematical equivalence, and a difference in terminology does not establish a new physical theory.

2.1 Quantum potentials and conformal or Weyl geometry

Madelung’s hydrodynamic formulation and Bohm’s causal interpretation already provide the amplitude–phase decomposition and quantum potential used in this paper [1, 2, 3]:

$$\Psi = Re^{iS/\hbar}, \quad Q = -\frac{\hbar^2}{2m} \frac{\Delta R}{R}. \quad (3)$$

These expressions are established background, not MFT results. Santamato’s Weyl-geometric construction supplies a closer precedent for interpreting quantum behavior geometrically through a variational principle [16]. Both motion and geometry enter that construction; its geometry should not be described as an independently fixed input. MFT instead starts its explicit amplitude construction from a prescribed conserved measure and a coframe volume ratio.

Shojai and Shojai identify the Bohmian quantum potential with the conformal degree of freedom of the spacetime metric and develop geometric interpretations involving gravity, scale and nonlocality [21]. This is a substantial antecedent to MFT’s proposed quantum–geometry relationship. Their Weyl/conformal and Bohmian mechanisms differ from the conserved-coframe prescription adopted here, but the general proposal that quantum effects can be encoded geometrically is shared. The distinctness of MFT cannot rest on that proposal alone.

Koch presents a geometric toy description of relativistic many-particle Bohmian equations using a conformal metric on configuration space, with the conformal scalar identified with the wave amplitude through explicit matching conditions [22]. This is especially relevant to the distinction between configuration-space geometry and physical spacetime. MFT uses $\rho_G = \rho_0/J$ and a separate coarse-graining target rather than Koch’s specific conformal matching construction. Neither this difference nor MFT’s broader scope proves physical inequivalence or superiority. A comparison of dynamics, measures and observables would be required; MFT has not supplied the necessary many-body projection.

2.2 Metric-measure geometry and density spaces

Perelman’s weighted functionals provide established mathematical examples in which curvature and measure weight are treated together [11]. Rahmanpour and Shojaie give a particularly close physical precedent: their metric-measure gravitational framework includes a geometric density function and proposes a connection to the quantum potential by comparison with Bohmian trajectories [17]. MFT’s more specific assumption is that a prescribed reference measure and the same coframe data jointly determine the local density and metric:

$$\gamma = F^T \gamma_0 F, \quad \rho_G = \rho_0/J, \quad J = dV_\gamma/dV_0. \quad (4)$$

This is a conditional restriction, not a demonstrated dynamical advance over metric-measure gravitation. The origin of the reference measure remains unspecified.

Khesin, Lenells, Misiolek and Preston study density spaces as quotients of diffeomorphism groups by volume-preserving subgroups and relate their geometry to the Fisher–Rao metric [23]. Their work establishes an important mathematical precedent for connecting deformation, volume and probability density. Thus an inverse-Jacobian density relation or square-root density map is not itself a novel MFT mechanism. The coframe generalization used here permits intrinsic curvature beyond a compatible flat pullback, but a link from that curvature to physical quantum dynamics still has to be constructed.

2.3 Madelung geometry, phase and quantum reconstruction

Reginatto and Hall–Reginatto reconstruct quantum equations under Fisher-information or exact-uncertainty assumptions [4, 5]. The identity

$$4|\nabla\sqrt{\rho}|^2 = |\nabla\rho|^2/\rho, \quad \rho > 0, \quad (5)$$

and the resulting amplitude-gradient term are therefore established antecedents. MFT’s use of this term becomes distinctive only if its dependence on the common coframe and the accompanying constrained metric variation yield additional consequences. Appendix C makes that variational distinction explicit without deriving quantum evolution.

Khesin, Misiolek and Modin establish the Kähler geometry of the Madelung transform under specified hypotheses, connecting wavefunction geometry with the cotangent bundle of density space and Fisher–Rao geometry [24]. This directly precedes MFT’s proposed connection among density, phase, symplectic structure and quantum states. It is a mathematical correspondence between defined structures, not a derivation of MFT’s macroscopic spacetime. MFT would need to show that its reduced state space realizes the required structures and domains.

Khesin and Modin’s July 2026 preprint extends this line of work to prequantum bundles and wavefunctions with noncritical zeros, and develops a momentum-map viewpoint on the Wallstrom phase-circulation condition [25]. These results are relevant to the nodes and phase-quantization limitations of the strictly positive conserved-coframe sector. They are cited as recent mathematical work, not as a completed solution of those problems within MFT. Importing their conclusions requires verifying their hypotheses for an actual MFT reduction.

Caticha’s entropic dynamics builds quantum dynamics from constrained inference and the geometry of probability and phase, using symplectic and metric preservation to obtain Schrödinger evolution [26]. Its inferential interpretation differs from MFT’s postulate that a common physical geometry is

primitive. An ontological distinction is not yet an observable distinction. Caticha’s construction is a useful benchmark for the explicit dynamical steps that MFT currently leaves open. Standard geometric quantization likewise provides line-bundle and symplectic machinery [9], but prequantization alone does not derive a physical Hilbert space, Hamiltonian or universal action scale.

2.4 Elasticity and emergent spacetime

Kleinert and Zaanen’s world-nematic-crystal model develops an elastic-medium and defect-based description of gravitational geometry, including a mechanism for suppressing torsion [27]. It is a direct antecedent to interpreting curvature, incompatibility and effective spacetime through elasticity. MFT does not specify the same microscopic medium or phase transition, and the world-crystal mechanism does not establish the particular measure identity used here. The comparison nevertheless prevents elasticity or defect-induced geometry from being presented as original to MFT.

Causal fermion systems formulate a pre-spacetime framework using Hilbert, operator and measure structures and a causal action principle [18]. Their relationship to MFT is architectural rather than an identity of starting objects or equations. MFT instead postulates coframe, connection, measure and ordering data and an unconstructed scale projection. For its gravitational endpoint, effective-field-theory reasoning organizes an Einstein–Hilbert kinetic term and higher-curvature corrections after suitable infrared assumptions [13]. This does not derive those assumptions, a common causal metric or universal coupling from the deep functional.

2.5 Internal planes, spectral geometry and gauge structure

The spectral-action constructions of Chamseddine–Connes and Chamseddine–Connes–Marcolli are major precedents for relating gauge, matter and gravitational sectors through generalized geometry [19, 20]. MFT’s finite-dimensional internal sector is much more limited: a simple two-form selects a plane, and an independently postulated operator assigns different eigenvalues to that plane and its complement. It does not reproduce a full spectral-action model.

The space of complex two-planes in $\mathbb{C}^5$ is the Grassmannian

$$\text{Gr}_2(\mathbb{C}^5) \cong SU(5)/S(U(2) \times U(3)). \quad (6)$$

This follows because $SU(5)$ acts transitively on such planes and the subgroup fixing a plane preserves its orthogonal complement, giving the block-unitary stabilizer with determinant one. Consequently the stabilizer is standard geometry, not a new group-theoretic discovery. A fixed simple two-form has the smaller stabilizer explained below; retaining the Abelian factor requires the phase-insensitive projector-sector assumption.

The $\bar{\mathbf{5}} \oplus \mathbf{10}$ branching and hypercharge normalization are standard $SU(5)$ -type representation theory [14], and the associated global quotient is established background [15]. MFT would contribute new physics only by justifying the ordering mechanism, effective gauge dynamics and chiral spectrum, rather than by reproducing this algebra. No physical grand-unified $SU(5)$ gauge theory is derived here.

2.6 Scope of the contribution

MFT does not claim priority for geometrizing probability or quantum effects, connecting quantum potentials to curvature, representing densities through deformations, describing gravity through elasticity,

or obtaining familiar internal symmetries from geometric structures. The proposed contribution is the particular arrangement of a conserved coframe–measure sector, a separate quantum-state reconstruction target and a scale-dependent spacetime projection, accompanied by explicit conditional calculations and limitations. The compact conformal example and its constrained variation within the conformal sector are the paper’s most concrete calculations; no priority claim is made for the standard identities used in them.

It remains unproved that this arrangement yields dynamics or observables different from existing approaches. A change of terminology, a larger list of sectors or a common-origin interpretation is insufficient to establish a new physical theory. The broader architecture is presented as a research proposal, while the explicit calculations are evaluated as toy constructions. The next substantive advance would be a controlled dynamical reduction showing what the shared geometry constrains beyond its separate ingredients.

3 Foundational Postulates and Claim Discipline

The four postulates below define MFT, not MGF in its full possible generality. MGF is the umbrella research framework; MFT is one explicit toy realization selected for calculation and falsifiable refinement.

3.1 Postulate I: a deep geometric state

MFT assumes that physical configurations descend from a deeper geometric object

$$\boxed{\mathfrak{G} = (\mathcal{X}, E, \nabla_G, \mu_G, \beta_G)} \quad (7)$$

Here $\mathcal{X}$ is a deep configuration manifold or generalized state space; E is a frame/deformation structure; ∇_G is a generalized connection; μ_G is a geometric measure; and β_G is an internal two-form order parameter used in the finite-dimensional gauge toy model. Equation (7) is a MFT definition, not an established physical structure. The components are not intended to represent independent substances. The model assumes that they are mutually constrained aspects of a single admissible geometry.

3.2 Postulate II: geometric measure

On a positive-definite effective sector with metric h , the probability-like density is defined by the Radon–Nikodym relation

$$\boxed{\rho_G = \frac{d\mu_G}{dV_h}} \quad (8)$$

This assumes $\mu_G \ll V_h$ and locally finite (or sigma-finite) measures; singular measures require a separate treatment. Equation (8) is standard measure-theoretic language applied as a MFT interpretive postulate. The stronger relation $\rho_G \propto J^{-1}$, developed below, is not assumed universally; it follows only in the conserved-deformation realization.

