

Making the gate: moving walkways from Galileo to Kerr

James M. Chappell, David L. Berkahn and Derek Abbott

*College of Engineering and Information Technology, School of
Electrical and Mechanical Engineering, Adelaide University, SA 5005,
Australia^{a)}*

(Dated: 25 September 2026)

Terence Tao's airport puzzle asks where a late passenger should spend a short burst of running, or a short pause to tie a shoe, when part of the terminal is a moving walkway. The Galilean answer is to run on the floor and to pause on the belt. The same comparison survives special relativity, with an extra Lorentz factor that makes the belt even less useful as a place to sprint. We recast the belt as the frame-dragging current of a Kerr black hole and ask where to site a terminal if the passenger's proper-time budget is fixed and the gate clock is a static clock in the spacetime. The pedagogical conclusions are: in order to have static concourses we need to site them outside the equatorial ergosphere; lay optional walkways along the zero-angular-momentum congruences; spend the running burst on the static floor; and place the lounges higher up the gravitational potential and not place the gate on the photon orbits. The paper is intended for an undergraduate course that has met special relativity and a first encounter with the Kerr geometry.

Keywords: special relativity, Kerr geometry, frame dragging, pedagogy, optimization

^{a)}Electronic mail: james.chappell@adelaide.edu.au

I. INTRODUCTION

A familiar race-to-the-gate problem, posed by Tao after a long walk through an airport terminal,¹ runs as follows. The path from A to B is a line. Some stretches are ordinary floor; some are moving walkways (referred to as travelators or belts) of speed u pointing toward the gate. The passenger walks at speed v relative to the local surface and can run at speed $v' > v$ for a limited time τ , or must pause for a time τ to tie a shoe. Which surface should host the run, and which the pause, if the aim is to minimise arrival time at B ?

The problem is a clean exercise in allocating a scarce resource—time spent at a different speed—across two backgrounds. It has been discussed in popular mathematics^{2,3} and is a natural homework problem in a first mechanics course. Tao also asked whether special relativity changes the answer.¹ It does not change the ranking. It does change the size of the gap.

Once the special-relativistic case is in hand, the same bookkeeping applies in a stationary spacetime. The “belt” is any congruence that carries the passenger toward the gate; the “floor” is any congruence that does not. Around a rapidly rotating gravitational source a natural travelator is the zero-angular-momentum-observer (ZAMO) congruence generated by general relativistic frame dragging. The space naturally rotates observers around the hole, at the local frame dragging rate, without imparting angular momentum.^{5,6} The natural floor is a family of static observers, which exists only outside the ergosphere (inside the ergosphere they are forced to co-rotate due to the strength of the frame dragging).

The remainder of the paper works this analogy in three stages. Section II records the Galilean calculation. Section III repeats it with Einstein velocity addition and a proper-time budget. Section IV places a terminal in the Kerr geometry, identifies the ZAMO travelator, and asks how close to the hole the building may sit. Section IV F states the resulting race-to-the-gate strategy. Section VI lists the reasons this is a teaching device and not a viable architectural proposal.

II. THE GALILEAN WALKWAY

Let the floor have length L_f and the belt length L_b , both measured in the airport frame. If we simply walk all the way to the gate, it will take time

$$T_0 = \frac{L_f}{v} + \frac{L_b}{v + u}. \quad (1)$$

Spend a burst of duration τ running on one surface and walk the rest, assuming the burst fits on that surface:

$$T_{\text{floor}} = T_0 - \frac{(v' - v)\tau}{v}, \quad (2)$$

$$T_{\text{belt}} = T_0 - \frac{(v' - v)\tau}{v + u}. \quad (3)$$

The time saved is larger on the floor because the distance saved due to running $(v' - v)\tau$ has more significance, over a slower baseline speed, used in the denominator. Another way to see the advantage, is that running multiplies the local speed by v'/v on the floor and by $(v' + u)/(v + u)$ on the belt, and so

$$\frac{v'}{v} > \frac{v' + u}{v + u} \quad (4)$$

whenever $u > 0$ and $v' > v$. The two arrival times coincide only in the degenerate cases $\tau = 0$, $u = 0$, or $v' = v$.

A pause of duration τ (shoe-tying) is conserving the inverse resource. Standing still on the floor covers nothing, whereas, standing still on the belt still covers $u\tau$. The pause therefore should be done on the belt. For example, for the extreme case of standing still on the floor for an unlimited time, you will never reach the gate, whereas standing still on the belt, you will still get to the gate at the speed of the travelator. A compact memory aid, that might be useful in class, is: *go fast on the slow surface; go slow on the fast surface.*²

If the belt runs backward at exactly walking speed, $u = -v$, walking on it produces zero ground speed and the passenger never finishes the belt after the burst ends. That singular case is excluded from Tao's original co-moving walkways and is mentioned only to show where Eq. (3) divides by zero.

III. SPECIAL RELATIVITY

The quantity to minimise is still airport-frame coordinate time: that is the clock the flight leaves by.¹ The natural model of “energy to run for τ seconds” is a fixed burst of passenger proper time τ . This is also called wristwatch time and properly represents the energy exerted by the passenger themselves.

