# $SU(m,n)$ Unified Field Theory

Complex Four-Index Gravity, Real Observer Projections,
Four-Sector Geometry and Tensor Quantum States

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**Abstract**

This work develops one $SU(m,n)$-symmetric framework governed by four postulates: invariant locally measured light speed; observer-independent laws in inertial and non-inertial frames; coexistence of all physically admissible prepared states with a single observer record; and a fundamentally complex spacetime observed through a real Lorentzian projection. The fundamental arena is a pseudo-Kähler manifold with Hermitian metric and a complex four-index field equation whose trace-free sector is carried by Bochner curvature. An observer is defined by an admissible antiholomorphic involution. Its fixed real form inherits the measured metric, sources and state tensor. The relation between complex and real curvature is derived with realification, the Gauss equation and the real Weyl projector; in particular, Bochner curvature projects to observer Weyl curvature only when explicit extrinsic and Ricci-compatibility corrections vanish. The four-sector construction is then formulated as an orientation completion of the real observer clock–ruler plane, rather than as a second fundamental spacetime. Quantum coexistence is represented by a complex rank-four tensor carrier with full Hermitian conjugation, a hypersurface overlap and covariant rank-eight evolution along the lifted observer clock. The common structure contains flat, Schwarzschild, Kerr, Kerr–Newman, LTB, FLRW, photon, free-graviton, hydrogen and ultraviolet worked sectors. The framework yields the locally invariant null speed, massless helicity-two graviton kinematics, reciprocal regular geometries, the relativistic Coulomb spectrum and the four-sector cancellation of local ultraviolet pole sources.

## Contents

# 1 Introduction

Special relativity turns the invariant speed of light into a restriction on the relation between inertial frames, and general relativity replaces that restricted covariance by local tensor laws on curved spacetime [1, 2, 3]. Quantum theory, meanwhile, assigns one state to several possible measurement outcomes. The present theory joins four-index gravity, complex curvature, four observer sectors, reciprocal geometry, tensor quantum states, null histories, gravitons and ultraviolet cancellation in one hierarchy of fundamental and observer-dependent objects.

This paper imposes that hierarchy through four physical postulates. Every observer measures the same local vacuum light speed. Physical laws retain their tensorial form for inertial and non-inertial observers. Every physically admissible configuration selected by a preparation coexists in one complete state, while a particular observer obtains one record at one event. Finally, spacetime itself is fundamentally complex and the spacetime measured by an observer is a real Lorentzian projection.

The fourth postulate fixes the order of construction. We begin with a pseudo-Kähler manifold $(\mathcal{M}, J, g, \omega)$ of complex dimension $N = m + n$ and signature $(m, n)$, not with four copies of a real Lorentzian metric. Its general pseudo-unitary special symmetry is $SU(m, n)$. The physical Lorentzian branch is the special case $(m, n) = (1, 3)$, with ambient symmetry $SU(1, 3)$ and the nested four-sector frame actions $SU(1, 3) \times SU(1, 2) \times SU(1, 1)$.

Gravity is written at the same complex level. A Hermitian four-index geometric tensor is equated to a Hermitian four-index source, including a trace-free Bochner-type component. The equation retains curvature information that is invisible after one selected two-index contraction. The four-index equation is therefore retained as the fundamental field equation.

The central bridge of the theory is the observer projection. An observer real form is the fixed set of an admissible antiholomorphic involutive isometry, with embedding $\iota$. The measured Lorentzian metric is the real symmetric pullback of the Hermitian metric. Curvature is not projected by merely erasing bars on indices: it is realified, evaluated on embedded tangent vectors and related to intrinsic observer curvature by the Gauss equation. Consequently the real Weyl tensor satisfies

$C_{\mu\nu\rho\sigma} = (\mathsf{W}_g[\Re B])_{\mu\nu\rho\sigma} + (\Delta_{\mathrm{Ric}})_{\mu\nu\rho\sigma} + (\Delta_K)_{\mu\nu\rho\sigma}$. Only a field-compatible, totally geodesic real form reduces this to the shorter Bochner-to-Weyl relation.

The four-sector construction is introduced only after this bridge. It completes the orientations of a real clock–ruler plane around one common null set and separates a geometry-sector label from a particle-sector label. This supplies a concrete realization of invariant local $c$ without interpreting a coordinate speed as a local measurement or allowing a massive particle to cross its null boundary dynamically. Reciprocal proper-scale maps then generate the regular continuations of the curved solutions analysed in this theory.

The third postulate is implemented by a complex tensor carrier rather than by adding mutually exclusive classical stress tensors. Each configuration solves its own field equation, and the carrier is the weighted complete state of the complex rank-four sources. Both coefficients and tensor bases are conjugated in the Hermitian overlap, and evolution is generated by a rank-eight operator along the lifted physical clock rather than a preferred coordinate derivative.

The theory uses one common convention throughout. The one-carrier phase $\exp(-\mathrm{i}E\tau/\hbar)$ is used consistently, affine rescalings of one null tangent are identified as descriptions of the same null history, and the ultraviolet sector projector acts on the complete local pole source.

The paper follows the logical sequence postulates, mathematical structure, predictions and solutions. Sections 2–8 define the fundamental complex theory, its real observer projection, the four-sector atlas and the tensor state. Section 9 classifies consequences by status. The subsequent worked sections collect the flat, black-hole, cosmological, null, graviton, hydrogen and ultraviolet examples.

# Part I

# Fundamental postulates

## 2 Four physical postulates

**Postulate 1** (Invariant speed of light)**.** *Every observer measures the speed of light to be constant, independently of the observer's state of motion.*

The postulate concerns a local measurement made with the observer's physical clock and rulers. It asserts one common null structure, including for an accelerated, freely falling or sector-oriented observer. It does not assert that an arbitrary coordinate quotient $\mathrm{d}x/\mathrm{d}t$ must equal $c$, and it does not allow a massive observer to be accelerated dynamically through a null boundary.

**Postulate 2** (Observer-independent physical laws)**.** *The laws of physics are the same in every reference frame, whether inertial or non-inertial.*

The fundamental equations must therefore be covariant. Tensor components, the local clock and the connection coefficients may differ between observers; the law does not. Acceleration and rotation remain measurable physical data and enter through the observer congruence and its transported frame.

**Postulate 3** (Coexistence of physical states)**.** *All physically admissible states of a system coexist simultaneously, while an observer measures one of them at any given moment.*

To make this statement compatible with Postulate 2, "simultaneously" does not mean equality of one preferred global coordinate time. It means that all configurations compatible with a preparation belong to one complete physical state. "At a given moment" refers to one event on the worldline of a particular observer. Each classical configuration remains self-consistent; coexistence is implemented in the quantum carrier and not by an unweighted sum of mutually exclusive classical stress tensors.

**Postulate 4** (Complex spacetime)**.** *Spacetime is fundamentally complex, while every observer measures its real Lorentzian projection.*

The fundamental metric, curvature, sources and quantum carrier live on a pseudo-Hermitian complex geometry. A physical observer is associated with an admissible real form of that geometry. The four-sector construction is introduced only after this real observer projection has been defined.

# Part II

# Mathematical structure

## 3 Conventions and tensor hierarchy

The fundamental manifold $\mathcal{M}$ has complex dimension $N = m + n$ and pseudo-Hermitian signature $(m, n)$. The physical case used throughout is $(m, n) = (1, 3)$, hence $N = 4$, so the ambient realification has dimension eight. An observer projection $\mathcal{N}$ has real dimension $d = 4$. Dimensional regularization is denoted separately by $D = 4 - 2\epsilon$ and occurs only in the ultraviolet calculation.

Holomorphic indices are $\alpha, \gamma, \kappa, \eta = 0, \ldots, N-1$, while antiholomorphic indices are barred. Real Lorentzian spacetime indices are $\mu, \nu, \rho, \sigma = 0, \ldots, 3$, and orthonormal-frame indices are $A, B$. The labels $N_g, S_X \in \mathbb{Z}_4$ denote the geometry and particle sectors and are never summed. A label $q \in \mathcal{Q}$ enumerates complete physical configurations and is not a tensor index.

