

MINIMAL PHOTON MASS AND LIGHT–MATTER INTERACTION: RELATIVISTIC FORMULATION, CONSTITUTIVE CLOSURE, AND EXPERIMENTAL CRITERIA FOR AN EFFECTIVE ELECTRODYNAMICS

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Abstract

We investigate a class of effective electrodynamic extensions in which a minimal photon rest mass is treated together with a constitutive sector independent of the mass parameter. Here, “minimal” denotes a parametrically small positive scale compatible with experimental upper bounds, not an experimentally established lower bound on m_γ . Starting from the relativistic dispersion relation of a massive vector field, we derive the corresponding group-velocity correction and formulate a constitutive closure for light–matter interaction that distinguishes intrinsic photon mass, medium-induced effective mass, and equivalent interaction energy. We impose energy–momentum conservation, thermodynamic admissibility, stability, recovery of limiting theories, and experimental identifiability. We further construct a conventional electromechanical null model including Maxwell stress, elasticity, electrostriction, photoelasticity, dissipation, and thermal response. The resulting framework shows that a local scalar coupling linear in electromagnetic energy density is not independently identifiable in the isotropic linear regime unless it carries an additional dynamic, tensorial, geometric, or nonlocal signature. We formulate differential observables and a hierarchy of null hypotheses for separating Proca-like effects from conventional medium response, and discuss constraints from quasi-static magnetic fields, dispersive propagation, fast radio bursts, and magnetohydrodynamic environments. The theory is presented as a falsifiable effective framework: compatibility with current bounds is not interpreted as detection, and the photon-mass sector is not used to infer macroscopic return, gravitational capture, or stellar feedback.

Keywords: Photon Mass; Proca Electrodynamics; Light–Matter Interaction; Constitutive Response; Volumetric Compression; Energy–Momentum; Identifiability; Differential Observable; Falsifiability.

1. Introduction

In this work, we formulate the hypothesis of a minimal nonzero photon rest mass as an effective extension of electrodynamics subject to quantitative observational consequences. In Maxwell electrodynamics, the photon in vacuum is described as a massless excitation. For the massive-vector sector, we adopt Proca electrodynamics as the reference framework, in which the mass term modifies both the field equations and the dispersion relation. We use this structure as a comparative basis for determining the experimental bounds associated with a possible photon mass [1–3].

We consider the available experimental bounds and their dependence on the inference method and auxiliary assumptions. We use the Particle Data Group compilation, which gathers constraints from time dispersion, planetary and astrophysical magnetic fields, the solar wind, and tests of electromagnetic laws [1]. We also incorporate constraints obtained from the Jovian magnetic field for a nonzero photon mass and light-vector sectors [2]. On this basis, we restrict the formulation to the experimentally admissible parameter space and define observables intended to discriminate the proposed dynamics from conventional electrodynamics.

We do not assume that a minimal photon mass has been experimentally established. We adopt $m_\gamma > 0$ as a working hypothesis and investigate its consistency conditions. The first requirement is relativistic. For a free particle or excitation of rest mass m_γ , energy E , and momentum p , the appropriate kinematic relation is

$$E^2 = p^2c^2 + m_\gamma^2c^4. \quad (1)$$

The group velocity associated with relation (1) is

$$v_g = \partial E / \partial p = pc^2/E < c, \text{ for } m_\gamma > 0. \quad (2)$$

For $m_\gamma > 0$, we obtain a kinematic correction governed by the ratio between the mass scale and the quantum energy. We relate this correction directly to propagation observables and require simultaneous compatibility with relativistic kinematics and with experimental bounds on dispersion and electromagnetic-field modifications [1,3].

For interaction with material media, we take as a preliminary starting point the energetic decomposition $E = (m_0 + p_{\text{medium}}\Delta V)c^2$, in which m_0 represents a minimal corpuscular contribution and $p_{\text{medium}}\Delta V$ a contribution associated with the local response of the medium. We explicitly distinguish the second term from rest mass and reformulate the expression as a constitutive hypothesis subject to derivation and testing. We thereby preserve the distinction among

interaction energy, in-medium effective mass, and rest mass, while maintaining dimensional, relativistic, and thermodynamic consistency.

We construct a minimal formulation to examine the hypothesis $m_\gamma > 0$ separately from phenomena already described by polarization, susceptibility, dispersion, absorption, scattering, and ordinary momentum transfer. We organize the analysis into six stages: (i) definition of quantities and notation; (ii) establishment of relativistic kinematics for the photon sector; (iii) formulation of the constitutive response of the medium; (iv) imposition of local and global energy–momentum conservation; (v) determination of limits and asymptotic regimes; and (vi) construction of predictions that discriminate the model from the Maxwellian description.

We organize the presentation as a deductive sequence. We first define the concepts, notation, and postulates; next, we construct the massive-vector sector, the constitutive closure, and the energy–momentum balances; we then derive quantitative consequences, examine orders of magnitude, and formulate criteria of identifiability and falsification. The central objective is to determine whether an additional contribution to light–matter interaction can produce an independent observable without being reabsorbed into conventional medium responses. As a partial result, we find that the isolated kinematic contribution of a photon mass within the bounds considered is extremely small for optical photons, whereas the constitutive sector leads to an explicit differential test between controlled mechanical states. We thus connect each physical interpretation to the corresponding equations, experimental bounds, and validity conditions.

Our objective is not to demonstrate a priori that m_γ differs from zero, but to construct an effective class that is sufficiently constrained to distinguish, through observables, three physically different situations: a contribution belonging to the free operator, a response induced by the medium, and an additional parametrization that merely renames already known constitutive effects. The contribution of this work therefore lies both in the consequences that remain admissible and in the regions of hypothesis space that the analysis itself renders non-discriminating or subject to exclusion.

2. Definitions, Concepts, Notation, and Postulates

To avoid ambiguities among concepts used in different physical contexts, we adopt the operational definitions below.

2.1 Minimal Photon Rest Mass

We denote by m_γ the hypothetical rest mass associated with the propagating vector sector. The central hypothesis at the first level of the model is $m_\gamma > 0$, subject to current experimental bounds. The qualifier “minimal” indicates only that we investigate a parametrically small positive-mass regime; it does not represent a deduced or measured lower bound. By definition, m_γ does not denote an effective plasma mass, polarization energy, an equivalent interaction mass, or energy stored in the medium.

2.2 Effective Mass

We denote by m_{eff} an emergent quantity that may appear in the dispersion relation of an excitation when propagation occurs in a medium or in the presence of interactions. The equality $m_{\text{eff}} = m_\gamma$ may be used only in a regime in which it is derived from the effective propagator; it is not adopted as a definition.

$$m_{\text{eff}}^2 = m_\gamma^2 + \Delta m^2_{\text{int}}(\omega, \mathbf{k}; \Xi), \quad (3)$$

where Δm^2_{int} represents an effective correction that depends on the angular frequency ω , the wave vector $\mathbf{k}$, and the set Ξ of constitutive parameters of the medium.

2.3 Equivalent Interaction Energy

We call interaction energy E_{int} the energetic contribution associated with coupling between the electromagnetic field and the medium. When it is convenient to express it in units of mass, we define only the equivalence

$$m_{\text{int,eq}} \equiv E_{\text{int}}/c^2. \quad (4)$$

In definition (4), we treat $m_{\text{int,eq}}$ exclusively as an energy equivalence expressed in units of mass and distinguish it from the photon rest mass.

2.4 Constitutive Response and Effective Compression

We use the term constitutive response to denote the local modification of the state of the medium induced by the field and by exchanges of energy and momentum. We reserve the term effective compression for a measurable or derivable variable of the constitutive model, and we identify it with mechanical pressure only through the corresponding stress tensor or equation of state.

2.5 Momentum Transfer

Momentum transfer denotes the change in momentum of the material or electromagnetic subsystem resulting from the interaction. The complete description must satisfy a conservation law for the total energy–momentum tensor. In covariant form, the structural condition is

$$\nabla_\mu T^{\text{total}}{}^{\mu\nu} = 0, \quad T^{\text{total}}{}^{\mu\nu} = T^\gamma{}^{\mu\nu} + T^{\text{medium}}{}^{\mu\nu} + T^{\text{int}}{}^{\mu\nu}. \quad (5)$$

2.6 Propagation, Phase Velocity, and Group Velocity

We distinguish the phase velocity $v_{\text{ph}} = \omega/k$ from the group velocity $v_{\text{g}} = \partial\omega/\partial k$. For a free massive vector sector, the dispersion relation modifies v_{g} according to Eq. (2). In dispersive media, both velocities additionally depend on the constitutive response; therefore, observed changes in propagation speed cannot be attributed directly to m_γ without subtracting the medium response.

2.7 Domain of Validity

We call the domain of validity the set of energy scales, lengths, densities, and parameters for which the effective formulation remains causal, stable, dimensionally consistent, and compatible with the experimental constraints used in its construction.

2.8 Notation Conventions

We use c for the relativistic limiting speed in vacuum; $\hbar$ for the reduced Planck constant; E for energy; p for the magnitude of linear momentum; ω for angular frequency; k for the magnitude of the wave vector; ρ for mass density; V for volume; ΔV for volume variation; and Ξ for the set of constitutive variables of the medium.

2.9 Minimal Postulates of the Model

P1 — Minimal photon hypothesis. We assume, as a testable hypothesis rather than an established result, a strictly positive photon rest mass m_γ that is sufficiently small to remain compatible with available experimental bounds [1,2]. The term “minimal” does not impose a positive lower bound: the zero value remains part of the null model and of the experimental comparison procedure.

P2 — Relativistic consistency. In the free regime, energy and momentum obey the relativistic dispersion relation (1); any constitutive correction must continuously reduce to that regime as the interaction with the medium tends to zero.