If the belt moves at u and the passenger moves at w relative to the belt, the ground speed, given by relativistic velocity addition using units where $c = 1$, is

$$W(u, w) = \frac{u + w}{1 + uw}. \quad (5)$$

Write $W = W(u, v)$ and $W' = W(u, v')$ for the walking and running cases. The Lorentz factor of a composed velocity is

$$\gamma(W(u, w)) = \gamma(u) \gamma(w) (1 + uw). \quad (6)$$

Walking everywhere now takes

$$T_0 = \frac{L_f}{v} + \frac{L_b}{W}. \quad (7)$$

A proper-time burst τ of running on the floor saves

$$\Delta T_{\text{floor}} = \frac{\gamma(v')(v' - v)\tau}{v}, \quad (8)$$

while the same burst on the belt saves

$$\Delta T_{\text{belt}} = \frac{\gamma(v')(v' - v)\tau}{\gamma(u)(v + u)}. \quad (9)$$

Their ratio,

$$\frac{\Delta T_{\text{floor}}}{\Delta T_{\text{belt}}} = \gamma(u) \left(1 + \frac{u}{v}\right), \quad (10)$$

is independent of the running speed and is strictly greater than one for $u > 0$, $v > 0$. The Galilean factor $1 + u/v$ is now multiplied by $\gamma(u)$. A fast belt is already cheap in airport time; converting walk to run on it is worth even less than in the Galilean assessment. In the ultra-relativistic limit $u \rightarrow c = 1$, $\Delta T_{\text{belt}} \rightarrow 0$: running on a lightlike belt does not move the departure board.

The shoe-tie dual is sharpened by time dilation. A pause of proper time τ on the belt lasts $\gamma(u)\tau$ on airport clocks and still covers distance $\gamma(u)u\tau$. The belt remains the best place to stand still.

IV. THE KERR TERMINAL

A. Introduction to the Kerr solution

Kerr spacetime is the geometry outside a spinning mass m with spin parameter $a = J/m$, $|a| \leq m$. We work in Boyer–Lindquist coordinates and, for the terminal, stay on the equator $\theta = \pi/2$. There the line element is

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 - \frac{4ma}{r} dt d\phi + \frac{r^2}{\Delta} dr^2 + r^2 d\theta^2 + \left(r^2 + a^2 + \frac{2ma^2}{r}\right) d\phi^2, \quad (11)$$

with $\Delta = r^2 - 2mr + a^2$. The cross term $g_{t\phi} = -2ma/r$ means that spacetime itself has an angular velocity relative to the distant stars,

$$\omega = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{2ma}{r^3 + a^2r + 2ma^2}. \quad (12)$$

That is the travelator. Completing the square in t and ϕ makes the same fact visible with the $d\phi - \omega dt$ term

$$ds^2 = -N_z^2 dt^2 + g_{\phi\phi} (d\phi - \omega dt)^2 + \frac{r^2}{\Delta} dr^2 + r^2 d\theta^2, \quad (13)$$

where the ZAMO lapse on the equator is

$$N_z(r) = \sqrt{-g_{tt} + g_{t\phi}^2/g_{\phi\phi}} = \sqrt{\Delta/g_{\phi\phi}}. \quad (14)$$

Two families of observers now play the role of floor and belt.

a. Static floor, $\Omega = d\phi/dt = 0$. These observers remain facing the same distant stars. They exist only where $g_{tt} < 0$, which on the equator is $r > r_E = 2m$ —outside the ergosphere, and independently of the spin of the source. Their clocks run at the static lapse

$$N_s(r) = \frac{d\tau}{dt} = \sqrt{-g_{tt}} = \sqrt{1 - \frac{2m}{r}} \quad (15)$$

relative to Killing time t , or the airline master clock at infinity.

b. ZAMO belt, $\Omega = \omega$, $L = u_\phi = 0$. Zero-angular-momentum observers ride the current (12). In the static floor's local frame that is a boost in the $+\phi$ direction at speed

$$u = \frac{2ma}{r\sqrt{\Delta}}, \quad (16)$$

so $u \rightarrow c$ as $r \rightarrow 2m$, and $u = c/6$ at the working radius $r = 4m$, assuming maximal spin $a = m$. A passenger with $d\phi = \omega dt$ is at rest on the belt. At the same radius the ZAMO

lapse N_z is slightly larger than N_s ($N_s \approx 0.707$, $N_z \approx 0.717$ at $r = 4m$, $a = m$): a belt wristwatch runs a little faster compared with a static clock on the floor.

Killing time t is the analog of airport-frame coordinate time in Sec. III. A pause of proper time τ *costs* $\Delta t = \tau/N$ of gate time, so the shoe-tie belongs where N is large. A running burst of the same τ *saves* more airport time where N is small, or where the passenger climbs against the drag—the same “run on the slow surface” rule as Sec. III. Holding a ring at fixed r still takes radial support, a closed structural ring or rockets, whether the deck is static or ZAMO. The geometry supplies only the azimuthal current. Importantly, we note that the frame dragging effect is akin to the Coriolis effect, and so it only arises when the passenger moves.