3.3 Postulate III: geometric probability on configuration space

Admissible histories or configurations $\Gamma \in \mathcal{C}_{\text{adm}}$ carry a formal probability measure

$$\boxed{\mathrm{d}\mathbb{P}_H[\Gamma] = Z^{-1} \exp\left[-\frac{\mathcal{J}_G[\Gamma]}{\epsilon_G}\right] \mathrm{d}\nu_0[\Gamma].} \quad (9)$$

The acronym HPSS is used here for *Hyperdimensional Probability State Space*. The parameter $\epsilon_G > 0$ controls measure concentration and ν_0 is a reference measure. In a continuum field space, Eq. (9) is formal unless a regularization or constructive measure is provided. Finite-mode truncations define probability distributions only when the reference measure and functional satisfy $0 < Z < \infty$; finite dimension and positivity alone do not guarantee this.

3.4 Postulate IV: scale projection

MFT assumes a scale-dependent map

$$\boxed{\mathcal{C}_\ell : \mathfrak{G} \longrightarrow \mathfrak{G}_{\text{eff}}(\ell),} \quad (10)$$

with different effective sectors dominating at different resolutions. The intended schematic behavior is

$$\mathfrak{G}_{\text{eff}}(\ell) \sim \begin{cases} \text{metric-measure/phase description,} & \ell \sim \ell_G \text{ or other fine scales,} \\ \text{Lorentzian metric and effective fields,} & \ell \gg \ell_G. \end{cases} \quad (11)$$

Equation (11) is the central unification hypothesis. MFT must ultimately derive or construct a concrete coarse-graining map; the present paper specifies the desired endpoints and several compatible toy mechanisms.

3.5 Claim taxonomy

Label	Meaning in this manuscript
Established	Standard mathematics or experimentally supported physics used as background.
MFT postulate	A defining assumption of the proposed framework.
Toy ansatz	A deliberately simple model choice used to test a mechanism.
Derived within assumptions	Algebraic or variational consequence of stated postulates/ansätze.
Conjecture / target	A result MFT would need for a stronger physical theory but has not obtained.

4 Deep Deformation and the Geometric-Measure Identity

4.1 Coframe geometry and the compatible-map limitation

Let $\mathcal{M}$ be an oriented real n -manifold with a fixed positive reference metric γ_0 , reference volume dV_0 , and local orthonormal coframe e_0^a . Introduce an invertible coframe deformation

$$e^a = F^a_b e_0^b, \quad F \in GL^+(n, \mathbb{R}). \quad (12)$$

The positive metric and volume ratio are

$$\boxed{\gamma = \delta_{ab} e^a \otimes e^b = F^T \gamma_0 F}, \quad (13)$$

$$J = \frac{dV_\gamma}{dV_0} = \sqrt{\frac{\det \gamma}{\det \gamma_0}} = \det F > 0. \quad (14)$$

The matrix formula is understood in compatible reference frames. For a map between general coordinate charts, the volume Jacobian includes the target metric at the image point and the source reference volume; a bare coordinate determinant is insufficient.

There is an essential distinction between an arbitrary coframe and a compatible deformation gradient. If $F^a_A = \partial_A \Phi^a$ and $\gamma = \Phi^* \delta$ for a local diffeomorphism into flat Euclidean space of the same dimension, then $\text{Riem}[\gamma] = 0$. A coordinate deformation cannot generate intrinsic curvature. A curved target similarly supplies pre-existing curvature rather than deriving it. The present curved toy sectors therefore allow coframes that are not gradients of such a map. This is an explicit enlargement of the compatible-deformation ansatz, not a consequence of it. For example, on a reference flat patch $e^a = e^{\sigma(x)} dx^a$ generally has $de^a \neq 0$ and can produce a curved conformal metric. Anholonomy alone is not a sufficient condition for curvature. Nor should de^a be confused with torsion: the Levi-Civita connection can cancel it in the torsion equation.

Fix a positive reference measure $\mu_0 = \rho_0 dV_0$ and impose the conserved-measure constraint on the same base,

$$d\mu_G = d\mu_0 = \rho_G dV_\gamma = \rho_0 dV_0. \quad (15)$$

Then

$$\boxed{\rho_G = \rho_0 / J}. \quad (16)$$

This is a Radon–Nikodym identity conditional on the prescribed reference measure, not a probability law derived from the metric alone. It fixes the measure density once F and μ_0 are specified, while leaving the unimodular shape data unconstrained. For a genuine material map the equivalent statement uses the pushforward measure with densities evaluated at corresponding points. The new coframe sector instead imposes conservation directly. The reference measure is additional input; its unique origin from deep geometry remains a target.

For finite total measure, conservation on the same base has a further consequence. For every fixed measurable region $B \subset \mathcal{M}$,

$$P_F(B) = \frac{\int_B \rho_G dV_\gamma}{\int_{\mathcal{M}} \rho_G dV_\gamma} = \frac{\int_B \rho_0 dV_0}{\int_{\mathcal{M}} \rho_0 dV_0}. \quad (17)$$

Thus the normalized measure of a fixed region is independent of the coframe deformation, even though its density relative to the changing metric volume varies. This identity does not by itself generate quantum

probability redistribution. A deformation-dependent observation map, moving outcome regions, or another justified reduction would be required to obtain varying effective outcome probabilities. Such a construction is not supplied here.

4.2 Volume, shape, and orientation

The polar decomposition writes

$$F = RU, \quad (18)$$

with $R \in SO(n)$ and U real positive symmetric, in reference orthonormal frames. The metric depends on U and is invariant under the left orthogonal factor. Local frame rotation is a redundancy of this metric description; it does not by itself supply a physical phase connection or nontrivial holonomy. A useful split is

$$F = J^{1/n} \tilde{F}, \quad \det \tilde{F} = 1. \quad (19)$$

Therefore

$$\gamma_{AB} = J^{2/n} \tilde{\gamma}_{AB} = \left(\frac{\rho_0}{\rho_G} \right)^{2/n} \tilde{\gamma}_{AB}. \quad (20)$$

Equation (20) is the clearest mathematical realization of the MFT geometric-measure identity. The volume variable J fixes the conserved measure density, the unimodular sector $\tilde{\gamma}$ carries shape/shear information, and the orientation/connection sector can carry holonomy information. MFT interprets the separation not as a division into independent fields but as a decomposition of one deformation.

4.3 Weighted metric-measure form

More generally, one can write

$$d\mu_G = \rho_* e^{-f_G} dV_\gamma, \quad \rho_G = \rho_* e^{-f_G}, \quad (21)$$

with a constant reference density ρ_* . For spatially constant ρ_0 in the conserved-coframe realization,

$$f_G = \ln J + \text{constant}. \quad (22)$$

Weighted Riemannian geometry and Perelman-type functionals provide established mathematical comparisons [11]; however, MFT does not treat f_G and γ as independent fundamental fields. Both are constrained projections of $\mathfrak{G}$, and therefore weighted-geometry formulae are used as scaffolding rather than as the microscopic MFT action.

5 Quantum-Measure Sector

5.1 Amplitude, normalization, and the scope of variation

Define a dimensionless amplitude by

$$A_G = \sqrt{\rho_G / \rho_*}, \quad \rho_* > 0. \quad (23)$$

For constant ρ_0 , $A_G \propto J^{-1/2}$. On a chosen effective positive sector with metric h , suppose $0 < \mathcal{N} = \int \rho_G dV_h < \infty$. Define $p = \rho_G/\mathcal{N}$, $a = \sqrt{p}$, so $\int a^2 dV_h = 1$. For an n -dimensional spatial volume, p has units L^{-n} and a has units $L^{-n/2}$. The normalized gradient energy is

$$E_Q[p; h] = \frac{\eta_G^2}{2m} \int |\nabla a|_h^2 dV_h = \frac{\eta_G^2 \rho_*}{2m\mathcal{N}} \int |\nabla A_G|_h^2 dV_h. \quad (24)$$

It has units of energy when η_G has units of action and m is a mass. For $p > 0$,

$$E_Q = \frac{\eta_G^2}{8m} \int \frac{|\nabla p|_h^2}{p} dV_h = \frac{\eta_G^2}{8m\mathcal{N}} \int \frac{|\nabla \rho_G|_h^2}{\rho_G} dV_h. \quad (25)$$

At fixed h , integration by parts with compactly supported variations, a closed manifold, or compatible boundary conditions gives

$$\left. \frac{\delta E_Q}{\delta p} \right|_h = Q_G = -\frac{\eta_G^2}{2m} \frac{\Delta_h \sqrt{p}}{\sqrt{p}} = -\frac{\eta_G^2}{2m} \frac{\Delta_h \sqrt{\rho_G}}{\sqrt{\rho_G}}. \quad (26)$$

Normalization introduces an additive Lagrange multiplier. Variation with respect to a itself instead yields $-\eta_G^2 \Delta_h a/m$. If the integration measure were weighted, the corresponding weighted divergence operator would replace Δ_h . These conventions remove an ambiguity in an unnormalized amplitude energy.

The fixed-metric derivative is the standard Madelung–Bohm/Fisher calculation [1, 2, 4, 5]. It is *not* the full variation of the MFT coframe sector: there both p and h depend on F , so $\delta E_Q/\delta F$ includes metric, volume and normalization terms. Equation (26) is an effective compatibility construction until a controlled reduction justifies holding h fixed. Likewise, the normalized wave amplitude lives on a specified configuration manifold; identifying that manifold's metric with physical three-space, especially for many-body states, remains an additional assumption.

