Topological String-Gradient Theory: Metric Replication of Space-Time and Cosmological Evolution

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Abstract

We present the Topological String-Gradient Theory (TSGT), a framework in which space-time is discrete at the Planck scale and gravity is the gradient of the spacing between discrete elements ("pixels"). Two kinematic postulates—a spatial pixel strain set by the Newtonian potential and an equal temporal strain—reproduce the weak-field Schwarzschild metric, including the full light deflection $\Delta\theta = 4GM/c^2b$; this reproduction is by construction. The discrete substrate is a random Poisson sprinkling of density L_p^{-4}, which preserves Lorentz invariance statistically and removes the preferred-frame problems of lattice formulations; lattice-derived predictions of earlier versions are retracted. If cosmic expansion creates new pixels rather than stretching existing ones—the only option compatible with an invariant L_p—then Poisson fluctuations in the pixel count fix the cosmological term to $\rho_\Lambda \simeq 2.8 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$, within a factor of two of the observed value; this argument is Sorkin's heuristic, following here from TSGT's own postulates. Written locally, the number–volume correspondence is the unimodular constraint; enforcing it in an Einstein–Hilbert action yields the trace-free Einstein equations, making TSGT provably equivalent to General Relativity at the classical level. The framework's independent content lies in the integer character of the pixel count and in the fluctuating dark-energy term it implies ($w \neq -1$). Unresolved problems, including the absence of a dynamical equation for the pixel density, are listed explicitly.

Contents

1 Introduction

Modern physics struggles to reconcile General Relativity (GR) with Quantum Mechanics, in large part because GR treats space-time as a smooth, infinitely divisible manifold [1, 2]. A recurring response is to assume that the manifold approximates a discrete substrate with granularity set by the Planck scale,

$$L_{\mathrm{p}} = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35} \text{ m}, \qquad E_{\mathrm{p}} = \sqrt{\frac{\hbar c^5}{G}} = 1.22 \times 10^{19} \text{ GeV}. \tag{1}$$

Topological String-Gradient Theory (TSGT) belongs to this family [3, 4]: space-time consists of discrete quantum nodes, and gravity is not a force acting within that structure but the geometric consequence of a gradient in the spacing of the structure itself.

This version of the paper differs substantially from earlier drafts, and it is worth stating the three changes at the outset because each of them costs the theory something.

Change 1: the substrate is random, not a lattice. Earlier versions assumed a regular cubic array of pixels. A regular array defines a preferred rest frame and preferred axes, and therefore breaks Lorentz and rotational invariance at a fundamental level—an assumption that is difficult to reconcile with the success of special relativity and that creates a severe naturalness problem. We therefore replace it with a random Poisson sprinkling (Section 3), which is discrete yet statistically Lorentz invariant. The cost is that the lattice-derived predictions of earlier drafts—energy-dependent photon speed, gravitational-wave dispersion, and propagation anisotropy—no longer follow, and we retract them (Section 3.3).

Change 2: expansion creates pixels, and that creates dark energy. If L_{p} is an invariant, cosmic expansion cannot stretch pixels; it must create them. Section 5 shows that this single requirement, combined with the Poisson statistics of Change 1, fixes the order of magnitude of the cosmological term to within a factor of two of its observed value. This replaces the withdrawn signatures with a quantitative one, and it is the first place in this framework where a number is computed and compared with data.

Change 3: the postulates follow from an action. Section 6 shows that the number–volume correspondence of Change 2, written locally, is the unimodular constraint on the metric determinant, and that enforcing it in the Einstein–Hilbert action reproduces the postulates of Section 2 rather than assuming them. This too costs the theory something, and the cost is the largest of the three: it makes the classical equivalence to General Relativity exact and provable, confining TSGT's independent content to the discreteness of the pixel count itself.

What is a result and what is not. Sections 2 and 4 are constructions: they show that TSGT is consistent with known gravity, not that it predicts it. Section 5 is the paper's only quantitative confrontation with observation, and even there the central argument is heuristic and is due to Sorkin [16, 17], not to the present author; TSGT's contribution is that the argument follows from its own postulates rather than being imported. Section 6 is a derivation rather than a construction, but what it derives is General Relativity, which is not new; its value is that it removes the postulates and sharpens where TSGT can still differ.