B. Geometry

The Kerr line element in Boyer–Lindquist coordinates describes a mass m and spin parameter $a = J/m$, with $|a| \leq m$.^{4,5,7} Several radii are important to consider if siting an airport terminal.

The outer event horizon (the point of no return) is at

$$r_+ = m + \sqrt{m^2 - a^2}. \quad (17)$$

When $a = 0$ (no rotation), $r_+ = 2m$, the standard Schwarzschild radius, and at maximal spin $a = m$, the inner and outer Kerr horizons coincide at $r_+ = m$. The outer event horizon, is the boundary where worldlines cannot escape to infinity. The equatorial ergosphere, outside which static (non orbiting) observers can exist, is at $r_E = 2m$. The innermost stable circular orbit (ISCO) for equatorial massive geodesics is⁵

$$\frac{r_{\text{ISCO}}}{m} = 3 + Z_2 \pm \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}, \quad (18)$$

with

$$\begin{aligned} Z_1 &= 1 + (1 - \chi^2)^{1/3} [(1 + \chi)^{1/3} + (1 - \chi)^{1/3}], \\ Z_2 &= \sqrt{3\chi^2 + Z_1^2}, \end{aligned} \quad (19)$$

and $\chi = a/m$. The upper sign is retrograde, the lower sign prograde (tighter orbit). Thus r_{ISCO} runs from $6m$ at $\chi = 0$ down to m for prograde orbits as $\chi \rightarrow 1$, and out to $9m$ for

Extremal Kerr, $\chi = 1$ ($a = m$)

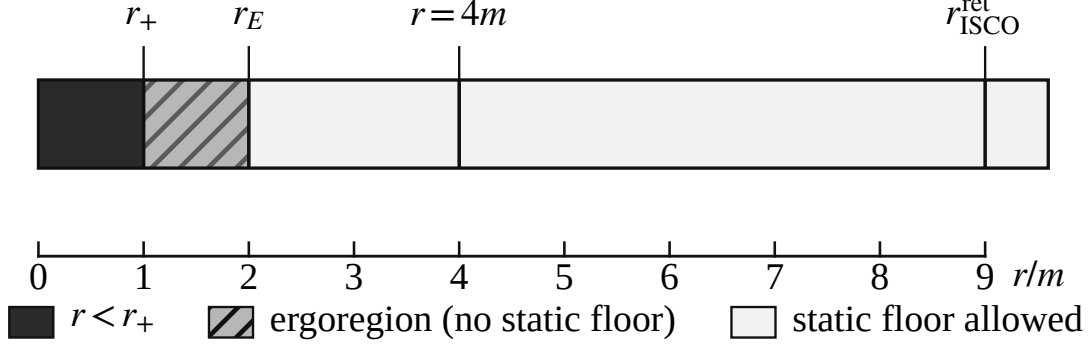


FIG. 1. Important equatorial radii to consider when siting a terminal. All worldlines inside the event horizon r_+ are trapped, and so this region needs to be strictly off-limits to passengers and staff, but would make an ideal waste disposal system. We need to be outside $r_E = 2m$, in order to have a static floor in the terminal. The ISCO ranges (stable orbits) from $r = m$ when prograde but requiring at least $r = 9m$ when retrograde. The photon orbits range from $r = m$ when prograde, out to $r = 4m$ when retrograde, and hence, the terminal should sit outside this radius to avoid extreme lensing effects. The magnitude of the tidal forces also needs to be a design consideration, for passenger comfort.

retrograde orbits. However, Thorne's photon-capture argument limits $\chi \approx 0.998$, so that the minimum prograde ISCO is then pushed out to $1.24m$.^{8,9}

The light rings are where photons can technically be in orbit. However, circular photon motion is unstable and will tend to decay or escape to infinity. The optics of the photon region are unusable as signage due to extreme lensing.⁵ Looking retrograde at $r = 4m$, you are staring down the light ring and so could see the back of your own head. Looking prograde you are not. Frame-dragging also aberrates and Doppler-shifts the two azimuths differently, so brightness, color, and angular size of the next gate do not match. A lounge with windows both ways would present two different airports: a planetarium of repeated gates one way, a single concourse the other.

