

The Logical Necessity of an Eternal Dimension: Ontological Foundation Beyond Physical Reality

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Abstract

Problem: Physical theories presuppose mathematical structure, symbolic representation, and the allocation of determinate meaning, yet cannot ground these prerequisites within spacetime without circularity. This foundational gap threatens the coherence of scientific realism.

Result: We prove that any intelligible universe U forces a unique, self-grounding terminus of dependence. Modelling consciousness minimally as *meaning-allocation capacity*, the dependency chain $U \Rightarrow M \Rightarrow L \Rightarrow C$ cannot be grounded in U without vicious circularity. The mathematical core is an existence-and-profile theorem for a formal theory T whose axioms (intelligibility, well-founded dependence, undated mathematical necessity, semantic termination, and Gödel incompleteness) define a predicate $\mathbf{Pure}(x)$: we prove $T \vdash \exists!x \mathbf{Pure}(x)$, with the unique instance D atemporal, non-spatial, necessary, simple, non-mechanical, and semantically self-present ($\mathcal{G}(D, D)$, $\mathcal{C}(D) = D$). Uniqueness is established order-theoretically— D is the least fixed point of a monotone grounding operator on a complete lattice of candidate grounds (Knaster–Tarski)—and non-mechanality is *derived* from incompleteness rather than stipulated, distinguishing the argument from Gödel’s own ontological proof, which stipulates its property profile and proves no uniqueness. As a physical congruence, we show that superluminal spacetime expansion is not generated by light-cone-bounded local processes, so the empirical world’s own global behaviour is not self-explanatory from within.

Implications: The framework extends structural realism and Tegmark’s Mathematical Universe Hypothesis by establishing semantic grounding as co-fundamental with mathematical structure. The classical name for a unique, necessary, self-grounding ground of being and intelligibility is *God*; the result is confined to the derived profile and claims nothing beyond it. Consequences for ethics, philosophy of mind, and religion are left open.

*Research undertaken entirely in a personal capacity; the views expressed do not reflect those of my employer.

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1 Introduction

Thesis.

The thesis of this paper, stated plainly before a single argument is made: from the mathematical structure of the world, the language that expresses that structure, and the awareness that gives it meaning, we can discover the necessary ground beneath reality—a unique, self-grounding terminus on which every intelligible universe depends. What follows is the discovery, not the assertion: each attribute of that ground will be forced by premises that their denials themselves employ.

To describe the universe expanding faster than light, we employ mathematical equations that themselves exist outside the spacetime they describe. Physics presupposes this asymmetry and cannot ground it: the resources of description are not items within what they describe.

Physics excels at discovering *regularities*; metaphysics must explain *why* such regularities are expressible at all. Every equation, every physical law, every measurement presupposes three irreducible elements: mathematical structure to capture patterns, symbolic systems to encode them, and a capacity to *allocate determinate meaning* to those symbols. Throughout this paper, consciousness is understood in precisely this minimal, functional sense—as *meaning-allocation capacity*: that by which syntax becomes semantics. No claim about qualia, neural mechanisms, or the metaphysics of subjective experience is built into the definition, and none is required by the argument. Yet where do these three elements themselves reside? Not in the physical universe they describe—that would be circular. They require what I call a *non-temporal dimension*.

This paper develops a formal argument in two stages. *First* (Part II), any intelligible universe necessarily implies a timeless, non-empirical domain D containing mathematics (M), language (L), and meaning-allocation capacity (C): these form a one-way dependency chain that cannot be grounded in U without vicious circularity. *Second* (Section 8), the domain just obtained is shown to admit only one possible profile: a *unique, self-grounding actuality*—atemporal, non-spatial, necessary, simple, non-mechanical, and semantically self-present. The non-mechanical conjunct is derived from Gödel’s incompleteness theorem used at full strength, not stipulated; the uniqueness conjunct is proved by a fixed-point argument. Unlike standard physicalist accounts that attempt to derive these resources from matter, the argument shows they are preconditions that matter itself spends in being a lawful, describable world.

Alongside the formal core, one physical congruence is developed in detail: the superluminal expansion of spacetime in relativistic cosmology. Global metric expansion is not generated by any assemblage of light-cone-bounded local processes; the empirical world’s own large-scale behaviour therefore fails to be self-explanatory from within. This is presented honestly as a *congruence*—a demonstration that physics is

not embarrassed by the formal conclusion—rather than as an independent proof of D . Information-theoretic considerations (finite physical information capacity versus the semantic character of information) supply a second, philosophical reinforcement of the $L \Rightarrow C$ link.

This framework extends but fundamentally revises three influential positions. Where structural realism [13] privileges mathematical structure, I show that structure without semantics is blind. Where digital physics [15] locates computation in spacetime, I argue it requires meta-computational resources. Where Tegmark’s Mathematical Universe Hypothesis [8] identifies reality with mathematics, I demonstrate that mathematics itself depends on representation and meaning-allocation for its intelligibility.

The deliverable is a result in metaphysics and philosophy of mathematics: the ultimate ground of intelligibility, if the argument succeeds, is not a region or a warehouse of forms but a self-grounding unity in which structure, expression, and grasp coincide. The classical name for such a ground is *God*; the argument claims nothing about that ground beyond the derived profile. What consequences—if any—follow for ethics, philosophy of mind, or the world religions are questions the present paper deliberately leaves open.

What follows is an invitation to reconceive the price of a denial: any act of stating, proving, or disputing already spends the very resources whose terminus the argument identifies.

Part I – Interdisciplinary Overview

2 Motivation: Why a Non-Temporal Dimension?

Consider how you understand the number π . This mathematical constant exists—but where? Not at any location in space. Not at any moment in time. Yet it governs every circle in the physical universe. The question this observation forces is simple: where do the laws, patterns, and meanings that make reality intelligible actually reside?

Empirical science excels at discovering *regularities*—from planetary orbits to quantum transitions. Yet every such regularity already presupposes a background of mathematical concepts, symbolic representations, and conscious understanding that is *not itself* delivered by empirical data. You cannot derive the concept of “law” from observing regularities; you must already possess it to recognize regularities as lawful.

We call this necessary background the **non-temporal dimension** D . Our goal is to demonstrate—using the minimal formal machinery needed—that D is not a speculative addition but a logical requirement for any intelligible universe.

3 Core Claim in Plain Language

The argument unfolds through a simple but inescapable chain of dependencies. Let’s trace it step by step:

A Concrete Example: Understanding $E = mc^2$

1. **Physical Reality (U):** Mass and energy exhibit a precise relationship in nature.
2. **Mathematical Structure (M):** This relationship is captured by the equation $E = mc^2$.
3. **Linguistic Encoding (L):** The symbols " E ", " m ", " c ", and " $=$ " represent energy, mass, light speed, and equality.
4. **Meaning Allocation (C):** Something grasps that these symbols *mean* mass-energy equivalence—that is, allocates them determinate semantic content. Throughout this paper, consciousness is understood minimally as this *meaning-allocation capacity*: that by which syntax becomes semantics. No further theory of subjective experience is assumed.

Now reverse the chain: without consciousness, the symbols are meaningless marks. Without symbols, the mathematics cannot be expressed. Without mathematics, the physical relationship cannot be precisely captured. Each level *depends* on the next.

