

Geometric Model of Atomic Orbits Based on Hyperbolic Functions and Interdisciplinary Invariant of Material Stability

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Abstract

An innovative relativistic approach to calculating the energy states of atomic orbits based on a system of hyperbolic functions (the Boltian model) is proposed. It is demonstrated that Sommerfeld's fine-structure constant strictly defines the spatial angle for the first energy state, whereas the conventional Bohr quantum formula is merely a special limiting case of the derived general geometric dependence. By employing a modified expression for the half-Lorentz factor, precise agreement of theoretical calculations with fundamental constants (Rydberg constant, Hartree energy) and experimental data for multi-electron systems is achieved. A universal spatial angle of material stability ($\approx 33.26^\circ - 34^\circ$) has been discovered. It acts as a shared invariant for the stability of atomic nuclei, the spatial architecture of DNA, and geomechanical macro-systems, pointing toward a unified geometric nature of the Universe.

Keywords: *atomic orbits, Boltian model, hyperbolic functions, fine-structure constant, Lorentz factor, Rydberg constant, angle of material stability, interdisciplinary invariant.*

1 Theoretical Foundations of the Model and Hyperbolic Geometry

To calculate the energy state of atomic orbits, the Boltian model is proposed, which is based on the hypothesis that calculations must account for the specific features of the system of hyperbolic functions. In this mathematical system, the angle is geometrically restricted and cannot exceed 45° .

Within the system of hyperbolic functions, the relativistic velocity is represented by the function $\text{sh}(2n\varphi)$. The primary physical meaning of the fine-structure constant introduced by A. Sommerfeld is purely geometric—it represents the ratio of the electron's velocity in the first orbit of the atom to the speed of light:

$$\alpha = \frac{v}{c} \tag{1}$$

In the proposed model, the fine-structure constant strictly determines the spatial angle for the first energy state ($n = 1$):

$$\alpha = \text{sh}(2 \cdot 1 \cdot \varphi) \quad (2)$$

Substituting the precision experimental value from CODATA yields the fundamental constant:

$$\alpha = \frac{1}{137.035999177} = \text{sh}(2\varphi) \quad (3)$$

From this, we analytically find the base angle value for further calculations:

$$\varphi = 0.00364861961388589763 \quad (4)$$

For an arbitrary orbit n , the relativistic relationship applies:

$$\frac{v}{c} = \text{sh}(2n\varphi) \quad (5)$$

2 Calculation of Energy States and Quantum Constants

The core geometric formula for calculating the energy state at the n -th orbit is formulated as follows:

$$E(n) = [\text{ch}(2n\varphi) - 1] \cdot m_e c^2 \quad (6)$$

where the rest energy of the electron is $m_e c^2 = 510998.95$ eV.

Numerical calculation of energy using this formula for the first two levels yields:

- For the first level: $E(1) = 13.6053308558$ eV
- For the second level: $E(2) = 54.422047906$ eV

Their combinational difference is:

$$\Delta E = E(2) - 2E(1) = 27.2113861946 \text{ eV} \quad (7)$$

The resulting value coincides with high precision with the theoretical Hartree energy ($E_H = 27.2113862459$ eV).

The proposed mathematical framework enables the high-precision calculation of energy states for any n -th orbit, for example:

$$E(5) = 340.16949689 \text{ eV} \quad (8)$$

$$E(7) = 666.803221946 \text{ eV} \quad (9)$$

3 Modified Lorentz Factor and Fine Structure

For complete relativistic consistency, the model incorporates a modified expression for the half-Lorentz factor k , which carries profound physical significance:

$$k = \sqrt{\frac{2}{1 + \sqrt{1 - (\text{sh}(2n\varphi))^2}}} \quad (10)$$

For the hydrogen atom at $\alpha = \text{sh}(2\varphi)$, the expansion of this coefficient takes the form:

$$k = \sqrt{\frac{2}{1 + \sqrt{1 - \alpha^2}}} \approx 1 + \frac{1}{8}\alpha^2 \quad (11)$$

This expression describes the classic Sommerfeld–Dirac fine splitting. Accordingly, squaring it yields the fine structure parameter:

$$k^2 = \frac{2}{1 + \sqrt{1 - \alpha^2}} \approx 1 + \frac{1}{4}\alpha^2 \quad (12)$$

Under an external magnetic field, the energy splitting is governed by the equation:

$$E = E(1) \cdot k^2 \cdot \left(1 \pm \frac{\mu B}{E(1)}\right) = E(1) + \frac{1}{4}\alpha^2 \cdot E(1) \pm \mu B \pm \frac{1}{4}\alpha^2 \cdot \mu B \quad (13)$$

This formula simultaneously and explicitly reveals the fine structure, the classical Zeeman effect, and the relativistic Zeeman effect.

4 Connection with the Rydberg Constant and Spectral Anomalies

The coefficient of the half-Lorentz factor can affect the energy state in different powers (1 and 2). Let us calculate the state energy for the first orbit at higher powers of the operator k .