P3 — Separation between intrinsic mass and medium response. The rest mass m_γ , the effective mass m_{eff} , and the interaction energy E_{int} are distinct quantities. Equalities among them require an explicit derivation and a specified domain of validity.

P4 — Conservation. Light–matter interaction is described through exchanges of energy and momentum between subsystems, with conservation of the total energy–momentum tensor imposed according to (5). No response term may create or eliminate energy or momentum without a corresponding flux or source.

P5 — compatibility thermodynamics. in mediums dissipatives, the formulation constitutiva must distinguir energy reversivelmente stored of energy dissipada and satisfy the condition of production non-negative of entropy in the domain macroscopic appropriate.

P6 — Experimental discrimination. A consequence is considered specific to the model only if it cannot be absorbed, within uncertainties, by conventional mechanisms of dispersion, polarization, absorption, reflection, scattering, plasma response, or material response. The model must provide at least one observable that can constrain m_γ or the parameters of the constitutive sector.

3. Physical–Mathematical Structure of the Model

We construct the free sector of the model from a massive-vector extension of electrodynamics. The purpose of this stage is not to identify the proposed hypothesis with every possible realization of Proca theory, but to establish a minimal consistency reference against which any additional constitutive term must be compared. We work initially in flat spacetime with the Minkowski metric $\eta_{\mu\nu}$ and subsequently indicate the covariant extension. The vector potential is A_μ and the field tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

3.1 Free Action and Mass Term

In the International System of Units, a convenient form of the Lagrangian density for the massive electromagnetic sector, coupled to an external current J_μ , is

$$\mathcal{L} = -(1/4\mu_0)F_{\mu\nu}F^{\mu\nu} + (1/2\mu_0)\mu_\gamma^2 A_\mu A^\mu - J_\mu A^\mu, \quad (6)$$

where μ_0 is the permeability of the vacuum and μ_γ is the inverse of the reduced Compton wavelength associated with the photon rest mass,

$$\mu_\gamma \equiv m_\gamma c/\hbar. \quad (7)$$

Equation (7) is particularly useful because it separates the mass dimension from the spatial scale introduced by the massive sector. In the limit $m\gamma \rightarrow 0$, we have $\mu\gamma \rightarrow 0$ and the Lagrangian density (6) continuously reduces to the Maxwell form coupled to the external current. Requiring this limit is a recovery condition for the established theory and is maintained throughout all constitutive extensions.

3.2 Field Equations

Applying the Euler–Lagrange equations to the potential A_ν , we obtain

$$\partial_\mu F^{\mu\nu} + \mu\gamma^2 A^\nu = \mu_0 J^\nu. \quad (8)$$

the geometric identity associated with the definition of $F_{\mu\nu}$ remains

$$\partial[\alpha F\beta\gamma] = 0. \quad (9)$$

Taking the divergence of Eq. (8) and using the antisymmetry of $F_{\mu\nu}$, we obtain

$$\mu\gamma^2 \partial_\nu A^\nu = \mu_0 \partial_\nu J^\nu. \quad (10)$$

For a conserved current, $\partial_\nu J^\nu = 0$, and $m\gamma \neq 0$, the condition

$$\partial_\nu A^\nu = 0 \quad (11)$$

follows from the equations of motion. In this regime, it is not introduced merely as a gauge choice: the explicit mass term breaks the local $U(1)$ gauge invariance of the standard Proca formulation and turns the Lorenz condition into a dynamical consequence for a conserved current. The description remains consistent as an effective theory of a massive vector field with three physical polarizations. An equivalent gauge-invariant formulation may be obtained by introducing a Stueckelberg field [11].

3.3 Dispersion Relation and Degrees of Freedom

In vacuum and in the absence of sources, we consider plane-wave solutions $A_\mu = \epsilon_\mu \exp(-ik_\alpha x^\alpha)$. Equations (8) and (11) provide the four-dimensional transversality condition $k_\mu \epsilon^\mu = 0$ and the relation

$$\omega^2 = c^2 k^2 + m\gamma^2 c^4 / \hbar^2. \quad (12)$$

Equation (12) is equivalent to the energy–momentum relation presented in (1), through $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$. Consequently,

$$v_{ph} = \omega/k = c\sqrt{1 + (m\gamma c/\hbar k)^2}, \quad (13)$$

$$v_g = \partial\omega/\partial k = c^2 k/\omega = c\sqrt{1 - m\gamma^2 c^4/E^2}. \quad (14)$$

For $m\gamma > 0$, the phase velocity may exceed c in certain regimes without being identified with the speed of information transport, whereas the group velocity remains below c in the free sector. In a massive-vector theory, we consider three physical polarizations: two transverse and one longitudinal. In the sufficiently small-mass limit, observables associated with the longitudinal mode must remain compatible with existing electromagnetic tests, yielding a constraint independent of the velocity measurement [1,3].

3.4 Low-Mass Expansion and Kinematic Signature

When $m\gamma c^2 \ll E$, Eq. (14) admits the expansion

$$v_g/c = 1 - 1/2 (m\gamma^2 c^4/E^2) + O(m\gamma^4 c^8/E^4). \quad (15)$$

For two components of energies E_1 and E_2 propagating over a distance L in the same regime, and neglecting at this stage medium dispersion and differential gravitational effects, the mass-induced difference in time of flight is

$$\Delta t_m \simeq (L m\gamma^2 c^3/2) |1/E_1^2 - 1/E_2^2|. \quad (16)$$

We use relation (16) as the necessary kinematic signature of the free sector. In observational applications, we separately model intrinsic source delays, plasma contributions, material dispersion, propagation geometry, and differential gravitational effects before inferring any value of $m\gamma$.

3.5 Constitutive Extension for Light–Matter Interaction

To describe the material response beyond the free sector, we introduce an additional constitutive sector and keep it distinct from the photon rest-mass term. We write the effective action as

$$S = \int d^4x [\mathcal{L}_\gamma + \mathcal{L}_{\text{medium}}(\Xi) + \mathcal{L}_{\text{int}}(A_\mu, F_{\mu\nu}, \Xi, \partial\Xi)], \quad (17)$$

We denote by Ξ the internal variables required to describe the medium, including, according to the physical regime, density, polarization, magnetization, strain, temperature, and other relevant degrees of freedom.

3.6 Reformulation of the Preliminary Energy Expression

We rewrite the preliminary expression $E = (m_0 + \rho_{\text{medium}}\Delta V)c^2$ so as to separate dimensional equivalence among quantities with dimensions of mass from their dynamical interpretation. We define

$$E_{\text{total}} = E_{\gamma, \text{free}} + E_{\text{int}}, \quad (18)$$

$$E_{\text{int}} \equiv \chi c \rho_{\text{medium}} \Delta V c^2, \quad (19)$$

We introduce χc as a dimensionless constitutive coefficient that may depend on the state of the medium, frequency, field intensity, and other admissible variables. We formally recover the preliminary expression when $\chi c = 1$ and the corresponding choice of $E_{\gamma, \text{free}}$ is made; we adopt this specialization only when it follows from the constitutive equations or from experimental calibration.

With Eqs. (18)–(19), we retain the explicit interpretation of $\rho_{\text{medium}} \Delta V$ as material mass contained in the interaction volume only when ρ_{medium} represents mass density and ΔV corresponds to a physically defined volume. Depending on the microscopic model, E_{int} may describe reversible work, polarization energy, strain, or another material contribution. The equivalence E_{int}/c^2 remains distinct from the intrinsic photon mass.

3.7 Effective Compression Variable

To make the notion of compression operational, we introduce the scalar volumetric strain ϵV in the regime in which a continuum description is valid,

$$\epsilon V \equiv -\Delta V/V_0, \quad (20)$$

with $\epsilon V > 0$ for compression under this convention. In the isotropic linear approximation, the volumetric elastic energy per unit volume may be represented by

$$u_{\text{int}} = 1/2 K \epsilon V^2, \quad (21)$$

where K is the effective bulk modulus of the medium. Thus, for the reference volume V_0 , $E_{\text{int}} = 1/2 K \epsilon V^2 V_0$. (22)

We use Eqs. (20)–(22) as a controlled constitutive realization for the isotropic continuum regime. In this realization, ΔV and p acquire measurable meanings and the energy bookkeeping is preserved. For media whose response is not adequately represented by an isotropic bulk modulus, we replace this closure with the corresponding tensorial constitutive law.

3.8 Energy and Momentum Conservation in the Coupled System

We adopt the decomposition into field, medium, and interaction sectors and impose conservation of the complete system independently of the representation used:

$$\partial_\mu T^{\text{total}}_{\mu\nu} = 0, \quad (23)$$

$$T^{\text{total}}_{\mu\nu} = T^{\gamma}_{\mu\nu} + T^{\text{medium}}_{\mu\nu} + T^{\text{int}}_{\mu\nu}. \quad (24)$$

In the finite region Ω , the $v = 0$ component gives the balance between the change in total internal energy and the flux through the boundary $\partial\Omega$. The spatial components give the exchange of momentum between field and matter. Within this structure, we define the effective force on the material subsystem through the rate of momentum transfer, without introducing a force independent of conservation.

$$F_{i,\text{med}} = dP_{i,\text{med}}/dt = -dP_{i,\text{field}}/dt - \Phi_{i,\partial\Omega}. \quad (25)$$

We use Eq. (25) as an operational criterion to separate radiation pressure, material response, and additional contributions produced by the constitutive sector.

3.9 Thermodynamic Constraint

When the medium is dissipative, we introduce the entropy density s and temperature T . In local continuum thermodynamics, the internal entropy production σs must satisfy

$$\sigma s \geq 0. \quad (26)$$

With condition (26), we constrain the constitutive parameters simultaneously by propagation observables, momentum transfer, and the second law of thermodynamics. We separate reversibly stored energy from irreversibly dissipated energy and require a corresponding energy source for any net cyclic work associated with the effective-compression term.