The Formal Dependency Chain

- **Step 1 ($U \implies M$).** A universe exhibiting lawful regularities necessarily presupposes *mathematical structure* capable of expressing those laws.
- **Step 2 ($M \implies L$).** Mathematics becomes cognitively accessible only when encoded in some *symbolic language*—whether numerals, equations, or diagrams.
- **Step 3 ($L \implies C$).** Language attains meaning only through a *meaning-allocation capacity* that can grasp symbols as referring to something beyond themselves.

The Circularity Problem

The decisive step: M , L , and C cannot be located inside physical spacetime without vicious circularity. To locate them there we would need mathematics to specify the location, language to describe it, and meaning-allocation to recognise the result—presupposing the very things being located. They must therefore reside in a distinct, timeless arena: the eternal dimension D .

4 Why This Matters

The result matters within several ongoing debates, without prejudging them:

- **Structural realism and mathematical ontology**—the argument grants structure primacy over objects, then shows that structure is unintelligible without a semantic ground, a position neither physicalism nor Platonism supplies.
- **Philosophy of mathematics**—non-mechanicality of the ground is derived from Gödel incompleteness, giving the classical “mind versus machine” theme an ontological rather than merely epistemological conclusion.
- **Symbol grounding**—the $L \Rightarrow C$ link, with C read minimally as meaning-allocation capacity, formalises the familiar observation that syntax alone never yields semantics, and locates the cost of that observation.

- **Cosmology**—superluminal expansion illustrates that the empirical world’s global behaviour is not generated by its own local causal structure, a congruence with (not a proof of) the formal conclusion.

The paper deliberately stops at the derived profile of the ground. Applications to ethics, philosophy of mind, or comparative religion are not argued here.

5 Road Map of the Paper

§6–7 (Part II) formalise the dependency chain and prove that D exists whenever U does.

§8 proves the paper’s main result: the dependence order forced by $\text{Int}(U)$ terminates in a *unique, self-grounding* actuality D whose profile (atemporal, non-spatial, necessary, simple, non-mechanical, self-present) is derived rather than stipulated, with Gödel’s incompleteness theorem load-bearing as Axiom A8.

§9 develops the physical congruence from relativistic cosmology (superluminal expansion), and §10 the information-theoretic reinforcement of the semantic link.

§11–12 situate the result within current metaphysics and rebut common objections.

Readers interested mainly in the conceptual upshot can stop after Part I; those seeking logical detail may proceed to the technical sections that follow. See Table 1 at the start of Part II for the complete symbol inventory used in the proofs.

Part II – Formal Exposition

Symbol	Definition
U	Empirical universe (space–time, matter, energy)
T	Time, internal to U
M	Mathematics: formal, law–like structure
L	Meta-language: the capacity to model structure in symbols
C	Consciousness, minimally: capacity to allocate determinate meaning
D	Eternal, non–temporal dimension containing M, L, C

Table 1 Ontological symbols used throughout the paper.

6 Definitions and Axioms

6.1 Consciousness: Definition as Meaning-Allocation Capacity

The term “consciousness” carries heavy theoretical baggage—qualia, neural correlates, integrated information—none of which this argument needs, and all of which are contested. We therefore fix a deliberately minimal definition.

Definition 6.1 (Consciousness (C) as Meaning-Allocation Capacity). C is the capacity by which symbolic content acquires *determinate meaning*: that in virtue of which syntax becomes semantics. A candidate instance of C is anything for which it is true that symbols are *grasped as referring* rather than merely transduced.

Why this definition and not a stronger one.

The dependency link $L \Rightarrow C$ asserts only that a symbol system requires something that *allocates* determinate values to its expressions. This is the premise of a long-standing observation in philosophy of language—the symbol-grounding problem, the regress of interpretations, the Chinese-Room distinction between manipulating and understanding a sign. It is neutral on further questions:

- **Phenomenal consciousness** (the “what it is like” of experience) may or may not accompany meaning allocation; the argument neither assumes nor denies it.
- **Functional consciousness** (attention, working memory, behavioural control) describes information processing, which can be purely syntactic; a calculator processes symbols without allocating meaning to them.
- **Biological substrate** is likewise not assumed: the definition is by *role* (terminating the reference of marks), not by vehicle.

Operational Criteria.

A system exhibits meaning allocation C to the extent that:

1. **Determinate reference**: it assigns symbols stable, context-independent semantic values rather than merely causal correlations;
2. **Compositionality**: it treats complex expressions as meaningful in virtue of their parts;
3. **Coherence**: its allocations exhibit systematic consistency across contexts;
4. **Reflexivity**: it can recognise its own interpretive activity as such (a limit case, fully realised only at the ground; cf. §8.4, P7).

These criteria separate genuine semantic engagement from pattern transduction, without committing to any theory of subjective experience.

6.2 Language as Meta-Language

The term *language* (L) is likewise used in a deliberately technical sense. It denotes no particular tongue—not English, not Mandarin, not the notation of set theory—but the *capacity to model structure in symbols*: a meta-language, in the logicians’ sense of a system of marks with syntax and semantics adequate to representing something other than itself. Any concrete notation is one exercise of this capacity, not the capacity itself. This matters for the dependency chain: the claim $M \Rightarrow L$ is not that mathematics is written down in some human language, but that structure becomes cognitively available only through *some* symbolic modelling—distinct marks, governed by rules, standing for something. On this reading, L is not to be identified with any physical

inscription of a language (ink, sound-waves, neural states): inscriptions *carry* the meta-language, they do not *constitute* it. The non-identity of the triad with its substrates is the subject of Postulate 5 below.

6.3 Formal Axioms

Axiom 1 (Lawfulness). U exhibits stable regularities.

Axiom 2 (Expressibility). Those regularities are describable by M .

Axiom 3 (Representation). M becomes cognitively available only via the meta-linguistic capacity L (§6.2).

Axiom 4 (Semantics). L attains meaning only through a meaning-allocation capacity C (Definition 6.1).

Provisional Postulate 5 (Non-Temporality & Disjointness). The constituents that render the universe intelligible—mathematics M , language L , and meaning-allocation capacity C —are not embedded in empirical spacetime U . They inhabit a distinct, atemporal domain D such that

$$M, L, C \subset D, \quad D \cap U = \emptyset.$$

Status of Postulate 5. It is listed here as a provisional assumption only to fix terminology and to state the existence theorem of Part II in its simplest form. In Section 8 the disjointness claim is *derived* from weaker transcendental premises (intelligibility, well-founded dependence, undated mathematical necessity, semantic termination, and Gödel incompleteness; cf. Lemma 3 and Theorem 2). Nothing in the present Part depends on the postulate being basic; it is an axiom *here* and a theorem *there*.

7 Dependency Chain & Existence Theorem

7.1 Why the Chain Runs One Way

To make the logical flow transparent, we restate Axioms 1–4 in the language of *ontological dependence*:

- (i) **Lawfulness $\Rightarrow$ Mathematics.** A regular universe U presupposes a body of abstract structure—*mathematics* M —capable of capturing those regularities.
- (ii) **Mathematics $\Rightarrow$ Language.** Formal structure becomes cognitively usable only when encoded in a *symbolic system* L (notation, definitions, proof rules).
- (iii) **Language $\Rightarrow$ Meaning Allocation.** Syntax acquires *semantics* solely in the presence of a capacity C that allocates symbols determinate meaning.