2 Core Mechanism: Gravity as a Metric Gradient

2.1 The spacing field

Let $\ell(x)$ denote the mean separation between neighbouring pixels near the point x, measured in the chart in which the Newtonian potential Φ below is defined. Far from all matter this takes its vacuum value $L_{\rm p}$; near a mass it is stretched:

$$\ell(x) = L_{\rm p}\big(1+\chi(x)\big), \qquad \chi \equiv -\frac{\Phi}{c^2}, \tag{2}$$

with Φ the Newtonian potential ($\Phi < 0$, hence $\chi > 0$). We further postulate that the local clock rate is retarded by the same fractional amount:

$$\tau(x) = \tau_{\rm p}\big(1-\chi(x)\big), \qquad \tau_{\rm p} \equiv L_{\rm p}/c. \tag{3}$$

Equations (2)–(3) are the two kinematic postulates of TSGT. Together they are equivalent to the isotropic weak-field metric

$$ds^2 = -\,(1-2\chi)\,c^2dt^2 + (1+2\chi)\,\delta_{ij}\,dx^i dx^j, \tag{4}$$

i.e. the linearised Schwarzschild solution [5, 6]. They are taken as postulates in this section; Section 6 derives both from an action principle.

What is stretched, and what is not. Equation (2) refers to a *mean* separation, and therefore does not require the pixels to be regularly arranged; this is what allows the construction of Section 3 to proceed without disturbing the present section. It requires one further clarification, however, because the phrase "the spacing is stretched" is easily misread. The quantity $\ell(x)$ is a *coordinate* separation: it measures how far apart neighbouring pixels lie when they are labelled by the chart in which Φ is defined. The *proper* density of pixels is not affected by the presence of matter at all. It retains its invariant value $L_{\rm p}^{-4}$ everywhere, which is precisely what Section 3 requires, and it is what makes the statistical Lorentz invariance of the substrate possible: a quantity that varied from place to place could not be a Lorentz invariant of the sprinkling.

Gravity in TSGT is therefore not a rarefaction of pixels in any invariant sense—there is no invariant sense in which pixels can be rarefied—but a variation in the coordinate volume that a fixed number of them occupies. Written covariantly, this is the statement $\sqrt{-g} = L_{\rm p}^4 n(x)$, with n the coordinate-space number density; it appears below as Eq. (28), and Eqs. (2)–(3) are its weak-field form in Newtonian gauge. The two are the same statement, and a reader who prefers the covariant one may take (28) as primitive and read the postulates of this section as its linearisation.

Coarse-graining and the gradient. Because the substrate is a Poisson sprinkling rather than a smooth field, the spacing $\ell(x)$ of Eq. (2) must be understood as a coarse-grained average over a region of linear size $\lambda_{\rm CG}$ satisfying $L_{\rm p} \ll \lambda_{\rm CG} \ll r$, where r is the scale on which χ varies appreciably. The number of pixels in such a region is $\mathcal{N} \sim (\lambda_{\rm CG}/L_{\rm p})^3 \gg 1$, so the Poisson fluctuation in the mean spacing is suppressed by $\mathcal{N}^{-1/2}$, and the coarse-grained field is smooth to the accuracy required by the Taylor expansion of Eq. (5). The gradient $\nabla\chi$ inherits a residual noise of order $L_{\rm p}/\lambda_{\rm CG}$ relative to the deterministic value, which vanishes in the continuum limit. A precise treatment of gradient fluctuations on the sprinkling itself is not available and is part of open problem OP3.

2.2 The gradient relation and the Newtonian limit

The name of the theory refers to the fact that acceleration is controlled not by the spacing itself but by how the spacing changes from one pixel to the next. Define

$$\Delta L(x) \equiv \ell(x + L_{\rm p}\hat{n}) - \ell(x) = L_{\rm p}^2\,(\hat{n}\cdot\nabla\chi) + \mathcal{O}(L_{\rm p}^3), \tag{5}$$

one power of $L_{\rm p}$ smaller than the spacing itself. The dynamical postulate is

$$\mathbf{g}\cdot\hat{n} = a_{\rm P}\,\frac{\Delta L}{L_{\rm p}}, \qquad a_{\rm P} \equiv \frac{c^2}{L_{\rm p}} \approx 5.6\times 10^{51}\ \mathrm{m\,s^{-2}}, \tag{6}$$

with $a_{\rm P}$ the Planck acceleration. The left-hand side appears as a projection because ΔL, as defined in (5), is the increment along the single direction $\hat{n}$; demanding that (6) hold for every $\hat{n}$ determines the vector completely and gives

$$\mathbf{g} = c^2 \nabla \chi = -\nabla \Phi, \tag{7}$$

so the acceleration automatically points along the gradient of the pixel spacing, towards larger spacing. For a point mass, $\chi = GM/(c^2 r)$ and (5) gives

$$\Delta L(r) = \frac{GML_{\rm p}^2}{c^2 r^2} \quad \Longrightarrow \quad g = \frac{c^2}{L_{\rm p}^2} \cdot \frac{GML_{\rm p}^2}{c^2 r^2} = \frac{GM}{r^2}. \tag{8}$$

Physical picture. Mass stretches the pixel distribution, in the coordinate sense set out above. A photon or a massive body crossing the resulting gradient follows the path of least geometric resistance—towards larger spacing—which macroscopically is indistinguishable from an attractive force [7].