The energy can be defined for $\sqrt{p} \in H^1$ even at some nodes, but Q_G is only pointwise defined where $p > 0$. Moreover, smooth finite $J > 0$ and strictly positive ρ_0 forbid exact zeros of ρ_G . Thus the minimal conserved-coframe sector excludes generic quantum nodes. A completion needs a justified projection/interference mechanism or an explicitly enlarged admissible class; it cannot simply equate every quantum density to ρ_0/J .

5.2 Weak-conformal curvature relation

The same deformation variable can be used to connect the amplitude sector to curvature in a restricted regime. Let

$$\gamma_{ij} = J^{2/n} \tilde{\gamma}_{ij}. \quad (27)$$

For a conformal transformation $\gamma = e^{2\sigma} \tilde{\gamma}$ with $J = e^{n\sigma}$, the scalar curvature is

$$\mathcal{R}[\gamma] = J^{-2/n} \left[\tilde{\mathcal{R}} - \frac{2(n-1)}{n} \tilde{\Delta} \ln J - \frac{(n-1)(n-2)}{n^2} |\tilde{\nabla} \ln J|^2 \right]. \quad (28)$$

Meanwhile, take constant ρ_0 and choose $\rho_* = \rho_0$, so $A_G = J^{-1/2}$. Any other constant normalization cancels from the following amplitude ratios:

$$\frac{\Delta A_G}{A_G} = -\frac{1}{2}\Delta \ln J + \frac{1}{4}|\nabla \ln J|^2. \quad (29)$$

Here the Laplacian in Eq. (29) may be taken with any specified metric, but must be the same on both sides. To compare with curvature, take $h = \gamma$ and use the exact conformal identity, with $u = \ln J$,

$$\frac{\Delta_\gamma A_G}{A_G} = J^{-2/n} \left[-\frac{1}{2}\tilde{\Delta}u + \frac{4-n}{4n}|\tilde{\nabla}u|^2 \right]. \quad (30)$$

Assume $u = O(\varepsilon)$ on a fixed length scale, derivatives scale accordingly, and $\tilde{\mathcal{R}} = O(\varepsilon^2)$. These are restrictions on generalized coframes, not on arbitrary compatible flat pullbacks. To first order, with all Laplacians below evaluated in the reference shape metric,

$$\mathcal{R} \simeq -\frac{2(n-1)}{n}\Delta \ln J, \quad Q_G \simeq \frac{\eta_G^2}{4m}\Delta \ln J. \quad (31)$$

Hence, for $n > 1$,

$$\boxed{Q_G \simeq -\frac{\eta_G^2 n}{8m(n-1)}\mathcal{R}.} \quad (32)$$

For $n = 3$,

$$\boxed{Q_G \simeq -\frac{3\eta_G^2}{16m} {}^{(3)}R, \quad \frac{\Delta A_G}{A_G} \simeq \frac{3}{8} {}^{(3)}R.} \quad (33)$$

Equation (33) is a conditional geometric identity, not a universal MFT prediction. It is a restricted weak-conformal consistency relation whose value is that both sides arise from the same J . A future experiment could only test it after a precise mapping from the laboratory geometry to the spatial metric and slicing used in the derivation.

6 Deep Compatibility Functional and HPSS

6.1 Why the deep object is not a spacetime action

A central discipline choice in MFT is to avoid identifying the deep selection functional with a Lorentzian physical action. A positive compatibility functional separates configuration weighting from effective time evolution. Positivity of this Euclidean-type object does not establish ghost freedom, unitarity or hyperbolicity of a later Lorentzian theory. Let

$$\Gamma = (E, \nabla_G, \mu_G, \beta_G) \quad (34)$$

be an admissible configuration. Introduce a characteristic deep scale ℓ_G . A minimal dimensionless functional family is

$$\boxed{\mathcal{J}_G[\Gamma] = \int_{\mathcal{X}} \frac{d\mu_G}{\ell_G^q} \left[a_T \ell_G^2 \|\mathcal{T}_G\|^2 + a_R \ell_G^4 \|\mathcal{R}_G\|^2 + 4a_\rho \ell_G^2 \|\nabla A_G\|^2 + a_\beta \ell_G^2 \|D_G \beta_G\|^2 + \widehat{V}_\beta(\beta_G) \right]} \quad (35)$$

where $\mathcal{T}_G = D_G E$ is a generalized torsion/deformation incompatibility and $\mathcal{R}_G = [\nabla_G, \nabla_G]$ is the deep curvature. Equation (35) is a MFT toy ansatz. Take $a_T, a_R, a_\rho, a_\beta \geq 0$ and $\widehat{V}_\beta \geq 0$. For a dimensionless convention, assume $d\mu_G$ has units L^q , the component norms of $\mathcal{T}_G, \mathcal{R}_G$ have units L^{-1}, L^{-2} , and A_G, β_G are dimensionless. Here μ_G is a geometric volume measure on the deep base, not a normalized probability measure. Define curvature by the connection curvature two-form (equivalently $[\nabla_X, \nabla_Y] - \nabla_{[X, Y]}$). The coefficients are not derived; an explicit base metric, fiber metric, connection and admissible variations remain necessary. This functional family is not yet an evaluated microscopic model.

Topology, boundary conditions, regularity, and measure constraints are treated as hard restrictions defining

$$\boxed{\mathcal{C}_{\text{adm}} = \{\Gamma : \text{specified topology, regularity, boundary, and measure constraints hold}\}.} \quad (36)$$

This is preferable to pretending that all topological obstructions can be represented by large but finite penalty terms.

6.2 Configuration measure and stability

The HPSS measure is Eq. (9), with formal partition function

$$Z = \int_{\mathcal{C}_{\text{adm}}} \exp[-\mathcal{J}_G/\epsilon_G] d\nu_0. \quad (37)$$

When $0 < Z < \infty$ and suitable compactness/coercivity conditions hold, small ϵ_G favors low-incompatibility sectors; gauge volumes and flat directions must be controlled. A stationary candidate satisfies

$$\delta \mathcal{J}_G[\Gamma_*] = 0, \quad (38)$$

with nonnegative second variation on admissible non-gauge directions (strict positivity for a nondegenerate local minimum). A self-adjoint operator additionally requires a field-space inner product and domain. Its eigenvalues do not define frequencies or particle masses without a kinetic structure. The Hessian

$$\boxed{\mathcal{H}_G = \frac{\delta^2 \mathcal{J}_G}{\delta \Gamma^2} \Big|_{\Gamma_*}} \quad (39)$$

obeys the spectral problem

$$\mathcal{H}_G u_n = \lambda_n u_n. \quad (40)$$

Stable normalizable modes are interpreted as candidate effective excitations. This is not yet a derivation of particles; it is the mathematical place where a particle spectrum could emerge.

7 Reduced State Geometry, Phase, and the Quantum of Action

7.1 Phase as holonomy

The metric alone loses local rotational/orientation information contained in the full deformation frame. MFT therefore assigns phase to a connection/holonomy sector rather than merely to a globally defined angle. Let $\mathcal{A}_G$ be an effective line-bundle connection on the reduced state space. In a local trivialization, parallel transport along a path γ is

$$U_G(\gamma) = \exp \left(i \int_{\gamma} \mathcal{A}_G \right). \quad (41)$$

For a closed loop this gives Abelian holonomy; for an open path it depends on endpoint gauge choices. A global wavefunction is a section, and the following polar expression is local away from nodes. The projected complex state is written

$$\boxed{\Psi_G = A_G \exp \left(\frac{i S_G}{\eta_G} \right)}. \quad (42)$$

Here $A_G^2 \propto \rho_G$ comes from measure geometry while S_G/η_G is a geometric phase variable. This decomposition mirrors standard polar representations of the wavefunction; a common geometric origin of its two factors is a target of the proposed reduction. The amplitude's configuration base and the reduced symplectic state space are distinct constructions. A prequantum section on the latter is not automatically a physical configuration-space wavefunction. A polarization, representation, or specified pullback relating their bases and bundles is required before the displayed local factors define one constructed quantum state.

7.2 Symplectic reduced-state postulate

The positive functional $\mathcal{J}_G$ does not itself generate Hamiltonian phase evolution. MFT therefore explicitly assumes that the stable reduced state space

$$\mathcal{P}_G = \mathcal{C}_{\text{stable}} / \sim \quad (43)$$

admits, in the quantum sector, a closed nondegenerate two-form ω_G . Geometric prequantization requires the normalized symplectic class to be integral under the usual hypotheses [9]:

$$\boxed{\left[\frac{\omega_G}{2\pi\eta_G} \right] \in H^2(\mathcal{P}_G, \mathbb{Z})}. \quad (44)$$

Thus for an integral two-cycle Σ ,

$$\int_{\Sigma} \omega_G = 2\pi n \eta_G. \quad (45)$$

Only if an integral cycle Σ_* has Chern pairing exactly +1, one may write

$$\boxed{\eta_G = \frac{1}{2\pi} \int_{\Sigma_*} \omega_G} \quad (46)$$

as a candidate elementary symplectic action flux. A primitive homology cycle need not have pairing one. If the symplectic form is exact, every closed-cycle period vanishes and integrality places no restriction on a nonzero action scale. A nonzero discrete period group and common normalization across sectors are additional assumptions. The observed quantum theory requires the matching $\eta_G = \hbar$, but Eq. (46) does not determine the numerical SI magnitude of $\hbar$. Moreover, prequantization is not the whole of geometric quantization; a polarization or analogous reduction is ordinarily required [9]. MFT leaves that construction open.

