

**Structural Governance of Gas Pressure Support in Galactic Disks:
A Two-Variable Scaling Relation Linking Rotational Organization
and Radial Structure**

Jessica Thyra Bower

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Abstract

The balance between ordered rotation and pressure support is a central feature of gaseous galactic disks, yet a one-variable measure such as V/σ does not fully describe how that balance changes within individual galaxies. This paper investigates whether gas pressure support is organized by rotational coherence alone or by a coupled dependence on dynamical organization and radial structure. The rotational organization parameter is defined as $\mathcal{R}_{\text{org}} = (V_{\text{rot}}/\sigma)^2$, and the fractional pressure-support measure as $F = (V_c^2 - V_{\text{rot}}^2)/V_c^2$. Across observational samples, increasing rotational organization is associated with decreasing pressure support at the population level. A candidate transition near $\mathcal{R}_{\text{org}} \simeq 84$ successfully identifies highly organized systems in an independent WHISP holdout sample, but it fails as a universal physical boundary when tested against regular, low- $\mathcal{R}_{\text{org}}$ H i disks. Radially resolved LITTLE THINGS data further show that a one-variable local relation between F and $\mathcal{R}_{\text{org}}$ cannot describe all galaxies: the sign of the within-galaxy trend varies from system to system. Introducing independently measured radial structure resolves much of this inconsistency. For the primary resolved sample, $F/(1 - F) \propto \mathcal{R}_{\text{org}}^{-0.91} (r/R_d)^{1.82}$, with $R^2 \simeq 0.87$ when galaxy-specific normalization is allowed. Leave-one-galaxy-out prediction improves relative to an organization-only model in 13 of 17 systems. An independent KAT-7 sample preserves the relative two-variable structure, although not its exact coefficient strength. Finally, the framework is tested against NIHAO hydrodynamic simulations in which the circular velocity is derived directly from the simulation potential, breaking the observational asymmetric-drift circularity. The locked two-variable predictor retains the expected positive direction in all four simulated galaxies and improves prediction relative to $\mathcal{R}_{\text{org}}$ alone. The result is interpreted as an empirical Structural Governance Framework: the equilibrium partition between ordered rotation and gas pressure support is jointly organized by rotational state and radial disk structure. The relation is not presented as a universal fundamental law or immutable constant.

Keywords: galaxy dynamics; H i kinematics; gas turbulence; pressure support; asymmetric drift; dwarf galaxies; disk structure; hydrodynamic simulations

1 Introduction

Galactic gas disks are not supported by ordered rotation alone. Random and turbulent motions contribute to radial dynamical support, particularly in dwarf irregulars, gas-rich low-mass systems, and outer disks where the characteristic velocity dispersion can be a non-negligible fraction of the rotation speed. The observed rotation velocity therefore need not equal the circular velocity required by the gravitational potential.

A common summary of the kinematic state is the ratio V_{rot}/σ . Large values indicate rotation-dominated gas, while smaller values indicate a larger relative contribution from disordered motions. The quantity is physically useful but intrinsically one-dimensional: it indicates how large ordered motion is relative to random motion, but it does not specify where in the disk the measurement is made or how the pressure field varies radially.

The present work asks whether this ratio is merely a useful classifier or whether it forms one axis of a more structured equilibrium relation. The analysis proceeds from population-level robustness to resolved radial dynamics, external observational validation, and finally simulation ground truth. The principal observational datasets include SPARC (Lelli et al., 2016), LITTLE THINGS (Hunter et al., 2012; Iorio et al., 2017), VLA-ANGST (Ott et al., 2012), THINGS (Walter et al., 2008; Ianjamasimanana et al., 2012), LVHIS (Koribalski et al., 2018), and SHIELD (Cannon et al., 2011). Independent validation uses WHISP rotation-curve quality information (Swaters et al., 2009), WHISP-derived kinematic quantities (Murugesan et al., 2020), KAT-7 dwarf-galaxy analyses (Namumba et al., 2017, 2018), and gas-rich ultra-diffuse galaxies modeled in three dimensions (Mancera Piña et al., 2020).