Extremal Kerr, $\chi = 1$ ($a = m$)

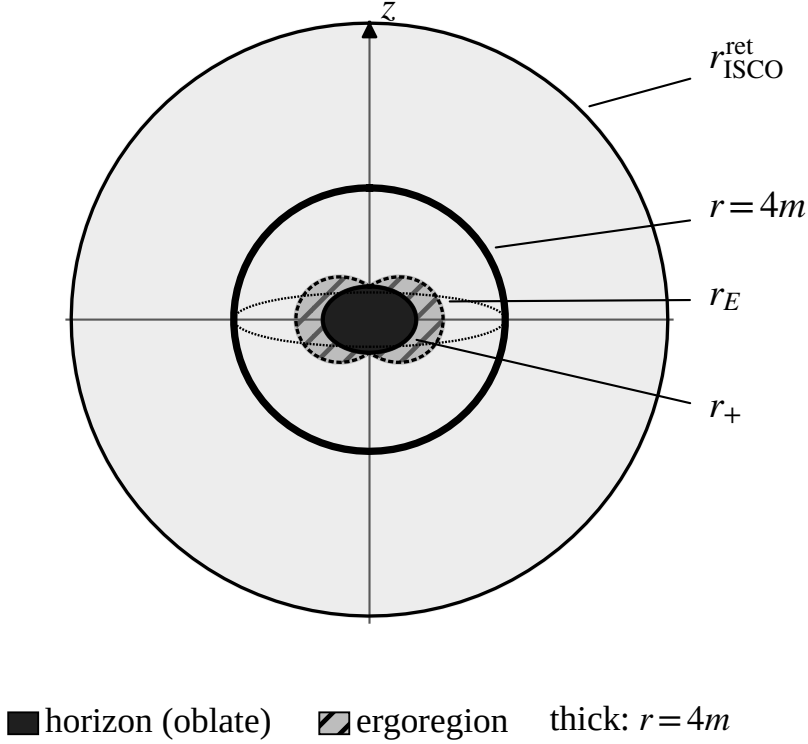


FIG. 2. Meridional section for extremal Kerr, $\chi = 1$ ($a = m$). The event horizon r_+ is an oblate ellipse (shaded black). The dashed ergosurface meets the horizon at the poles and reaches $r_E = 2m$ on the equator; static observers exist only outside it. The thick curve is the proposed terminal location $r > 4m$. Retrograde circular geodesics are stable only outside $r_{\text{ISCO}}^{\text{ret}} = 9m$, whereas prograde ones are stable down to $r_{\text{ISCO}}^{\text{pro}} = m$. Photon orbits lie between $r = m$ (prograde) and $r = 4m$ (retrograde). Siting should be just outside $r = 4m$ to avoid optical issues with the retrograde photon orbit.

C. Frame dragging as a walkway

The metric component $g_{t\phi}$ (on the equator) implies that spacetime itself has angular velocity $\omega(r)$, as shown in Eq. (12), which is a natural travelator, as measured relative from infinity. ZAMOs will follow that current^{5,6}, provided the walkway boosts them to this velocity and provides radial support. The difference between the static observers and the ZAMO rotating frame, is a travelator the passenger can step onto.

Now, a ring-shaped terminal held fixed with respect to the distant stars is not spun up by the frame dragging: the gravitomagnetic force on an initially static object is purely radial, so the ring is not spun up; the current just streams past underneath it.¹⁰ This type of terminal can therefore provide a static floor. Optional moving walkways laid along the ZAMO congruence are then literal travelators, with no motors beyond the geometry, provided the passengers are boosted into this frame. This boost is therefore analogous to the boost passengers currently feel when stepping onto a regular airport travelator.

We wish to place the terminal safely away from the dangerous radii, and so, as a design choice, we might select the radius $r = 4m$ with maximal spin, which is at twice the radius of the ergosphere r_E . We then find the ZAMO travelator speed, at $r = 4m$, of

$$u = \frac{2ma}{r\sqrt{\Delta}} = \frac{1}{6}, \quad (20)$$

with $\Delta = r^2 - 2mr + a^2 = 9m^2$. This is about 50,000 km/s, and is the locally measured speed of the ZAMO travelator relative to the static floor. That is a very fast belt even by modern airport standards and a significant velocity u in the special-relativistic formulae of Sec. III. Using the lapse for the ZAMO observer, as shown in Eq. (14), at $r = 4m$, we find the lapse $N_z = 0.7171$ or a 1.4% faster clock on the belts, compared with the static lapse $N_s = \sqrt{1 - \frac{2m}{r}}|_{r=4m} = \frac{1}{\sqrt{2}} \approx 0.7071$. This slightly faster clock on the moving belt is a minor relativistic correction to the main saving of the ZAMO observers on the belt.

A proper-time burst spent where N_z is smaller saves more airport time, which is the same “run on the slow surface” rule with gravity as part of the background.

Kerr spacetime never changes, whenever we look, the hole is still the same shape, still spinning the same way. That “nothing about the geometry depends on the time label” is a symmetry. The time coordinate that runs along that symmetry is called Killing time, t , after the mathematician Wilhelm Killing. Far from the hole, Killing time is ordinary time on a clock that sits still relative to the distant stars. It is the airports master clock at infinity, although we allow the airlines to use a different local gate boarding time.

D. How to step onto the ZAMO belt

On the static floor you face the same distant stars. Your angular velocity is $\Omega = 0$. Frame dragging is trying to sweep you along, so you are continuously thrusting against the swirl.