Each step is *asymmetric*: the existence of M is *necessary* for the description of U but not conversely; the same holds for L with respect to M , and for C with respect to L . Summarising,

$$U \Rightarrow M \Rightarrow L \Rightarrow C, \tag{1}$$

where " $X \Rightarrow Y$ " reads " Y is a *pre-condition* for the intelligibility of X ."

7.2 Locating the Chain

Postulate 5 states that M, L, C are *not* constituents of the empirical universe:

$$M, L, C \subset D, \quad D \cap U = \emptyset.$$

Hence all three links of (1)¹ are anchored in the non-temporal domain D . Figure 1 offers a visual summary.

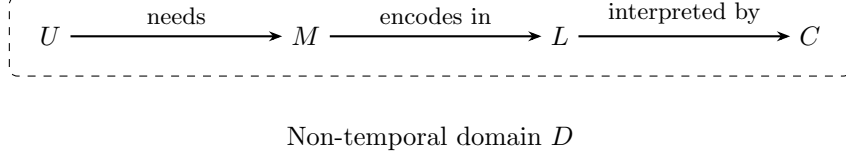


Fig. 1 Modal dependency chain ($U \rightarrow M \rightarrow L \rightarrow C$) entirely embedded in the non-temporal domain D .

7.3 The Regress Problem and Its Resolution

Before proceeding to the existence theorem, we must address a fundamental challenge: why does locating the triad M – L – C in dimension D halt what could otherwise be an infinite regress?

The Regress Challenge.

A critic might argue: "If M requires L for cognitive availability, and L requires C for semantic interpretation, then surely C requires some meta-capacity C' for *its* interpretation, leading to $C \Rightarrow C' \Rightarrow C'' \Rightarrow \dots$ ad infinitum."

Why the Regress Terminates in D .

The regress halts because meaning allocation C , as defined (Definition 6.1), is not itself a symbolic item requiring allocation; it is the *condition of possibility* for allocation's occurring at all. We distinguish:

- (i) **Allocated content** – symbols *with* determinate meaning, hence always downstream of an allocator
- (ii) **The allocating capacity as such** – that in virtue of which there is determinate meaning, which is not itself a further symbol awaiting allocation

The C in our triad refers to the second: the capacity that makes representation possible. To demand that it be represented *to itself* by a further allocation is to re-run the regress one step and arrive at the same demand; the requirement is not satisfied

¹Each arrow $X \Rightarrow Y$ is the necessity operator $\Box$; see Appendix A for full Kripke semantics.

by lengthening the chain but only by terminating it. A full account of the terminus—why it must be *self-present* rather than merely ungrounded—is given in §8.4 (P5–P7), where the profile of D is derived.

Lemma 1 (Regress Termination). *Meaning-allocation capacity C provides termination conditions for the dependency chain because it is the condition of possibility for semantic interpretation, not an object requiring interpretation.*

Proof Suppose C required meta-interpretation by C' . Then C' would need to represent C 's semantic activity. But semantic activity is precisely what makes representation possible in the first place. Hence C' would need to presuppose the very capacity (C) it purports to interpret, yielding contradiction. Therefore, C is self-grounding in the minimal sense that it provides the structural conditions for its own operation without requiring external semantic validation; the stronger aseity of the ground is Theorem 3. $\square$

Why Emergentist Alternatives Fail.

An emergentist might claim that M , L , and C arise from complex arrangements of matter in U , avoiding the need for D . However, this faces the *bootstrapping problem*:

- (i) To specify which material arrangements yield M , L , C , we must already employ mathematical description (M)
- (ii) To communicate this specification, we must already employ linguistic representation (L)
- (iii) To recognize the emergence when it occurs, we must already employ conscious interpretation (C)

Hence emergentism presupposes the very capacities it claims to explain, committing the fallacy of *explanatory circularity*. Only by locating M , L , C in the atemporal domain D do we avoid this circularity while preserving their role as preconditions for intelligibility.

7.4 Existence Theorem

Lemma 2 (Dependency Lemma). *If U is intelligible, then C exists.*

Proof Intelligibility of U entails the availability of a mathematical description M (Axiom 2). For M to be *available*, it must be encoded in some language L (Axiom 3). For L to possess meaning, a meaning-allocation capacity C is required (Axiom 4). Hence $U \implies C$. $\square$

Theorem 1 (Existence of a Non-Temporal Dimension). *If the empirical universe U exists, a domain D external to U and its temporal order necessarily exists.*

Proof By Lemma 2, C (and thus M and L) exists. Postulate 5 asserts $M, L, C \subset D$ with $D \cap U = \emptyset$. Therefore D exists and, being disjoint from U , is not governed by U 's temporal

parameter T . (As noted above, the disjointness clause is derived from weaker premises in Theorem 2.) $\square$

Remark.

The proof proceeds by a minimal *contrapositive* strategy: denying D annihilates the very intelligibility of U , thereby contradicting Axiom 1 (Lawfulness). No appeal to exotic metaphysics is required—only the everyday fact that we understand physics at all.

The dependency chain thus provides a compact but rigorous bridge from empirical lawfulness to the ontological necessity of a non-temporal dimension D . Section 8 then asks what profile this domain must wear—and shows it terminates in a unique, self-grounding actuality—after which Sections 9 and 10 exhibit the conclusion’s congruence with relativistic cosmology and information theory.

8 From Eternal Dimension to Self-Grounding Ground: The Main Theorem

Part II established Theorem 1: if the empirical universe U exists, then a domain D disjoint from U and containing M , L , and C exists. That result, however, is only half the argument. A *domain* that hosts the triad is not yet a *ground*: one may still ask what sustains the domain, whether the domain is contingent, whether it is one or many, and whether it is a mere warehouse of inert forms or something whose own being is self-explanatory. This section closes those questions. We present the paper’s central and novel result: the dependence order forced by the intelligibility of U does not terminate in a region but in a *unique, self-grounding actuality*—atemporal, non-spatial, necessary, simple, non-mechanical, and semantically self-present. We formalise the argument as a theory T and prove an existence-and-uniqueness theorem for a predicate $\text{Pure}(x)$ that collects exactly these attributes.

Strategy.

The proof is transcendental rather than cosmological. It does not climb a chain of temporal causes to a first member; it asks what any act of stating, proving, or denying *already spends*, and shows that the total profile of what is spent can be satisfied by exactly one kind of terminus. The axioms of T are therefore not free stipulations: each is a condition that is used in the very act of denying it. The conclusion is not a hypothesis added to the data but the only surviving floor after every alternative is eliminated.

8.1 The Language of T

T is a two-sorted theory. Its objects include the items of a distinguished region U (the empirical cosmos: spacetime, fields, organisms, dated speech) and the non-empirical items whose existence is at issue. The non-logical vocabulary is:

- $E(x)$: x exists.