The rotational organization parameter is defined as

$$\mathcal{R}_{\text{org}} \equiv \left(\frac{V_{\text{rot}}}{\sigma} \right)^2, \quad (1)$$

and the fractional rotational shortfall as

$$F \equiv 1 - \left(\frac{V_{\text{rot}}}{V_c} \right)^2. \quad (2)$$

At the population level, increasing $\mathcal{R}_{\text{org}}$ is associated with decreasing pressure support. However, the resolved LITTLE THINGS analysis shows that the local radial relation is not universal. The mathematical reason follows directly from radial force balance: pressure support depends not only on V_{rot}/σ , but also on the radial pressure gradient.

This motivates a two-variable framework in which rotational organization supplies the kinematic axis and normalized radius supplies a structural axis. The term *structural governance* is used to describe this coupled regulation of the equilibrium support state. The phrase is intentionally distinct from a claim of a universal fundamental law: the data support a reproducible structural dependence, while the exact numerical coefficients vary between samples.

2 Data, Definitions, and Mathematical Framework

2.1 Rotational Organization

The central kinematic quantity is

$$\mathcal{R}_{\text{org}} = \frac{V_{\text{rot}}^2}{\sigma^2}. \quad (3)$$

The squared form is convenient because rotation and pressure terms enter the dynamical equations in squared-velocity form. If the population relation is written $\sigma = AV^b$, then

$$\mathcal{R}_{\text{org}} = A^{-2}V^{2(1-b)}. \quad (4)$$

Thus a shallow σ – V scaling necessarily generates a strong organization sequence with increasing rotational velocity.

2.2 Pressure-Support Fraction

For each resolved ring,

$$F = \frac{V_c^2 - V_{\text{rot}}^2}{V_c^2} = 1 - \frac{V_{\text{rot}}^2}{V_c^2}. \quad (5)$$

The corresponding odds-like pressure-support variable is

$$Y \equiv \frac{F}{1 - F}. \quad (6)$$

Only rings with $0 < F < 1$ are used in logarithmic analyses.

2.3 Radial Force Balance

The radial equilibrium equation can be written

$$V_c^2 = V_{\text{rot}}^2 + V_A^2, \quad (7)$$

where the pressure-support contribution is

$$V_A^2 = -\sigma^2 \frac{d \ln(\Sigma \sigma^2)}{d \ln r}. \quad (8)$$

Defining

$$D \equiv -\frac{d \ln(\Sigma \sigma^2)}{d \ln r}, \quad (9)$$

so that $V_A^2 = \sigma^2 D$ and $V_c^2 = V_{\text{rot}}^2 + \sigma^2 D$. Dividing by σ^2 gives

$$\frac{V_c^2}{\sigma^2} = \mathcal{R}_{\text{org}} + D. \quad (10)$$

Substitution into the definition of F yields the exact identity

$$F = \frac{D}{\mathcal{R}_{\text{org}} + D}, \quad (11)$$

and therefore

$$\frac{F}{1 - F} = \frac{D}{\mathcal{R}_{\text{org}}}. \quad (12)$$

Taking logarithms,

$$\ln Y = \ln D - \ln \mathcal{R}_{\text{org}}. \quad (13)$$

This identity explains why a universal local one-variable law is not generally possible: D is an independent radial structural quantity.

Differentiation makes the point explicit:

$$\frac{d \ln Y}{d \ln r} = \frac{d \ln D}{d \ln r} - \frac{d \ln \mathcal{R}_{\text{org}}}{d \ln r}. \quad (14)$$

The sign of the local pressure-support trend is therefore determined by the competition between the radial growth of the pressure gradient and the radial growth of rotational organization.

2.4 Structural Reduction

Expanding the gradient term,

$$D = -\frac{d \ln \Sigma}{d \ln r} - 2 \frac{d \ln \sigma}{d \ln r}. \quad (15)$$

For an ideal exponential disk, $\Sigma(r) = \Sigma_0 e^{-r/R_d}$, so

$$-\frac{d \ln \Sigma}{d \ln r} = \frac{r}{R_d}. \quad (16)$$

This motivates a reduced two-variable empirical form

$$\frac{F}{1-F} = A_g \mathcal{R}_{\text{org}}^{-\alpha} \left(\frac{r}{R_d} \right)^\beta, \quad (17)$$

or

$$\ln \left(\frac{F}{1-F} \right) = c_g - \alpha \ln \mathcal{R}_{\text{org}} + \beta \ln \left(\frac{r}{R_d} \right), \quad (18)$$

where $c_g = \ln A_g$ is galaxy-specific. The one-variable hypothesis is recovered when $\beta = 0$. The principal structural question is therefore whether $\beta \neq 0$ and whether inclusion of the radial term improves prediction outside the fitted galaxy.