As measured from infinity you actually have negative angular momentum. A ZAMO has $L = 0$ and angular velocity $\Omega = \omega$, the local dragging rate. You are then co-rotating with the current, so the azimuthal thruster can be switched off. Getting onto the belt means adding angular momentum until L hits zero. In the static floor's local frame that is a boost in the $+\phi$ direction, up to the belt speed. The paper's architecture is two rings at nearly the same radius: an outer, star-fixed floor (the terminal), an inner walkway that slides around it at speed ω .

You transfer from one to the other. Geometry is the motor after that: once you co-move with the ZAMOs, you are just riding the current.

E. How close may the building sit?

Three concentric constraints apply for locating the terminal, around a maximal spin $a = m$ source.

1. *Outside the ISCO for a stable geodesic docking area.* This is because circular massive motion is only stable outside the ISCO with $r_{\text{ISCO}} = m$ for prograde orbits. Inside the ISCO an orbiting free-falling spaceplane plunges.
2. *Outside the ergosphere for a static floor.* Static observers only exist outside $r_E = 2m$ on the equator, and the limit r_E is independent of the spin. Therefore, a terminal that wants both a floor and a belt will need to sit outside r_E . Frame dragging is a velocity dependent effect (similar to the Coriolis effect) and so provided the radial position is held constant, there is no azimuthal force. Hence, it will need rockets (or form a closed structural ring around the source) in order to hold a constant radial position.
3. *Outside the photon region.* The gate must remain visible and so extreme lensing makes this region unusable, and so require $r > 4m$. Lightlike belts are the $u \rightarrow c$ limit of Sec. III: running there does not move the gate clock.

Note that a supermassive rotating source will be required if anyone is to be actually able to physically walk along the concourse. Tidal accelerations scale as M/r^3 and are survivable near $r \sim 2m$ only when M is of galactic-nucleus size.¹¹ We find that we can design a workable terminal around intermediate-mass black holes, with masses $10^4 M_\odot$.

The estimate below is the usual radial stretch across a $h = 2$ metre person,

$$\Delta a \approx \frac{2mh}{r^3}. \quad (21)$$

Kerr corrections change the prefactor by an order-unity amount depending on spin and whether the observer is static or ZAMO. They do not change which mass range hurts.

HEAD-TO-TOE STRETCH, IN g

mass	at $2m$	at $4m$	at $10m$
$10 M_\odot$ (stellar)	$2.1 \times 10^7 g$	$2.6 \times 10^6 g$	$1.7 \times 10^5 g$
$10^3 M_\odot$	$2100 g$	$260 g$	$17 g$
$10^4 M_\odot$	$21 g$	$2.6 g$	$0.17 g$
$10^5 M_\odot$	$0.21 g$	$0.026 g$	$0.002 g$
Sgr A* $4.3 \times 10^6 M_\odot$	$1.1 \times 10^{-4} g$	$1.4 \times 10^{-5} g$	$9 \times 10^{-7} g$

A stellar-mass hole appears unworkable for an airport—at a radius of $4m$ around a mass of $10 M_\odot$ the tidal stretch is a few million g ; you are turned to spaghetti tens of thousands of kilometres outside any of these radii.

The intermediate-mass black holes, the HLX-1 / M82 X-1 class, of around **$10^4 M_\odot$** appear most suitable, the tides are tolerable, and so can naturally become part of the architecture.

The terminal is also a feasible size for a closed circular architecture. For $10^4 M_\odot$, $m \approx 15,000$ km, so $r = 4m$ is about 59,000 km—nine Earth radii. Sgr A* at the same r/m is 0.17 AU.

F. Making the gate

The objective is to arrive before a fixed reading of the gate clock. The passenger has a proper-time budget τ at running speed.

The optimal policy is the same at every level of the analogy.

1. Walk, do not run, on any ZAMO section heading toward the gate.
2. Spend the entire burst on the static floor, or on any stretch that must be climbed against the lapse or against the drag.