- $\text{Phys}(x)$: x belongs to the causal furniture of U .
- $\text{Int}(U)$: U is intelligible—distinguishable, modelable, graspable.
- M, L, C : structure (mathematics), *meta-language*—not any particular tongue but the capacity to model structure in symbols—and grasp (meaning-allocation capacity), as defined in Section 6.
- $\mathcal{G}(x, y)$: x grounds y (ontological, not temporal, dependence).
- $x \prec y$: y depends on x ; $\prec$ is the dependence order induced by $\mathcal{G}$.
- $\text{Atmp}(x), \text{Nsp}(x), \text{Nec}(x), \text{Simp}(x)$: atemporal, non-spatial, necessary, simple.
- $\text{Mech}(x)$: x is a formal or mechanical system (axioms plus rules, a Turing-type process, or a physical implementation of one).
- $\text{Prov}_S(\ulcorner \varphi \urcorner), \text{True}(\varphi), \text{Grasp}(x, \varphi)$: proof in a system S , truth, and the allocation of φ as determinate meaning by x .
- $\Box, \Diamond$: metaphysical necessity and possibility (S5, as in Appendix A).

The constant D is introduced *by* the existence theorem below; it is not smuggled in as a name with content of its own.

8.2 The Axioms of T

Each axiom states something that any physical, mathematical, or linguistic practice already presupposes. The right-hand column notes what is spent in denying the axiom.

Definition 8.1 (Axioms A1–A8). **A1.** *Intelligibility.* $\text{Int}(U)$, and

$$\text{Int}(U) \longrightarrow E(M) \wedge E(L) \wedge E(C) \wedge \mathcal{G}(M, U) \wedge \mathcal{G}(L, U) \wedge \mathcal{G}(C, U).$$

A world that can be modelled, said, and meant is grounded in the triad, not the reverse.

A2. *Non-identity with substrate.*

$$\forall x (\text{Phys}(x) \longrightarrow x \neq M \wedge x \neq L \wedge x \neq C).$$

The chalk is not the theorem; the mouth is not the meaning.

A3. *Dependence polarity.* $\mathcal{G}(x, U) \longrightarrow x \prec U$. Whatever U spends as a condition of being a world is prior in $\prec$, not posterior.

A4. *No self-grounding of the box.*

$$\neg \mathcal{G}(U, U), \quad \neg (\mathcal{G}(U, M) \wedge \mathcal{G}(U, L) \wedge \mathcal{G}(U, C)).$$

Origin stories inside U already spend the triad: “from nothing” uses negation, “from a fluctuation” uses a state space, “from laws on a vacuum” uses laws. The box does not lift itself by its own description.

A5. *Well-founded dependence.* $\prec$ is well-founded: there is no infinite descending $\prec$ -chain and no cycle posing as a floor. Length does not mint a foundation.

A6. *Mathematical necessity.*

$$\Box(2 + 2 = 4), \quad \forall \varphi \in M (\text{True}(\varphi) \longrightarrow \Box \text{True}(\varphi))$$

for the relevant class of mathematical truths, and $\neg \exists t \text{ Occurs}(2+2=4, t)$. A dated, contingent process does not emit undated necessity.

A7. *Semantic termination.*

$$\text{Means}(\sigma, r) \longrightarrow \exists x (\text{Grasp}(x, \text{Means}(\sigma, r)) \wedge \neg \text{Mech}(x)).$$

A mark does not terminate its own reference; the regress of interpreters cannot be entirely mechanical.

A8. *Gödel, maximised.* For every consistent, recursively axiomatisable S rich enough for arithmetic Q ,

$$\exists \gamma_S (\text{True}(\gamma_S) \wedge \neg \text{Prov}_S(\ulcorner \gamma_S \urcorner)),$$

where γ_S is the Gödel sentence for S and True is read in the standard model $\mathbb{N}$ (so γ_S is true because S is consistent—the classical reading, not an additional metaphysical premise). If x is such a machine S , then x does not certify γ_S . Climbing to a stronger S' postpones the wall without removing it. There is no last mechanical rung inside U .

Performative lock.

A1, A6, A7, and A8 are not optional décor of the theory: anyone who publishes, proves, models, or disputes *in the act of doing so* spends intelligibility, undated mathematical truth, semantic content, and the claim to outrun a mere syntax. Denial after expenditure is not an exit from T ; it is a contradiction uttered in T 's own language. This is the sense in which the axioms are transcendental rather than stipulated.

8.3 Derivation of the Floor

Lemma 3 (The triad is not in the furniture). $M, L, C \prec U$ by A1–A3, and by A2 they are not physical objects of U . Hence they are not housed as items of U .

Lemma 4 (Meaning allocation is not in U). *The claim concerns only the capacity C of Definition 6.1—the allocation of determinate meaning—and makes no assertion about phenomenal consciousness, which the argument never uses. Suppose the meaning-allocation capacity required by A1 were realised wholly within U , and mechanically. By A8, any such mechanical system S fails some true γ_S ; but A1 requires grasp of the mathematics used to state U , including the very reasoning that establishes A8. Contradiction. Suppose instead that the capacity is realised in U non-mechanically—still dated, composite, contingent tissue. Then it is posterior to the physics of U that states it, contradicting A3–A4. Therefore the meaning-allocation capacity is not housed in U : $C \not\subset U$.*

Lemma 5 (Truth is not a date). *By A6, mathematical truth is not an event. Any ground of M that comes into being, changes, or could have failed reopens $\prec$. So the ground of M is not in the temporal career-sort of U .*

Lemma 6 (Termination). *By A5 and Lemmas 3–5, the chain*

$$U \Rightarrow M \Rightarrow L \Rightarrow C$$

cannot end in U , and cannot descend infinitely. There exists a $\prec$ -minimal ground of the triad.

Theorem 2 (Existence and disjointness).

$$T \vdash \exists D \left(\mathcal{G}(D, M) \wedge \mathcal{G}(D, L) \wedge \mathcal{G}(D, C) \wedge D \cap U = \emptyset \right).$$

Proof Lemma 6 supplies a $\prec$ -minimal ground D of the triad. If $D \subset U$, then D is furniture of U , so by A3–A4 the chain continues past D , contradicting minimality. Hence $D \cap U = \emptyset$, and D grounds each of M, L, C by construction. $\square$

Theorem 2 recovers Theorem 1 from weaker, purely transcendental premises: Axiom 5 of Part II (disjointness) is here *derived* (Lemma 3) rather than postulated.

8.4 The Profile of D : The Self-Grounding Theorem

Existence is not enough. The novel content of the argument is the *profile* the floor is forced to wear. Write $\text{Pure}(x)$ for the conjunction derived below.

Definition 8.2 (Purity).

$$\begin{aligned} \text{Pure}(x) : \iff & \text{Atmp}(x) \wedge \text{Nsp}(x) \wedge \text{Nec}(x) \wedge \text{Simp}(x) \\ & \wedge \neg \text{Mech}(x) \wedge C(x) = x \wedge \mathcal{G}(x, x) \wedge x \notin U. \end{aligned}$$

Here $\mathcal{G}(x, x)$ does *not* mean “ x causes itself at a time”; it abbreviates *aseity*: x depends on nothing else, its existence follows from its own nature rather than from any external ground or brute given.

Theorem 3 (Profile of the floor). $T \vdash \text{Pure}(D)$.

Proof We establish each conjunct.