2.5 Observational Samples and Provenance

The 52-galaxy measured-dispersion population sample combines ordinary-disk systems from SHIELD, LITTLE THINGS, VLA-ANGST, THINGS, and LVHIS, with source-specific velocity and dispersion rules frozen before the robustness tests (Cannon et al., 2011; Hunter et al., 2012; Ott et al., 2012; Walter et al., 2008; Ianjamasimanana et al., 2012; Koribalski et al., 2018). SPARC supplies high-quality rotation-curve information and official quality classifications for the matched analysis (Lelli et al., 2016).

The primary resolved pressure-support analysis uses the 17 dwarf irregular galaxies modeled in three dimensions by Iorio et al. (2017). Their products supply $V_{\text{rot}}(r)$, $\sigma_v(r)$, the asymmetric-drift-corrected $V_c(r)$, and radial profiles. Five author-flagged outer rings are excluded before analysis: DDO 50 at 390 arcsec, DDO 210 at 100 arcsec, and DDO 216 at 165, 180, and 195 arcsec. The corrected sample contains 244 rings, of which 234 satisfy $0 < F < 1$.

The external KAT-7 validation uses NGC 6822 (Namumba et al., 2017) and Sextans A and B (Namumba et al., 2018). These analyses provide ring-level rotation and dispersion information together with pressure-corrected circular velocities.

3 Rotational Organization as a Robust Population-Level Relation

The survey-controlled population model is

$$\ln \sigma_i = a_{s(i)} + b \ln V_i + \epsilon_i, \quad (19)$$

with survey-specific intercepts. The measured-dispersion sample gives $b \simeq 0.159$ with an approximate 95% interval of 0.075–0.244. The likelihood-ratio probability for adding V after survey control is approximately $p_{LR} \simeq 0.0015$. Because $\mathcal{R}_{\text{org}} \propto V^{2(1-b)}$, the central slope implies $\mathcal{R}_{\text{org}} \propto V^{1.68}$.

Whole-survey deletion leaves b in the narrow range 0.138–0.204. Individual-galaxy influence tests are similarly stable: Cook-distance removal gives $b \simeq 0.155$, Huber regression 0.157, Tukey regression 0.152, and leave-one-galaxy-out slopes approximately 0.144–0.172.

A matched SPARC analysis asks whether measured dispersion contributes information after rotation speed is already known. For the primary 20-galaxy subset, adding σ improves the ordered quality model with likelihood-ratio $p = 0.0206$; the extended 28-galaxy sample gives $p = 0.0393$. This supports the interpretation of $\mathcal{R}_{\text{org}}$ as more than rotation speed in disguise.

A homogeneous Stilp-based asymmetry test gives $\rho(a, V/\sigma) \simeq -0.549$, $p \simeq 0.0067$, although the relation weakens after baryonic-mass control and is therefore treated as mass-confounded rather than causal. The relevant H i superprofile methodology and dwarf-galaxy turbulence measurements are described by Stilp et al. (2013a, 2013b).

Stratified within-survey permutation testing with 100,000 permutations gives $p_{1s} \simeq 0.00109$, $p_{2s} \simeq 0.00308$. A 50,000-realization bootstrap gives a median slope $b = 0.1604$ and an approximate 95% interval of 0.080–0.239. A more restrictive matched-pair test is underpowered and is retained as having no scientific conclusion.

Errors-in-variables analysis of the 14-galaxy formal-error subset gives $b_{\text{EIV}} \simeq 0.290$, 95% interval $\simeq 0.164$ –0.416. The proportional-scaling hypothesis $b = 1$ is strongly rejected ($p \simeq 6.7 \times 10^{-9}$). Quadratic and broken-slope alternatives reveal no significant hidden transition toward

$b \simeq 1$.

Taken together, the first nine tests establish a robust population organization sequence: gas dispersion rises with rotation speed, but far more slowly than linearly, so $\mathcal{R}_{\text{org}}$ increases strongly across the population.

4 The $\mathcal{R}_{\text{org}} \simeq 84$ Scale: Useful Threshold, Not Fundamental Constant

A candidate high-organization scale emerged near $\mathcal{R}_c \simeq 84$, corresponding to $V_{\text{rot}}/\sigma \simeq 9.17$. Internally, the threshold correctly classifies 18/20 (90%) galaxies in the primary sample and 26/28 (92.9%) in the extended sample. Bootstrap transition estimates span roughly $\mathcal{R}_c \simeq 65\text{--}115$, already suggesting a regime rather than an immutable point.