8 Born-Rule Reconstruction and Entanglement

8.1 Projector measure

Let stable quantum modes span a complex Hilbert space $\mathcal{H}_Q$ and let mutually exclusive outcomes correspond to orthogonal projectors P_i satisfying $\sum_i P_i = I$. Assume a normalized, nonnegative, countably additive, noncontextual probability assignment on projectors. In separable Hilbert spaces of dimension at least three, Gleason's theorem constrains such measures to the trace form [6]

$$\mu(P) = \text{Tr}(\hat{\rho}P). \quad (47)$$

For a normalized pure state $\hat{\rho} = |\Psi\rangle\langle\Psi|$ and rank-one outcome projectors $P_i = |i\rangle\langle i|$ with orthonormal outcome vectors,

$$P(i|\Psi) = |\langle i|\Psi\rangle|^2. \quad (48)$$

MFT therefore presents the Born rule as a *reconstruction conditional on projector-measure assumptions*, not as an assumption-free derivation from deformation geometry alone.

The deeper HPSS measure and the Born conditional measure should not be identified naively. A more careful target is

$$P(i|\Psi, \mathbf{M}) = \frac{\mathbb{P}_H(\mathcal{C}_{i,\mathbf{M}} \cap \mathcal{C}_\Psi)}{\mathbb{P}_H(\mathcal{C}_\Psi)}, \quad \mathbb{P}_H(\mathcal{C}_\Psi) > 0. \quad (49)$$

Here the measurable outcome events partition the preparation event for a fixed measurement $\mathbf{M}$. Exact-state preparation events can have zero measure; then a regular conditional probability/disintegration or finite preparation window is needed. Agreement with Eq. (48) remains unproved. Gleason's projector theorem alone excludes dimension two and does not identify the reconstructed density operator with a particular coframe amplitude; both issues require additional structure.

8.2 Entanglement benchmark

For a two-mode singlet state

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle), \quad (50)$$

standard quantum mechanics gives

$$E(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b}. \quad (51)$$

Appropriate CHSH settings yield $|S| = 2\sqrt{2}$, exceeding the local Bell bound $|S| \leq 2$ [7, 8]. MFT does not attempt to restore local hidden variables in emergent spacetime. Instead, it interprets the nonfactorizable Hilbert state as evidence that deep configuration geometry need not factorize when its

effective spacetime projections are spatially separated:

$$\boxed{\text{spatial separation in } 3 + 1 \text{ dimensions} \not\Rightarrow \text{factorization in deep configuration geometry.}} \quad (52)$$

Nonfactorization alone does not imply a Bell violation: ordinary classical common-cause distributions can also correlate outcomes. A completion must specify how its conditional response laws fail Bell factorization (or explicitly identify another changed Bell assumption), recover the quantum correlations and preserve operational no-signaling. The singlet benchmark below assumes those quantum laws rather than deriving them.

9 Relativistic Scalar Compatibility Check

Before connecting the measure sector to full quantum field theory, it is useful to check whether the amplitude/phase variables are at least compatible with the standard relativistic scalar equation. This subsection is a consistency check, not a derivation of relativistic quantum mechanics.

Take the Klein–Gordon equation in signature $(-+++)$,

$$\boxed{\left(\square_g - \frac{m^2 c^2}{\eta_G^2}\right) \Psi_G = 0,} \quad (53)$$

where $\square_g = g^{\mu\nu} \nabla_\mu \nabla_\nu$. Substitute the polar form

$$\Psi_G = \sqrt{\rho_G} \exp(iS_G/\eta_G). \quad (54)$$

Separating imaginary and real parts gives respectively

$$\boxed{\nabla_\mu (\rho_G \nabla^\mu S_G) = 0} \quad (55)$$

and

$$\boxed{g^{\mu\nu} \nabla_\mu S_G \nabla_\nu S_G + m^2 c^2 - \eta_G^2 \frac{\square_g \sqrt{\rho_G}}{\sqrt{\rho_G}} = 0.} \quad (56)$$

The first is a covariant continuity-type equation, while the second is a relativistic Hamilton–Jacobi equation modified by an amplitude-curvature term in configuration space. Setting $\eta_G = \hbar$ reproduces the familiar polar decomposition of the Klein–Gordon equation.

This check accomplishes only a limited task. It shows that the same pair (ρ_G, S_G) used in the nonrelativistic geometric-measure sector can parameterize a standard relativistic scalar wave equation. It does not establish that MFT has derived Eq. (53), nor does it solve the known interpretive difficulty that the usual Klein–Gordon current is not a positive-definite single-particle probability density. A complete MFT theory would therefore have to pass to a quantum-field interpretation or supply a more fundamental positive observable measure on the reduced state space.

The slow-envelope limit gives another consistency check. Writing

$$\Psi_G = e^{-imc^2 t/\eta_G} \psi_G \quad (57)$$

and assuming envelope variations slow compared with the rest-frequency scale yields the nonrelativistic Schrödinger equation to leading order when $\eta_G = \hbar$ and the background is sufficiently weak. This is

standard asymptotic physics. MFT uses the limit only to demand that its effective scalar sector sit in the correct relativistic-to-nonrelativistic hierarchy.

The significance for the architecture is that the quantum-potential structure and the relativistic dispersion structure need not be introduced as unrelated mechanisms. If the reduced state geometry supplies a universal η_G , while the coarse Lorentzian sector supplies the effective metric $g_{\mu\nu}$, then Eq. (53) becomes a natural target equation coupling two projections of the same deep geometry. Demonstrating that target from the microscopic compatibility functional remains open.

10 Time and the Emergent Lorentzian Sector

A complete realized history may be represented as a curve through a configuration space,

$$\Gamma_* : t \mapsto Q^A(t), \quad (58)$$

where Q^A collects coarse degrees of freedom. This motivates one ordering parameter but does not derive a physical timelike dimension; any ordinary dynamical system can be parameterized by a curve. The physically stronger requirement is the emergence of a hyperbolic principal operator. An invertible real coframe transformation of a positive metric cannot itself change signature: for every nonzero real vector v ,

$$v^T F^T \gamma_0 F v = (Fv)^T \gamma_0 (Fv) > 0. \quad (59)$$

Consequently a Lorentzian effective metric must arise through additional dynamical or reduction structure, rather than an ordinary frame change of the positive metric. The following effective action assumes time and spatial coordinates; their emergence from the deep sector is not derived.

Consider a perturbative mode ϕ_G with effective quadratic action

$$S_{\text{pert}} = \frac{1}{2} \int dt d^3x \left[M_G (\partial_t \phi_G)^2 - K_G |\nabla \phi_G|^2 \right]. \quad (60)$$

For coefficients that may depend on position and time, variation gives

$$\partial_t (M_G \partial_t \phi_G) - \nabla \cdot (K_G \nabla \phi_G) = 0. \quad (61)$$

For constant positive M_G, K_G , this reduces to

$$\partial_t^2 \phi_G - c_G^2 \nabla^2 \phi_G = 0, \quad \boxed{c_G^2 = K_G / M_G}. \quad (62)$$

The principal differential operator has the Minkowski-type sign structure

$$-\frac{1}{c_G^2} \partial_t^2 + \nabla^2. \quad (63)$$

MFT treats this as a candidate effective mechanism for Lorentzian causal structure, not as a derivation of exact Lorentz invariance. A viable theory would require all observed matter and gauge sectors to share the same principal causal metric and would need to suppress any preferred-frame effects to experimental limits.

11 Dimensionality as an Illustrative Selection Model

MFT does not presently derive three spatial dimensions. Nevertheless, a finite toy model can illustrate how dimensional rank might be treated as a discrete variable in the HPSS measure rather than as an unexplained immutable input. Let d denote candidate spatial rank and consider the twisted bundle

$$\mathcal{E}_d = \frac{\mathbb{R}_\theta \times \mathbb{R}^d}{(\theta, \mathbf{q}) \sim (\theta + L, -\mathbf{q})}. \quad (64)$$

For this *specific full-inversion twist*, the determinant is

$$\det(-I_d) = (-1)^d. \quad (65)$$

Thus the twist reverses orientation only for odd d . This is not a general theorem that nonorientability requires odd dimension; it is a property of the chosen inversion map.

A second illustrative requirement is a nonzero two-form on the spatial fiber itself, excluding $d = 1$. This is stronger than requiring holonomy, which can be nontrivial even on a one-dimensional base. Under these two toy assumptions the admissible sequence is

$$d = 3, 5, 7, \dots \quad (66)$$

To introduce a mathematically neutral complexity cost, use the scalar Laplacian on S^{d-1} , whose eigenvalues are

$$\lambda_\ell^{(d)} = \ell(\ell + d - 2). \quad (67)$$

The first nonconstant shell has $\lambda_1 = d - 1$ with degeneracy $g_1 = d$. A simple spectral cost is therefore

$$\mathcal{C}_d = g_1 \lambda_1 = d(d - 1). \quad (68)$$

Let

$$\mathcal{I}_D(d) = \zeta_D d(d - 1), \quad \zeta_D > 0, \quad (69)$$

and define a discrete dimensional measure

$$\boxed{P_D(d) = Z_D^{-1} \exp[-\mathcal{I}_D(d)/\kappa_D], \quad d \in \{3, 5, 7, \dots\}.} \quad (70)$$

Assume $\kappa_D > 0$. Then $Z_D < \infty$, and $d = 3$ is the maximum-weight sector, not the only possible dimension at finite concentration.

The value of this calculation is methodological, not evidentiary. It demonstrates how a discrete dimension label can be placed inside the same probability-state-space language used elsewhere in MFT and how explicit topological and spectral assumptions can produce measure concentration at a particular rank. It does not explain why the full-inversion twist, two-form requirement, or chosen spectral cost are fundamental. Accordingly, the final theory retains only the abstract selection statement

$$\boxed{d_* = \arg \min_{d \in \mathcal{D}_{\text{adm}}} \mathcal{I}_D(d),} \quad (71)$$

while Eq. (70) is presented as an illustrative realization. Any future claim that MFT derives $d = 3$ would require a deeper functional from which the admissibility conditions and cost follow rather than

being selected to make three minimal.