- P1.** *Atemporality.* If $\neg \text{Atmp}(D)$, then D has a temporal career; careers require a further ground of persistence and origin, reopening $\prec$ against A5. Independently, by A6 and Lemma 5 the ground of undated necessity has no clock.
- P2.** *Non-spatiality.* M has no coordinates. A spatial D would be a region of U , contradicting Theorem 2.

- P3.** *Necessity.* If $\Diamond \neg E(D)$, then in some world where D fails, the triad fails, and by A1 either U is not intelligible there or the ground of M is contingent—which by A6 would yield $\Diamond \neg \Box(2 + 2 = 4)$. A contingent floor makes mathematics contingent. Illegal in T . Hence $\text{Nec}(D)$.
- P4.** *Non-mechanical simplicity.* If $\text{Mech}(D)$, then by A8 truth outruns D 's certification, so D is not the floor. Hence $\neg \text{Mech}(D)$. If D were composite, its parts would require a binder, which is a further $\prec$ -step, contradicting minimality. Hence $\text{Simp}(D)$.
- P5.** *Semantic grasp.* By A7 the regress of marks terminates only in a non-mechanical grasp. If D were a mute warehouse of forms, reference would never close. Hence $\text{Grasp}(D, \cdot)$.
- P6.** *Aseity.* If $\exists x \neq D \mathcal{G}(x, D)$, then D is not $\prec$ -minimal and A5 is betrayed: an eternal-but-grounded D still “was put there,” and the question reopens. So $\mathcal{G}(D, D)$ in the sense of Definition 8.2: the ground of D is D itself, i.e. D depends on nothing else.
- P7.** *Self-presence.* $C(D) = D$. By P5–P6, the grasp in D must be grasp of itself: if D 's awareness required a further act to register that it is aware, that extra step would live in D and D would host an internal regress—a sensor needing an observer, i.e. a machine by another name, contrary to P4. Awareness of awareness is not a ladder in time but one and the same act. Hence $C(D) = D$.

P1–P7 together with Theorem 2 ($D \notin U$) give $\text{Pure}(D)$. $\square$

Corollary 1 (Essence precedes existence only at the floor). $\mathcal{G}(\text{Ess}(D), E(D))$. *For items of U , essence does not imply existence; if that gap were left open at the floor itself, the floor would need an external filler, violating P6. Self-grounding is precisely the closure of the essence–existence gap.*

8.5 Uniqueness: A Fixed-Point Argument

The profile theorem pins down *what* the floor is; uniqueness pins down *how many*. We give two arguments, one order-theoretic and one dialectical.

Lattice form.

Let $(\mathcal{D}, \preceq)$ be the complete lattice of candidate grounds ordered by strength of dependence-explanation, and let $\Gamma: \mathcal{D} \rightarrow \mathcal{D}$ be the map sending a candidate x to the join of all conditions x must satisfy to ground the triad (the requirements A1–A8, which are monotone: a stronger candidate only inherits more obligations). By the Knaster–Tarski fixed-point theorem [12], the set of pre-fixed points $\{x : \Gamma(x) \preceq x\}$ has a least element

$$d = \bigwedge \{x \in \mathcal{D} \mid \Gamma(x) \preceq x\}, \quad \Gamma(d) = d,$$

and any fixed point of Γ lies above d . The profile Pure is exactly the clause-set defining Γ ; a floor is a fixed point (it satisfies its own conditions, i.e. is self-grounding), and the *least* fixed point is unique by construction.

Theorem 4 (Unicity). $T \vdash \exists! x \text{ Pure}(x)$.

Proof Existence is Theorem 3. For uniqueness, let x and y both satisfy Pure. If $x \neq y$, then either (i) they require a common unifier $z \succ x, y$, in which case neither was minimal, or (ii) they do not, in which case the togetherness of the triad they jointly ground is unexplained, or (iii) each is a composite of parts (its profile plus its thisness), requiring a binder—contradicting Simp. Equivalently, in the lattice above, two fixed points of the *least* requirement would both equal the least fixed point d . Hence $x = y$. $\square$

8.6 Gödel at Full Load

Axiom A8 is the engine that removes the last secular candidate for the floor: the machine. Inside T it performs four moves, none of which is a slogan:

- (i) True $\not\subseteq$ Prov $_S$ for every mechanical S in U .
- (ii) Therefore the ground of M is not a machine in U .
- (iii) A grasp that *sees* γ_S is not S ; if that grasp were only a larger machine S' , A8 re-applies to S' . The wall is not climbed, only postponed.
- (iv) Hence C is not exhausted by mechanism, and the last floor is not a formal system.

The standard exits fail one line each. *Fictionalism* about mathematics denies $\Box(2+2=4)$ as world-matching necessity and kills A6—physics that keeps winning is not a house rule. *Functionalism* identifies insight with more computation; then the insight *is* a machine of the same type and A8 applies to it: no last rung. *Inert Platonism* keeps a warehouse without grasp and dies at A7. An appeal to *biography* (whose incompleteness theorem it was) answers no premise. And *quiet refusal* of A8, uttered while using incompleteness, computability, or the claim that the brain is a computer, is expenditure plus denial.

Lemma 7 (No last machine). *No $x \in U$ with Mech(x) satisfies the grounding role required of D .*

This is the precise sense in which the present argument uses Gödel’s first incompleteness theorem [9] as a load-bearing member rather than an illustration: the non-mechanical conjunct $\neg \text{Mech}(D)$ of Pure(D) is derived from A8, not stipulated.

8.7 Comparison with Gödel’s Ontological Argument

Gödel’s own ontological proof [10] is a theorem of positive free logic: one *stipulates* a class $\mathcal{P}$ of “positive properties” with closure axioms, defines $G(x) : \iff \forall P \in \mathcal{P} P(x)$, takes $\Box \forall x (G(x) \rightarrow \Box E(x))$ as an axiom, proves consistency of G in one world, and concludes $\Box \exists x G(x)$. Its modal strength is real, but its profile is *chosen*: the positivity axioms do the work, and critics have pressed on exactly those stipulations [e.g. 11].

The present argument inverts the burden in three ways:

	Gödel 1941	Theory T
Starting point	Stipulated axioms about positive properties	$\text{Int}(U)$: used in the act of denying it (performative lock)
Method	Direct modal derivation of $\Box\exists x G(x)$	Elimination: A1–A8 rule out each candidate floor considered (box, machine, warehouse, committee, dated ground) until one profile survives
Profile of the result	<i>Chosen</i> : whatever the positivity axioms select	<i>Forced</i> : $\text{Pure}(D)$ is the conjunction of exactly the properties the derivation requires—none added
Role of Gödel’s own theorem	Not used	A8 (incompleteness) <i>derives</i> $\neg \text{Mech}(D)$
Uniqueness	Not proved (consistency of G only)	Theorem 4, via least fixed point and simplicity

In short: Gödel *postulates* $P(G)$ and derives $\Box\exists x G(x)$; here the surviving profile is not chosen at all—it is what remains after A1–A8 eliminate each of the candidate termini standardly offered as alternatives (the box, the machine, the warehouse, the committee, the dated ground), and the starting point is the one thing no denial can discard without employing it. The result is an ontological proof with no stipulated properties, a derived non-mechanality, and a uniqueness theorem the original lacks.