The threshold was frozen at 84 before scoring an 18-galaxy WHISP holdout. The holdout uses WHISP-derived kinematic quantities from Murugesan et al. (2020) and the rotation-curve quality classes of Swaters et al. (2009). It should be noted that some historical quality information was encountered incidentally during source retrieval; accordingly, this test is best described as locked-threshold external validation rather than a fully blinded prospective experiment. The threshold and prediction rule were nonetheless frozen and not retuned to improve the outcome. The resulting confusion matrix is shown in Table 1.

Table 1: Confusion Matrix for the Frozen $\mathcal{R}_{\text{org}} \geq 84$ Rule Against the WHISP Holdout

	Observed high quality	Observed lower quality
$\mathcal{R}_{\text{org}} \geq 84$	9	0
$\mathcal{R}_{\text{org}} < 84$	2	7

This gives accuracy = 88.9%, sensitivity = 81.8%, specificity = 100%, with Matthews correlation coefficient $MCC \simeq 0.798$ and one-sided Fisher exact probability $p \simeq 0.00113$. The threshold is therefore a strong classifier of highly organized systems, understood within the caveat above.

It is not, however, a universal rotation/nonrotation boundary. Mancera Piña et al. (2020) modeled six gas-rich ultra-diffuse galaxies as thin, regularly rotating H i disks. Two stress-test ex-

amples are $\mathcal{R}_{\text{org}}(\text{AGC } 122966) \simeq 27.9$ and $\mathcal{R}_{\text{org}}(\text{AGC } 334315) \simeq 12.8$, both far below 84 despite regular rotation. $\mathcal{R}_{\text{org}} \sim 84$ is therefore interpreted as an empirical high-organization reliability scale, not a fundamental constant.

This conclusion follows naturally from the exact equilibrium form $F = D/(\mathcal{R}_{\text{org}} + D)$. At fixed $\mathcal{R}_{\text{org}}$, the pressure-support fraction still varies with D . A one-dimensional threshold is therefore a projection of a higher-dimensional state surface.

5 Pressure Support and the Failure of a One-Variable Local Law

The resolved LITTLE THINGS analysis uses the corrected 244-ring sample from Iorio et al. (2017). With the frozen threshold $\mathcal{R}_c = 84$, 236 rings lie below the threshold and eight lie above it. Median pressure support is $\tilde{F}_{\mathcal{R} < 84} = 0.0760$, versus $\tilde{F}_{\mathcal{R} \geq 84} = 0.0071$. A Mann-Whitney comparison gives $p_{1s} = 1.91 \times 10^{-4}$, $p_{2s} = 3.81 \times 10^{-4}$.

Yet only DDO 87 and DDO 101 contribute high- $\mathcal{R}$ rings. When those galaxies cross the threshold internally, F moves slightly upward rather than downward. The pooled segregation therefore does not represent a universal local discontinuity.

The continuous pooled relation remains strong. Fitting F against $\log_{10} \mathcal{R}_{\text{org}}$ gives $b = -0.1622$, with clustered $p = 1.90 \times 10^{-4}$ and a whole-galaxy bootstrap interval of approximately $[-0.2088, -0.0750]$. A fixed knot at 84 is not favored by BIC; smoother continuous descriptions perform better.

The one-variable local model fails more directly galaxy by galaxy. Of the 17 systems, six have negative within-galaxy F - $\mathcal{R}_{\text{org}}$ slopes and 11 have positive slopes. The random-slope mean is not significant. Thus $\partial F / \partial \ln \mathcal{R}_{\text{org}} < 0$ across the population, while $dF_g / d \ln \mathcal{R}_{\text{org}}$ has no universal sign within galaxies.

The exact identity explains this heterogeneity. Let $s_D = d \ln D / d \ln r$ and $s_R = d \ln \mathcal{R}_{\text{org}} / d \ln r$. Then $s_Y = s_D - s_R$. The sign predicted from the competition between D and $\mathcal{R}_{\text{org}}$ matches the observed direction in all 17 galaxies. This 17/17 result is mechanistic rather than independent validation because the observational V_c is itself constructed from the asymmetric-drift correction.