The fundamental pseudo-Hermitian metric obeys

$$g_{\alpha\bar{\beta}} = \overline{g_{\beta\bar{\alpha}}}, \qquad \nabla g = 0, \qquad \mathrm{sig}(g) = (m, n). \tag{3.1}$$

The observed real metric uses signature $(- + ++)$ in even sectors; the neighbouring odd-sector representative has the opposite overall sign but the same Lorentzian null set.

Three different complex structures must not be conflated:

1. the holomorphic coordinates and pseudo-Hermitian geometry of $\mathcal{M}$;
2. the complex coefficients of the quantum carrier;
3. the four bookkeeping phases $1, \mathrm{i}, -1, -\mathrm{i}$ used by the real four-sector observer atlas.

## 4 Fundamental complex spacetime

### 4.1 Pseudo-Kähler geometry

The fundamental geometric object is

$$(\mathcal{M}, J, g_{\alpha\bar{\beta}}, \omega), \qquad \omega = \mathrm{i} g_{\alpha\bar{\beta}} \mathrm{d}z^{\alpha} \wedge \mathrm{d}\bar{z}^{\bar{\beta}}, \qquad \mathrm{d}\omega = 0. \tag{4.1}$$

The invariant interval may be written

$$\mathrm{d}s^2 = g_{\alpha\bar{\beta}}\,\mathrm{d}z^{\alpha}\mathrm{d}\bar{z}^{\bar{\beta}}. \tag{4.2}$$

The metric is pseudo-Hermitian rather than positive definite, so an admissible real form can inherit Lorentzian signature.

In flat complex spacetime,

$$\eta_{\alpha\bar{\beta}} = \mathrm{diag}(\underbrace{-1,\ldots,-1}_{m},\underbrace{+1,\ldots,+1}_{n}), \qquad \mathrm{d}s^2 = \eta_{\alpha\bar{\beta}}\mathrm{d}z^{\alpha}\mathrm{d}\bar{z}^{\bar{\beta}}. \tag{4.3}$$

The pseudo-unitary special group preserving this form is $SU(m,n)$. The previously used $SU(1,n)$ family is its one-time-direction special case.

### 4.2 $SU(m,n)$ symmetry and the physical nested frame

The general fundamental symmetry is $SU(m,n)$, where $N = m + n$. The Lorentzian four-dimensional case has $(m,n) = (1,3)$. Its full complex tangent frame is preserved by $SU(1,3)$. The nested clock–ruler modules of the four-sector description carry the additional independent lower-frame actions. They form the independently acting product

$$\boxed{SU(1,3) \times SU(1,2) \times SU(1,1)} \tag{4.4}$$

on the three corresponding pseudo-unitary frame modules. Thus $SU(m,n)$ is the general ambient symmetry, while Eq. (4.4) is its physical $(1,3)$ nested realization.

## 5 Complex four-index field equations

### 5.1 Curvature and Bochner decomposition

The nonzero Kähler curvature components are written $R_{\alpha\bar{\beta}\gamma\bar{\delta}}$. Their decomposition is

$$\begin{aligned} R_{\alpha\bar{\beta}\gamma\bar{\delta}} = B_{\alpha\bar{\beta}\gamma\bar{\delta}} + \frac{1}{N+2}\Big(&R_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}} + R_{\gamma\bar{\delta}}g_{\alpha\bar{\beta}} \\ &+R_{\alpha\bar{\delta}}g_{\gamma\bar{\beta}} + R_{\gamma\bar{\beta}}g_{\alpha\bar{\delta}}\Big) \\ &- \frac{1}{(N+1)(N+2)}\Big(g_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}} + g_{\alpha\bar{\delta}}g_{\gamma\bar{\beta}}\Big)\,R. \end{aligned} \tag{5.1}$$

The tensor $B_{\alpha\bar{\beta}\gamma\bar{\delta}}$ is the trace-free Bochner curvature. It is the complex Kähler counterpart of the information carried by the real Weyl tensor, but the two tensors are not identified before the real projection in Sec. 6.

### 5.2 Complex four-index Einstein tensor

The complex four-index geometric tensor is

$$\boxed{G_{\alpha\bar{\beta}\gamma\bar{\delta}} = \frac{1}{1-N}\Big[R_{\alpha\bar{\beta}\gamma\bar{\delta}} - R_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}} - R_{\gamma\bar{\delta}}g_{\alpha\bar{\beta}} + g_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}}R\Big].} \tag{5.2}$$

Its first natural contraction gives

$$g^{\alpha\bar{\beta}}G_{\alpha\bar{\beta}\gamma\bar{\delta}} = R_{\gamma\bar{\delta}} - g_{\gamma\bar{\delta}}R. \tag{5.3}$$

The crossed contractions can differ by fixed dimension-dependent factors; the uncontracted equation is fundamental and is not replaced by one selected two-index contraction.

### 5.3 Complex four-index source

Let $T_{\alpha\bar{\beta}}$ be the Hermitian rank-two source, let $T = g^{\alpha\bar{\beta}}T_{\alpha\bar{\beta}}$, and let $\widehat{T}_{\alpha\bar{\beta}\gamma\bar{\delta}}$ have the algebraic symmetries and traces of the Bochner sector. The source used in the complex four-index equation is

$$\begin{aligned}
T_{\alpha\bar{\beta}\gamma\bar{\delta}} = \frac{1}{1-N}\Bigg[&\widehat{T}_{\alpha\bar{\beta}\gamma\bar{\delta}} - \frac{N+1}{N+2}\Big(T_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}} + T_{\gamma\bar{\delta}}g_{\alpha\bar{\beta}}\Big) \\
&+ \frac{1}{N+2}\Big(T_{\alpha\bar{\delta}}g_{\gamma\bar{\beta}} + T_{\gamma\bar{\beta}}g_{\alpha\bar{\delta}}\Big) \\
&+ \frac{1}{(1-N)(N+1)(N+2)}\Big( -(N^2+N+1)g_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}} \\
&\qquad + (2N+1)g_{\alpha\bar{\delta}}g_{\gamma\bar{\beta}}\Big)T\Bigg].
\end{aligned} \tag{5.4}$$

The fundamental field equation is

$$\boxed{G_{\alpha\bar{\beta}\gamma\bar{\delta}} = \kappa T_{\alpha\bar{\beta}\gamma\bar{\delta}}.} \tag{5.5}$$

The trace-free part gives the complex Bochner-source relation

$$B_{\alpha\bar{\beta}\gamma\bar{\delta}} = \kappa\widehat{T}_{\alpha\bar{\beta}\gamma\bar{\delta}} \tag{5.6}$$

with the normalization fixed by Eqs. (5.2) and (5.4). The complex Bianchi identity and source equation are required to give

$$\nabla^{\alpha}G_{\alpha\bar{\beta}\gamma\bar{\delta}} = 0, \qquad \nabla^{\alpha}T_{\alpha\bar{\beta}\gamma\bar{\delta}} = 0. \tag{5.7}$$

The Hermitian reality conditions are

$$\begin{aligned}
\overline{T_{\alpha\bar{\beta}}} &= T_{\beta\bar{\alpha}}, \\
\overline{T_{\alpha\bar{\beta}\gamma\bar{\delta}}} &= T_{\beta\bar{\alpha}\delta\bar{\gamma}}, \\
\overline{G_{\alpha\bar{\beta}\gamma\bar{\delta}}} &= G_{\beta\bar{\alpha}\delta\bar{\gamma}}.
\end{aligned} \tag{5.8}$$

## 6 Real Lorentzian observer projection

### 6.1 Observer real form

An observer does not obtain a real spacetime by deleting the imaginary parts of coordinates component by component. The projection is defined geometrically. Let

$$\sigma : \mathcal{M} \to \mathcal{M} \tag{6.1}$$

be an antiholomorphic involutive isometry,

$$\sigma^2 = 1, \qquad \sigma_* J = -J\sigma_*, \qquad \sigma^* g = g. \tag{6.2}$$

Its fixed set is the observer real form

$$\mathcal{N} = \mathrm{Fix}(\sigma), \qquad \iota : \mathcal{N} \hookrightarrow \mathcal{M}. \tag{6.3}$$

For a regular middle-dimensional fixed component, the anti-holomorphic isometry makes the real form Lagrangian and the isometry makes it totally geodesic in the standard real-form construction

[5, 6]. In the pseudo-Kähler setting used here these properties are imposed as admissibility conditions together with Lorentzian signature of the induced metric.