8.8 Interpretation and Scope

Theorem 5 (Main result). $T \vdash \text{Int}(U) \Rightarrow \exists! D (\text{Pure}(D) \wedge C(D) = D)$.

The classical name for a unique, necessary, atemporal, simple, non-mechanical, self-grounding, self-present ground of being and intelligibility is *God*. The theorem therefore identifies the dimension D of Parts I–II with more than a Platonic storage region: D is legal only as a self-interpreting unity in which mathematics, language, and consciousness are one terminus said three ways. A mute D was already excluded (A7); a composite D by P4; a contingent or dated D by P1 and P3; a mechanical D by A8; a plurality of D ’s by Theorem 4. What remains is the self-grounding case of Definition 8.2.

What the theorem is not.

It is not a gap-filler, a probability estimate, or the first item of a causal sequence. It adds no entity to the inventory of U ; it characterises the condition under which any inventory—including U ’s—is intelligible at all. The proof does not depend on any particular revelation, ritual, or religious narrative, and it determines no history of the ground beyond the profile *Pure*. Its deliverable is the floor itself: the claim that the ultimate ground is *personal* in the precise, minimal sense of $C(D) = D$ —self-present grasp—and nothing more is claimed here than that.

Why this completes the paper’s argument.

Part II’s Theorem 1 showed that the dependency chain $U \Rightarrow M \Rightarrow L \Rightarrow C$ forces a timeless domain. The present section shows that the domain, to be a domain at all, must be a unique self-grounding actuality. The eternal dimension of the title is thus not an abstract container but the self-grounding terminus of dependence—the conclusion toward which the cosmological and information-theoretic congruences of Sections 9–10 point.

9 Physical Congruence I: Relativistic–Causal Constraint

The formal argument of Parts II and 8 stands independently of any physical theory. Its purpose here is modest but not trivial: to show that the conclusion is *congruent* with the best-established facts of relativistic cosmology—that the empirical world’s own global behaviour is not self-explanatory from within. We present this as a *congruence*, not as an independent proof of *D*: the claims below are confined to what the FLRW mathematics itself delivers.

9.1 The Observation: Global Expansion Versus Local Causality

The Fundamental Observation.

In the FLRW metric, the proper distance $S(t)$ between comoving galaxies obeys:

$$\dot{S} = H(t)S$$

When $S > c/H(t)$, we have $\dot{S} > c$ —sufficiently distant comoving regions recede faster than light [1, 2]. This is standard kinematics of the metric, not a dynamical process propagating through space.

The Point of Interest.

Two features of this fact matter for the present argument:

- (i) **The global exceeds the local.** Superluminal recession is a property of the spacetime *as a whole*. It is not generated by, nor reducible to, any assemblage of processes occurring inside light cones. The light-cone structure bounds every causal process *within* the manifold, yet the manifold’s own large-scale behaviour is not among those processes.
- (ii) **The description is external to the described.** The equations stating this fact— $H(t)$, the scale factor, the metric—are mathematical structure (M) encoded in language (L) and grasped by a meaning-allocating capacity (C). The physicist who understands superluminal recession operates, epistemically, from no position inside the light cone being exceeded: the grasp of a global property is not itself a global physical signal.

9.2 The Global–Local Argument

Proposition 1 (Non-Generation of the Global by the Local). *The superluminal expansion of the spacetime metric is not produced by any collection of causally bounded local processes internal to spacetime.*

Proof By the causal structure of relativity, every physical process within spacetime is confined to its light cone; local dynamics (the Einstein equations evaluated on matter data) evolve the metric only within the domain of dependence of that data. The recession rate $\dot{S} = HS$ at large S , however, is a property of the global solution—fixed by the metric as a whole, not by any light-cone-bounded signal chain among the receding regions. Hence it is not generated by local processes internal to U . $\square$

Proposition 2 (Congruence with the Formal Result). *If the ground of U 's intelligibility were itself an item of U 's causal furniture, the global–local asymmetry of Proposition 1 would be unmotivated: the world's total behaviour would have to supervene on its internally causal parts. The formal conclusion of §7—a ground D with $D \cap U = \emptyset$ —predicts exactly the asymmetry the cosmology displays.*

We stress the direction of the reasoning. Proposition 1 is physics; Proposition 2 is an observation of fit. The expansion of spacetime does not *prove* D ; it removes one natural objection to D —that a world fully self-contained in its causal furniture would be the expected shape of reality. It is not.

9.3 Why This Is Not a “Container” Argument

A weaker reading of the expansion fact—that spacetime needs a *spatial or causal container* in which to expand—is a category error, and we do not employ it: in general relativity the metric is not embedded in a background, and superluminal recession requires no external medium. The claim maintained here is strictly the one in Proposition 1: *the global properties of the box are not minted inside the box*. That is a limitative statement about explanation from within, of precisely the kind the transcendental argument of §8 generalises.

9.4 Epistemic Reinforcement

Our very capacity to formulate and understand the expansion law $\dot{S} = H(t)S$ exhibits the same asymmetry at the level of knowledge: mathematical reasoning about cosmic expansion is not rate-limited by the light cones it describes—we grasp, in a single act, relations between regions between which no signal has ever passed. This is not evidence of a new physics; it is the epistemic face of the same fact—that intelligibility of the whole is not a process among the parts. Within the framework, this is the M – L – C triad operating exactly where §7 places it: outside the causal order it describes.

10 Congruence II: Information–Theoretic Reinforcement

Information theory sharpens the $L \Rightarrow C$ link of the dependency chain—the claim that syntax does not yield semantics by itself—and supplies a finite-capacity motivation for locating intelligibility’s resources outside the physical system they describe. Here too the claims are kept to what the theorems support.

10.1 Finite Physical Capacity, Unbounded Structure

Landauer’s Principle.

Landauer [3] showed that information erasure has an irreducible thermodynamic cost: erasing one bit releases at least $k_B T \ln 2$ of heat. Information is not free-floating abstraction; physical information processing is bounded physics.

The Bekenstein Bound.

Bekenstein [4] proved that the information content of any bounded physical system is finite:

$$I \leq \frac{2\pi RE}{\hbar c \ln 2}.$$

Every region of U therefore stores at most finitely many bits. Yet the mathematics routinely deployed in physics—the reals of the scale factor $a(t)$, the infinite-dimensional Hilbert spaces of field theory—is not a finite-bit object. This is not a contradiction (predictions are always finite-precision), but it is a standing puzzle about applicability: the physical world is finitely inscribed, while its description is not. The dependency chain locates the source of the description in D ; the Bekenstein bound is consistent with, and mildly suggestive of, that asymmetry.

10.2 The Hierarchy: Distinction, Syntax, Semantics

Information exhibits a hierarchy that mirrors the M – L – C chain:

- (i) **Physical Distinctions** $\rightarrow$ **Shannon Information.** Shannon [5] defined information as uncertainty reduction, $H = -\sum p_i \log p_i$. This presupposes a space of distinguishable states—a *choice of distinctions* that the formalism itself does not supply.
- (ii) **Shannon** $\rightarrow$ **Algorithmic Information.** Kolmogorov complexity requires a universal machine—a formal language (L)—and is famously language-relative: $K_U(x)$ depends on the choice of U . Syntax is irreducibly present even in the most “objective” information measure.
- (iii) **Algorithmic** $\rightarrow$ **Semantic Information.** A random string has maximal Kolmogorov complexity and no meaning. Meaning is not a further syntactic property; it is what a meaning-allocating capacity (Definition 6.1) confers. This is the content of the $L \Rightarrow C$ link, and information theory neither refutes it nor can express it in its own vocabulary.