3.10 Dimensional Analysis and Admissibility Conditions

The fundamental equations introduce three distinct scales: the mass scale $m\gamma$, the length scale $\bar{\lambda}\gamma = \hbar/(m\gamma c)$, and the constitutive scales of the medium. The combination $\mu\gamma = m\gamma c/\hbar$ has dimensions of inverse length, whereas $\mu\gamma^2 A_\mu A^\mu$ has the same dimensions as the kinetic term $F_{\mu\nu}F^{\mu\nu}$ after normalization of the Lagrangian density. In the material sector, $p_{\text{medium}}\Delta V$ has dimensions of mass and $p_{\text{medium}}\Delta V c^2$ has dimensions of energy. The coefficient χc in Eq. (21) is dimensionless.

We define the minimal admissibility conditions: (A1) continuous recovery of Maxwell theory when $m\gamma \rightarrow 0$ and the additional constitutive sector is switched off; (A2) absence of kinetic-energy terms with signs incompatible with stability in the effective domain; (A3) causality of the constitutive response; (A4) total conservation of energy and momentum; (A5) non-negative entropy production in dissipative regimes; (A6) parameters within independent experimental bounds; and (A7) existence of at least one prediction that is not degenerate with the conventional electromagnetic response.

3.11 Initial Testable Consequences

We separate the observables into two groups. The first contains quantities directly associated with the free mass sector: dispersion according to (12), the group-velocity correction according to (15), effects associated with the longitudinal mode, and modifications of static fields on scales comparable to the corresponding Compton wavelength. The second contains observables associated with the constitutive sector: stored energy, momentum transfer, mechanical or thermal response, and medium-dependent dispersion.

We apply the discrimination criterion to this separation. When an observed effect can be reproduced with $m_\gamma = 0$ and physically admissible conventional constitutive parameters, we do not classify it as discriminating evidence for a nonzero photon rest mass.

In the next stage, we specify a minimal class for $\mathcal{A}_{\text{int}}$ and derive its explicit contribution to Δm^2_{int} in (3), to the interaction energy in (18), and to the tensor $T_{\text{int}}^{\{\mu\nu\}}$. From these relations, we evaluate the mechanical interpretation of the contribution represented by $\rho_{\text{medium}}\Delta V$ and determine the conditions under which it produces a distinct measurable signature.

4. Propagation and Interaction with Material Media

We formalize the separation between the intrinsic mass sector and the material response through the effective field propagator. In a homogeneous medium, we write the linearized equation in Fourier space as

$$[D_0^{-1}(k)]^{\{\mu\nu\}} A_\nu(k) - \Pi^{\{\mu\nu\}}(k; \Xi) A_\nu(k) = \mu_0 J^\mu(k), \quad (27)$$

where $[D_0^{-1}]^{\{\mu\nu\}}$ represents the inverse operator associated with the free Maxwell–Proca sector and $\Pi^{\{\mu\nu\}}$ is the polarization or self-energy tensor induced by the medium response. We use this decomposition to prevent a medium-induced pole shift from being automatically identified with an intrinsic rest mass.

4.1 Free Operator and Propagation Poles

For a conserved current and within the physical sector, we schematically represent the denominator of the free propagator as

$$\mathcal{D}_0(k) = k_\mu k^\mu - \mu^2, \quad (28)$$

with $k_\mu k^\mu = \omega^2/c^2 - |\mathbf{k}|^2$ in the adopted metric convention. The free poles satisfy $\mathcal{D}_0(k)=0$ and reproduce Eq. (12). In the presence of a medium, the physical poles are shifted by the proper components of the self-energy. For each sector $a \in \{T, L\}$, we write

$$\mathcal{D}a(\omega, \mathbf{k}; \Xi) = \omega^2/c^2 - \mathbf{k}^2 - \mu\gamma^2 - \Pi a(\omega, \mathbf{k}; \Xi) = 0. \quad (29)$$

In Eq. (29), Πa has dimensions of inverse length squared in this normalization. We therefore identify an effective dispersive correction without automatically converting it into a rest mass. Only in a limit in which Πa is approximately constant, real, and independent of the state of the medium could its effect be locally parameterized as a quadratic mass shift; even then, we keep the physical origins of the terms distinct.

4.2 Real Part, Imaginary Part, and Dissipation

We decompose the causal response as $\Pi a = \text{Re } \Pi a + i \text{Im } \Pi a$. The real part shifts the dispersion relation, whereas the imaginary part describes attenuation or gain in the linear regime. For passive media in equilibrium, we select the sign convention so that the dissipative contribution is compatible with non-negative entropy production.

4.3 Medium-Off Limit

Imposing $\Pi^{\mu\nu} \rightarrow 0$ when the constitutive variables responsible for the response are removed, we recover

$$\omega^2 \rightarrow c^2 \mathbf{k}^2 + m\gamma^2 c^4/\hbar^2. \quad (30)$$

We use (30) as a consistency test for any subsequent parameterization. If a contribution persists when the medium is switched off, it must be incorporated into the fundamental sector or eliminated as a parameterization artifact; if it disappears with the medium, we classify it as an induced response.

4.4 Minimal Constitutive Construction and Volumetric Compression

We construct a minimal realization to relate the effective-compression variable introduced in (20) to a measurable electromechanical response. We consider a material element with reference volume V_0 , volumetric strain ϵV , and effective bulk modulus K . The reversibly stored energy is given by (22). The mechanical pressure conjugate to the volumetric strain follows from

$$p_{\text{comp}} = \partial E_{\text{int}} / \partial (V_0 \epsilon V) = K \epsilon V, \quad (31)$$

in the isotropic linear regime. With this definition, we use the term compression only when ϵV and K have an established constitutive interpretation. For media whose relevant response is predominantly electromagnetic, we replace (31) with the appropriate free-energy density as a function of polarization, magnetization, strain, and temperature.

4.5 First-Order Effective Coupling

To investigate the influence of electromagnetic intensity on the mechanical variable, we introduce the lowest-order effective energy density

$$u(\varepsilon V, F) = \frac{1}{2} K \varepsilon V^2 + u_{EM}(F) - g \varepsilon V \mathcal{J}(F), \quad (32)$$

where g is a constitutive coefficient and $\mathcal{J}(F)$ is an admissible scalar invariant constructed from the field and the properties of the medium. In the simplest isotropic case, we may use a combination proportional to the local electromagnetic energy density. The mechanical equilibrium condition $\partial u / \partial \varepsilon V = 0$ yields

$$\varepsilon V^* = (g/K) \mathcal{J}(F). \quad (33)$$

Substituting (33) into (32), we obtain an effective correction of second order in the coupling. Thus, compression is not postulated as a universal consequence of photon passage; it emerges only in the constitutive realization specified by the material parameters and boundary conditions.

4.6 Relation to the $\rho_{medium} \Delta V$ Parameterization

From the definition $\varepsilon V = -\Delta V / V_0$, we write $|\Delta V| = V_0 |\varepsilon V|$. The combination $\rho_{medium} \Delta V$ has dimensions of mass, but its energetic contribution depends on the physical mechanism responsible for the strain. Using (33), we obtain the scale

$$|\rho_{medium} \Delta V| = \rho_{medium} V_0 |g \mathcal{J}(F) / K|. \quad (34)$$

Equation (34) converts the originally phenomenological term into a quantity linked to measurable parameters. We do not identify $\rho_{medium} \Delta V$ with an additional photon mass. We use this combination as a measure of the energetic contribution associated with the material response.

4.7 Kinematic Closure of Compression

To eliminate arbitrariness in the volumetric variable, we adopt C as the positive compression of a material element with reference volume V_0 and current volume V ,

$$C \equiv (V_0 - V) / V_0, \quad V = V_0(1 - C).$$

Mantidthe mass of the elemento, $\rho V = \rho_0 V_0$, of modo that
 $\rho = \rho_0 / (1 - C), \quad C = 1 - \rho_0 / \rho.$

Using material continuity $D\rho/Dt + \rho \nabla \cdot \mathbf{v} = 0$, we obtain
 $DC/Dt = -(1 - C) \nabla \cdot \mathbf{v},$

and, in the small-deformation regime, $DC/Dt \simeq -\nabla \cdot \mathbf{v}$ and $C \simeq -\nabla \cdot \mathbf{u}$. Thus, in the minimal model, C is not an independent degree of freedom from density or the displacement field. We do not simultaneously introduce redundant variables without an explicit constraint.

4.8 Minimal Interaction Energy and Conjugate Quantity

We write the total energy density as $u_{\text{tot}} = u_{\text{EM}} + u_{\text{mat}} + u_{\text{int}}$ and adopt, as a minimal local realization for the additional sector,

$$u_{\text{int}}(C, u_{\text{EM}}) = \frac{1}{2} K_I C^2 - g_I C u_{\text{EM}},$$

where K_I has dimensions of energy density and g_I is dimensionless when u_{EM} is an energy density. The scalar quantity conjugate to the compression is

$$X_C \equiv \partial u_{\text{int}} / \partial C = K_I C - g_I u_{\text{EM}}.$$

In the quasi-static limit and in the absence of additional mechanical constraints, $X_C = 0$ implies $C_{\text{eq}} = (g_I / K_I) u_{\text{EM}}$. We treat this relation as an effective constitutive hypothesis rather than a new universal law: its validity must be established for the material and regime under consideration.

4.9 Formal Non-Redundancy Criterion

Let u_0 be the energy of the reference model, including the calibrated conventional material response. We regard the additional sector as observationally independent only when there is no admissible reparameterization $\Xi_0 \rightarrow \Xi_0'$ capable of satisfying, within experimental uncertainties, $u_0(\Xi_0') = u_0(\Xi_0) + u_{\text{int}}$.

$$u_0(\text{and}, B, \rho, T, \chi, C; \Xi_0') = u_0(\text{and}, B, \rho, T, \chi, C; \Xi_0) + u_{\text{int}}.$$

If this equality can be realized without violating independent measurements of the conventional parameters, u_{int} represents only a reparameterization of the known response in that domain and not a physically discriminating contribution.