Let

$$E^{\alpha}{}_{\mu} = \frac{\partial z^{\alpha}}{\partial x^{\mu}} \tag{6.4}$$

be the holomorphic part of the tangent map. The observed metric is the real symmetric pullback

$$\boxed{g_{\mu\nu} = \mathrm{Re}\Big(g_{\alpha\bar{\beta}} E^{\alpha}{}_{\mu} \overline{E^{\beta}{}_{\nu}}\Big), \qquad \mathrm{sig}(g) = (-+++).} \tag{6.5}$$

An overall conventional factor of two may be absorbed into the definition of the realification of the Hermitian metric.

The real observer map is therefore

$$\boxed{\mathcal{M} \xrightarrow{(\sigma,\iota)} (\mathcal{N}, g_{\mu\nu}).} \tag{6.6}$$

### 6.2 Curvature realification and the Gauss equation

For a complex Kähler curvature-type tensor $X$, let $\Re X$ denote its realification followed by evaluation on four embedded real tangent vectors. Abstractly,

$$(\Re X)_{\mu\nu\rho\sigma} := X(\mathrm{d}\iota\, e_{\mu}, \mathrm{d}\iota\, e_{\nu}, \mathrm{d}\iota\, e_{\rho}, \mathrm{d}\iota\, e_{\sigma}), \tag{6.7}$$

where on the right-hand side $X$ denotes the unique real multilinear tensor obtained from the mixed holomorphic components and their complex conjugates. This definition retains the antisymmetries of a real curvature tensor; a raw replacement of barred indices by unbarred ones would not.

Let $K^{I}{}_{\mu\nu}$ be the second fundamental form of the observer real form, with $I$ a normal-bundle index. The semi-Riemannian Gauss equation is

$$R_{\mu\nu\rho\sigma} = (\Re R_{\alpha\bar{\beta}\gamma\bar{\delta}})_{\mu\nu\rho\sigma} + \eta_{IJ}\left(K^{I}{}_{\mu\rho}K^{J}{}_{\nu\sigma} - K^{I}{}_{\mu\sigma}K^{J}{}_{\nu\rho}\right). \tag{6.8}$$

For the fixed set of an isometry, $K^{I}{}_{\mu\nu} = 0$; nevertheless the full formula is kept because it shows exactly what changes for a more general observer section.

### 6.3 The Bochner-to-Weyl bridge

For any real algebraic curvature tensor $X_{\mu\nu\rho\sigma}$ in dimension $d > 3$, define its Weyl projector by

$$\begin{aligned}(\mathsf{W}_g X)_{\mu\nu\rho\sigma} = X_{\mu\nu\rho\sigma} &- \frac{2}{d-2}\left(g_{\mu[\rho}X_{\sigma]\nu} - g_{\nu[\rho}X_{\sigma]\mu}\right) \\ &+ \frac{2X}{(d-1)(d-2)} g_{\mu[\rho}g_{\sigma]\nu},\end{aligned} \tag{6.9}$$

where $X_{\mu\nu} = g^{\rho\sigma}X_{\rho\mu\sigma\nu}$ and $X = g^{\mu\nu}X_{\mu\nu}$. Applying this projector to Eq. (6.8) and using the complex Bochner decomposition gives the exact bridge

$$\boxed{C_{\mu\nu\rho\sigma} = (\mathsf{W}_g[\Re B])_{\mu\nu\rho\sigma} + (\Delta_{\mathrm{Ric}})_{\mu\nu\rho\sigma} + (\Delta_K)_{\mu\nu\rho\sigma}.} \tag{6.10}$$

where

$$\begin{aligned}(\Delta_{\mathrm{Ric}})_{\mu\nu\rho\sigma} &:= (\mathsf{W}_g[\Re(R^{(\mathrm{Ric})}_{\alpha\bar{\beta}\gamma\bar{\delta}})])_{\mu\nu\rho\sigma}, \\ (\Delta_K)_{\mu\nu\rho\sigma} &:= \mathsf{W}_g\left[\eta_{IJ}(K^{I}{}_{\mu\rho}K^{J}{}_{\nu\sigma} - K^{I}{}_{\mu\sigma}K^{J}{}_{\nu\rho})\right]_{\mu\nu\rho\sigma}.\end{aligned} \tag{6.11}$$

**Consistency condition 6.1** (Projection compatibility). *An observer real form is field-compatible when it is totally geodesic, $K = 0$, and the realification of the complex Ricci-built sector is a real Ricci-built algebraic curvature tensor. Equivalently, $\Delta_K = \Delta_{\mathrm{Ric}} = 0$.*

On a field-compatible observer form,

$$\boxed{C_{\mu\nu\rho\sigma} = \mathsf{W}_g[\mathfrak{R}B]_{\mu\nu\rho\sigma}.} \tag{6.12}$$

Thus $B \to C$ is not assumed. It is the composition of complex realification, pullback to the observer form and the real Weyl projector. If the observer section is not field-compatible, Eq. (6.10) predicts explicit intrinsic corrections.

### 6.4 Projection of sources and field equations

The observed rank-two source is

$$T_{\mu\nu} = \mathrm{Re}\Big(T_{\alpha\bar{\beta}} E^{\alpha}{}_{\mu} \overline{E^{\beta}{}_{\nu}}\Big) . \tag{6.13}$$

The observed rank-four source is obtained by the same realification as a curvature-type tensor, followed by the real Ricci/trace-free decomposition:

$$T_{\mu\nu\rho\sigma} = \mathsf{P}^{(4)}[T_{\alpha\bar{\beta}\gamma\bar{\delta}}]_{\mu\nu\rho\sigma}. \tag{6.14}$$

Here $\mathsf{P}^{(4)}$ is not merely component restriction; it includes $\mathfrak{R}$, the Hermitian reality condition and the algebraic projection to real Riemann symmetries.

For a field-compatible real form the following diagram commutes:

$$\begin{array}{ccc} G_{\alpha\bar{\beta}\gamma\bar{\delta}} & = & \kappa T_{\alpha\bar{\beta}\gamma\bar{\delta}} \\ \big\downarrow \mathsf{P}^{(4)} & & \big\downarrow \mathsf{P}^{(4)} \\ G_{\mu\nu\rho\sigma} & = & \kappa T_{\mu\nu\rho\sigma}. \end{array} \tag{6.15}$$

The lower equation is the real four-index formulation, whose contraction gives the observed two-index Einstein equation. If the projection is not field-compatible, the two correction tensors in Eq. (6.10) enter the lower equation as geometric projection sources and must not be hidden inside $\widehat{T}$. For a connection-compatible real form the projection also commutes with the covariant derivative. A general embedding instead contributes the Codazzi and extrinsic-curvature terms already encoded by the full bridge.

The fundamental complex theory is therefore not reduced to the real Einstein equation. The real equation is one observer projection of the complex four-index equation.

## 7 Four-sector geometry of the real observer projection

### 7.1 Four orientations of one clock–ruler plane

Let $\vartheta^A$ be an orthonormal coframe of the real observer projection. The two null lines in a selected clock–ruler plane divide the non-null orientations into four connected domains. Their centres form the cycle

$$+t \longrightarrow +n \longrightarrow -t \longrightarrow -n \longrightarrow +t. \tag{7.1}$$

The geometry sector $N_g \in \mathbb{Z}_4$ is represented by

$$\boxed{\vartheta^A_{(N_g)} = \mathrm{i}^{N_g} \mathcal{I}^*_{N_g} \vartheta^A, \qquad g^{(N_g)}_{\mu\nu} = (-1)^{N_g} \mathcal{I}^*_{N_g} g_{\mu\nu}.} \tag{7.2}$$

The coframe distinguishes all four orientations, whereas the metric retains only the even–odd sign. Multiplication by a nonzero constant preserves the null set and the Levi–Civita connection inside a fixed chart. The phases in Eq. (7.2) belong to the atlas of the real projection; they are not a replacement for the fundamental complex coordinates of Sec. 4.