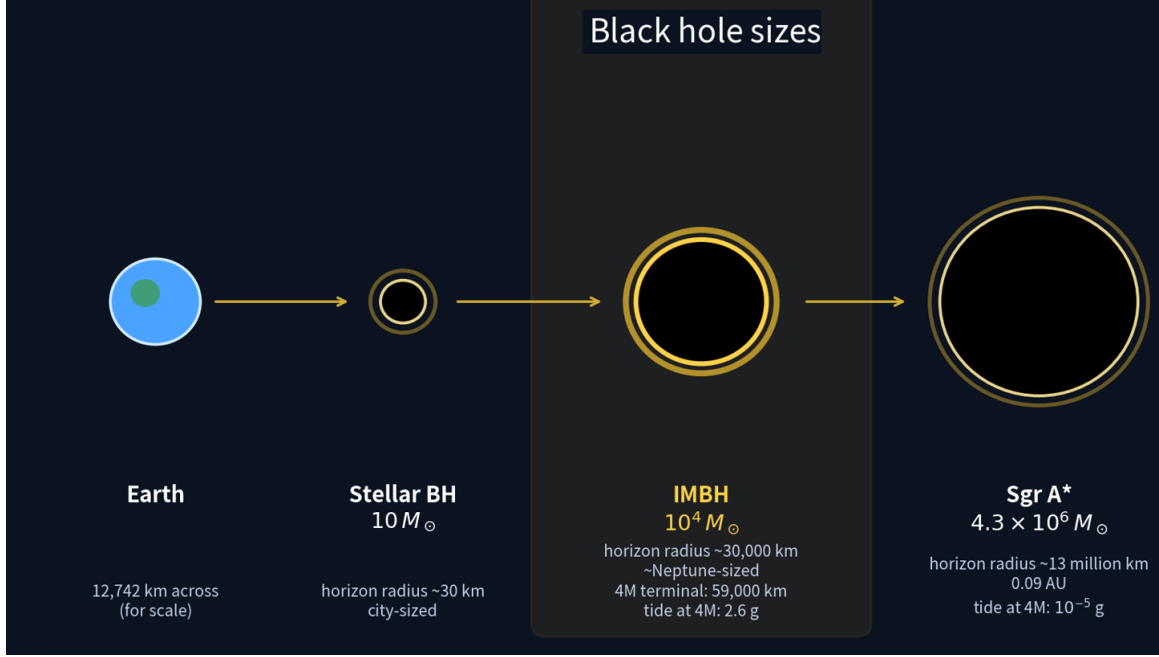


FIG. 3. Stellar mass black holes have very large tidal forces for any nearby terminal location. The intermediate mass black holes appear the most suitable with acceptable tidal forces of 2.6 g at $r = 4m$. Sgr A*, the black hole at the centre of the milky wave has very small tidal forces but with its large size, it would not be possible to build a circular terminal around it.

3. If a pause is required, take it on the ZAMO travelator, where the geometry continues to carry the passenger.
4. Do not enter a retrograde belt, and definitely do not enter a belt with $u = -v$ in the local static frame: walking there cancels the current and the remaining belt is never finished.

A short calculation that can be left as an exercise: for a static observer the coordinate time saved by a local running burst of proper time τ on a patch of lapse N scales as $\Delta t \propto 1/N$. The burst belongs where N is smallest among the patches the passenger will in any case traverse on foot.

V. FURTHER DESIGN DETAILS

A. Siting airport lounges

A Kerr airport lounge is not just a place for a comfortable seat. Its key job is to give passengers as much proper time τ as possible before a departure time that is fixed on the gate clock. For a static observer the lapse N_s increases with r and is largest far from the hole, as shown in Eq. (15). Motion relative to the static frame multiplies by a further γ^{-1} , so lounges conserve the most time if they are static. The lounge therefore belongs on the high side of the terminal: larger r than the jetway, on the static floor.

This is the opposite of the sprint rule and the opposite of loitering near the horizon.^{13,15}

If the gate is bolted at radius r_g , a lounge at $r_L > r_g$ yields

$$\tau_L = N_s(r_L) \Delta t > N_s(r_g) \Delta t = \tau_{\text{gate}} \quad (22)$$

for the same wait Δt in Killing time. Passengers age slightly more, sleep slightly longer, but more importantly, finish their drinks. The gain is modest unless the lounge sits at a distinctly larger r than the gate. Hence, the first-class lounge would be on the highest floor, sitting several levels “up” the potential. At a given floor level the ZAMO clocks are the faster ones, and so we put the lounge at a fixed non-rotating position.

The dual statement is the shoe-tie again. A fixed proper interval spent in a deep lounge consumes more Killing time and can make one late. That is, we have the simple design rule: do not put the bar next to the horizon.

Airport	Kerr terminal
Floor (slow baseline)	Static observers, $\Omega = 0$
Belt (fast baseline)	ZAMO congruence, $\Omega = \omega(r, \theta)$
Run where extra distance is expensive in clock time	Spend τ where the lapse is small
Pause on the belt	Pause on the ZAMO rail; geometry keeps carrying you

The angular velocity of circular *geodesics*

$$\Omega = \pm \frac{\sqrt{m}}{r^{3/2} \pm a\sqrt{m}}, \quad (23)$$

stable only outside the ISCO.

B. Where to put the gates

The lounge maximises passenger proper time. The gate defines the deadline. Whether a deeper gate helps depends on which clock the airline uses.

If last call is a reading of a master clock at infinity, or of any other clock that does not move with the gate, then putting the jetway closer to the horizon does nothing to the deadline. The board still flips at a fixed t . A slow clock on the wall is only decor.

If instead the aircraft pushes back when the *local* gate clock reads a booked time, a deeper gate does buy coordinate time. A static clock at radius r advances as

$$d\tau_{\text{gate}} = N_s(r_g)dt = \sqrt{-g_{tt}(r_g)} dt. \quad (24)$$

Smaller r means smaller $\sqrt{-g_{tt}}$, so that clock reaches “10:00” at a later t . Passengers waiting in an outer lounge then enjoy

$$\tau_L = \frac{N_s(r_L)}{\sqrt{-g_{tt}(r_g)}} \tau_{\text{gate}} > \tau_{\text{gate}} \quad (25)$$

before the same local last call. The gate has been turned into a shoe-tie: the deadline itself has been parked on the slow surface.