4.9.1 Tensorial Isolation of Electrostriction in the Null Model P_0

To make the content of the null model explicit, we first consider a linear dielectric in the small-strain regime, with strain tensor $u_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$. We write the reference electromechanical free energy as

$$F_0 = F_{el}(u_{ij}, T, \dots) - \frac{1}{2} \epsilon_0 \epsilon_{kl}(u_{ij}, \omega) E_k E_l.$$

The dependence of permittivity on strain defines the fourth-rank electrostrictive tensor $A_{klij} = \partial \epsilon_{kl} / \partial u_{ij}$. The stress contribution associated with electrostrictive coupling is, under the adopted convention,

$$\sigma_{ij}^{es} = -\frac{1}{2} \epsilon_0 E_k A_{klij} E_l.$$

For a linear isotropic medium, we may parameterize A_{klij} by two independent coefficients,

$$A_{klij} = (a_1/2)(\delta_{ki}\delta_{lj} + \delta_{kj}\delta_{li}) + a_2\delta_{kl}\delta_{ij},$$

$$\sigma_{ij}^{es} = -\frac{1}{2} \epsilon_0 [a_1 E_i E_j + a_2 |E|^2 \delta_{ij}].$$

With $C \simeq -u_{kk}$ and hydrostatic pressure $p = -\sigma_{ii}/3$, the conventional volumetric contribution contains a term proportional to $|E|^2$ and therefore to u_{EM} . Consequently, the local scalar coupling $-g_I C u_{EM}$ has, in an isotropic linear medium, the same first-order algebraic signature as conventional electrostriction.

For this reason, we treat constant g_I as a diagnostic parameter. Independent physical content requires at least a structure that cannot be absorbed by electrostrictive coefficients or calibrated permittivity: for example, a dispersive dependence $g_I(\omega, k)$, a tensorial polarization-dependent response, geometric anisotropy, memory, retardation, or a joint correlation among independently measured observables.

This negative result has independent content: the scalar, local, isotropic, and linear realization of the coupling does not, by itself, provide a signature of new physics. We therefore do not treat a formally adjustable residual as evidence; we require any extension to produce an observable structure that survives electrostrictive calibration and redefinition of the constitutive parameters of the reference model.

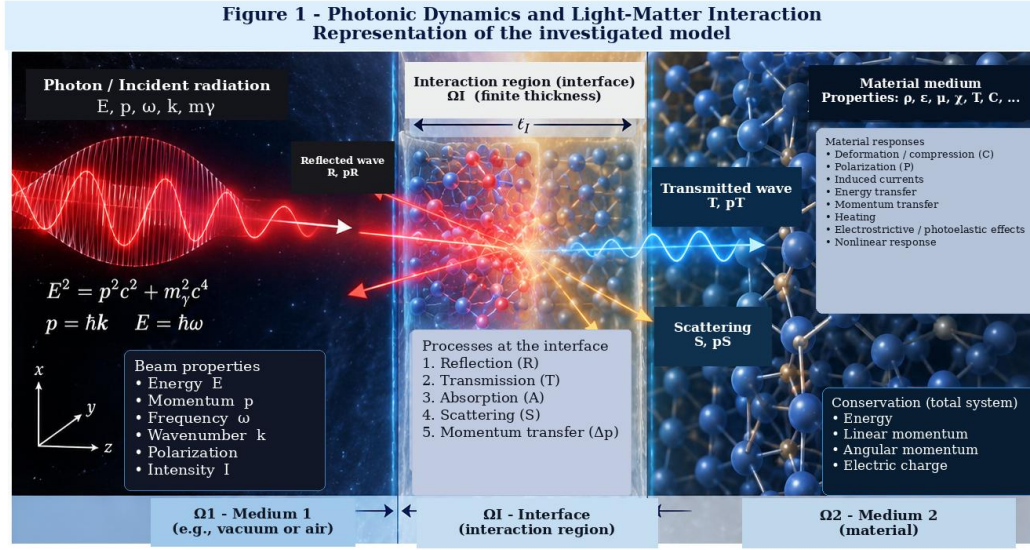


Figure 1 – Schematic representation of propagation and light–matter interaction in the effective model. The illustration is conceptual and does not constitute observational evidence of photon mass.

4.10 Energy–Momentum Balance and Effective Force

We integrate the local conservation law (23) over the spatial volume Ω with boundary $\partial\Omega$. For the total energy U_Ω and energy flux S through the boundary, we obtain

$$dU_\Omega/dt = - \oint_{\partial\Omega} S \cdot n \, dA + P_{\text{ext}}, \quad (35)$$

where P_{ext} represents power supplied by explicitly included external sources. For the total momentum P_Ω , we write

$$dP_{i\Omega}/dt = - \oint_{\partial\Omega} T_{ij} n_j \, dA + F_{i,\text{ext}}. \quad (36)$$

We define the effective force on the material subsystem as the rate of change of its momentum after accounting for momentum fluxes through the boundary. This definition includes radiation pressure and conventional electromagnetic forces in the appropriate limit.

4.11 Criterion for the Use of the Term Drag

We reserve the term drag for a dissipative or resistive contribution that produces net momentum transfer in a defined direction relative to motion or flux. We denote a possible additional contribution by F_{drag} only when this operational criterion is satisfied.

$$F_{\text{drag},i}(t) = - \int_0^\infty \Gamma_{ij}(\tau; \Xi) v_j(t-\tau) d\tau, \quad (37)$$

where Γ_{ij} is a causal response kernel. For an instantaneous isotropic response, $\Gamma_{ij}(\tau) = \gamma \delta_{ij} \delta(\tau)$, we recover $F_{\text{drag}} = -\gamma v$. The nonlocal form of (37) represents material memory without modifying the photon rest mass. Thermodynamic admissibility requires the dissipative part of the response to be compatible with non-negative entropy production.

4.12 Pressure, Drag, and Source Reaction

We distinguish three mechanisms: pressure associated with normal momentum flux; drag associated with the dissipative component of the response; and source reaction associated with global momentum conservation during emission. These mechanisms may coexist but are not equivalent. For total emission with radiated momentum P_{rad} , conservation in the closed source–field–medium system imposes

$$\Delta P_{\text{source}} + \Delta P_{\text{rad}} + \Delta P_{\text{medium}} = 0. \quad (38)$$

Equation (38) provides the starting point for subsequent applications to extended sources. In this article, we restrict its use to momentum bookkeeping in light–matter interaction; we do not infer from it gravitational return, stellar recirculation, or the evolution of astrophysical sources, which are independent dynamical problems.

5. Quantitative Consequences and Observables

We organize the model tests in a hierarchy that separates fundamental parameters from constitutive parameters. In the free sector, we estimate m_γ exclusively from observables whose functional dependence follows from the dispersion relation (12) or from the associated modification of quasi-static fields. In the material sector, we estimate the parameters of Δm^2_{int} and of the constitutive energy independently.

5.1 Vacuum Dispersion Test

For two energies E_1 and E_2 , the mass contribution to the propagation delay follows the E^{-2} dependence indicated in (16). We define the residual estimator after subtracting intrinsic source delays, plasma contributions, and other conventional effects.

5.2 Medium-Dependence Test

The signature produced by $\Pi_a(\omega, k; \Xi)$ must vary when Ξ is controllably changed, whereas the rest mass m_γ remains invariant. This difference provides a separation test. If the observed

correction disappears in the low-density limit or when the relevant constitutive coupling is removed, we classify it as a medium effect rather than a rest-mass effect.

5.3 Mechanical Momentum-Transfer Test

For force or pressure experiments, we first calculate the electromagnetic momentum flux predicted by conventional theory and the mechanical response of the material. We introduce the additional sector only after this stage and require its parameters to explain a residual that cannot be absorbed by the reference model.

5.4 Explicit Refutation Conditions

We regard specific realizations of the model as refuted when they exhibit at least one of the following properties: violation of energy–momentum conservation; a dynamical instability that cannot be removed within the declared domain; negative entropy production in a passive medium; requirement of an $m\gamma$ above independently established bounds; or a predicted residual absent at the sensitivity of a properly controlled experiment.

5.5 Experimental Constraints and Orders of Magnitude

We confront the fundamental sector with experimental bounds before assigning a magnitude to constitutive effects. The Particle Data Group compilation gathers constraints obtained by distinct methods and explicitly records their dependence on auxiliary assumptions [1]. We therefore do not treat all reported bounds as statistically homogeneous.

5.6 Compton Scale and Dimensionless Mass Parameter

We associate with the photon mass the reduced Compton scale $\bar{\lambda}\gamma = \hbar/(m\gamma c)$.

$$\bar{\lambda}\gamma = \hbar/(m\gamma c). \quad (39)$$

For a mode of energy $E = \hbar\omega$, we define the dimensionless parameter

$$\varepsilon\gamma(E) \equiv m\gamma c^2/E. \quad (40)$$

In the regime $\varepsilon\gamma \ll 1$, we expand the group velocity obtained from the dispersion relation and find

$$v_g/c = \sqrt{1 - \varepsilon\gamma^2} = 1 - \varepsilon\gamma^2/2 + O(\varepsilon\gamma^4). \quad (41)$$

The first nonzero kinematic correction is therefore quadratic in $m\gamma/E$. This structure fixes the sensitivity scale of time-of-flight tests and prevents the introduction of a correction linear in $m\gamma$ without an independent dynamical derivation.