At $k^2 = 1.000013313193087$, we obtain:

- $E = E(1) \cdot k^2 = 13.60551198620 \text{ eV}$
- $E = E(1) \cdot k^4 = 13.605693119010529 \text{ eV}$

According to the latest CODATA fundamental data, the Rydberg constant is $E_R = 13.60569312299 \text{ eV}$. The deviation of the calculated value from the experimental one is minimal:

$$\Delta E = E_R - E = 3.97947 \cdot 10^{-9} \text{ eV} \quad (14)$$

For most energy states, the general relationship is defined as $E(v) = E(n) \cdot k$. For instance, for the Boron ion:

$$E(v) = E(5) \cdot k = 340.2260856436 \text{ eV} \quad (15)$$

which directly correlates with the experimental value $E = 340.22580 \text{ eV}$. A specific quantum anomaly is observed exclusively for Helium, where the dependency becomes inverted:

$$E(v) = \frac{E(2)}{k} \quad (16)$$

5 Radiation Formula and Limiting Transition to the Bohr Model

The general formula for calculating radiation energy within the geometric model has the following form:

$$E_{\text{rad}} = \left[\text{ch} \left(\frac{2n\varphi}{n^2} \right) - 1 \right] \cdot m_e c^2 = \left[\text{ch} \left(\frac{2\varphi}{n} \right) - 1 \right] \cdot m_e c^2 \quad (17)$$

We can show that the universally accepted Bohr quantum formula is merely a special case (the first term of the Taylor series expansion) of this general hyperbolic relationship:

$$E_{\text{rad}} = \left[\text{ch} \left(\frac{2\varphi}{n} \right) - 1 \right] \cdot m_e c^2 \approx \left[1 + \frac{1}{2} \left(\frac{2\varphi}{n} \right)^2 - 1 \right] \cdot m_e c^2 = \frac{1}{2} \left(\frac{2\varphi}{n} \right)^2 \cdot m_e c^2 = \frac{13.6}{n^2} \text{ eV} \quad (18)$$

where $2\varphi = \alpha$. In this case, the exact ratio between the state energy and the radiation energy scales rigidly through the principal quantum number:

$$\frac{E(n)}{E_{\text{rad}}} = n^4 \quad (19)$$

6 Spatial Mass Correlations and Stability Limits of Nuclear Structures

Since the model is based on a purely geometric construction of physical relationships, it allows us to analytically express the fundamental mass ratio of the proton to the electron through the fine-structure constant:

$$\frac{m_p}{m_e} = \frac{4\pi + \phi^*}{\alpha} = 1836.145 \quad (20)$$

where α is the fine-structure constant, and the sum $(4\pi + \phi^*) = 13.3989996$ represents a dimensionless constant from high-energy physics ($e^{13.3989996}$).

From this ratio, we determine the anomalous angular shift:

$$\phi^* = 0.832628985640827 \quad (21)$$

Dividing it by the fundamental angle of the model φ gives:

$$\frac{\phi^*}{\varphi} = 228.203834 \quad (22)$$

The resulting number directly points to the region of transuranic elements, specifically to the 114th element of the periodic table (Flerovium). Through a trigonometric transformation, we determine the critical angle of energy translation:

$$228.203834 \cdot x \Rightarrow 2\text{arctg}(e^{\phi^*}) - 90^\circ = 42.991129^\circ \quad (23)$$

This angle correlates with the half-life and stability of superheavy nuclei around the "island of stability":

- For **element 114**, the lifetime is **2.6 seconds**;
- For **element 115** (Moscovium), the lifetime drops sharply to **0.22 seconds** (the geometric angle exceeds 43°);
- For **element 118** (Oganesson), the lifetime is only **0.9 milliseconds** (the angle exceeds 44°).

The breaking point, where the stable closed geometry of the nucleus begins to irreversibly radiate energy (natural radioactivity), is clearly traced at the boundary of stable isotopes. The calculation for Lead ($n = 82$) using the relativistic connection formula $E(v) = E(82) \cdot k$ yields a limiting spatial angle of 33.26° .

7 Interdisciplinary Invariants of Material Stability

The discovered critical angle of material stability ($\approx 33.26^\circ - 34^\circ$) acts as a universal code of spatial architecture in the Universe, manifesting self-similarly at the micro-, macro-, and bio-levels of organization:

- **Micro-world (Nuclear Physics):** Upon exceeding an angle of 33.56° (the calculated threshold for Lead is 33.26°), the atomic nucleus loses its capacity to hold internal energy in a static state, triggering the mechanisms of radioactive decay in heavy elements.
- **Bio-world (Genetics):** The DNA molecule in its most conformationally stable B-form utilizes precisely an angle of $33^\circ - 34^\circ$ for the inclination of its helical structure, balancing on the upper limit of spatial stability without risking structural disruption of the strands.
- **Macro-world (Geomechanics):** The angle of repose for dry sand, granular media, and disperse soils is $33^\circ - 34^\circ$. When trying to forcibly exceed this angle, the internal friction forces break down, and the macro-system transitions abruptly into kinetic motion (landslide, avalanche formation).

8 Conclusions

1. **Relativistic effects of the micro-world represent the natural geometry of non-Euclidean space.** The half-Lorentz factor in the proposed framework functions as a geometric transition operator between interconnected systems of hyperbolic functions.
2. The proposed **Boltian model**, based on the system of hyperbolic functions, allows for the precision description of the energy states of atomic orbits via fundamental geometric constants. It is demonstrated that the classical Bohr quantum formula is merely its simplified limiting case.
3. The existence of a single angular invariant ($33.26^\circ - 34^\circ$) for the stability of atomic nuclei, the topology of the DNA molecule, and the mechanical stability of macro-systems proves a common geometric foundation for the organization of matter across all scales of the Universe.

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