That trick has costs. The last walk is inward, where $dt = d\ell/(Nv)$ is expensive unless the inbound path is a belt—ZAMO, or a descending radial ride. Inside the ergosphere there is no static clock at all; a gate there can only keep ZAMO time. Problems with redshift of the public-address system, extreme lensing and the presence of photon orbits remain as in Sec. IV—a gate on the photon orbit, is still a bad jetway.

The clean pedagogical layout is therefore stacked in radius: lounges at large r on the static deck (fast clocks, more local time); concourse and travelators using frame dragging in the middle; gates inward of the lounge, but outside m and the ISCO.

It is easier for spaceplanes to dock and depart if the gates are an orbiting geodesic outside the ISCO. There is also an additional relativistic time dilation advantage. Also passengers do not need to walk to the gate, but simply wait for the plane to move under the lounge and take the elevator down.

Airlines should not be allowed to promote a near-horizon gate as “more time to board” unless the actual closing time is given by the clock at the gate.

There should also be a critical warning for all arriving passengers: climbing out of a Thorne-limit potential well is a rocket problem, not a taxi problem.

C. Classroom use

The Galilean calculation belongs in a first mechanics course, immediately after constant-velocity kinematics. Section III is a single problem set once students have Einstein velocity addition and the Lorentz factor. Section IV is a first application of Kerr after the Schwarzschild solution is introduced: students are not asked to derive the metric, only to read ω , N_s and N_z off it and calculate relative clock rates and to decide whether a given location admits a static floor.

a. Worked example. Take a floor and a belt of equal length $L_f = L_b = 300$ m, walking speed $v = 1.5 \text{ m s}^{-1}$, running speed $v' = 3.0 \text{ m s}^{-1}$, belt speed $u = 1.5 \text{ m s}^{-1}$, and a running burst of duration $\tau = 20$ s that fits on either surface. Walking all the way takes

$$T_0 = \frac{L_f}{v} + \frac{L_b}{v+u} = 200 \text{ s} + 100 \text{ s} = 300 \text{ s}. \quad (26)$$

Running on the floor saves $(v' - v)\tau/v = 20$ s; running on the belt saves $(v' - v)\tau/(v + u) = 10$ s. Arrival times are 280 s and 290 s. The ratio of the savings is $1 + u/v = 2$, independent of v' .

Now repeat the allocation in units with $c = 1$, taking dimensionless speeds $v = 0.20$, $v' = 0.40$, the Kerr travelator speed of Sec. IV, $u = 1/6$, and a unit proper-time burst $\tau = 1$. Equations (8)–(10) of the paper give

$$\Delta T_{\text{floor}} = 1.091, \quad \Delta T_{\text{belt}} = 0.587, \quad \frac{\Delta T_{\text{floor}}}{\Delta T_{\text{belt}}} = \gamma(u) \left(1 + \frac{u}{v}\right) = 1.859. \quad (27)$$

The Galilean factor $1 + u/v = 1.833$ is still there; $\gamma(u) = 1.014$ multiplies it. Running on the belt is even less useful than Galileo would predict, which is the whole of the special-relativistic correction.

b. Suggested problems.

1. *Derive the special-relativistic ratio.* Starting from Einstein addition $W(u, w) = (u + w)/(1 + uw)$ and the composed Lorentz factor $\gamma(W(u, w)) = \gamma(u)\gamma(w)(1 + uw)$, show that a proper-time burst τ at running speed v' saves

$$\Delta T_{\text{floor}} = \frac{\gamma(v')(v' - v)\tau}{v}, \quad \Delta T_{\text{belt}} = \frac{\gamma(v')(v' - v)\tau}{\gamma(u)(v + u)},$$

and therefore

$$\frac{\Delta T_{\text{floor}}}{\Delta T_{\text{belt}}} = \gamma(u) \left(1 + \frac{u}{v}\right),$$

independent of v' . What happens to ΔT_{belt} as $u \rightarrow 1$?

2. *The $r = 4m$ terminal.* On the equator of a maximally spinning hole, $a = m$, at radius $r = 4m$, compute Δ , ω , the static lapse N_s , the ZAMO lapse N_z , and the locally measured belt speed $u = 2ma/(r\sqrt{\Delta})$. Confirm $u = 1/6$ and $N_z/N_s = \gamma(u)$. Using the burst of Problem 1 with $v = 0.20$ and $v' = 0.40$, how much more Killing time is saved by spending τ on the static floor than on the ZAMO belt? [The lapse split is only 1.4%; the kinematic factor $1 + u/v$ does most of the work.]
3. *Where may a static ticket counter sit?* For spin $\chi = a/m = 0.998$ (Thorne's limit), evaluate the prograde ISCO from Eqs. (12)–(13) and compare it with the equatorial ergosphere $r_E = 2m$. Can a static ticket counter exist at that ISCO? At the same spin, can a static counter exist at the paper's working radius $r = 4m$? Optional: plot $r_{\text{ISCO}}(\chi)/m$ for prograde and retrograde orbits on $0 \leq \chi \leq 0.998$, draw the horizontal line $r = 2m$, and mark the region that admits both a static floor and a stable circular geodesic.