5.7 Propagation Delay over the Finite Distance

For propagation over a distance D , we define Δt_γ as the difference between the time of flight of the massive mode and D/c . Retaining the first nonzero term of expansion (41), we obtain $\Delta t_\gamma(E;D) \simeq (D/2c)(m\gamma^2 c^2/E)^2$. (42)

For two modes of energies E_1 and E_2 emitted simultaneously in the idealized model, the differential contribution is

$$\Delta t_{12} \simeq (D m\gamma^2 c^3/2)(1/E_1^2 - 1/E_2^2). \quad (43)$$

We treat (43) as a kinematic signature of the free sector rather than as a direct estimator from raw data. In astrophysical applications, we separate intrinsic source delays, plasma dispersion, gravitational effects, and other contributions before associating any residual with the parameter $m\gamma$ [1,3].

5.8 Hierarchy between fundamental and Constitutive Scales

We introduce two dimensionless ratios to organize the perturbative domain:

$$\varepsilon_\gamma = m\gamma^2 c^2/E, \quad \varepsilon_{\text{int}} = |E_{\text{int}}|/E. \quad (44)$$

The weak-perturbation regime satisfies $\varepsilon_\gamma \ll 1$ and $\varepsilon_{\text{int}} \ll 1$. When only ε_γ is small but ε_{int} is not, propagation is dominated by the medium response and does not permit a direct inference of the rest mass. When both are small, the two sectors may be treated perturbatively and compared through their distinct scaling laws.

5.9 Observable Structure and Identifiability

We organize the experimental comparison through the observable vector $O = (O_1, \dots, O_n)$ and the parameter vector $\theta = (m\gamma, \theta_{\text{int}})$, where θ_{int} collects the constitutive coefficients. For a specified experimental configuration, we write

$$O_a = O_a(0) + \Delta O_a(\gamma)(m\gamma) + \Delta O_a(\text{int})(\theta_{\text{int}}) + \Delta O_a(\text{cross}). \quad (45)$$

The decomposition (45) does not assume absolute physical independence among the sectors; it defines an inference strategy. The term $O_a(0)$ contains the conventional prediction in the domain considered, $\Delta O_a(\gamma)$ contains the correction from the mass sector, and $\Delta O_a(\text{int})$ contains the constitutive correction.

5.10 Sensitivity Matrix

we define localmente the sensitivity matrix

$$J_{ai}(\theta) = \partial O_a / \partial \theta_i. \quad (46)$$

We regard a parameter subset as identifiable when the corresponding columns of J are linearly independent in the experimental domain after inclusion of the relevant uncertainties. Degeneracy between $m\gamma$ and a plasma parameter, for example, prevents exclusive attribution of an observed delay to photon mass. Joint analysis of observables with distinct dependences on frequency, density, composition, and geometry reduces this type of degeneracy.

5.11 Consistency across Experiments

For N independent experiments, we denote by Θ_j the parameter region compatible with experiment j . We define the joint admissible region as

$$\Theta_{\text{adm}} = \bigcap_{j=1}^N \Theta_j. \quad (47)$$

We accept a specific realization of the model only when Θ_{adm} is nonempty and remains compatible with the stability, causality, conservation, and thermodynamic conditions established in the preceding sections. This criterion prevents independently retuned constitutive parameters from being used to conceal an inconsistency among experiments.

5.12 Quantitative Predictions and Asymptotic Limits

In this section, we quantify the consequences that follow directly from the free mass sector and separate them from constitutive corrections. We use the conservative order-of-magnitude reference adopted in the experimental-constraint section and do not interpret compatibility as detection.

5.13 Compton Scale Associated with the Jovian Bound

Substituting $m\gamma c^2 = 2.5 \times 10^{-18}$ eV into Eq. (39), we find

$$\bar{\lambda}\gamma = \hbar c / (m\gamma c^2) \approx 7,89 \times 10^{10} \text{ m} \approx 0,527 \text{ UA}. \quad (48)$$

The scale (48) does not represent a universal interaction range for every possible realization of the model; it corresponds to the reduced Compton scale associated with the mass bound used. For smaller masses, $\bar{\lambda}\gamma$ increases inversely with $m\gamma$.

5.14 Group-Velocity Correction

For a photon of energy E , the relative deficit of group velocity with respect to c , in the regime $\varepsilon\gamma \ll 1$, is

$$(c - v_g)/c \approx \frac{1}{2}(m\gamma c^2/E)^2. \quad (49)$$

For $E = 1$ eV and $m\gamma c^2 = 2.5 \times 10^{-18}$ eV, we obtain

$$(c - v_g)/c \lesssim 3,13 \times 10^{-36}. \quad (50)$$

For $E = 1$ keV, the same ratio is reduced by a factor of 10^6 , resulting in a value of order 10^{-42} . We therefore find a kinematic hierarchy strongly dependent on E^{-2} . In the optical range and at higher energies, the time-of-flight effect associated exclusively with a mass at this scale is extremely small over Solar-System distances.

5.15 Propagation Delay: Solar and Interstellar Scales

Applying Eq. (42) to a mode with $E = 1$ eV propagating over one astronomical unit, $D \approx 1.496 \times 10^{11}$ m, we find

$$\Delta t_\gamma(1 \text{ eV}; 1 \text{ UA}) \lesssim 1,56 \times 10^{-33} \text{ s}. \quad (51)$$

Keeping the same values of E and m_γ and taking $D = 1$ pc, we obtain approximately

$$\Delta t_\gamma(1 \text{ eV}; 1 \text{ pc}) \lesssim 3,22 \times 10^{-28} \text{ s}. \quad (52)$$

These orders of magnitude show that, for the Jovian bound used here, the time-of-flight signature in optical photons does not by itself provide a competitive channel on solar or nearby stellar scales. Sensitivity increases at lower energies and larger distances, but observational interpretation then requires simultaneous control of plasma dispersion and intrinsic source delays. Consequently, we do not use an observed delay as direct evidence of m_γ without modeling the competing terms.

5.16 Exact Form and High-Energy Asymptotic Limit

The perturbative expansion used in Eqs. (41)–(43) follows from the exact expression

$$v_g(E) = c\sqrt{1 - (m_\gamma c^2/E)^2}, \quad E \geq m_\gamma c^2. \quad (53)$$

in the limit and $/(m_\gamma c^2) \rightarrow \infty$, we verify

$$\lim v_g(E) = c, \quad \lim \Delta t_\gamma(E; D) = 0. \quad (54)$$

The asymptotic recovery of luminal propagation at high energies is an internal condition of the relativistic kinematics adopted here. In the complementary limit $E \rightarrow m_\gamma c^{2+}$, the group velocity tends to zero while the phase velocity grows, as follows directly from the massive dispersion relation.

5.17 Scaling of Constitutive Corrections

For the material sector, we write the total energy in the perturbative regime as

$$E = E_0 + E_{\text{int}} + O(E_{\text{int}}^2/E_0), \quad |E_{\text{int}}|/E_0 \ll 1, \quad (55)$$

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_{\text{int}} + \mathbf{O}(\mathbf{E}_{\text{int}}^2/\mathbf{E}_0), \quad |\mathbf{E}_{\text{int}}|/\mathbf{E}_0 \ll 1, \quad (55)$$

and we retain $\epsilon_{\text{int}} = |\mathbf{E}_{\text{int}}|/\mathbf{E}$ as a parameter independent of $\epsilon\gamma$. This separation allows us to compare two classes of correction with different scaling laws. The free sector produces, at the first nonzero order, contributions proportional to $m\gamma^2/E^2$; the constitutive sector may follow different frequency, density, polarization, geometry, or thermodynamic-state dependences.

$$\delta a \equiv [Oa(\theta) - Oa(0)]/Oa(0), \quad (56)$$

and the ratio of detectabilidad

$$S_a \equiv |Oa(\theta) - Oa(0)|/\sigma_a, \quad (57)$$

where σ_a represents the total uncertainty associated with the observable, including statistical and systematic components. We regard a prediction as quantitatively discriminating only when its parametric dependence is not degenerate with conventional effects and when S_a can reach the scale required by the proposed experiment.

5.19 Consequences for the Experimental Strategy

Results (48)–(57) separate two experimental regimes. For the fundamental sector, we prioritize observables whose spatial extent is sensitive to the Proca modification and dispersion observables in which the E^{-2} dependence can be isolated. For the constitutive sector, we prioritize controlled experiments in which density, composition, temperature, mechanical state, geometry, or polarization are varied while the free-sector parameters remain fixed.

The order-of-magnitude analysis also constrains the mechanical interpretation. For values of m compatible with the Jovian bound considered here, the direct kinematic correction to the velocity of optical photons is extremely small. Therefore, any measurable macroscopic effect involving pressure, force, or momentum transfer in the laboratory cannot be attributed automatically to rest mass merely from its magnitude. Such attribution requires a functional dependence specific to $m\gamma$ that survives subtraction of conventional electromagnetic and material mechanisms.

6. Experimental Constraints, Identifiability, and Falsification

In this section, we confront the photon-mass sector with independent classes of experimental and astrophysical constraints. We keep separate the inferences that depend on dispersive propagation, large-scale magnetic fields, and magnetohydrodynamics. This separation

is necessary because the numerical limits do not share the same auxiliary assumptions, spatial support, or inference structure.