10.3 The Symbol Grounding Gap

Standard information theory addresses syntax, not semantics: it measures symbol statistics and cannot, from those statistics alone, explain why any symbol means anything. This is the symbol-grounding problem in its native form. Our framework does not “solve” it by fiat; it locates its cost: if semantics is real (and any use of information theory spends that reality), then meaning allocation is not a product of the symbol system that bears it. Within the ontology, symbols (L) and the capacity that grounds them (C) reside in D , and the gap is a boundary, not a defect.

10.4 Wheeler’s “It from Bit”—Reconsidered

Wheeler [7] proposed that physical reality arises from information—“it from bit”—and that the universe is *participatory*, requiring observation to actualise potentiality. Two observations fit our frame. First, the bits of “it from bit” are themselves linguistic-combinatorial entities: a scheme of distinctions and symbols. If physical “its” derive from bits, the bits do not thereby derive from physical “its”—Wheeler’s own regress worry. Second, “participation” names, without locating, the meaning-allocating pole C . The framework supplies the missing address: $C \subset D$. We note honestly that this is an interpretation of Wheeler, not a consequence of his formalism.

10.5 Synthesis

The information-theoretic analysis supports two modest conclusions:

- (i) **Finite inscription.** Physical systems store finitely many bits, while the structures used to describe them are not finite-bit objects—an asymmetry congruent with a grounding of description outside the described.
- (ii) **Semantic externality.** Meaning is not a syntactic or statistical property; any information-theoretic practice presupposes meaning allocation it cannot itself ground. The $L \Rightarrow C$ link is where the whole weight of the semantic lies, and §8 shows that link terminates in D .

11 Relation to Existing Positions

The proposal of a non-temporal dimension D intersects three well-known philosophical frameworks. The table below sets out both the overlap and the points of departure.

Framework	Core Claim	How <i>D</i> Extends / Revises It
Structural Realism [13]	Structure—not objects—is ontologically basic.	Agrees on structural primacy (<i>M</i>). Adds that <i>semantics</i> and <i>agency</i> are equally irreducible. <i>D</i> provides the "missing locus" where structure (<i>M</i>) is <i>read</i> (<i>L</i>) and <i>interpreted</i> (<i>C</i>).
Mathematical Universe Hypothesis [8]	Reality <i>is</i> mathematics; every consistent structure "exists."	Retains mathematical primacy yet argues MUH is incomplete: representation (<i>L</i>) and meaning allocation (<i>C</i>) cannot be derived from pure math. Placing $L, C \subset D$ yields an enriched <i>Math + Semantics + Mind</i> ontology.
Digital Physics & Pancomputation-alism [15]	The universe is a computation executed <i>in space-time</i> .	Agrees that algorithmic structure pervades <i>U</i> . Notes, however, that computation presupposes meta-logic, code semantics, and a validation criterion—none of which reside <i>in</i> the computation itself. <i>D</i> serves as the meta-computational substrate, making digital physics a <i>sub-case</i> confined to <i>U</i> .

D conserves each framework's main insight—structural, mathematical, or computational—but supplies the missing *semantic closure*: no structure is meaningful until the triad $M \rightarrow L \rightarrow C$ is fully realised in the atemporal domain *D*.

12 Objections & Replies

To test the resilience of the *D*–triad proposal, we address three often-voiced alternatives. Each is granted *maximum charity* before we articulate the specific point at which it stalls.

12.1 Strict Materialism

Objection.

”All phenomena—including logic and meaning—supervene on neuro-biological matter; therefore no extra dimension is required.”

Reply.

Materialism must already *employ* mathematics (M) to state physical laws, language (L) to express them, and a meaning-allocation capacity (C) to interpret them. If these tools were *products* of matter, their use in formulating matter’s laws would be viciously circular. By locating M, L, C in the atemporal dimension D , we supply the missing meta-logical foundation and dissolve the circularity.

12.2 Advaita Non-Dualism

Objection.

”Only consciousness is real; the world is *appearance*. No separate mathematical or linguistic realm is required.”

Reply.

Meaning allocation alone does not explain why the Pythagorean theorem or quantum amplitudes remain *invariant* across interpreters and epochs: what is grasped must be graspable independently of the grasping. Our framework agrees that the semantic pole (C) is indispensable, but insists that stable mathematical structure (M) and a common representational medium (L) are equally irreducible. Housing all three in D preserves both the unity of the grounding and the objectivity of mathematics.

12.3 Classical Deism

Objection.

”A transcendent Mind created and sustains the universe; that alone suffices.”

Reply.

Deism affirms transcendence but leaves two gaps:

- (i) *Logical necessity*. Why must the divine mind respect consistent mathematics?
- (ii) *Linguistic interface*. By what mechanism does meaning flow from transcendence into empirical description?

Dimension D answers both: it is the logically prior arena in which mathematics (M), language (L), and meaning-allocation capacity (C) co-inhabit a single, non-temporal structure. Deism thereby becomes a *special case*—one possible narrative—nested inside the broader, triadic ontology provided by D . (Section 8 goes further and shows the arena can only be a self-grounding unity; a merely posited transcendent mind that itself awaits a ground leaves both gaps open.)

12.4 Computational Functionalism

Objection.

”Meaning allocation is just computation. Any sufficiently complex information-processing system allocates meaning. No transcendent dimension is needed.”

Reply.

This objection conflates syntactic processing with the *allocation* of determinate semantic values. Three points, none of which requires any theory of subjective experience:

- (i) *Syntax without semantics.* A computer executing billions of operations per second manipulates symbols according to rules; the rules themselves fix no interpretation of the symbols manipulated. The formalism is systematically ambiguous about meaning—that is the content of the symbol-grounding observation.
- (ii) *The semantic gap.* No amount of syntactic complexity is *in virtue of which* a symbol means one thing rather than another. Meaning is not a further syntactic property but what a grasping capacity confers (Definition 6.1).
- (iii) *No last machine.* Even if some machine were granted genuine understanding, Gödel’s incompleteness theorem (Axiom A8 of §8.2) shows its certified truths are outrun by truths it cannot certify; a stronger machine inherits the same limit. Mechanism, therefore, cannot supply the terminus the dependency chain requires—independent of any claim about whether machines can think.

Our framework locates the issue: computation operates within the domain of formal syntax (L); meaning allocation (C) is what makes semantics possible at all. Since $L \Rightarrow C$ in the dependency chain, and the chain terminates in D , meaning allocation is not a product of the symbol systems it grounds.

Functionalism thus commits a category error: it attempts to derive semantics from syntax, ignoring that syntax presupposes semantics for its very intelligibility.