12 Gravity and the General-Relativistic Limit

12.1 Weak-field volume mode

The Newtonian limit provides a useful matching calculation. In weak-field GR with negligible anisotropic stress, one may write schematically [12]

$$g_{00} \simeq -(1 + 2\Phi_N/c^2), \quad g_{ij} \simeq (1 - 2\Phi_N/c^2)\delta_{ij}. \quad (72)$$

The spatial volume ratio is then

$$J_s = \sqrt{\frac{\det h}{\det h_0}} = (1 - 2\Phi_N/c^2)^{3/2} \simeq 1 - \frac{3\Phi_N}{c^2}. \quad (73)$$

Define

$$\sigma_G = \ln J_s. \quad (74)$$

To first order,

$$\boxed{\Phi_N \simeq -\frac{c^2}{3}\sigma_G.} \quad (75)$$

Using the conserved-measure toy relation gives

$$\frac{\delta\rho_G}{\rho_0} \simeq \frac{3\Phi_N}{c^2}. \quad (76)$$

This is a translation of the standard weak-field metric into MFT measure language, not an independent derivation of Newtonian gravity.

A simple static toy action for the volume mode is

$$S_\sigma = \int dt d^3x \left[-\frac{K_\sigma}{2} |\nabla\sigma_G|^2 + a_G \rho_m c^2 \sigma_G \right]. \quad (77)$$

Variation gives

$$K_\sigma \nabla^2 \sigma_G + a_G \rho_m c^2 = 0. \quad (78)$$

If the lapse responds as $N = e^{-a_G \sigma_G}$, then $\Phi_N = -a_G c^2 \sigma_G$. Matching Eq. (75) sets $a_G = 1/3$ as an infrared matching condition. Substitution yields

$$\nabla^2 \Phi_N = \frac{a_G^2 c^4}{K_\sigma} \rho_m. \quad (79)$$

Matching the Poisson equation $\nabla^2 \Phi_N = 4\pi G \rho_m$ identifies

$$\boxed{G = \frac{a_G^2 c^4}{4\pi K_\sigma}, \quad K_\sigma = \frac{c^4}{36\pi G} \quad (a_G = 1/3).} \quad (80)$$

This relation is derived within the toy action and matching assumptions. MFT has not derived K_σ

microscopically.

12.2 Infrared Einstein limit

In this subsection use natural units $c = \hbar = 1$, $M_{\text{eff}}^2 = (8\pi G)^{-1} > 0$, and $\Lambda = \Lambda_{\text{eff}}/M_{\text{eff}}^2$; Λ_{eff} is a vacuum energy density, whereas Λ is geometric curvature. Appropriate boundary terms are understood. The full tensor theory should not be obtained by inserting a weighted action engineered to reproduce Einstein's equation. Instead MFT requires coarse-graining to yield an effective Lorentzian metric $g_{\mu\nu}$. If the infrared theory is local, diffeomorphism invariant, approximately Lorentz invariant, and metric dominated, then standard effective-field-theory reasoning organizes the gravitational action as [13]

$$S_{\text{grav}}^{\text{IR}} = \int \sqrt{-g} d^4x \left[-\Lambda_{\text{eff}} + \frac{M_{\text{eff}}^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right]. \quad (81)$$

Below the derivative-expansion cutoff, assuming higher-curvature terms are perturbatively suppressed and no additional unsuppressed light gravitational fields, the Einstein–Hilbert term supplies the leading metric kinetic term. The cosmological term remains at leading order and need not be small. Variation with a matter action gives

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} + \text{suppressed corrections.} \quad (82)$$

MFT's unification claim is therefore not that Eq. (82) has been microscopically derived. It is that the coarse metric entering Eq. (82) and the fine-scale quantum measure entering Eq. (42) descend from the same $\mathfrak{G}$.

13 Internal Spectral Geometry and Gauge Structure

13.1 Rank-two internal order

Let the relevant finite-dimensional internal fiber be

$$V_G \simeq \mathbb{C}^N \quad (83)$$

with a Hermitian inner product. Introduce a two-form order parameter $\beta_G \in \Lambda^2 V_G^*$ and the toy potential

$$V_\beta = b_1(\|\beta_G\|^2 - v_\beta^2)^2 + b_2\|\beta_G \wedge \beta_G\|^2, \quad b_{1,2} > 0, \quad v_\beta > 0. \quad (84)$$

A zero-energy global minimum obeys $\|\beta_G\| = v_\beta$ and $\beta_G \wedge \beta_G = 0$, selecting a decomposable two-form and hence a rank-two plane in the present toy sector. Such minima exist for the internal dimensions considered here. This is a consequence of the ansatz, not a classification of all stationary points or a fundamental principle fixing the potential.

Write B for the skew matrix of β_G in an orthonormal basis. For a nonzero simple form, $P_2 = B^\dagger B / v_\beta^2$ is a rank-two Hermitian projector when $\|\beta_G\| = v_\beta$ in the exterior-algebra norm. This construction is independent of the form phase and transforms by unitary conjugation. For $N = 5$, choose orthonormal

vectors u, v spanning the selected plane and define

$$\mathcal{J}_{\text{int}} = |u\rangle\langle v| - |v\rangle\langle u|. \quad (85)$$

Then

$$\mathcal{J}_{\text{int}}^\dagger \mathcal{J}_{\text{int}} = P_2, \quad (86)$$

where P_2 projects onto the selected two-plane. An independently postulated Hermitian splitting operator

$$\boxed{\mathcal{H}_{\text{int}} = \lambda_3 I + \Delta\lambda P_2} \quad (87)$$

has spectrum

$$\boxed{\text{Spec}(\mathcal{H}_{\text{int}}) = \{\lambda_3, \lambda_3, \lambda_3, \lambda_2, \lambda_2\}, \quad \lambda_2 = \lambda_3 + \Delta\lambda.} \quad (88)$$

This operator is not the Hessian of V_β , whose variations live in a two-form space. Its eigenvalues require separate positivity assumptions for a stability interpretation. Thus the $3 + 2$ degeneracy is generated by rank-nullity once the five-dimensional fiber and rank-two order are specified.

13.2 Why five is a minimal phenomenological input

The choice $N = 5$ is not derived from first principles. It can, however, be motivated by two consistency arguments. A rank-two plane in $\mathbb{C}^N$ leaves a complement of dimension $N - 2$, with determinant-one stabilizer

$$S(U(2) \times U(N - 2)). \quad (89)$$

The smallest N that yields two unequal non-Abelian unitary factors is $N = 5$, producing $2 + 3$. This is a phenomenological minimal-distinct-factor criterion, not a theorem that nature must choose five.

A second consistency check uses the exterior-algebra package

$$\overline{V}_N \oplus \Lambda^2 V_N, \quad (90)$$

whose dimension is

$$D(N) = N + \frac{N(N - 1)}{2} = \frac{N(N + 1)}{2}. \quad (91)$$

The value $D = 15$, counting the 15 Weyl field components in the five gauge multiplets of one Standard Model family without a right-handed neutrino, in $SU(5)$ -type packaging, gives $N = 5$. This uses empirical and representational input and is therefore a consistency motivation rather than an independent derivation.

13.3 Residual symmetry and Abelian generator

If $\lambda_2 \neq \lambda_3$, unitary transformations commuting with $\mathcal{H}_{\text{int}}$ preserve the two eigenspaces, giving

$$\text{Stab}(\mathcal{H}_{\text{int}}) = U(3) \times U(2). \quad (92)$$

Restricting the parent unitary frame group to $SU(5)$ is an additional choice of fixed complex volume form, not a consequence of preserving Hermitian volume. Under that choice,

$$\boxed{S(U(3) \times U(2)) \cong \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}}. \quad (93)$$

The global quotient structure is established group theory and appears in Standard Model gauge-group analyses [15]. The symmetry just computed is the commutant of P_2 , not the stabilizer of a fixed two-form. For example, with $\beta = v_\beta e^4 \wedge e^5$, a block matrix $\text{diag}(A, D) \in SU(5)$ preserves β itself only when $\det D = 1$; then $\det A = 1$ as well. Thus

$$\text{Stab}_{SU(5)}(\beta) = SU(3) \times SU(2), \quad \text{Stab}_{SU(5)}(P_2) = S(U(3) \times U(2)). \quad (94)$$

To retain the Abelian factor, the effective ordering variable must be the plane projector or the projective class $[\beta]$, so that its phase is discarded by an explicitly assumed reduction. If a physical fixed two-form condensate survives, the larger group cannot be claimed as the unbroken symmetry. This revision adopts a *projector-sector assumption*; constructing a phase-insensitive microscopic dynamics remains open. The auxiliary $\mathcal{J}_{\text{int}}$ depends on the chosen plane basis, while P_2 does not.

The phase reduction also needs a dynamical justification. Holding the connection fixed, a local phase change gives

$$D_G(e^{i\theta} \beta_G) = e^{i\theta} (D_G \beta_G + i d\theta \otimes \beta_G). \quad (95)$$

Although P_2 is unchanged, the term $\|D_G \beta_G\|^2$ in the deep functional generally changes. A compensating gauge transformation, a kinetic term defined on the quotient, or a controlled elimination of the phase is therefore needed. Replacing the order parameter by its projector is not, by itself, a derivation of equivalent dynamics.

An Abelian generator commuting with the blocks is

$$Y_G = aP_3 + bP_2. \quad (96)$$

Tracelessness requires $3a + 2b = 0$. Choosing the conventional normalization $a = -1/3$ gives

$$\boxed{Y_G = -\frac{1}{3}P_3 + \frac{1}{2}P_2 = \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right)}. \quad (97)$$

The relative eigenvalues follow from the $3 + 2$ split and tracelessness; the overall normalization/sign convention remains conventional.