6 Radial Structure Restores the Local Relation

To determine whether the omitted pressure-gradient structure can be represented by an independent structural coordinate, the fit

$$\ln Y = c_g - \alpha \ln \mathcal{R}_{\text{org}} + \beta \ln(r/R_d) \quad (20)$$

was performed on the 234 rings satisfying $0 < F < 1$. The primary fit gives $\alpha = 0.9066$, $\beta = 1.8165$, $R^2 = 0.8695$. Thus

$$\frac{F}{1-F} \propto \mathcal{R}_{\text{org}}^{-0.91} \left(\frac{r}{R_d} \right)^{1.82}. \quad (21)$$

A cleaner 10-galaxy subset gives $\alpha = 0.9718$, $\beta = 1.6648$. The organization exponent is therefore close to the inverse first-power dependence expected from the exact equilibrium identity. The radial term is strongly positive in both sample definitions.

At fixed organization, $\partial \ln Y / \partial \ln(r/R_d) = \beta > 0$, whereas at fixed structure, $\partial \ln Y / \partial \ln \mathcal{R}_{\text{org}} = -\alpha < 0$. The two terms therefore act in opposing directions. Constant- Y contours obey $d \ln \mathcal{R}_{\text{org}} / d \ln(r/R_d) = \beta/\alpha \simeq 2.00$ for the primary fit; approximately, $\mathcal{R}_{\text{org}} \propto (r/R_d)^2$ along contours of constant pressure-support odds.

Leave-one-galaxy-out prediction provides the key portability test. The organization-only model gives $RMSE_{1D} \simeq 0.161$, while the two-variable model gives $RMSE_{2D} \simeq 0.138$. The structural model performs better in 13 of 17 held-out galaxies, a roughly 14% reduction in RMSE.

Combining the exact identity and the empirical fit gives an implied reduced representation of the gradient term, $D \propto \mathcal{R}_{\text{org}}^{1-\alpha} (r/R_d)^\beta$. For the primary sample, $D \propto \mathcal{R}_{\text{org}}^{0.093} (r/R_d)^{1.82}$, and for the clean sample, $D \propto \mathcal{R}_{\text{org}}^{0.028} (r/R_d)^{1.66}$. The residual organization dependence is weak, consistent with the interpretation that normalized radius captures much of the missing structural information.

For external validation, rounded exponents $\alpha_{\text{lock}} = 1.00$, $\beta_{\text{lock}} = 1.75$ are locked, defining

$$X = -\ln \mathcal{R}_{\text{org}} + 1.75 \ln(r/R_d). \quad (22)$$

7 Feedback, Turbulence, and Large-Scale Stirring

The structural relation uses σ , but the relation itself does not uniquely identify what drives that dispersion. Immediate local stellar feedback, gravity, accretion, shear, interactions, and large-scale bulk flows can all contribute.

Hunter et al. (2021) provide an independent LITTLE THINGS test of the relationship between recent star formation and H i turbulence. They subtract azimuthally averaged radial profiles and compare local excess star formation with local excess H i kinetic energy density and velocity dispersion. They find poor local correspondence between excess star formation and H i turbulence. The mean excess velocity dispersion in star-forming regions is only about $\Delta\sigma \simeq 0.37 \text{ km s}^{-1}$, and the inferred local coupling of supernova energy to H i turbulence is approximately 0.5%, about a factor of two too small to account for all galaxy-scale turbulence in their analysis (Hunter et al., 2021).

The appropriate conclusion is narrow: immediate local stellar feedback is not the dominant observed explanation for the full large-scale H i dispersion field. Delayed, nonlocal, or phase-dependent feedback remains possible. The data also leave room for large-scale gravitational and structural stirring. Thus the Structural Governance Framework should be read primarily as an equilibrium relation describing how an existing dispersion field contributes to support, rather than as a unique theory of how that dispersion is generated.

8 External Observational Validation

The independent KAT-7 sample uses NGC 6822 (Namumba et al., 2017), Sextans A, and Sextans B (Namumba et al., 2018). After the same $0 < F < 1$ filter, 25 rings remain: 13 in NGC 6822, seven in Sextans B, and five in Sextans A.

The locked predictor is $X = -\ln \mathcal{R}_{\text{org}} + 1.75 \ln(r/R_d)$, and the validation model is $\ln Y = c_g + \gamma X$. The fitted external strength is $\gamma = 0.482 \pm 0.220$ with approximate 95% interval 0.025–0.940. The predicted direction therefore survives externally, but exact unit amplitude does not.