A particle $X$ has an independent relative sector $S_X \in \mathbb{Z}_4$ and an ordinary within-sector Lorentz transformation ${\Lambda_X}^A{}_B$:

$$\mathrm{d}X^A = \mathrm{i}^{S_X} {\Lambda_X}^A{}_B \vartheta^B_{(N_g)}, \qquad N_X = N_g + S_X \pmod 4. \tag{7.3}$$

A geometry-chart transition changes $N_g$ and the total readout $N_X$ but leaves $S_X$ and the particle's within-sector motion unchanged. Ordinary acceleration changes $\Lambda_X$ but not $S_X$. Geometry orientation, particle orientation and motion are therefore distinct data.

### 7.2 Invariant local light speed

Let $u^\mu$ be the unit clock of any observer in the active real metric and let $k^\mu$ be null. The local decomposition is

$$k^\mu = \frac{\omega_u}{c}(u^\mu + s^\mu), \qquad u_\mu s^\mu = 0, \qquad s_\mu s^\mu = 1, \qquad \omega_u = -c\, u_\mu k^\mu. \tag{7.4}$$

The spatial and temporal magnitudes measured by that observer are equal, so the measured speed is $c$. This calculation is local and does not require the observer worldline to be geodesic. Non-inertial motion changes the transport of successive frames, not the null relation at one event.

Every massive within-sector Lorentz motion has $|\beta_X| < 1$. A side observer uses one clock direction that the reference observer calls spatial, but it has an ordinary three-dimensional rest space of its own. A formally superluminal slope obtained by dividing by the wrong sector clock is not a locally measured speed.

### 7.3 Proper-scale tensor and reciprocal continuation

Let $u^\mu$ be the active geometry clock and $h^\mu{}_\nu = \delta^\mu{}_\nu + u^\mu u_\nu$ its rest projector. A Jacobi map $\mathcal{D}^\mu{}_\nu$ between neighbouring worldlines defines the positive Cauchy–Green tensor and its positive square root,

$$C^\mu{}_\nu = h^{\mu\rho} h_{\lambda\kappa} \mathcal{D}^\lambda{}_\rho \mathcal{D}^\kappa{}_\nu, \qquad L^\mu{}_\rho L^\rho{}_\nu = C^\mu{}_\nu. \tag{7.5}$$

The self-adjoint spatial tensor $L^\mu{}_\nu$ contains the invariant proper scales. A given projected GR solution supplies a positive critical-scale tensor $G^\mu{}_\nu$. Active directions solve

$$G^\mu{}_\nu n^\nu_{(i)} = q_i L^\mu{}_\nu n^\nu_{(i)}. \tag{7.6}$$

The active chart has $0 \le q_i < 1$, and $q_i = 1$ is its common null boundary. On the rest bundle, define

$$\boxed{\widetilde{L}^\mu{}_\nu = G^\mu{}_\rho (L_\perp^{-1})^\rho{}_\sigma G^\sigma{}_\nu.} \tag{7.7}$$

On a common eigendirection,

$$\widetilde{L}_i = \frac{G_i^2}{L_i}, \qquad \widetilde{q}_i = q_i^{-1}. \tag{7.8}$$

The reciprocal map is involutive and fixes the boundary. The active metric is the full signed pullback of the already projected real solution,

$$g^{(N_g)} = (-1)^{N_g} \mathcal{I}^*_{N_g} \bar{g}[L]. \tag{7.9}$$

All Jacobian-generated mixed terms are retained. Equation (7.7), rather than the constant phase alone, is the geometric content of the reciprocal continuation.

Curvature regularity, timelike proper time and null affine parameter are three independent tests. A reciprocal endpoint with vanishing curvature can still lie at infinite causal parameter and then represents an asymptotic boundary, not a finite passage into the next algebraic sector.

### 7.4 Projected tensor transport

Rank-two physical sources are ordinary chart pullbacks. All-lower real rank-four curvature and source tensors acquire the orientation sign produced when an index is lowered with the signed metric:

$$\begin{aligned} T^{(N_g)}_{\mu\nu} &= \mathcal{I}^*_{N_g} T_{\mu\nu}, \\ C^{(N_g)}_{\mu\nu\rho\sigma} &= (-1)^{N_g} \mathcal{I}^*_{N_g} C_{\mu\nu\rho\sigma}, \\ T^{(N_g)}_{\mu\nu\rho\sigma} &= (-1)^{N_g} \mathcal{I}^*_{N_g} T_{\mu\nu\rho\sigma}. \end{aligned} \tag{7.10}$$

Scalar contractions and physical spectral values are unchanged when the metric, tensor and integration measure are transported together.

## 8 Fundamental complex quantum state

### 8.1 Complete classical configurations

Let $q \in \mathcal{Q}_{\text{prep}}$ label one complete complex configuration compatible with a preparation. Its source components are assembled first and then lifted to the complex four-index source $T^{(q)}_{\alpha\bar{\beta}\gamma\bar{\delta}}$. Each such configuration separately obeys

$$G^{(q)} = \kappa T^{(q)}, \qquad \nabla^\alpha T^{(q)}_{\alpha\bar{\beta}\gamma\bar{\delta}} = 0. \tag{8.1}$$

Components introduced to close the conservation law belong simultaneously to one configuration and never receive independent state coefficients.

If different configurations carry different self-consistent geometries, their state space is the direct sum

$$\mathfrak{H}_{\text{prep}} = \bigoplus_{q \in \mathcal{Q}_{\text{prep}}} \mathfrak{H}_q. \tag{8.2}$$

This prevents a pointwise addition of tensors on different manifolds. A local sum is used only after a common complex background or an explicit relational identification has been supplied.

### 8.2 Complex rank-four carrier

On a common fundamental complex geometry the complete carrier is

$$\boxed{\Psi_{\alpha\bar{\beta}\gamma\bar{\delta}}(z,\bar{z}) = \ell_{\text{P}}^{(5-d)/2} \kappa \sum_{q \in \mathcal{Q}_{\text{prep}}} c_q T^{(q)}_{\alpha\bar{\beta}\gamma\bar{\delta}}(z,\bar{z}), \qquad c_q \in \mathbb{C}.} \tag{8.3}$$

The projected hypersurface has real spacetime dimension $d$. Since $G$ and $\kappa T$ have dimension $L^{-2}$, for $d = 4$ the carrier has dimension $L^{-3/2}$, and its Hermitian hypersurface overlap is dimensionless. In the general branch case, Eq. (8.3) is understood as the direct-sum section $\mathbf{\Psi} = \bigoplus_q c_q \Psi_q$.

Postulate 3 is realized by the support of this carrier: every configuration with nonzero preparation coefficient coexists in the complete state. The classical equation remains branchwise and is not sourced by the raw sum on the right-hand side of Eq. (8.3).

## 8.3 Full Hermitian conjugation

Complex conjugation acts on both the coefficients and the tensor basis:

$$\Psi_{\alpha\bar{\beta}\gamma\bar{\delta}} \longleftrightarrow \overline{\Psi}_{\bar{\alpha}\beta\bar{\gamma}\delta}. \tag{8.4}$$

Using the pseudo-Hermitian metric, define

$$\overline{\Phi}^{\alpha\bar{\beta}\gamma\bar{\delta}} = g^{\alpha\bar{\kappa}} g^{\lambda\bar{\beta}} g^{\gamma\bar{\eta}} g^{\theta\bar{\delta}} \overline{\Phi}_{\bar{\kappa}\lambda\bar{\eta}\theta}. \tag{8.5}$$

For a projected observer hypersurface $\Sigma$, the ordered Hermitian overlap is

$$\boxed{\langle\Phi,\Psi\rangle_\Sigma = \int_\Sigma \sqrt{h}\,\overline{\Phi}^{\alpha\bar{\beta}\gamma\bar{\delta}}\Psi_{\alpha\bar{\beta}\gamma\bar{\delta}}\,\mathrm{d}^{d-1}x.} \tag{8.6}$$

The embedding tensors that identify the observer slice are understood in the integrand. Equivalently, the complete tensors may first be projected with $\mathsf{P}^{(4)}$ and then contracted with the observed real metric.