13.4 Matter representations

Standard $SU(5)$ -type representation algebra [14] gives

$$\bar{\mathbf{5}} \rightarrow (\bar{\mathbf{3}}, 1)_{1/3} \oplus (1, 2)_{-1/2} \quad (98)$$

and

$$\Lambda^2 \mathbf{5} \rightarrow (\bar{\mathbf{3}}, 1)_{-2/3} \oplus (3, 2)_{1/6} \oplus (1, 1)_1. \quad (99)$$

These decompositions are not novel MFT results. They demonstrate that the geometrically generated $3+2$ block has a known route to one-family Standard Model-like representation content. A gauge-singlet state may be appended for a right-handed-neutrino-like sector.

The existence of the stabilizer does not by itself produce dynamical gauge bosons. MFT must additionally assume that local changes of internal frame become redundancies of the effective description, allowing an effective connection

$$\mathcal{A}_\mu^{\text{eff}} \in \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1). \quad (100)$$

Promoting this symmetry to gauge dynamics is therefore an effective MFT assumption, not a consequence of rank-nullity alone.

14 Chirality, Generations, and Mass

14.1 Representation-resolved chirality target

A finite-dimensional splitting does not derive fermions or chirality. On a suitable compact even-dimensional spin or spin-c manifold without boundary, define the chiral component $\mathcal{D}_{\text{int},r}^+$ of a twisted Dirac operator in each gauge representation r . With boundary or noncompact geometry, Fredholm and boundary/asymptotic conditions require separate treatment. Its index is

$$\boxed{\text{Ind } \mathcal{D}_{\text{int},r}^+ = \dim \ker \mathcal{D}_{\text{int},r}^+ - \dim \ker \mathcal{D}_{\text{int},r}^-} \quad (101)$$

The relevant family target is schematically

$$\boxed{\text{Ind } \mathcal{D}_{\text{int},r}^+ = 3s_r \quad \text{for each required family sector } r,} \quad (102)$$

where s_r encodes the intended four-dimensional chirality convention after reduction. A single total index of three does not establish three Standard Model families. Indices count net modes; extra vectorlike pairs can remain. The internal dimension, topology, twisting bundles, representation multiplicities and relation between internal and four-dimensional chirality must be specified before the Atiyah–Singer theorem [10] can be applied. No such spectrum is calculated here.

14.2 Higgs-like internal deformation and masses

In this subsection use natural units $c = \hbar = 1$. Let an effective internal geometric mode H_G transform as the required electroweak doublet. At the effective level one may use the standard symmetry-breaking potential

$$V(H_G) = -\mu_H^2 H_G^\dagger H_G + \lambda_H (H_G^\dagger H_G)^2. \quad (103)$$

Take $\mu_H^2 > 0$ and $\lambda_H > 0$ for a stable symmetry-breaking potential, with $v_G^2 = \mu_H^2 / \lambda_H$ in the convention $\langle H_G^\dagger H_G \rangle = v_G^2 / 2$. This is standard effective field theory, not a derived microscopic MFT potential. The MFT interpretation is that H_G is a normal deformation mode of the same internal geometry. If internal left- and right-handed modes are $\chi_{L,i}$ and $\chi_{R,j}$, a candidate geometric overlap is

$$\boxed{Y_{ij}^{(G)} = g_G \int_{\mathcal{K}_G} d\mu_G \bar{\chi}_{L,i} \mathbf{h}_0 \chi_{R,j},} \quad (104)$$

where h_0 is the normalized internal profile of the Higgs mode, not the four-dimensional Higgs field or its vacuum expectation value. Mode normalizations and the higher-dimensional coupling must give dimensionless $Y_{ij}^{(G)}$; g_G here denotes a schematic overlap coefficient, not a derived equality to a gauge coupling. Gauge indices must be contracted to a singlet, with the conjugate doublet used where required. Factoring the four-dimensional field as $H_G(x)h_0(y)$, the standard effective relation is

$$(M_F)_{ij} = \frac{v_G}{\sqrt{2}} Y_{ij}^{(G)}. \quad (105)$$

The attraction of Eq. (104) is that mass ratios can in principle arise from geometric mode overlaps. No numerical fermion masses or Yukawa matrices are calculated here.

15 Constants and Couplings as Geometric Data

MFT organizes dimensionful and dimensionless constants differently. Dimensionful constants should ultimately be tied to invariant scales or fluxes; dimensionless couplings are more naturally associated with ratios, trace normalizations, spectral data, or overlap integrals. This is an organizing principle rather than a completed calculation.

The universal action scale is interpreted through Eq. (46). Newton's constant is related in the weak volume-mode toy model to the inverse stiffness K_σ through Eq. (80). A unified internal connection with common high-scale normalization would give equal canonically normalized non-Abelian and GUT-normalized Abelian couplings at the symmetry scale. For the generator (97),

$$\text{Tr } Y^2 = \frac{5}{6}. \quad (106)$$

Normalizing $T_Y = \sqrt{3/5} Y$ gives $\text{Tr } T_Y^2 = 1/2$, and the conventional relation

$$g_1 = \sqrt{\frac{5}{3}} g_Y \quad (107)$$

follows. Under a common unified normalization, the familiar high-scale relation $\sin^2 \theta_W = 3/8$ results. This is standard SU(5)-type normalization and is not a novel prediction of MFT. MFT would need to derive the common internal kinetic normalization and the symmetry scale before this becomes more than a consistency check.

If an internal spectral operator has eigenvalues with dimensions of inverse length squared,

$$\mathcal{H}_{\text{int}} \chi_n = \lambda_n \chi_n, \quad (108)$$

then, for $\lambda_n \geq 0$ and an appropriate kinetic normalization, a separated relativistic mode can have an effective mass scale

$$m_n \sim \frac{\hbar}{c} \sqrt{\lambda_n}, \quad (109)$$

under the corresponding Klein–Gordon-type reduction. Equation (109) is a standard separation-of-variables mechanism used here as a toy target, not a calculated MFT mass spectrum.

16 Cosmological Regimes

The cosmological discussion uses natural units $c = \hbar = 1$ and is intentionally limited. For a homogeneous FLRW effective metric,

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j, \quad (110)$$

an isotropic spatial deformation has $J_s = a^3$, so $\sigma_G = \ln J_s = 3 \ln a$. If a particular geometric measure component is conserved, it dilutes as a^{-3} ; MFT does not assert that every probability-like quantity obeys that scaling.

A coarse scalar coordinate ϕ_G associated with a measure/deformation mode may be represented effectively by

$$S_{\text{cos}} = \int \sqrt{-g} d^4x \left[\frac{M_{\text{eff}}^2}{2} R - \frac{1}{2} (\nabla \phi_G)^2 - V_G(\phi_G) \right]. \quad (111)$$

For a homogeneous field,

$$\rho_\phi = \frac{1}{2} \dot{\phi}_G^2 + V_G, \quad p_\phi = \frac{1}{2} \dot{\phi}_G^2 - V_G. \quad (112)$$

Potential domination gives $w \simeq -1$, while rapid oscillation about a zero-energy minimum in an approximately quadratic potential, with frequency much larger than the Hubble rate and negligible decay, gives an average $w \simeq 0$. These are standard scalar-field results. MFT merely notes that background and excitation modes of one deeper geometry could occupy both regimes. The present model does not explain the observed dark-energy density, dark-matter abundance, inflationary perturbation spectrum, or cosmological-constant hierarchy.

A speculative early-universe interpretation is that a high-symmetry deep geometry undergoes a transition into a stable $3 + 1$ -dimensional effective phase. The earlier inversion-twisted dimensional toy model can illustrate measure concentration toward $d = 3$, but it is not retained as a fundamental derivation of dimensionality.

17 Falsifiability and Quantitative Research Targets

A theory that can accommodate every result after the fact has little explanatory value. MFT therefore distinguishes between universal constraints and realization-specific predictions. An explicit conditional kinematic submodel is the weak-conformal relation (33). Its algebra follows from definitions and the conformal ansatz. An empirical test would have to independently identify the physical amplitude, spatial metric, reference measure and regime of coarse-graining; otherwise this is an identity check rather than a new observable prediction. A discrepancy would reject at least one of those physical identifications or assumptions, not the established conformal formula.

A second class of targets arises from the infrared expansion (81). Higher-curvature corrections should be suppressed at low curvature. Their coefficients are not calculated, so this is currently a parameterized research program rather than a numerical prediction. Likewise the common causal metric required by the Lorentzian sector implies that leading species-dependent propagation speeds must vanish or be extremely suppressed. The weak equivalence principle similarly becomes a nontrivial consistency requirement if all matter excitations are projections of the same coarse metric geometry.

The internal spectral model is falsifiable at the mathematical level before it is experimentally useful. A rigorous MFT completion must produce a consistent five-dimensional internal sector, a consistent phase-insensitive rank-two projector sector, an effective gauge connection, anomaly-compatible chiral

matter, and an index structure capable of producing observed generation content. Failure to construct these objects without adding arbitrary compensating sectors would count against the architecture.

18 Limitations and Open Problems

MFT is architecturally broad but mathematically incomplete. The most important unresolved problems are the following.