The exact locked-amplitude model gives $RMSE_{\gamma=1} = 0.829$, while allowing only the

overall strength γ to scale gives $RMSE_{\text{scaled}} = 0.737$. The organization-only comparator gives $RMSE_{\mathcal{R}} = 0.792$. Thus the two-variable structure improves prediction when its overall response strength is allowed to recalibrate, but exact amplitude transfer is not validated.

Only after this locked test were the two exponents allowed to vary. The diagnostic external fit gives $\alpha_{\text{ext}} \simeq 0.50$, $\beta_{\text{ext}} \simeq 0.94$, so $\beta_{\text{ext}}/\alpha_{\text{ext}} \simeq 1.87$, close to the locked structural ratio 1.75. The external data therefore preserve the relative organization-versus-radius geometry more strongly than the absolute response amplitude.

9 Simulation Ground Truth

9.1 Breaking Asymmetric-Drift Circularity

The observational response uses an asymmetric-drift-corrected V_c , so the strongest test requires a circular velocity known independently from the gas-pressure correction. This analysis uses NIHAO hydrodynamic simulations presented by Dutton et al. (2019).

Dutton et al. explicitly distinguish the gas rotation curve, gas dispersion profile, and a circular velocity derived from the gravitational potential in the disk plane. Their potential-derived quantity satisfies $V_{\text{pot}}^2 = -R \partial\Phi/\partial R$, denoted $V_{\text{circ,true}}$ for this test. This breaks the observational circularity because the reference circular velocity is obtained from the simulation potential Φ , not reconstructed from the same asymmetric-drift term used in the response (Dutton et al., 2019).

The four reconstructed systems are g2.64e10, g2.34e10, g1.59e11, and g1.37e11. The raw reconstruction contains 41 matched radial measurements. A blind reconstruction procedure digitized $V_{\text{rot}}(r)$, $\sigma(r)$, and $V_{\text{circ,true}}(r)$ before any outcome quantity was calculated.

A repeat-digitization QA check re-read 36 individual curve values. All 36 fell within the assigned digitization uncertainties; the median absolute repeat difference was 1.00 km s^{-1} , the RMS difference 0.96 km s^{-1} , and the maximum discrepancy 2.0 km s^{-1} . No central value was changed after QA.

9.2 Explicit Structural-Scale Substitution

A common published optical exponential scale length R_d could not be verified for all four NIHAO systems. The simulation test therefore uses an explicitly named structural substitute, R_{struct} , rather than silently relabeling another size as R_d . For g2.34e10, g2.64e10, and g1.59e11, projected stellar half-mass radii from Cardona-Barrero et al. (2023) are used: $R_{\text{struct}} = 1.76, 2.46, 6.22$ kpc, respectively. For g1.37e11, the face-on full-galaxy stellar half-mass radius $R_{50} = 3.19$ kpc from Obreja et al. (2016) is used.

These stellar half-mass radii are used as independent structural scales, not as directly fitted optical exponential disk scale lengths. The simulation test is therefore a ground-truth test of the two-variable structural form, not an exact one-for-one validation of the observational r/R_d normalization.

9.3 Frozen Predictor and Response

The simulation response is $F_{\text{true}} = 1 - (V_{\text{rot}}/V_{\text{circ,true}})^2$, with $Y_{\text{true}} = F_{\text{true}}/(1 - F_{\text{true}})$. The locked predictor is $X_{\text{sim}} = -\ln \mathcal{R}_{\text{org}} + 1.75 \ln(r/R_{\text{struct}})$. The prespecified filter $0 < F_{\text{true}} < 1$ leaves 35 of the 41 frozen rings for logarithmic scoring.

The primary grouped model, $\ln Y_{\text{true}} = c_g + \gamma X_{\text{sim}}$, gives $\gamma = 0.6935 \pm 0.1375$ with an ordinary 95% interval of approximately 0.413–0.974 and $R^2 = 0.9168$. The grouped central-value RMSE is $RMSE_{2D} = 0.564$ versus $RMSE_{1D} = 0.653$ for the organization-only comparator. BIC likewise favors the structural model, 76.96 versus 87.26.

Even fixing the structural response strength exactly to unity gives $RMSE_{2D,\gamma=1} = 0.608$ versus $RMSE_{1D,\gamma=1} = 1.024$. A pooled diagnostic that uses simulation identity for clustered uncertainty rather than removing simulation-specific normalization gives $\gamma_{\text{pooled}} = 0.9994 \pm 0.1011$, numerically close to unity. Because this mixes within- and between-simulation information, the galaxy-normalized estimate $\gamma \simeq 0.694$ remains the conservative primary result.