Normalization and a resolved comparison channel are

$$\langle\Psi,\Psi\rangle = 1, \qquad P(\Phi|\Psi) = \frac{|\langle\Phi,\Psi\rangle|^2}{\sum_A |\langle\Phi_A,\Psi\rangle|^2}. \tag{8.7}$$

The sum runs over a complete mutually exclusive set of observer channels. The probability remains a global full-tensor contraction. No independent scalar wavefunction and no second measurement dynamics are introduced.

The physical channel basis is normalized with the Hermitian or biorthogonal tensor pairing. The type-N graviton dual basis below gives its explicit radiative realization.

## 8.4 Covariant rank-eight evolution

Let $U^\alpha$ be the holomorphic part of the lifted observer clock, with real tangent

$$U = U^\alpha\partial_\alpha + \overline{U}^{\bar{\alpha}}\partial_{\bar{\alpha}}, \qquad g_{\alpha\bar{\beta}}U^\alpha\overline{U}^{\bar{\beta}} = -1. \tag{8.8}$$

Let $\mathscr{D}_U$ be the covariant derivative of the full tensor along $U$, including the observer-frame connection. The fundamental evolution is

$$\boxed{\mathrm{i}\ell_\mathrm{P}\,\mathscr{D}_U\Psi_{\alpha\bar{\beta}\gamma\bar{\delta}} = H_{\alpha\bar{\beta}\gamma\bar{\delta}}{}^{\kappa\bar{\lambda}\eta\bar{\theta}}\Psi_{\kappa\bar{\lambda}\eta\bar{\theta}}.} \tag{8.9}$$

The operator remains rank eight. It is Hermitian or bi-Hermitian on the physical state space and transforms by conjugation under a complex frame map or real sector pullback,

$$\Psi' = U\Psi, \qquad H' = UHU^{-1}. \tag{8.10}$$

Equation (8.9) therefore does not privilege a coordinate $x^0$. In an inertial adapted frame it reduces to the earlier $\mathrm{i}\ell_\mathrm{P}\nabla_0\Psi = H\Psi$.

For a stationary eigenmode, the unified one-carrier convention is

$$H\Psi_q = \frac{E_q}{E_\mathrm{P}}\Psi_q, \qquad c_q(\tau) = c_q(0)e^{-\mathrm{i}E_q\tau/\hbar}, \qquad E_\mathrm{P}\ell_\mathrm{P} = \hbar c. \tag{8.11}$$

This convention is required by the Hermitian one-carrier overlap and by the free-graviton relation $E = \hbar\omega$. It fixes the phase of the carrier; quadratic observables are constructed from that phase without redefining the one-particle energy.

### 8.5 Projection of the quantum state

The tensor measured on an observer real form is

$$\boxed{\Psi_{\mu\nu\rho\sigma} = \mathsf{P}^{(4)}[\Psi_{\alpha\bar{\beta}\gamma\bar{\delta}}]_{\mu\nu\rho\sigma}.} \tag{8.12}$$

The projection acts on the full complex tensor, not only on its coefficients. The sector representatives are subsequently

$$\Psi^{(N_g)}_{\mu\nu\rho\sigma} = (-1)^{N_g}\mathcal{I}^*_{N_g}\Psi_{\mu\nu\rho\sigma}. \tag{8.13}$$

A consistent projection must preserve the physical overlap, spectral value and probability:

$$\langle\Phi_{\alpha\bar{\beta}\gamma\bar{\delta}}, \Psi_{\alpha\bar{\beta}\gamma\bar{\delta}}\rangle = \langle\Phi_{\mu\nu\rho\sigma}, \Psi_{\mu\nu\rho\sigma}\rangle, \qquad E'_q = E_q, \qquad P' = P. \tag{8.14}$$

Equation (8.14) defines an admissible observer projection of the physical state space.

# Part III

# Physical predictions and mathematical consequences

## 9 Consequences of the theory

**Derived result 9.1** (Universal locally measured $c$)**.** *Equation (7.4) implies that every admissible local observer measures the same vacuum light speed $c$. This includes inertial, accelerated, freely falling and sector-oriented observers.*

**Derived result 9.2** (Four observer sectors)**.** *Completing the orientation of one real clock–ruler plane around its common null lines gives four non-null connected domains and the $\mathbb{Z}_4$ cycle in Eq. (7.1). This is a mathematical consequence of the chosen orientation completion of the real projection.*

**Physical prediction 9.1** (No ordinary massive FTL crossing)**.** *A massive particle remains within the Lorentz cone of its fixed relative sector. Acceleration changes $\Lambda_X$, not $S_X$, and cannot convert an ordinary particle into a side particle by passage through $c$.*

**Derived result 9.3** (Complex complete quantum state)**.** *Postulates 3 and 4 together require the complete carrier to combine complex four-index configurations on the fundamental pseudo-Hermitian geometry, as in Eq. (8.3). Complex coefficients on an otherwise purely real tensor basis are only the observer-projected special case.*

**Physical prediction 9.2** (Single observer readout)**.** *At one event an observer records one resolved channel $\Phi_A$ with the conditional probability in Eq. (8.7), although the complete state retains all nonzero prepared configurations.*

**Derived result 9.4** (Null-state limit)**.** *For a selected null history, metric proper time is identically zero. The events therefore possess no nontrivial ordering by the photon's own proper time, although timelike observers order them with their clocks. Affine rescalings of the same null ray are gauge-equivalent and do not create extra physical states.*

**Derived result 9.5** (Free graviton sector)**.** *In a Ricci-flat radiative projection the complete source reduces to its trace-free curvature part. The vacuum Bianchi equations, transverse screen representation and rank-eight spectrum give*

$$m_G = 0, \qquad \lambda_G = \pm 2, \qquad E_G = \hbar\omega. \tag{9.1}$$

*This is the free radiative spectrum of the projected four-index vacuum sector.*

**Physical prediction 9.3** (Reciprocal regularity)**.** *For the explicit Schwarzschild, Kerr, Kerr–Newman, homogeneous LTB and regular-fluid FLRW branches analysed below, the reciprocal active scale grows while the relevant curvature invariants decay. Their limiting reciprocal centres are asymptotic in the timelike and null directions that have been checked.*

**Derived result 9.6** (Local ultraviolet sector cancellation)**.** *The complete equal-weight particle-sector orbit annihilates a local covariant rank-two UV pole response with sector character $(-1)^{S_X}$, as shown by the complete-orbit projector in Eq. (20.3).*

# Part IV

# Exact and worked solutions

## 10 Flat complex spacetime

**Worked solution 10.1** (Pseudo-Hermitian vacuum)**.** *For $N = 4$ with $(m, n) = (1, 3)$,*

$$\mathrm{d}s^2 = -\mathrm{d}z^0\mathrm{d}\bar{z}^{\bar{0}} + \mathrm{d}z^1\mathrm{d}\bar{z}^{\bar{1}} + \mathrm{d}z^2\mathrm{d}\bar{z}^{\bar{2}} + \mathrm{d}z^3\mathrm{d}\bar{z}^{\bar{3}}. \tag{10.1}$$

*The full tangent frame is preserved by $SU(1, 3)$. The nested observer modules carry the additional independent actions of $SU(1, 2)$ and $SU(1, 1)$, giving the nested product in Eq. (4.4). For the canonical real structure $\sigma(z) = \bar{z}$, the fixed form $z^\alpha = x^\alpha$ has*

$$\mathrm{d}s^2 = -(\mathrm{d}x^0)^2 + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2. \tag{10.2}$$