6.1 Reference Bounds and Methodological Hierarchy

To prevent opportunistic selection of a single bound, we define three classes of constraint: C_1 , local or planetary tests; C_2 , propagation and plasma tests; and C_3 , large-scale astrophysical fields. For each class C_j , we denote by B_j the set of auxiliary assumptions and write the allowed region as $\Theta_j = \Theta(O_j | B_j)$. The global region is interpreted as a physical intersection only when the sets B_j are compatible with the specific realization of the model.

$$\Theta_{\text{adm}} = \bigcap_j \Theta(O_j | B_j), \quad \text{with explicit compatibility among the sets } B_j. \quad (58)$$

6.2 Jovian Constraint and Associated Spatial Scale

$$\Theta_{\text{adm}} = \bigcap_j \Theta(O_j | B_j), \quad \text{with explicit compatibility among the sets } B_j. \quad (58)$$

The corresponding experimental condition is expressed as

$$m\gamma c^2 < 2.5 \times 10^{-18} \text{ eV} \Rightarrow \mu\gamma = m\gamma c/\hbar < 1/\bar{\lambda}\gamma. \quad (59)$$

In the present model, we impose (59) directly on the intrinsic parameter $m\gamma$. We do not transfer this bound to Δ_{mint} , to χ_{int} , or to the constitutive parameters of the medium, since these quantities describe physically distinct sectors. Identifying these parameters produces an artificial constraint and violates the separation established in P3.

6.3 Dispersive Propagation and Plasma Degeneracy

Temporal dispersion of transient signals provides a second class of test. For two energies E_1 and E_2 , the free sector predicts, in the relativistic regime, a time-of-flight difference proportional to $m\gamma^2(E_1^{-2} - E_2^{-2})$, as derived above. In radio signals, however, cold-plasma dispersion generates the same leading inverse-frequency dependence, requiring an explicit plasma null model.

We write the observed delay between two channels as

$$\Delta_{\text{tobs}} = \Delta_{\text{tplasma}} + \Delta_{\text{t}\gamma}(m\gamma) + \Delta_{\text{tint}}(\theta_{\text{int}}) + \Delta_{\text{tsys}}. \quad (60)$$

Equation (60) constitutes the identifiability criterion for our formulation. Estimation of $m\gamma$ is admissible only when the component $\Delta_{\text{t}\gamma}$ cannot be absorbed, within uncertainties, by Δ_{tplasma} , Δ_{tint} , or systematic terms. Thus, we do not use an isolated chromatic delay as evidence of photon mass.

6.3.1 Plasma Null Model and Cosmological Separation by Redshift

In a cold, homogeneous, unmagnetized plasma, the leading transverse dispersion relation may be written as $\omega^2 = c^2 k^2 + \omega_p^2$, with $\omega_p^2 = n_e e^2 / (\epsilon_0 m_e)$. When the free massive sector is included simultaneously, the local relation becomes

$$\omega^2 = c^2 k^2 + \omega_p^2 + (m\gamma c^2 / \hbar)^2.$$

Along a single line of sight and in the low-mass/high-frequency regime, delays produced by plasma and by $m\gamma$ have the same dominant chromatic dependence, proportional to v_{obs}^{-2} . This degeneracy is exact at leading order in that restricted description and cannot be broken by frequency scaling alone.

For a cosmological source, expansion modifies the two contributions differently. In a spatially homogeneous FLRW model, we write $H(z) = H_0 E(z)$. The mass-induced delay between two observed frequencies ν_l and ν_h , in the limit $m\gamma c^2 \ll h\nu$, may be written as [9,10]

$$\Delta\tau = [1/(2H_0)](m\gamma c^2 / \hbar)^2 H\gamma(z)(\nu_l^{-2} - \nu_h^{-2}),$$

$$H\gamma(z) = \int_0^z dz' / [(1+z')^2 \text{ and } (z')].$$

For the intergalactic medium, the mean dispersion contribution contains, in addition to the same v_{obs}^{-2} dependence, a cosmological kernel associated with electron density, which in the homogeneous case may be represented schematically by

where $f_e(z)$ represents the effective free-electron fraction and other baryonic factors used in the intergalactic-medium DM model. The kernels $H\gamma(z)$ and $F_p(z)$ are not proportional. Consequently, a sample of well-localized FRBs distributed in redshift can statistically reduce the degeneracy between the two coefficients. The separation is neither complete nor automatic: host-galaxy, Milky Way, local-environment, and intergalactic-plasma contributions must be jointly modeled and marginalized.

6.4 Magnetohydrodynamic Tests and Solar Wind

Solar-wind tests probe Maxwell–Proca modifications in a regime different from time-of-flight measurements. The Particle Data Group compilation records a limit of order 10^{-18} eV associated with Ryutov’s magnetohydrodynamic analysis and emphasizes the dependence of such constraints on the assumed spatial extent and plasma model [1,8]. We therefore use these bounds as method-dependent constraints rather than as statistically interchangeable measurements.

For the constitutive sector, the presence of plasma imposes an additional requirement. Any effective-compression term or material response must be confronted with conventional magnetohydrodynamics before being interpreted as a new contribution. Denoting the conventional prediction by O_MHD and the proposed correction by ΔO_{int} , we require, as a conservative operational bound in regimes without a statistically significant deviation,

$$|\Delta O_{int}| \leq |O_{obs} - O_{MHD}| + \sigma_{tot} \quad (61)$$

throughout any regime in which no statistically significant deviation is present. Inequality (61) converts null observations into direct bounds on the constitutive parameters.

6.5 Exclusion Criterion for the Mass Sector

We define $m_{\gamma, \max}(C_j)$ as the largest value compatible with experimental class C_j under its assumptions B_j . For a realization of the model in which those assumptions are valid, we exclude any parameter satisfying

$$m_{\gamma} > \min_j m_{\gamma, \max}(C_j). \quad (62)$$

Equation (62) does not authorize indiscriminate combination of bounds that depend on incompatible physical mechanisms. When an alternative realization modifies large-scale behavior, we reassess the propagation operator and recalculate the corresponding bound rather than directly importing a value obtained under universal Proca assumptions.

6.6 Exclusion Criterion for the Constitutive Sector

For the parameters θ_{int} , we define the discrepancy functional

$$\chi^2(\theta) = \sum_{a,\beta} [O_{a,obs} - O_a(\theta)] C^{-1}_{a\beta} [O_{\beta,obs} - O_{\beta}(\theta)], \quad (63)$$

where C represents the experimental and theoretical covariance matrix. In a specific statistical analysis, the allowed region is defined by the adopted confidence criterion or posterior distribution. At this stage, we use (63) as the general comparison structure and do not assign statistical significance without explicitly specified data and likelihood functions.

The separation $\theta = (m\gamma, \theta_{\text{int}})$ makes it possible to test whether an improvement in fit attributed to the constitutive sector remains after marginalization over $m\gamma$ and over conventional medium parameters. This procedure prevents a parametric degeneracy from being interpreted as detection of new physics.

6.7 Outcome of the Comparison at the Present Stage

With the reference bounds considered, we find that the kinematic photon-mass sector produces extremely small corrections for optical photons over solar distances, according to (50)–(52). Therefore, the hypothesis $m\gamma > 0$, by itself and within the allowed interval used here, does not support large-amplitude macroscopic mechanical effects in optical propagation. Any effect of this nature requires an additional constitutive sector that is quantitatively specified and independently testable.

We also find that the physically relevant region cannot be defined by a single numerical value of $m\gamma$. Planetary, magnetohydrodynamic, galactic, and propagation bounds probe different combinations of scale and assumptions. Consequently, we retain as an admissibility condition simultaneous compatibility with the set of constraints applicable to the chosen realization, without promoting any particular indirect measurement to the status of a detection.

6.8 Robustness, Identifiability, and Computational Protocol

In this section, we subject the formulation to an additional internal-control stage designed to separate formal consistency, parameter identifiability, and effective predictive capability. We consider the parameter vector $\theta = (m\gamma, \theta_{\text{int}}, \theta_{\text{conv}})$, where θ_{int} collects the constitutive parameters introduced by the model and θ_{conv} collects the conventional parameters required to describe the medium and experimental apparatus. The analysis is constructed so that no agreement obtained by compensation among parameters is interpreted as isolated evidence for $m\gamma > 0$.

6.9 Test Hierarchy and Null Model

We adopt as null model H_0 conventional electrodynamics supplemented by the established material response for the experimental system under consideration. We define H_1 as the extension in which $m\gamma$ is allowed to vary while $\theta_{\text{int}} = 0$, and H_2 as the extension in which both $m\gamma$ and the additional constitutive parameters θ_{int} are allowed to vary.

$$H_0: m\gamma = 0, \theta_{\text{int}} = 0; \quad H_1: m\gamma \geq 0, \theta_{\text{int}} = 0; \quad H_2: m\gamma \geq 0, \theta_{\text{int}} \neq 0. \quad (64)$$

For each data set D , we compare the predictions $O(\theta)$ with the measured observables and keep the noise, calibration, and conventional-response parameters explicit. The H2 extension acquires additional empirical content only when it improves the description of the data without violating the independent constraints established in Sections 5–6.

6.10 Dimensionless Control Parameters

We introduce dimensionless parameters to control the approximation regimes and prevent extrapolations beyond the domain of validity. For the mass sector, we use $\varepsilon_m = m\gamma c^2/E$. For the constitutive sector, we define $\varepsilon_{\text{int}} = |\Delta m^2_{\text{int}}|/m_{\text{ref}}^2$, with m_{ref} chosen according to the dynamical scale of the experiment. For dissipation, we use $\varepsilon_{\text{diss}} = |\text{Im } \Pi|/|\text{Re } D^{-1}|$ whenever the self-energy description Π is applicable.

$$\varepsilon_m \equiv m\gamma c^2/E, \quad \varepsilon_{\text{int}} \equiv |\Delta m^2_{\text{int}}|/m_{\text{ref}}^2, \quad \varepsilon_{\text{diss}} \equiv |\text{Im } \Pi|/|\text{Re } D^{-1}|. \quad (65)$$

6.11 local Identifiability and Degeneracy

We use the sensitivity matrix defined in Section 5.9 to construct the local information matrix $F = J^T C^{-1} J$. When F has full rank in the investigated parameter subspace, the parameters are locally identifiable within the adopted approximations. Very small eigenvalues of F indicate nearly degenerate directions in which different combinations of $m\gamma$, θ_{int} , and θ_{conv} produce observables that are indistinguishable within uncertainties.

$$F_{ij} = \sum_{\alpha, \beta} (\partial O_{\alpha} / \partial \theta_i) C^{-1}_{\alpha\beta} (\partial O_{\beta} / \partial \theta_j). \quad (66)$$

We define the condition number $\kappa(F)$ as a complementary numerical diagnostic. Large values of $\kappa(F)$ signal unstable inference and require reparameterization, additional observables, or physically independent priors. We do not interpret estimates obtained in numerically ill-conditioned regions as robust determinations of fundamental parameters.