1. **Concrete deep geometry.** The manifold/bundle structure of $\mathcal{X}$, the precise nature of the generalized frame E , and the compatibility conditions relating E , ∇_G , and μ_G remain to be specified beyond the toy level.
2. **Measure construction.** The continuum HPSS expression is formal. A rigorous regularized or constructive measure is required before configuration-space probabilities can be treated as mathematically complete.
3. **Coarse-graining map.** The map $\mathcal{C}_\ell$ is postulated. MFT does not yet derive the emergence of a unique Lorentzian metric, exact local Lorentz invariance, or diffeomorphism invariance from the microscopic functional.
4. **Quantum dynamics.** The amplitude and phase sectors have geometric interpretations, but a full derivation of the Schrödinger equation or relativistic quantum field theory from the reduced symplectic structure is not supplied. Geometric prequantization alone is insufficient.
5. **Numerical $\hbar$.** Topological integrality can motivate a candidate action unit under extra period assumptions, but the measured value of $\hbar$ is not calculated.
6. **Spatial dimensionality.** The model contains an illustrative dimensional selection mechanism but no first-principles derivation of three large spatial dimensions.
7. **Gauge dynamics.** The $3 + 2$ stabilizer mechanism yields an internal symmetry group, not automatically a dynamical gauge theory. The promotion of local frame redundancy to Yang–Mills dynamics must be derived.
8. **Chirality and generations.** The index-theoretic target $\text{Ind } \mathcal{D}_{\text{int},r}^+ = 3s_r$ is not calculated. This is one of the most important mathematical milestones for a future version.
9. **Anomalies and consistency.** Gauge and gravitational anomaly cancellation, unitarity, positivity, renormalization behavior, and global bundle consistency have not been established for a complete MFT model.
10. **Phenomenology.** No new particle mass, cross section, cosmological parameter, or experimentally measured deviation from GR/QM is currently derived from first principles.

These limitations are not peripheral caveats; they define the research program required to convert a toy architecture into a physical theory.

19 Discussion

MGF’s organizing hypothesis is that probability and spacetime geometry have a common origin. MFT explores this through a restricted coframe construction with a prescribed measure. In the conserved-

deformation realization, the determinant of the same deformation that builds the metric also controls the local measure density. This provides a conditional relation between metric volume and the density of a prescribed measure. Equation (17) shows why that relation does not yet establish a common physical origin for quantum outcome probabilities and gravitational dynamics. The volume/shape/orientation decomposition then suggests a useful allocation of effective roles: volume deformation controls local measure, shape deformation controls tensor geometry, and connection holonomy controls phase and internal transport.

The framework also benefits from separating deep compatibility from effective dynamics. Many speculative unification proposals lose clarity by writing a single action containing recognizable Einstein, Yang–Mills, Higgs, and matter terms and then describing the combined expression as a derivation. MFT instead takes the harder position that the deep functional should contain only geometric incompatibility data, while effective field actions must emerge after stable-mode projection and coarse-graining. This exposes the additional constructions required before the architecture becomes a dynamical theory.

The internal rank-two toy model is an example of this discipline. The $3 + 2$ block is conditional on the chosen internal dimension and spectral ansatz; a simple two-form selects a two-plane in a minimally motivated five-dimensional fiber, and the complement then has dimension three. The projector commutant reproduces the familiar global group form only after the fixed-complex-volume and phase-discarding assumptions; the fixed two-form stabilizer is smaller. The fact that the associated representation algebra matches standard $SU(5)$ -type packaging is encouraging but should not be confused with a derivation of the Standard Model. The model must still explain why the internal fiber is realized physically, why its local stabilizer becomes gauge redundancy, why the correct chiral zero modes occur, and why the observed coupling and mass data follow.

Similarly, the quantum sector is best regarded as a reconstruction program. The measure identity supplies a positive amplitude candidate; a separately postulated connection supplies a phase sector; prequantization constrains periods without fixing a universal action unit; and Gleason’s theorem constrains projector probabilities once the Hilbert and noncontextual measure structures are assumed. The observation map, quantum dynamics, and representation linking these constructions remain open. None of these steps independently proves quantum mechanics from geometry, but together they turn the vague claim that “quantum theory is geometric” into a sequence of auditable mathematical targets.

The gravity sector is intentionally conservative. This version supplies no derivation of a Lorentzian metric or Einstein dynamics from the deep functional. A physical completion must show that a common geometry yields that metric and has the required low-energy limit. Effective-field-theory reasoning then explains why the Einstein–Hilbert term supplies the leading metric kinetic term under the stated infrared assumptions; the cosmological term also remains at leading order, while the weak-field volume-mode calculation provides a tangible relation between geometric compliance and Newton’s constant within a restricted toy model.

20 Conclusion

Metrarchic Field Theory is presented as a geometric-probability toy theory within the broader Metrarchic Geometry Framework, rather than as a completed theory of everything. Its central proposition is that metric geometry and probability measure are scale-dependent manifestations of a common deformation structure. A prescribed conserved-measure realization gives $\rho_G = \rho_0/J$, while the same deformation produces the metric through $\gamma = F^T \gamma_0 F$. Probabilities of fixed reference-base regions nevertheless

remain unchanged. These conditional identities motivate the proposed synthesis, without demonstrating its dynamics or a quantum observation map: fine-scale measure gradients can carry quantum-like amplitude structure, orientation and connection sectors can carry phase, and coarse shape/curvature sectors can approach relativistic metric dynamics.

The framework is organized around a formal nonnegative deep compatibility functional, a formal configuration-space probability measure, a reduced symplectic quantum sector, and a scale-dependent coarse-graining map. The resulting architecture is compatible with known quantum-potential and Fisher-information structures, geometric prequantization, Bell-nonfactorizable correlations, effective-field-theory general relativity, and a finite-dimensional internal spectral model whose projector sector and postulated splitting operator give a conditional $3 + 2$ block symmetry. The manuscript explicitly distinguishes these compatibility results from stronger derivations that have not yet been achieved.

The most valuable future work is therefore mathematical rather than ontological: construct a concrete deep bundle geometry; regularize the HPSS measure; derive the reduced symplectic/Hilbert structure; prove emergence of a universal Lorentzian metric; build an internal Dirac operator with the required index; establish anomaly consistency; and compute at least one distinctive observable from parameters fixed by the geometry. Success on those tasks could move MFT from a partially specified research architecture toward a testable physical theory while simultaneously sharpening the broader MGF research program. Failure would identify precisely where the unification idea breaks. The present calculations do not establish that the architecture yields new dynamics beyond the related approaches reviewed above. In either case, the framework is intended to be judged by the same standard as any useful toy theory: clarity of assumptions, correctness of derivations, recovery of known limits, and the ability of its mathematical structure to be falsified or improved.

A Notation and Status Ledger

Symbol	Meaning	Status
$\mathfrak{G}$	Deep geometric state	MFT postulate
E, F	Coframe and invertible deformation matrix	Standard geometry used in MFT
J	Local deformation Jacobian	Standard geometry
ρ_G	Density $d\mu_G/dV$	Definition; probability interpretation is MFT
$\rho_G = \rho_0/J$	Conserved-measure deformation relation	Derived in restricted sector
A_G	Geometric amplitude $\sqrt{\rho_G/\rho_*}$	MFT definition
∇_G	Generalized deep connection	MFT postulate
$\mathcal{A}_G$	Effective phase connection	MFT quantum construction
ω_G	Reduced-state symplectic form	MFT assumption
η_G	Elementary geometric action scale	Conditional flux interpretation; matched to $\hbar$
$\mathcal{J}_G$	Positive deep incompatibility functional	Toy ansatz
ϵ_G	HPSS concentration parameter	Toy parameter
$\mathcal{H}_G$	Hessian/stability operator	Derived from $\mathcal{J}_G$ once specified
β_G	Internal two-form order parameter	Toy ansatz

Symbol	Meaning	Status
$\mathcal{H}_{\text{int}}$	Finite internal splitting operator	Postulated operator; not derived Hessian
$\mathcal{D}_{\text{int}}$	Internal Dirac operator	Open construction
$\mathcal{C}_\ell$	Scale-dependent coarse-graining map	MFT postulate/open construction

B Weak-Conformal Derivation Check

For completeness, let $\gamma = e^{2\sigma}\tilde{\gamma}$ in n dimensions. The standard conformal scalar-curvature transformation is

$$R[\gamma] = e^{-2\sigma} \left[\tilde{R} - 2(n-1)\tilde{\Delta}\sigma - (n-1)(n-2)|\tilde{\nabla}\sigma|^2 \right]. \quad (113)$$

Since $J = e^{n\sigma}$, $\sigma = (1/n)\ln J$, giving Eq. (28). For $A_G = J^{-1/2} = e^{-n\sigma/2}$, direct differentiation gives

$$\frac{\tilde{\Delta}A_G}{A_G} = -\frac{1}{2}\tilde{\Delta}\ln J + \frac{1}{4}|\tilde{\nabla}\ln J|^2. \quad (114)$$

In the weak, locally shape-flat limit, $J^{-2/n} \simeq 1$, $\tilde{R} \simeq 0$, and quadratic gradient terms are subleading. Therefore

$$R \simeq -\frac{2(n-1)}{n}\tilde{\Delta}\ln J, \quad Q_G \simeq \frac{\eta_G^2}{4m}\tilde{\Delta}\ln J, \quad (115)$$

which yields Eq. (32). Appendix C gives an exact compact example and the nonlinear remainder. No stronger domain of validity is claimed.