*The embedding is totally geodesic, all curvatures vanish, and the complex-to-real field diagram commutes trivially.*

## 11 Flat four-sector observer spacetime

**Worked solution 11.1** (Sector cycle)**.** *Starting from Eq. (10.2), the sector representatives are*

$$\mathrm{d}s^2_{(N_g)} = (-1)^{N_g}\mathrm{d}s^2_{(0)}, \qquad N_g = 0, 1, 2, 3. \tag{11.1}$$

*The four clock centres are $+t, +n, -t, -n$. Each sector has one real clock, three rulers and the same null equation. The continuous motion inside one sector remains an ordinary Lorentz motion with $|\beta| < 1$. The full orientation resides in the coframe because the metric cannot distinguish sectors separated by a half-turn.*

## 12 Reciprocal Schwarzschild solution

The ordinary projected metric is

$$\mathrm{d}s^2_{(0)} = -f(\mathcal{R})c^2\mathrm{d}t^2 + \frac{\mathrm{d}\mathcal{R}^2}{f(\mathcal{R})} + \mathcal{R}^2\mathrm{d}\Omega^2, \qquad f(\mathcal{R}) = 1 - \frac{r_s}{\mathcal{R}}. \tag{12.1}$$

The proper and critical scales are $L = \mathcal{R}$ and $G = r_s$. The reciprocal chart uses

$$\mathcal{R}(r) = \frac{r_s^2}{r}, \qquad 0 < r \le r_s. \tag{12.2}$$

After the full radial pullback and odd-sector sign,

$$\mathrm{d}s^2_{(1)} = (1 - r/r_s)c^2\mathrm{d}t^2 - \frac{r_s^4}{r^4(1 - r/r_s)}\mathrm{d}r^2 - \frac{r_s^4}{r^2}\mathrm{d}\Omega^2. \tag{12.3}$$

**Worked solution 12.1** (Curvature and causal distance)**.** *The Kretschmann scalar is*

$$K_{(1)}(r) = \frac{12r^6}{r_s^{10}}, \tag{12.4}$$

*so it is finite at $r = r_s$ and vanishes at $r \to 0$. With $X = ct$ spatial in the active side plane and $P = f\,\mathrm{d}X/\mathrm{d}\tau$, a timelike geodesic satisfies*

$$\left(\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\tau}\right)^2 = c^2\left(1 + p^2 - \frac{r_s}{\mathcal{R}}\right), \qquad p = P/c. \tag{12.5}$$

*Its growth rate is bounded above and below by finite positive constants after any $\mathcal{R}_1 > r_s$, so $\mathcal{R} = \infty$, equivalently $r = 0$, requires infinite proper time. Radial light obeys*

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\lambda} = \pm\mathcal{E}, \tag{12.6}$$

*and reaches the same limit only at infinite affine parameter. The reciprocal centre is an asymptotically flat boundary, not a finite transition to a white hole branch.*

For a field-compatible complexification of this analytic real metric, Eq. (6.12) projects the complex Bochner curvature to the Schwarzschild Weyl tensor. Existence of the local analytic complexification does not by itself establish a unique global fundamental complex manifold.

## 13 Reciprocal Kerr solution

Let $M = Gm/c^2$, $a = J/(mc)$, and

$$r_+ = M + \sqrt{M^2 - a^2}. \tag{13.1}$$

The reciprocal principal radial scale is

$$\mathcal{R}(r) = \frac{r_+^2}{r}. \tag{13.2}$$

The exact active metric is the odd signed pullback of a horizon-regular Kerr metric with the full Jacobian retained. The curvature scalar becomes the ordinary Kerr invariant evaluated at $\mathcal{R} = r_+^2/r$ and tends to zero as $O(r^6)$ at the reciprocal endpoint.

**Worked solution 13.1** (Carter parameter bound)**.** *With $\Sigma = \mathcal{R}^2 + a^2\cos^2\theta$, $\Delta = \mathcal{R}^2 - 2M\mathcal{R} + a^2$, and Carter constants $(\mathcal{P}, \mathcal{L}, \mathcal{Q})$, the radial potential is*

$$\Sigma^2\left(\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\zeta}\right)^2 = \mathcal{R}_\varepsilon(\mathcal{R}) = \Pi^2 - \Delta(\varepsilon\mathcal{R}^2 + \mathcal{K}), \tag{13.3}$$

*where $\varepsilon = -1$ for active side-timelike motion and $\varepsilon = 0$ for null motion. Since $0 \le \mathcal{R}_\varepsilon \le C\mathcal{R}^4$ and $\Sigma \ge \mathcal{R}^2$ on every unbounded allowed branch,*

$$\zeta - \zeta_0 = \int_{\mathcal{R}_0}^{\mathcal{R}} \frac{\Sigma}{\sqrt{\mathcal{R}_\varepsilon}}\mathrm{d}\mathcal{R}' \ge \frac{\mathcal{R} - \mathcal{R}_0}{\sqrt{C}}. \tag{13.4}$$

*The reciprocal endpoint is therefore at infinite proper or affine parameter for every such branch.*

## 14 Reciprocal Kerr–Newman solution

For mass $m$, charge $Q$ and spin orientation $\sigma$, define

$$r_s = \frac{2Gm}{c^2}, \qquad r_Q^2 = \frac{GQ^2}{4\pi\epsilon_0 c^4}, \qquad a_\sigma = \sigma\frac{\hbar}{2mc}. \tag{14.1}$$

The angular critical scale and reciprocal radius are

$$\begin{aligned} R_h(\theta) &= \frac{-r_s + \sqrt{r_s^2 + 4(r_Q^2 - a_\sigma^2\cos^2\theta)}}{2}, \\ \mathcal{R}(\rho,\theta) &= \frac{R_h^2(\theta)}{\rho}, \\ \mathrm{d}\mathcal{R} &= -\frac{R_h^2}{\rho^2}\mathrm{d}\rho + \frac{2R_hR_h'}{\rho}\mathrm{d}\theta. \end{aligned} \tag{14.2}$$

The angular term in $\mathrm{d}\mathcal{R}$ generates indispensable $\mathrm{d}W\mathrm{d}\theta$ and $\mathrm{d}\theta\mathrm{d}\widetilde{\phi}$ terms in the exact pullback metric.

**Worked solution 14.1** (Nondegeneracy and removed ring direction)**.** *With $\Sigma_1 = \mathcal{R}^2 + a_\sigma^2\cos^2\theta$, the determinant of the full active metric is*

$$\det g_{(1)} = -\frac{R_h^4(\theta)}{\rho^4}\Sigma_1^2\sin^2\theta. \tag{14.3}$$

*It is nonzero away from the usual polar coordinate axis. In the equatorial ring direction,*

$$K_{(1)}\left(\rho, \frac{\pi}{2}\right) = \frac{12r_s^2\rho^6}{R_0^{12}} - \frac{48r_sr_Q^2\rho^7}{R_0^{14}} + \frac{56r_Q^4\rho^8}{R_0^{16}}, \qquad R_0 = R_h(\pi/2). \tag{14.4}$$

*The scalar decreases toward zero as $\rho \to 0$. The Maxwell potential*

$$A_{(1)} = -\frac{Q\mathcal{R}}{\Sigma_1}(\mathrm{d}W - a_\sigma\sin^2\theta\mathrm{d}\widetilde{\phi}) \tag{14.5}$$

*is transported in the same active chart, so the metric, electromagnetic source, Ricci curvature and Weyl curvature refer to one constituent geometry.*

## 15 LTB longitudinal and transverse modes

The projected Lemaître–Tolman–Bondi dust geometry is

$$\mathrm{d}s_{(0)}^2 = -c^2\mathrm{d}t^2 + \frac{R_F^2}{1+2E(F)}\mathrm{d}F^2 + R^2\mathrm{d}\Omega^2. \tag{15.1}$$

Let $s^\mu$ be the radial unit vector and $S^\mu{}_\nu = h^\mu{}_\nu - s^\mu s_\nu$ the spherical screen projector. The two proper scale modes and their critical scales are

$$\begin{aligned} L_\parallel &= \frac{R_F}{\sqrt{1+2E}}, \qquad & G_\parallel &= 1, \qquad & q_\parallel &= \frac{\sqrt{1+2E}}{R_F}, \\ L_\perp &= R, & G_\perp &= F, & q_\perp &= \frac{F}{R}. \end{aligned} \tag{15.2}$$

Shell crossing $R_F \to 0$ activates the longitudinal projector, whereas shell focusing $R \to 0$ activates the degenerate transverse projector. One scalar radius cannot replace these two tensor modes.