6.12 Stability and Positivity Tests

For each constitutive realization, we verify the absence of uncontrolled exponentially growing modes over the frequency and wavenumber domain considered. In the free Proca sector with $m\gamma^2 > 0$, the frequency satisfies $\omega^2 = c^2 k^2 + m\gamma^2 c^4 / \hbar^2$. In the coupled sector, we require the physical poles of the propagator to remain compatible with stability and causality and the dissipative part of the response to respect the non-negative entropy production established in Postulate P5.

$$\omega^2(k) = c^2 k^2 + m\gamma^2 c^4 / \hbar^2 + \Delta \omega^2_{\text{int}}(k, \omega; \Xi). \quad (67)$$

6.13 Reproducible Numerical Protocol

We implement the computational test in sequence: define the grid or sampling of θ ; calculate the dispersion relation and corresponding observables; reject points that violate stability, causality, conservation, or thermodynamic constraints; apply independent experimental bounds; and evaluate the discrepancy function only in the remaining region. To prevent the numerical search from producing spurious solutions, we repeat the analysis with different discretization scales and initial conditions and verify convergence of the observables.

the admissible region global is defined by the intersection

$$\Omega_{\text{adm}} = \Omega_{\text{rel}} \cap \Omega_{\text{cons}} \cap \Omega_{\text{therm}} \cap \Omega_{\text{stab}} \cap \Omega_{\text{exp}}. \quad (68)$$

where Ω_{rel} represents relativistic consistency, Ω_{cons} conservation, Ω_{therm} thermodynamic admissibility, Ω_{stab} stability, and Ω_{exp} experimental compatibility. If $\Omega_{\text{adm}} = \emptyset$ for a specific parameterization, we regard that realization as excluded. If $\Omega_{\text{adm}} \neq \emptyset$, we retain only the conclusion of compatibility and do not convert it into detection.

6.14 Recovery of Limiting Theories

$$\lim(m\gamma \rightarrow 0, \theta_{\text{int}} \rightarrow 0) H_2 = \text{Maxwell}; \quad \lim(m\gamma \rightarrow 0) H_2 = \text{medium}; \quad \lim(\theta_{\text{int}} \rightarrow 0) H_2 = \text{Proca}. \quad (69)$$

6.15 Model-Specific Prediction and Minimum Novelty Criterion

$$\lim(m\gamma \rightarrow 0, \theta_{\text{int}} \rightarrow 0) H_2 = \text{Maxwell}; \quad \lim(m\gamma \rightarrow 0) H_2 = \text{medium}; \quad \lim(\theta_{\text{int}} \rightarrow 0) H_2 = \text{Proca}. \quad (69)$$

This criterion constrains subsequent development: an original contribution of the model must appear as a functional structure, scale dependence, or correlation among observables that cannot be absorbed by conventional parameters. In this way, we tie theoretical novelty to the possibility of experimental discrimination.

6.16 Experimental Strategy

We organize the empirical evaluation of the model through a hierarchy of observables that separates the free photon-mass sector, the conventional electromagnetic response, and the additional constitutive sector. We use this separation to prevent a medium effect from being reinterpreted as evidence of rest mass or a small Proca correction from being amplified by a non-identifiable material parameterization. The protocol is formulated in terms of null hypotheses, measurable residuals, and explicit exclusion criteria.

6.17 Primary Observable of the Free Sector

For approximately free propagation, we adopt as the primary observable the arrival-time difference between two energy components $E1$ and $E2$ emitted by the same source, after subtraction of instrumental, intrinsic-source, plasma, and other conventional delays.

$$\Delta t \simeq (L m \gamma^2 c^3 / 2) (1/E1^2 - 1/E2^2). \quad (70)$$

The quadratic dependence on $m\gamma$ and approximate inverse-square dependence on energy constitute the kinematic signature of the free sector. In astrophysical sources, this dependence is degenerate with plasma effects over certain frequency ranges; we therefore use samples with independent distance or redshift information and explicit plasma modeling.

6.18 Quasi-Static Field Test

The presence of a Proca mass also modifies the spatial structure of quasi-static fields. We introduce the reduced Compton wavelength associated with the photon,

$$\bar{\lambda}_\gamma = \hbar/(m\gamma c). \quad (71)$$

and use the ratio $\xi = L/\bar{\lambda}_\gamma$ as the control parameter for systems of characteristic extent L . Long-range corrections become relevant only when ξ ceases to be negligible. Planetary magnetostatic and magnetohydrodynamic data probe this regime differently from propagation tests. The analysis of Jupiter's field using Juno mission data provides a contemporary example of a constraint obtained by direct fitting of the spatial field structure [2], whereas solar-wind magnetohydrodynamics provides a reference constraint widely compiled by the Particle Data Group [1,8].

6.19 Differential Test in Material Media

For the constitutive sector, we propose a differential protocol in which the same source and the same spectral range are measured in two material configurations A and B, while keeping as many instrumental parameters as possible in common. We write the observable as

$$\Delta OAB = O(E,k; m\gamma, \Xi A) - O(E,k; m\gamma, \Xi B). \quad (72)$$

Because $m\gamma$ belongs to the free operator, its common contribution tends to cancel in differences constructed to isolate the material response, whereas terms depending on ΞA and ΞB remain. We use this property to test the constitutive parameterization without automatically converting a material difference into an estimate of photon mass. The model is rejected in this class

of experiments when the proposed response function fails to describe dispersion, absorption, energy balance, and mechanical response simultaneously.

6.20 Differential Observable in Two Mechanical States

$$R_C \equiv [P_meas(C_2) - P_meas(C_1)] - [P_0(C_2) - P_0(C_1)],$$

To make the constitutive test more restrictive, we prepare the same material in two mechanical states C_1 and C_2 while keeping, within uncertainties, composition, temperature, optical geometry, frequency, and incident power fixed. We define

6.21 Momentum Transfer and Calorimetric Balance

where P_0 is the prediction of the reference model, explicitly including elasticity, the Maxwell stress tensor, electrostriction, photoelasticity, dissipation, thermal response, and other independently determined electromechanical contributions. An additional signature requires $R_C \neq 0$ with reproducible dependence on intensity, frequency, polarization, mechanical state, geometry, and interaction-region thickness. Because a local scalar coupling with constant g_I is degenerate with electrostriction in the isotropic linear regime, physical independence requires an additional dynamic, tensorial, geometric, or nonlocal signature.

Table 1 – Operational interpretation of the differential observable R_C .

Operational result	Interpretation
R_C compatible with zero	No evidence for an additional constitutive contribution in the tested regime.
Nonzero R_C , but absorbable into P_0	Incompletely calibrated or modeled conventional response; not a signature of a new sector.
Nonzero, reproducible, non-reabsorbable R_C	Candidate additional contribution, still subject to conservation, stability, thermodynamic, and cross-experiment consistency tests.
Pre-specified dispersive, tensorial, geometric, or memory dependence	Potentially discriminating signature if incompatible with H_0 and reproduced without ad hoc retuning.
Incompatibility among independent observables	Parametric realization rejected in the experimental domain considered.

$$P_{in} = P_{ref} + P_{trans} + P_{diss} + dU/dt. \quad (73)$$

Momentum transfer is calculated from the flux of the energy–momentum tensor through the control boundary. Agreement among force, reflection, transmission, and calorimetry is a necessary condition for any additional mechanical interpretation. We do not use the mere existence of radiation pressure, heating, or shadowing as evidence for $my > 0$, because these phenomena are

present in conventional electrodynamics. Instead, we seek a quantitative residual with a scaling law specific to the proposed sector.

6.22 Decision Statistics and Exclusion Region

For a data set D and the models H_0 , H_1 , and H_2 defined in Section 6.9, we write the generalized quadratic discrepancy function

$$\chi^2(\theta) = [D - O(\theta)]^T C^{-1} [D - O(\theta)]. \quad (74)$$

and we perform inference only within Ω_{adm} . Model comparison must include the penalty corresponding to increased parameter dimensionality and the propagation of systematic uncertainties. We define a region as excluded when no admissible parameter can reproduce the data within the previously fixed statistical criterion. We define a region as compatible when this condition is satisfied; compatibility is not equivalent to detection.

6.23 Explicit Falsification Criteria

We regard a specific realization of the model as falsified when at least one of the following conditions occurs: (i) the dispersion relation violates independent bounds applicable to the same regime; (ii) the constitutive response violates conservation, stability, or thermodynamic admissibility; (iii) the proposed residual is absent at the sensitivity required by the model; or (iv) the residual is fully absorbed by physically admissible parameters of the null model.

Conversely, we regard a signature as potentially discriminating only when the same parameter set θ reproduces independent observables and produces a nonzero residual relative to H_0 whose frequency, distance, polarization, or material dependence was predicted before fitting. This requirement prevents post hoc parameter adjustment from being interpreted as confirmation.