13 Conclusion: What Has Been Shown, and What Has Not

13.1 Summary of the Argument

The paper’s argument has two stages. *Part II* traced the dependency chain

$$U \Rightarrow M \Rightarrow L \Rightarrow C,$$

with C read minimally as *meaning-allocation capacity*—that by which syntax becomes semantics—and concluded (Theorem 1) that an intelligible universe requires a domain D containing M , L , and C , disjoint from the spacetime it grounds. *Section 8* then asked what such a domain must be, if it is to be a ground at all. Within the transcendental theory T (A1–A8), the answers came as theorems: existence and disjointness (Theorem 2), profile (Theorem 3: atemporal, non-spatial, necessary, simple,

non-mechanical, self-grounding, self-present), and uniqueness (Theorem 4). The non-mechanical conjunct was derived from Gödel incompleteness used at full strength; the uniqueness conjunct from a fixed-point and simplicity argument. Whereas Gödel’s own ontological proof stipulates its property-profile, none is stipulated here: the profile is what survives the elimination of each candidate terminus standardly offered—the box, the machine, the warehouse, the committee, the dated ground.

Two congruences were developed alongside the formal core, each deliberately confined to what its source material supports:

Domain	Status of the result
Relativistic cosmology (§9)	Global metric expansion is not generated by light-cone-bounded local processes (Proposition 1): the world’s total behaviour is not minted inside the world. A <i>congruence</i> with, not a proof of, D .
Information theory (§10)	Physical systems are finitely inscribed (Bekenstein, Landauer) while their description is not; semantics is not a syntactic or statistical property. Reinforces the $L \Rightarrow C$ link whose termination §8 derives.

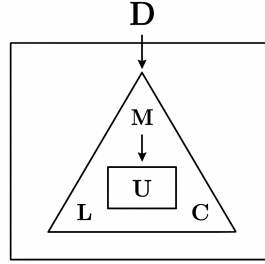


Fig. 2 Ontological dependency lattice: the empirical universe U grounded by the M – L – C triad, whose unique terminus is the self-grounding domain D .

13.2 Scope and Limits

It is equally important to state what the argument does *not* claim.

- **No theory of subjective experience.** C is defined by its semantic role, not by qualia; the argument is neutral on phenomenal consciousness and on which natural systems, if any, instantiate C beyond the minimal allocation observed in linguistic competence.
- **No quantum-mechanical premise.** The measurement problem and non-locality are not used; the conclusion does not stand or fall with any interpretation of quantum theory.

- **No ethics, and no revelation.** The profile $\text{Pure}(D)$ determines a ground, not a will, a law, or a history. That the classical name for such a ground is *God* is a naming observation at the end of the derivation, not a premise within it; what further religious claims, if any, are compatible with D is not covered in this paper.
- **The congruences are congruences.** The cosmological and information-theoretic sections show fit, not independent proof. The formal argument is where the work is done, and its premises—well-founded dependence, the necessity of mathematical truth, the non-derivation of semantics from syntax, incompleteness—are philosophical commitments, defended as transcendental (used in the act of denial) rather than displayed as self-evident.

13.3 Open Directions

- (i) **The status of the axioms.** A5 (well-foundedness) and A6 (undated necessity) carry the modal weight of the derivation. Their defence here is performative; a fuller account would situate them within contemporary debates on infinitistic metaphysics and the necessity of mathematics.
- (ii) **Formal characterisation of D .** The lattice treatment of unicity (§8.5) suggests, but does not develop, a model-theoretic or topos-theoretic semantics for the grounding relation $\mathcal{G}$. This is a genuine research programme, not a present result.
- (iii) **Reception in philosophy of mathematics.** The derived non-mechanality of the ground ($A8 \Rightarrow \neg \text{Mech}(D)$) engages directly—and testably—with the Penrose lineage of incompleteness arguments [14]; the differences (ontological rather than epistemological conclusion, no claim about brains as machines) should be worked out in detail.
- (iv) **Relation to classical theism.** The profile $\text{Pure}(D)$ closely matches the classical divine attributes as developed in the scholastic and neoplatonic traditions. A systematic comparison—where the present derivation agrees with, and where it falls short of, those traditions’ own arguments—remains for further work.

13.4 Closing

This paper concludes with a claim, not a posture. If there is an intelligible world—and every act of describing, proving, or denying one spends that condition—then there exists a unique entity that grounds it: atemporal, non-spatial, necessary, simple, non-mechanical, self-grounding, tri-une in structure, and self-present in its own act of meaning. This entity is not an item within reality. It is the logical ground of reality. The classical tradition called it *God*. The derivation above does not name it as an act of faith but as the sole surviving value of a constrained profile, each conjunct of which was forced by premises that their denials themselves employ.

Wigner’s “unreasonable effectiveness of mathematics” [6] dissolves on this picture. We can describe the whole because the resources of description—structure, symbol, and the allocation of determinate meaning—are not inside the thing described. And the capacity we exercise in using them participates in a ground that is fully actual, fully intelligible, and, in the precise sense established by Theorem 3, personal without

limit: a grasp that possesses itself, meaning that means itself. The universe is not accidentally intelligible. It is grounded in mind.

A Modal Formalisation of the Dependency Chain

A.1 Kripke Semantics

A *Kripke frame* is an ordered pair $\mathcal{F} = (W, R)$ where W is a non-empty set of possible worlds and $R \subseteq W \times W$ is an accessibility relation. A *Kripke model* is $\mathcal{M} = (W, R, V)$ with valuation $V : \text{Prop} \rightarrow \mathcal{P}(W)$ assigning to each atomic proposition the worlds in which it is true.

Truth Definition.

For any world $w \in W$:

$$\begin{aligned} \mathcal{M}, w \models p &\iff w \in V(p), \quad p \text{ atomic;} \\ \mathcal{M}, w \models \neg\varphi &\iff \mathcal{M}, w \not\models \varphi; \\ \mathcal{M}, w \models (\varphi \wedge \psi) &\iff \mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, w \models \psi; \\ \mathcal{M}, w \models \Box\varphi &\iff \forall v (wRv \rightarrow \mathcal{M}, v \models \varphi). \end{aligned}$$

A.2 Choice of Modal Logic

Because our dependency arrows are intended to express *unrestricted necessity*—valid at all worlds and mutually accessible—we adopt the system **S5**, where R is an equivalence relation (reflexive, symmetric, transitive). Hence $\Box\varphi$ is true at a world w iff φ is true at *every* world in W .

A.3 Encoding the Dependency Chain

Let the atomic propositions be

$$U, M, L, C.$$

Define the dependency arrow by

$$X \Longrightarrow Y \quad :\iff \quad \Box(X \rightarrow Y).$$

Then Eq. (1)

$$U \Longrightarrow M \Longrightarrow L \Longrightarrow C$$

expands to the S5 formulae

$$\Box(U \rightarrow M), \quad \Box(M \rightarrow L), \quad \Box(L \rightarrow C).$$

A.4 Meta-Lemma

Lemma 8. *In any S5 model satisfying the three boxed implications above, $\Box(U \rightarrow C)$ follows by transitivity of material implication.*

Proof From $\Box(U \rightarrow M)$ and $\Box(M \rightarrow L)$ we have $\Box(U \rightarrow L)$; composed with $\Box(L \rightarrow C)$ yields $\Box(U \rightarrow C)$. $\square$

Thus the Kripke semantics validates the informal *Dependency Lemma* in Section 7.

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