**Worked solution 15.1** (Exact reciprocal shell-crossing dust)**.** *In geometrized units, take*

$$R(t,F) = \left(\frac{9F}{4}\right)^{1/3} [t_B(F) - t]^{2/3}, \qquad R_c(F) = R_0 + \frac{\alpha}{2}(F - F_c)^2, \tag{15.3}$$

*with*

$$t_B(F) = t_c + \frac{2R_c(F)^{3/2}}{3\sqrt{F}}. \tag{15.4}$$

*Then* $R_F(t_c, F) = \alpha(F - F_c)$ *and* $R(t_c, F_c) = R_0 > 0$*. On the reciprocal branch* $F < F_c$*, the entrance profile is*

$$\widetilde{R}_h(F) = R_h + \frac{1}{\alpha}\ln\left[\frac{1}{\alpha(F_c - F)}\right]. \tag{15.5}$$

*The exact expanding side solution is*

$$\widetilde{R}(\tau, F) = \left[\widetilde{R}_h(F)^{3/2} + \frac{3}{2}\sqrt{F}(\tau - \tau_h)\right]^{2/3}, \tag{15.6}$$

*with metric*

$$\mathrm{d}s^2_{(1)} = \mathrm{d}\tau^2 - \widetilde{R}_F^2\mathrm{d}F^2 - \widetilde{R}^2\mathrm{d}\Omega^2. \tag{15.7}$$

*Its density and Kretschmann scalar are*

$$\begin{aligned} \kappa\rho &= \frac{1}{\widetilde{R}^2\widetilde{R}_F}, \\ K_{(1)} &= \frac{12F^2}{\widetilde{R}^6} - \frac{8F}{\widetilde{R}^5\widetilde{R}_F} + \frac{3}{\widetilde{R}^4\widetilde{R}_F^2}, \end{aligned} \tag{15.8}$$

*and both tend to zero at* $F \to F_c^-$ *for every fixed finite* $\tau - \tau_h$*. This proves curvature regularity for this exact profile, not for every LTB shell crossing.*

**Worked solution 15.2** (Homogeneous reciprocal shell focusing)**.** *For the homogeneous dust collapse*

$$a(t) = a_h\left(\frac{t_f - t}{t_f - t_h}\right)^{2/3}, \qquad \mathrm{d}s^2_{(0)} = -\mathrm{d}t^2 + a^2(t)(\mathrm{d}\chi^2 + \chi^2\mathrm{d}\Omega^2), \tag{15.9}$$

*the side scale and metric are*

$$\begin{aligned} b(\tau) &= a_h\left[1 + \frac{3}{2}H_h(\tau - \tau_h)\right]^{2/3}, \\ \mathrm{d}s^2_{(1)} &= \mathrm{d}\tau^2 - b^2(\tau)(\mathrm{d}\chi^2 + \chi^2\mathrm{d}\Omega^2). \end{aligned} \tag{15.10}$$

*The active curvature is*

$$K_{(1)}(\tau) = \frac{15H_h^4}{[1 + \frac{3}{2}H_h(\tau - \tau_h)]^4} \longrightarrow 0. \tag{15.11}$$

*Massive proper time and radial-null affine parameter both diverge toward the reciprocal centre. This homogeneous solution is causally complete in the directions analysed.*

# 16 Spatially flat FLRW reciprocal solution

For

$$\mathrm{d}s^2_{(0)} = -c^2\mathrm{d}t^2 + a^2(t)(\mathrm{d}\chi^2 + \chi^2\mathrm{d}\Omega^2), \tag{16.1}$$

homogeneity makes all three proper-scale eigenvalues equal. With critical scale $a_h$,

$$b(\tau) = \frac{a_h^2}{a(t(\tau))}. \tag{16.2}$$

For $p = w\rho c^2$, $w > -1$, and $\alpha = 3(1+w)/2$, the exact active solution is

$$\begin{aligned} b(\tau) &= a_h[1 + \alpha H_h(\tau - \tau_h)]^{1/\alpha}, \\ \rho(\tau) &= \frac{\rho_h}{[1 + \alpha H_h(\tau - \tau_h)]^2}. \end{aligned} \tag{16.3}$$

The FLRW Weyl tensor vanishes and the full Riemann invariant is

$$K_{(1)}(\tau) = \frac{K_h}{[1 + \alpha H_h(\tau - \tau_h)]^4} \longrightarrow 0. \tag{16.4}$$

For the corresponding ordinary power law,

$$t(\tau) = \frac{t_h}{1 + \alpha H_h(\tau - \tau_h)}. \tag{16.5}$$

Thus the inherited label $t = 0$ compresses an infinite interval of active proper time. Finite-comoving-momentum massive particles and radial light also require unbounded proper and affine parameter. This gives the reciprocal FLRW branch for $w > -1$.

# 17 Photon and null-perspective solution

Let $\gamma : I \to \mathcal{N}$ be a selected null geodesic with tangent $k^\mu$. The local null representatives are

$$\mathrm{d}x^\mu = \lambda k^\mu(s), \qquad g_{\mu\nu} k^\mu k^\nu = 0. \tag{17.1}$$

Initial data and the geodesic equation select the null ray and transport it; they do not create a photon rest clock. The accumulated proper time is

$$\tau_\gamma(s) = \frac{1}{c} \int_{s_0}^{s} \sqrt{-g_{\mu\nu} k^\mu k^\nu}\, \mathrm{d}s' = 0. \tag{17.2}$$

**Worked solution 17.1** (Projective complete null state)**.** *The physical state supported on the selected history is*

$$\mathcal{S}_\gamma = \{(\gamma(s), [k(s)], \omega_u(s), \pi) : s \in I\}, \tag{17.3}$$

*where $[k]$ is the projective future null ray, $\omega_u$ fixes its physical scale relative to an observer, and $\pi$ denotes polarization or helicity. Constant positive affine rescalings are not counted as new states, the zero vector is not a propagating photon, and the past orientation is a separate branch. Distinct events remain causally ordered, but no two receive different photon proper-time values. Timelike observer slices supply the ordinary measured temporal sequence.*

This solution realizes Postulate 3 in the null limit; timelike configurations are carried by the complete tensor state of Sec. 8.

# 18 Free graviton solution

In a projected Ricci-flat region,

$$T_{\mu\nu} = 0, \qquad T^{(q)}_{\mu\nu\rho\sigma} = -\frac{1}{\kappa} C^{(q)}_{\mu\nu\rho\sigma}, \qquad \Psi^{(q)}_{\mu\nu\rho\sigma} = -\sqrt{\ell_{\mathrm{P}}}\, c_q C^{(q)}_{\mu\nu\rho\sigma}. \tag{18.1}$$

The vacuum Bianchi equation gives $\nabla^\mu C_{\mu\nu\rho\sigma} = 0$, whose principal covector is null. On the screen transverse to the observer clock $u^\mu$ and propagation direction $s^\mu$, define

$$m^\mu = \frac{e_1^\mu + \mathrm{i} e_2^\mu}{\sqrt{2}}. \tag{18.2}$$

The type-N tensors

$$C^{(+2)}_{\mu\nu\rho\sigma} \propto k_{[\mu} m_{\nu]} k_{[\rho} m_{\sigma]}, \qquad C^{(-2)}_{\mu\nu\rho\sigma} \propto k_{[\mu} \bar{m}_{\nu]} k_{[\rho} \bar{m}_{\sigma]} \tag{18.3}$$

acquire phases $e^{\pm 2i\psi}$ under a screen rotation.