Within simulations, all four slopes have the predicted positive sign. Leave-one-simulation-out aggregate shape RMSE is 0.796 for the locked model versus 0.978 for organization alone.

Strict absolute prediction gives 0.865 versus 1.470, with the locked model winning in three of four held-out simulations.

A post-unblinding free-exponent diagnostic gives $\alpha_{\text{sim}} \simeq 0.748$, $\beta_{\text{sim}} \simeq 0.994$, so $\beta_{\text{sim}}/\alpha_{\text{sim}} \simeq 1.33$. The signs persist but the exact observational exponent ratio does not.

A 5,000-draw sensitivity analysis sampling assigned digitization and structural-scale uncertainties gives median grouped $\gamma \simeq 0.621$, a 95% sensitivity range of roughly 0.19–1.11, and a positive slope in 99.94% of draws. The exact locked two-variable model outperforms the exact organization-only form in about 76% of uncertainty realizations.

The simulation verdict is therefore a qualified ground-truth validation: the two-variable structural dependence survives when pressure support is measured against independently known simulated gravity, but exact universal coefficient strength is not established.

10 Discussion

10.1 Structural Governance as an Equilibrium Statement

The central result is not simply that a second predictor improves a regression. The exact radial equilibrium identity already shows that pressure support depends on two mathematically distinct components: $F/(1 - F) = \mathcal{R}_{\text{org}}^{-1}D$, where the first term is kinematic organization and the second is radial pressure structure. The empirical relation shows that a large part of the difficult-to-measure structural term can be represented by normalized radial position: $D \Rightarrow A_g(r/R_d)^\beta$. Thus the local state is better represented as a surface $Y = Y(\mathcal{R}_{\text{org}}, r/R_d)$ than as a one-dimensional curve.

This explains the apparent paradox in the resolved data. Galaxies that are globally more organized have lower average pressure support, while individual galaxies can follow different radial paths across the two-dimensional surface. A pooled one-variable correlation is therefore a projection of a higher-dimensional equilibrium.

10.2 Meaning of the Empirical Exponents

The primary observational exponents are $\alpha \simeq 0.91$, $\beta \simeq 1.82$, while the clean subset gives $\alpha \simeq 0.97$, $\beta \simeq 1.66$. The organization exponent being close to unity is physically suggestive because the exact equilibrium identity predicts an inverse first-power dependence if the remaining structure is absorbed into D . The radial exponent should not, however, be interpreted as a new fundamental force-law exponent. It is an effective reduced description of surface-density gradients, dispersion gradients, disk thickness, and departures from a single ideal exponential component.

10.3 Reinterpreting the Scale Near 84

The independent WHISP result demonstrates that $\mathcal{R}_{\text{org}} \sim 84$ is useful. The UDG and radial tests demonstrate that it is not fundamental. Within the two-variable framework this is expected: sufficiently high organization strongly suppresses pressure support over a wide range of plausible structural states, so a high- $\mathcal{R}$ cut can act as a reliable classifier even though lower- $\mathcal{R}$ systems can remain regular rotators when their structural term is favorable. The threshold should therefore be reported as a high-organization reliability scale rather than a physical constant.

10.4 What Is and Is Not Governed

The framework describes how the observed gas kinematic state contributes to dynamical support. It does not uniquely identify the origin of σ . Immediate local star formation appears too weakly coupled to the large-scale H I velocity field to account for all of the dispersion (Hunter et al., 2021), but delayed feedback, nonlocal feedback, gravitational instability, accretion, shear, and interactions remain possible contributors. A useful separation is: what generates σ , versus how does σ contribute to radial support. The present paper primarily addresses the second question.

10.5 External Portability and Coefficient Variation

The KAT-7 and NIHAO tests both preserve the predicted structural direction while weakening the case for exact coefficient universality. In KAT-7, the relative exponent ratio remains close to the locked observational value even though the overall response strength is smaller. In NIHAO,

the free simulation exponents retain the expected signs but yield a shallower radial balance.

This suggests a useful conceptual separation, $\ln Y = c_g + \gamma[-\ln \mathcal{R}_{\text{org}} + q \ln(r/R_s)]$, where q describes relative organization-versus-radius balance and γ describes response strength. Current data are insufficient to establish either as universal, but this parameterization makes clear why structural geometry can be portable even when amplitude is not.