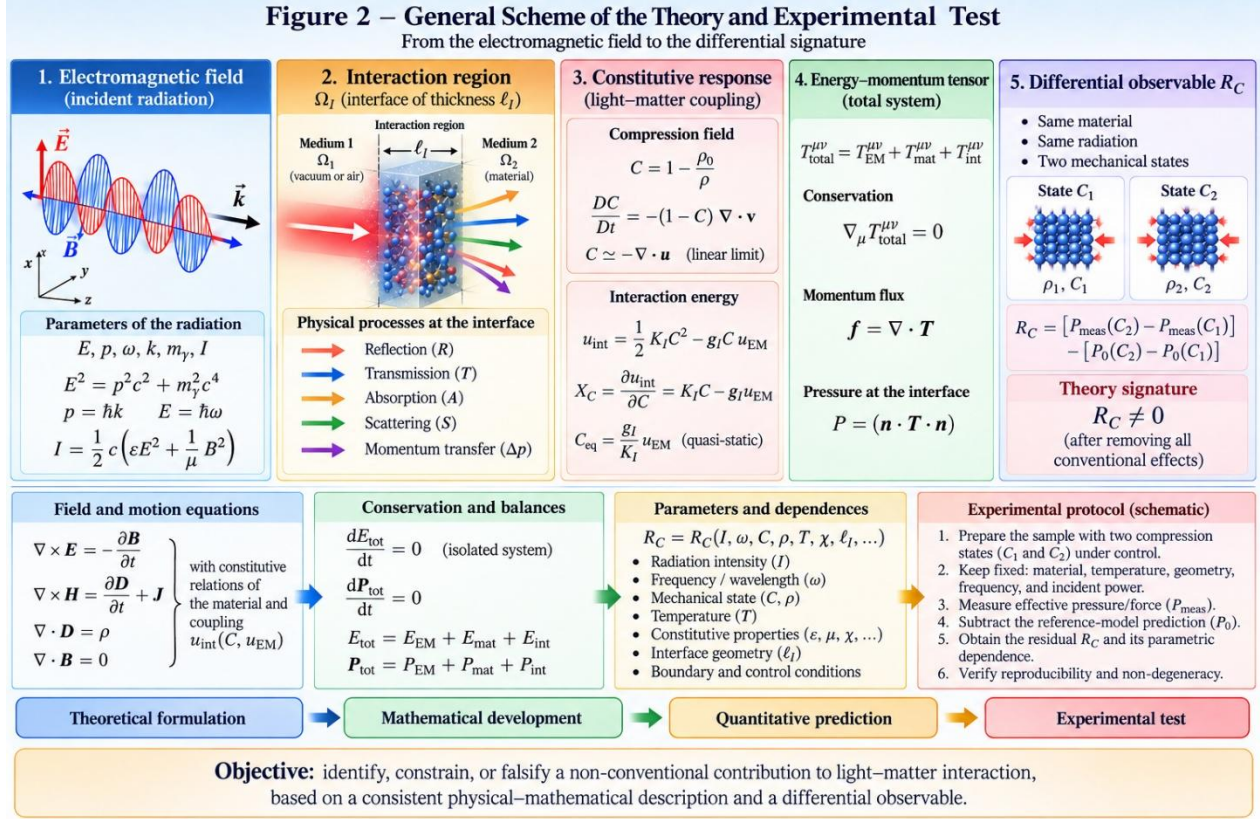


Figure 2 – Scheme of the physical-mathematical closure and the proposed differential protocol for discriminating the constitutive sector.

7. Discussion, Scope, and Limitations

The construction developed here separates two questions that are frequently conflated in effective models: the possibility of a nonzero photon rest mass and the modification of propagation caused by matter. In the first case, the alteration belongs to the free operator and persists in the medium-off limit. In the second, the alteration disappears or changes as the constitutive parameters approach their vacuum regime. This separation allows us to confront the fundamental sector with propagation and quasi-static-field tests while treating material response independently.

In the electromagnetic domain, Proca theory provides the minimal reference for a massive vector field [4]. The presence of the mass term modifies the long-range structure of the field and introduces a physical longitudinal polarization. In material media, the photon self-energy and response functions also produce modified dispersion relations, including in magnetized plasmas [5]. For this reason, we use persistence in the medium-off limit as the operational criterion for distinguishing intrinsic mass from an environmental effective mass.

The formulation also does not, by itself, establish a dynamics of radiation or matter returning to the source. In particular, $m\gamma > 0$ does not imply significant gravitational capture, energetic recirculation, or macroscopic feedback. Emission reaction expressed through global momentum conservation and possible constitutive drag are phenomena distinct from gravitational capture, mass recirculation, or stellar evolution. We keep these extensions outside the scope of this article and reserve them for a separate dynamical problem in which emission, return, capture, and escape fluxes must be modeled explicitly.

A further question of general scope remains: would a non-degenerate local signature, formulated in terms of conservative observables, remain identifiable when embedded in an open, magnetized, multiscale system? In that regime, the problem becomes one of determining whether dynamical information can survive propagation, dispersion, and energy redistribution without violating the established balance laws. The present formulation does not presume the answer; it provides the minimum criteria that any such application must satisfy.

7.1 Domain of Validity

Beyond its specific application to the photon-mass problem, the framework developed here possesses a broader methodological scope. Throughout this work, we have explicitly separated null-model contributions, constitutive responses, and genuinely discriminating observables, while requiring that any proposed additional sector satisfy identifiability, conservation, stability, and experimental consistency criteria. This construction is not restricted to electrodynamics. More generally, it provides a methodological strategy for the investigation of multiscale physical systems in which several mechanisms contribute simultaneously to the same observable and where the central scientific challenge is to determine whether an apparent residual corresponds to an independently testable physical process or merely to an alternative parameterization of already established physics.

Therefore, an additional contribution of the present work is methodological. The explicit distinction between null-model structure, auxiliary constitutive sectors, and genuinely discriminating observables provides a transferable framework for the analysis of multiscale systems characterized by strong degeneracies among physical mechanisms. Although developed here in the context of effective electrodynamics, the principles of identifiability, model reduction, and residual validation are independent of the particular physical realization considered and may

therefore be adapted to other fields in which competing transport, interaction, or propagation processes coexist.

8. Conclusions and Final Considerations

We restrict the present model to regimes in which a continuous effective description of the medium is valid and energy scales remain below the breakdown scale of the adopted constitutive parameterization. We do not extrapolate material parameters to cosmological vacuum, stellar interiors, or strong-field regimes without a specific derivation. This restriction keeps the fundamental photon hypothesis, material response, and possible astrophysical applications logically distinct.

As a direct consequence of the free sector, we derive the quadratic correction to the group velocity and the propagation delay proportional to $m\gamma^2/E^2$. We use this dependence to define a class of discriminating observables. In the material sector, we formulate the response in terms of parameters independent of $m\gamma$ and establish identifiability criteria designed to prevent plasma, polarization, dispersion, or dissipation effects from being improperly absorbed into the mass parameter.

Comparison with the experimental literature strongly constrains the parameter space. Results based on the Jovian magnetic field, the solar wind, propagation tests, and large-scale fields have different sensitivities and depend on specific auxiliary assumptions [1–3,7]. We incorporate these differences into the domain of validity rather than selecting a single bound as universally applicable to every possible realization of the model.

The resulting formulation defines a testable program and includes a non-redundancy result: in the linear isotropic regime, a local scalar term proportional to C_{u_EM} can be reabsorbed into the conventional electrostrictive response and therefore does not by itself constitute evidence for a new sector. An admissible realization must simultaneously satisfy the dispersion relation, the Maxwell limit, energy–momentum-tensor conservation, dynamical stability, the thermodynamic condition, and a nonempty intersection of the parameter regions obtained from independent experiments. We formalize this requirement through Ω_{adm} and incorporate tests of identifiability, numerical conditioning, and recovery of limiting theories. An empty intersection refutes the realization under consideration; a nonempty intersection establishes compatibility, not detection.

With this structure, we formally separate the fundamental photon problem from extensions involving astrophysical sources, open systems, and feedback processes. This separation is

necessary so that no macroscopic application uses the photon-mass hypothesis as a previously established conclusion: any connection across scales must arise only through quantitatively derived or constrained parameters, observables, and conservation laws. The present formalism is therefore self-contained within its domain of validity and provides boundary conditions for broader applications.

We also establish a three-level experimental strategy: free propagation, quasi-static fields, and the differential response of material media. At each level, we define observables, dimensionless parameters, conservation balances, and decision criteria. This structure allows the hypothesis to be constrained or excluded by negative results and prevents already-explained electromagnetic phenomena from being used as indirect confirmation.

The usefulness of the formalism does not depend on a positive detection. If propagation observables, quasi-static fields, and material response are fully described by the null model, the admissible space for m_γ and for the additional constitutive sector will be reduced, and degenerate realizations will be excluded as independent explanations. A null result therefore remains physically informative: it establishes bounds, eliminates redundant parametrizations, and identifies which additional structures would be required to produce a genuinely discriminating signature.

Therefore, We further emphasize that the present constitutive framework does not imply equivalence among distinct electromagnetic environments. Solar radiation fields, laboratory optical systems, coherent laser sources, astrophysical plasmas, and propagation through structured media are characterized by different spectral distributions, coherence properties, geometrical configurations, thermodynamic conditions, and interaction mechanisms. Consequently, constitutive parameters, effective degrees of freedom, and admissible approximations must be established within the specific physical domain under investigation. The validity of a description calibrated in one regime cannot be assumed a priori in another without an explicit derivation or independent empirical verification.

Author Contributions

XXXXXXXXXX conceived the study, developed the theoretical framework, performed the mathematical analysis, conducted the investigation, prepared the figures, wrote the original manuscript, and reviewed and edited the final version. The author has read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

Declaration of AI-assisted editorial preparation

The scientific theory, hypotheses, conceptual framework, research design, original concepts, analyses, results, and scientific interpretations presented in this work were developed by the author. AI Was used exclusively as an auxiliary tool for grammatical and spelling correction, language editing and translation, formatting, LaTeX preparation, and editorial revision. All AI-assisted editorial modifications were reviewed and approved by the author, who assumes full responsibility for the accuracy, originality, integrity, and validity of the manuscript.

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