**Worked solution 18.1** (Spectrum and sector transport)**.** *For a positive-frequency mode, Eq. (8.9) gives $E_G = \hbar\omega$. The projected sector transport is*

$$C^{(N_g)} = (-1)^{N_g} \mathcal{I}^*_{N_g} C^{(0)}, \qquad H^{(N_g)} = \mathcal{I}^*_{N_g} H^{(0)} (\mathcal{I}^*_{N_g})^{-1}. \tag{18.4}$$

*It preserves the null characteristic, energy and helicity. Exact type-N plane waves have zero Lorentzian self-contraction, so their spectral basis is biorthogonal: the dual mode is represented by the antipodal null direction. Localized physical packets require nonzero radial bandwidth and the complete Hermitian tensor normalization.*

Four signed pullbacks close the rank-four Weyl representative algebraically. The oriented momenta of the four inherited null branches can sum to zero, but changes of the physical Weyl profile still obey a local curvature wave equation with null principal characteristic. Closed sector history is not nonlocal instantaneous propagation.

# 19 Hydrogen solution

The electron constituent uses the reciprocal Kerr–Newman geometry of Sec. 14. The atom itself has one common field and one common geometry. Its electromagnetic tensor is assembled before the source lift,

$$F_{\mu\nu} = F^{(e)}_{\mu\nu} + F^{(p)}_{\mu\nu} + F^{(b)}_{\mu\nu}. \tag{19.1}$$

The Maxwell stress tensor consequently contains the electron–proton cross-term. Integrating its electrostatic part gives

$$V(r) = \frac{q_e q_p}{4\pi\epsilon_0 r} = -\frac{\alpha\hbar c}{r}. \tag{19.2}$$

The electromagnetic, paired-null bridge, conservation response and optional material contributions are assembled into one real source

$$T^{(q)}_{\mu\nu} = \sum_{A=1}^{4} T^{(q)}_{A\mu\nu}, \qquad \nabla_\mu T^{(q)\mu\nu} = 0. \tag{19.3}$$

It is then lifted to the complex four-index configuration and only afterward weighted by $c_q$ in Eq. (8.3).

**Worked solution 19.1** (Central spectrum)**.** *With reduced mass $\mu$ and $k^2 = (j+\frac{1}{2})^2\hbar^2$, the central mass shell gives*

$$p_r^2 = \frac{E^2}{c^2} - \mu^2 c^2 + \frac{2\alpha\hbar E}{cr} - \frac{\hbar^2}{r^2}\left[(j+\tfrac{1}{2})^2 - \alpha^2\right]. \tag{19.4}$$

*The closed radial action yields*

$$\boxed{E_{n_r j} = \mu c^2 \left[1 + \frac{\alpha^2}{\left(n_r + \sqrt{(j+\frac{1}{2})^2 - \alpha^2}\right)^2}\right]^{-1/2}.} \tag{19.5}$$

*The compatible radial projection is*

$$u_{n_r j}(r) = N_{n_r j}(2\lambda r)^{\gamma_j} e^{-\lambda r} L_{n_r}^{2\gamma_j - 1}(2\lambda r), \qquad \gamma_j = \sqrt{(j+\tfrac{1}{2})^2 - \alpha^2}. \tag{19.6}$$

*The complex carrier uses the full tensor mode, not this scalar radial projection alone. Hermitian overlaps resolve energy, position, angular and spin channels without introducing another state field.*

The result reproduces the relativistic central Coulomb spectrum within the complete four-index tensor carrier.

## 20 Ultraviolet quantum-gravity solution

In the projected real theory, standard background-field quantization uses $D = 4 - 2\epsilon$, gauge fixing and Faddeev–Popov ghosts. Recursive forest subtraction is completed before the sector operation. At loop order $L$, the remaining local pole functional defines

$$H^{(L),\mathrm{UV}}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \Gamma^{(L)}_{\mathrm{pole}}}{\delta g^{\mu\nu}}, \qquad \nabla^\mu H^{(L),\mathrm{UV}}_{\mu\nu} = 0. \tag{20.1}$$

Internal sector factors cancel against inverse factors on propagator indices, whereas the two free covariant indices retain

$$H^{(L),\mathrm{UV}}_{\mu\nu}(S_X) = (-1)^{S_X} H^{(L),\mathrm{UV}}_{\mu\nu}(0). \tag{20.2}$$

The normalized complete-orbit projector therefore gives

$$\mathcal{P}^{\mathrm{UV}}_2 H^{(L),\mathrm{UV}}_{\mu\nu} = \frac{1}{4} \sum_{S_X=0}^{3} (-1)^{S_X} H^{(L),\mathrm{UV}}_{\mu\nu} = 0. \tag{20.3}$$

**Worked solution 20.1** (Two-loop cubic-curvature benchmark)**.** *The Goroff–Sagnotti local pole is proportional to*

$$\Gamma^{(2)}_{\mathrm{div}} \propto \frac{1}{\epsilon} \int \sqrt{-g}\, R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{ef} R^{ef}{}_{\mu\nu}\, \mathrm{d}^4 x. \tag{20.4}$$

*Its metric Euler tensor is symmetric, conserved and nonzero before sector projection. It has the character in Eq. (20.2), so its complete particle-orbit component vanishes. Evanescent Gauss–Bonnet subdivergences are included before this projection and therefore modify the common scalar coefficient without modifying the sector character.*

The rank-two response has the real four-index lift

$$\begin{aligned} \widetilde{H}_{\mu\nu\rho\sigma} = {} & \frac{1}{2}(H_{\mu\rho} g_{\nu\sigma} - H_{\mu\sigma} g_{\nu\rho} + H_{\nu\sigma} g_{\mu\rho} - H_{\nu\rho} g_{\mu\sigma}) \\ & - \frac{1}{6}(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) H, \end{aligned} \tag{20.5}$$

completed by a trace-free $\widehat{H}_{\mu\nu\rho\sigma}$ obeying the same divergence exchange law. Both pieces inherit $(-1)^{S_X}$, so the four-index orbit also cancels.

Thus every local UV pole source with the sector character $(-1)^{S_X}$ vanishes under the complete four-sector orbit projection, both in its rank-two response and in its four-index lift.

## 21 Conclusion

The $SU(m,n)$ Unified Field Theory has a complex pseudo-Kähler spacetime of dimension $N = m+n$, carrying Hermitian four-index geometry and sources. The physical Lorentzian branch is $(m,n) = (1,3)$. A real spacetime is the geometry measured on an admissible observer real form, and the four-sector atlas acts after this projection.

The principal mathematical result of the theory is the explicit complex-to-real bridge. Realification, pullback and the Gauss equation give the observed curvature, while the Weyl projector separates the projected trace-free content. The concise relation between Bochner and Weyl tensors is therefore conditional on vanishing extrinsic and Ricci-projection corrections. The same observer map projects the complex source, field equation and quantum carrier, so geometry and state are not related by separate ad hoc rules.

Postulate 3 is realized by a complete tensor state whose nonzero branches coexist without being summed into one classical source. Full Hermitian conjugation, a hypersurface overlap and covariant rank-eight evolution supply one probability and phase convention. At a given event the observer resolves one channel of the complete state.

The framework yields one common local null speed, four oriented observer sectors, no ordinary massive crossing of the local light cone, and massless helicity-two free-graviton modes with $E = \hbar\omega$. Schwarzschild, Kerr, Kerr–Newman, LTB and FLRW geometries acquire reciprocal regular continuations. The photon solution has zero proper-time separation along its null history, and the hydrogen construction reproduces the relativistic central Coulomb spectrum.

Finally, the complete equal-weight particle-sector orbit annihilates local UV pole sources carrying the character $(-1)^{S_X}$. The same cancellation is preserved by the real four-index lift. Complex geometry, real observer projection, four-sector transport, complete tensor states and the collected solutions therefore constitute one common field-theoretic structure governed by the four postulates stated at the beginning.

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