10.6 A More Direct Future Structural Coordinate

The exact pressure-gradient identity suggests a more fundamental coordinate than optical R_d . Defining a local pressure scale length $R_P^{-1} \equiv -d \ln(\Sigma \sigma^2)/dr$, then $D = r/R_P$ and therefore $F/(1-F) = (r/R_P)/\mathcal{R}_{\text{org}}$. This form has exact unit powers. The empirical $(r/R_d)^\beta$ term may therefore be acting as a portable observational proxy for the more direct pressure-structure coordinate r/R_P . Future work should compare R_d , R_{HI} , stellar half-mass radius, and directly measured R_P using larger samples with independent gravitational potentials.

10.7 Limitations and Falsification Criteria

The primary resolved sample contains 17 galaxies; high- $\mathcal{R}$ rings are concentrated in only two. The external KAT-7 sample contains three galaxies and 25 usable rings. The NIHAO test contains four simulations and uses digitized published profiles rather than author-supplied radial tables. Its structural normalization is a published stellar half-mass radius, not the exact observational optical R_d .

These limitations argue against a universal-law claim. The framework would be weakened substantially if larger independent resolved samples found $\beta \simeq 0$, if independent potential-based measurements showed no improvement over organization alone, or if held-out systems failed to preserve a common positive structural direction. Conversely, convergence toward $\alpha \simeq 1$ and a stable structural coordinate across larger independent samples would strengthen the case for a more universal reduced relation.

11 Conclusions

The completed analysis supports the following conclusions.

First, rotational organization $\mathcal{R}_{\text{org}} = (V_{\text{rot}}/\sigma)^2$ is a robust population-level dynamical coordinate. Gas velocity dispersion rises with rotation speed far more slowly than linearly, causing organization to increase strongly across the galaxy population.

Second, the empirical scale $\mathcal{R}_{\text{org}} \sim 84$ is useful as a high-organization classifier but is not a universal rotation/nonrotation transition or new fundamental constant.

Third, the one-variable local relation fails. The exact equilibrium identity $F/(1 - F) = D/\mathcal{R}_{\text{org}}$ shows why: the pressure-gradient structure D is an independent dynamical degree of freedom.

Fourth, normalized radial position restores much of the missing organization. The primary observational relation is $F/(1 - F) \propto \mathcal{R}_{\text{org}}^{-0.91} (r/R_d)^{1.82}$, with clean-sample support near $\mathcal{R}_{\text{org}}^{-0.97} (r/R_d)^{1.66}$. The two-variable model improves leave-one-galaxy-out prediction and performs better in 13 of 17 held-out galaxies.

Fifth, the independent KAT-7 sample preserves the predicted structural direction and relative organization-versus-radius balance, while exact coefficient strength is not validated.

Sixth, the NIHAO simulation test breaks the observational circularity. With $V_{\text{circ,true}}^2 = -R \partial\Phi/\partial R$ supplied independently by the simulated gravitational potential, the locked structural predictor retains the expected direction in all four simulations and outperforms organization alone on the frozen central reconstruction. This constitutes a qualified ground-truth validation of the two-variable structure.

The appropriate present description is therefore not a universal fundamental law but an empirical structural scaling relation embedded in a two-variable pressure-support framework. Its physical interpretation is referred to as the Structural Governance Framework: the equilibrium support state of disk gas is jointly constrained by rotational organization and radial structural position.

12 Data and Code Availability

The analysis was conducted as a sequence of frozen tests in which samples, exclusions, thresholds, model forms, and validation predictions were fixed before the corresponding outcomes were scored. The simulation reconstruction was frozen before any outcome variable was calculated. Analysis calculations were performed in Python using standard scientific-computing libraries, including NumPy, pandas, SciPy, and statsmodels.

13 Use of AI Tools

The author directed this research program, including the candidate organization variable, the decision to test the $\mathcal{R}_{\text{org}}$ scale against systems chosen specifically to break it, and the two-variable structural form recovered from the exact pressure-gradient identity. The author personally retrieved and reviewed every FITS file used in this analysis, including each moment-0, moment-1, and moment-2 map. AI systems were used for statistical computation, literature retrieval, and drafting, with results cross-checked across systems before acceptance. The author is responsible for all conclusions.

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