C Explicit Compact Coframe Example and Constrained Variation

This example makes the kinematic curvature-measure sector concrete without claiming to solve the deep dynamics. Work on a flat reference torus $\mathbb{T}^n$, $n > 1$, of side L , reference volume $V_0 = L^n$, with periodic coordinates. Let

$$u(x) = \varepsilon \cos(kx^1), \quad k = 2\pi/L, \quad e^a = e^{u/n}dx^a, \quad \gamma_{ij} = e^{2u/n}\delta_{ij}. \quad (116)$$

Then $J = e^u > 0$, $\rho_G = \rho_0 e^{-u}$, and for constant $\rho_0 > 0$, $\mathcal{N} = \rho_0 V_0$. The normalized amplitude is

$$a = V_0^{-1/2} e^{-u/2}, \quad \int a^2 dV_\gamma = 1. \quad (117)$$

For $n > 1$, the transverse coframes generally obey $de^a \neq 0$, so this is not a compatible flat pullback. Let Δ_0, ∇_0 refer to the flat reference metric. Exact evaluation gives

$$R_\gamma = e^{-2u/n} \left[-\frac{2(n-1)}{n}\Delta_0 u - \frac{(n-1)(n-2)}{n^2}|\nabla_0 u|^2 \right], \quad (118)$$

$$Q_G = \frac{\eta_G^2}{m} e^{-2u/n} \left[\frac{1}{4}\Delta_0 u - \frac{4-n}{8n}|\nabla_0 u|^2 \right]. \quad (119)$$

Consequently, within this shape-flat coframe sector,

$$\boxed{Q_G + \frac{\eta_G^2 n}{8m(n-1)} R_\gamma = -\frac{\eta_G^2}{4mn} e^{-2u/n} |\nabla_0 u|^2.} \quad (120)$$

This gives the explicit quadratic remainder in the weak relation, rather than extending it to all amplitudes. In three dimensions its leading terms are

$$R_\gamma = \frac{4}{3} \varepsilon k^2 \cos(kx^1) + O(\varepsilon^2), \quad Q_G = -\frac{\eta_G^2}{4m} \varepsilon k^2 \cos(kx^1) + O(\varepsilon^2). \quad (121)$$

Here Q_G is a defined compatibility expression; no Schrödinger evolution has been inferred.

The same example supplies the constrained variation of the normalized gradient energy within the fixed-shape conformal sector. Because both the amplitude and the volume depend on u ,

$$E_Q[u] = C \int_{\mathbb{T}^n} e^{-2u/n} |\nabla_0 u|^2 dV_0, \quad C = \frac{\eta_G^2}{8mV_0}. \quad (122)$$

For periodic variations with fixed reference measure, integration by parts yields

$$\boxed{\frac{\delta E_Q}{\delta u} = C e^{-2u/n} \left[-2\Delta_0 u + \frac{2}{n} |\nabla_0 u|^2 \right].} \quad (123)$$

The shape metric, connection and internal fields are not varied in this calculation. The functional derivative is with respect to dV_0 ; its units are energy per reference volume. It differs from varying only p at fixed metric. The stationary condition for this isolated energy implies $\Delta_0(e^{-u/n}) = 0$; periodicity forces u constant. Thus this positive gradient energy alone has no nonconstant stationary solution on the torus. The sinusoidal configuration is an admissible kinematic example, not a stable vacuum. Nontrivial stationary structure needs additional terms or constraints.

A separate finite-mode probability check is immediate: restrict $\varepsilon \in [-A, A]$ with $A > 0$, choose normalized reference measure $d\nu_0 = d\varepsilon/(2A)$, and take a dimensionless cost $\mathcal{J}(\varepsilon) = \alpha \varepsilon^2$, $\alpha > 0$. Then for $\epsilon_G > 0$, $e^{-\alpha A^2/\epsilon_G} \leq Z \leq 1$, so the Gibbs measure exists. This regularized illustrative cost is not an evaluation of the full deep functional, nor is this ensemble measure the amplitude density on the torus.

D Rank-Two Internal Splitting Check

A complex skew two-form in a five-dimensional Hermitian vector space can be represented in a canonical basis by two possible nonzero skew blocks,

$$\beta = \lambda_1 e^1 \wedge e^2 + \lambda_2 e^3 \wedge e^4. \quad (124)$$

Then

$$\beta \wedge \beta = 2\lambda_1 \lambda_2 e^1 \wedge e^2 \wedge e^3 \wedge e^4. \quad (125)$$

The constraints $\beta \neq 0$ and $\beta \wedge \beta = 0$ force one block to vanish and one to remain, producing rank two. The associated endomorphism $\mathcal{J}_{\text{int}}$ acts nontrivially only on the selected two-plane, so $\mathcal{J}_{\text{int}}^\dagger \mathcal{J}_{\text{int}} = P_2$ has eigenvalues 1, 1, 0, 0, 0. Adding a multiple of the identity produces Eq. (88). The result is exact linear algebra within the finite-dimensional toy construction.

E Newtonian Point-Mass Check

The weak volume-mode model permits a simple point-mass consistency check. For a point source of mass M , standard Newtonian gravity gives

$$\Phi_N(r) = -\frac{GM}{r}. \quad (126)$$

Using the infrared matching relation $\Phi_N = -c^2\sigma_G/3$,

$$\boxed{\sigma_G(r) = \frac{3GM}{c^2 r}}. \quad (127)$$

To first order,

$$J_s = e^{\sigma_G} \simeq 1 + \frac{3GM}{c^2 r}, \quad \frac{\rho_G}{\rho_0} = J_s^{-1} \simeq 1 - \frac{3GM}{c^2 r}. \quad (128)$$

The measure density here is the coarse MFT geometric-measure variable and must not be confused with ordinary material mass density. The acceleration of a slow test body in the lapse ansatz $N = e^{-\sigma_G/3}$ is

$$\ddot{\mathbf{x}} = \frac{c^2}{3} \nabla \sigma_G = -\nabla \Phi_N, \quad (129)$$

so the toy model reproduces the standard point-mass acceleration after the matching conditions are imposed. This is a consistency check on sign and normalization, not an independent prediction.

F Bell/CHSH Benchmark Calculation

For the singlet density operator

$$\rho_{AB} = |\Psi^-\rangle\langle\Psi^-| = \frac{1}{4} \left[I \otimes I - \sum_{i=1}^3 \sigma_i \otimes \sigma_i \right], \quad (130)$$

measure spin along unit vectors $\mathbf{a}$ and $\mathbf{b}$ with observables

$$A(\mathbf{a}) = \mathbf{a} \cdot \boldsymbol{\sigma}, \quad B(\mathbf{b}) = \mathbf{b} \cdot \boldsymbol{\sigma}. \quad (131)$$

Using $\text{Tr} \sigma_i = 0$ and $\text{Tr}(\sigma_i \sigma_j) = 2\delta_{ij}$,

$$E(\mathbf{a}, \mathbf{b}) = \text{Tr}[\rho_{AB} A(\mathbf{a}) \otimes B(\mathbf{b})] = -\mathbf{a} \cdot \mathbf{b}. \quad (132)$$

Choose

$$\mathbf{a} = \hat{z}, \quad \mathbf{a}' = \hat{x}, \quad \mathbf{b} = \frac{\hat{z} + \hat{x}}{\sqrt{2}}, \quad \mathbf{b}' = \frac{\hat{z} - \hat{x}}{\sqrt{2}}. \quad (133)$$

Then

$$E(\mathbf{a}, \mathbf{b}) = -\frac{1}{\sqrt{2}}, \quad E(\mathbf{a}, \mathbf{b}') = -\frac{1}{\sqrt{2}}, \quad (134)$$

$$E(\mathbf{a}', \mathbf{b}) = -\frac{1}{\sqrt{2}}, \quad E(\mathbf{a}', \mathbf{b}') = +\frac{1}{\sqrt{2}}. \quad (135)$$

The CHSH combination

$$S = E(\mathbf{a}, \mathbf{b}) + E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) - E(\mathbf{a}', \mathbf{b}') \quad (136)$$

therefore satisfies

$$\boxed{|S| = 2\sqrt{2}.} \quad (137)$$

Summing the joint probability

$$P(s, t | \mathbf{a}, \mathbf{b}) = \frac{1}{4} [1 - st \mathbf{a} \cdot \mathbf{b}] \quad (138)$$

over the remote outcome gives $P(s | \mathbf{a}) = 1/2$, independent of $\mathbf{b}$, so the standard no-signaling marginal is preserved. MFT has not derived the singlet state from its microscopic functional; the calculation establishes only that the assumed effective quantum formalism gives the standard benchmark. No specific microscopic MFT response model has been constructed by this calculation.

G Internal U(1) Normalization Check

With the block generator

$$Y = \text{diag} \left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2} \right), \quad (139)$$

one obtains

$$\text{Tr } Y^2 = 3 \left(\frac{1}{9} \right) + 2 \left(\frac{1}{4} \right) = \frac{5}{6}. \quad (140)$$

If non-Abelian generators are normalized by $\text{Tr}(T_a T_b) = \frac{1}{2} \delta_{ab}$, define

$$T_Y = \sqrt{\frac{3}{5}} Y, \quad (141)$$

so that $\text{Tr } T_Y^2 = 1/2$. If a single high-scale geometric connection couples with strength g_G to this normalized generator, while conventional hypercharge is written $g_Y Y$, then

$$g_Y = \sqrt{\frac{3}{5}} g_G, \quad g_1 \equiv \sqrt{\frac{5}{3}} g_Y = g_G. \quad (142)$$

Assuming also $g_2 = g_G$, the symmetry-scale weak mixing angle is

$$\sin^2 \theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{3}{8}. \quad (143)$$

This is the familiar SU(5)-type high-scale normalization, not a unique MFT prediction. Its role in the manuscript is to show that the rank-two $3+2$ geometry is algebraically compatible with the conventional normalization once a common internal kinetic scale is assumed.

AI-Assisted Preparation Disclosure

This manuscript was developed with AI-assisted drafting, mathematical cross-checking, organization, and selected literature checks. The Metrarchic Geometry Framework and the Metrarchic Field Theory conceptual architecture are attributed to the author, and responsibility for the scientific claims,

interpretations, and any remaining errors rests with the author. AI assistance is not presented as an independent source of scientific authority.

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