The three qualitative prompts already in the text—why extra distance is worth more on the floor; where to tie a shoe on a backward belt $u = -v$; whether a static counter can sit at the Thorne-limit ISCO—are suitable for a ten-minute discussion after the problems. Solutions to Problems 1–3 are in the supplementary material.

VI. LIMITATIONS AND CLASSROOM USE

Nothing in the construction is an argument for building Terminal 5 on Sgr A* (Heathrow Terminal 5 is notorious for its long satellites, travelators and an underground tunnel). Real passengers also change their walking speed on belts;¹² that human-factors layer is omitted here. An accretion flow, a jet, and a photon region are hostile neighbours.^{11,13,14} The Thorne cap, the infinite-throat structure of near-extremal Kerr,⁹ and the distinction between Boyer–Lindquist r and proper radial distance should all be flagged when the ISCO is drawn on the board.

The value of the example is that one elementary variational question—where to spend τ —survives three successive geometries and keeps the same answer. Students who have met velocity addition can derive Eqs. (8)–(10) in a single problem set. Students who have met Kerr can then be asked to identify ω in Eq. (??) with u , to decide whether a given ring

admits a static floor, and to explain why the photon orbit is not a suitable gate.

Suggested discussion prompts:

- Why does the extra distance $(v' - v)\tau$ save more clock time on the floor than on the belt?
- If there are only retrograde travelers available, where is the least-bad place to tie a shoe in that case?
- Compare advantages/disadvantages of attaching the plane docking gate to the fixed jetway, versus parking the planes in a geodesic orbit under the jetway, in terms of relative clock times and ease of docking and boarding.

ACKNOWLEDGMENTS

This note grew from discussions at the Adelaide University staff club over Terence Tao's walkway puzzle and its possible relativistic extensions. The Kerr airport siting problem is a teaching device; it should not be seen as a realistic proposal for a civil-aviation authority.

REFERENCES

- ¹T. Tao, “An airport-inspired puzzle,” <https://terrytao.wordpress.com/2008/12/09/an-airport-inspired-puzzle/> (2008).
- ²G. Antonick, “Terence Tao’s airport-inspired puzzle,” New York Times Wordplay (27 July 2015).
- ³R. Eastaway, “Heathrow Airport and the Travelator Problem,” <https://robeastaway.com/blog/travelator> (2015).
- ⁴R. P. Kerr, “Gravitational field of a spinning mass as an example of algebraically special metrics,” Phys. Rev. Lett. **11**, 237 (1963).
- ⁵J. M. Bardeen, W. H. Press, and S. A. Teukolsky, “Rotating black holes: locally nonrotating frames, energy extraction, and scalar synchrotron radiation,” Astrophys. J. **178**, 347 (1972).
- ⁶C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973), Chaps. 33–34.

- ⁷J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Addison-Wesley, San Francisco, 2003), Chap. 15.
- ⁸K. S. Thorne, “Disk-accretion onto a black hole. II. Evolution of the hole,” *Astrophys. J.* **191**, 507 (1974).
- ⁹T. Jacobson, “Where is the extremal Kerr ISCO?,” *Class. Quantum Grav.* **28**, 187001 (2011).
- ¹⁰L. F. O. Costa and J. Natário, “Frame-dragging: meaning, myths, and misconceptions,” *Universe* **7**, 388 (2021); also arXiv:2109.14641.
- ¹¹A. Loeb, “Living near a supermassive black hole,” *Sci. Am. blog*, 11 March 2019.
- ¹²M. Srinivasan, “Optimal speeds for walking and running, and walking on a moving walkway,” *Chaos* **19**, 026112 (2009).
- ¹³J. D. Schnittman, “Life on Miller’s planet: the habitable zone around supermassive black holes,” arXiv:1910.00940.
- ¹⁴P. Bakala, J. Dočekal, and Z. Turoňová, “Habitable zones around almost extremely spinning black holes (black sun revisited),” *Astrophys. J.* **889**, 41 (2020).
- ¹⁵T. Opatrný, L. Richterek, and P. Bakala, “Life under a black sun,” *Am. J. Phys.* **85**, 14 (2017).
- ¹⁶N. A. Doughty, “Surface properties of Kerr–Newman black holes,” *Am. J. Phys.* **49**, 720 (1981).
- ¹⁷J. Enderlein, “A heuristic way of obtaining the Kerr metric,” *Am. J. Phys.* **65**, 897 (1997).
- ¹⁸E. Gallo and D. Marolf, “Resource Letter BH-2: Black Holes,” *Am. J. Phys.* **77**, 294 (2009).
- ¹⁹E. F. Taylor and J. A. Wheeler, *Exploring Black Holes: Introduction to General Relativity* (Addison-Wesley Longman, San Francisco, 2000).