Honest status of Eq. (8). This is a consistency check, not a prediction. Postulate (2) identifies the pixel strain with the Newtonian potential, so recovering Newton is guaranteed. The claim TSGT is entitled to make is narrower: a single relation expressed purely in Planck units, Eq. (6), is sufficient to reproduce the entire weak-field sector. New physics can appear only in the discreteness corrections.

2.3 Light deflection and the optical analogy

From (4) the coordinate speed of light is $c_{\rm eff} = c(1 - 2\chi)$, so the vacuum behaves as a medium of refractive index

$$n_{\rm opt}(r) \simeq 1 + \frac{2GM}{c^2 r}, \tag{9}$$

(the subscript distinguishes it from the pixel density $n(x)$ of Section 6), and Fermat's principle applied to (9) gives

$$\Delta\theta = \frac{4GM}{c^2 b}, \tag{10}$$

in agreement with GR and with Cassini-class measurements at the 10^{-5} level [8, 9].

The temporal postulate (3) is therefore not optional. Half of (10) comes from the stretching of space and half from the retardation of clocks; a purely spatial deformation would predict $\Delta\theta = 2GM/c^2 b$, exactly half the observed value, and would be excluded by five orders of magnitude.

2.4 Regime of validity

The linearisation (5) assumes $\Delta L \ll L_{\rm p}$, and fails where $g \sim a_{\rm P}$, that is at

$$r_* = \sqrt{\frac{GML_{\rm p}}{c^2}} = \sqrt{\frac{r_s L_{\rm p}}{2}}, \qquad r_s = \frac{2GM}{c^2}, \tag{11}$$

the geometric mean of the Schwarzschild radius and the Planck length. For one solar mass, $r_* \simeq 1.5 \times 10^{-16}$ m. Since pixels have finite size, TSGT cannot support unbounded density, so the framework is structurally incompatible with curvature singularities; what replaces them is not determined by the present formulation and is recorded as open problem OP6.

2.5 Comparison with Planck-density thresholds

It is worth distinguishing the criterion (11) from the one used in Planck-star models [32], where the transition is triggered when the matter density reaches the Planck density ρ_{P}. The two are not equivalent. TSGT's criterion is a statement about the gradient of the spacing—how rapidly geometry changes from one pixel to the next—rather than about how much matter is packed into a volume. Since $r_* \propto M^{1/2}$ while the Planck-density radius scales as $M^{1/3}$, the two thresholds diverge as $M^{1/6}$:

Mass	r_* (TSGT)	r (Planck density)	ρ_*/ρ_{P}
$\approx 1.2 \times 10^{-9}$ kg ($\approx 0.06\, M_{\mathrm{P}}$)	criteria coincide		~ 1
Earth mass	2.7×10^{-19} m	6.5×10^{-25} m	1.4×10^{-17}
Solar mass	1.5×10^{-16} m	4.5×10^{-23} m	2.5×10^{-20}
$10\, M_\odot$	4.9×10^{-16} m	9.7×10^{-23} m	7.9×10^{-21}
$4 \times 10^{6}\, M_\odot$	3.1×10^{-13} m	7.2×10^{-21} m	1.2×10^{-23}

Table 1: TSGT's rupture threshold compared with the Planck-density criterion, with $r_\rho \equiv (3M/4\pi\rho_{\mathrm{P}})^{1/3}$ the radius at which a uniform sphere of mass M attains ρ_{P}. The two coincide at a mass of order the Planck mass and separate as $M^{1/6}$ above it: for a solar mass, TSGT predicts the transition at a density some twenty orders of magnitude below the Planck density.

The two criteria therefore agree only near the Planck mass, and for any macroscopic body TSGT predicts that the substrate fails long before Planck density is attained (Fig. 1b). Whether this earlier failure is physically meaningful, or whether the two criteria simply measure different quantities that need not coincide, is not settled here.

3 Discreteness Without a Lattice

3.1 Why a regular array fails

A regular array of pixels is the intuitive picture of discreteness, and it is the one adopted in earlier drafts of this work. It cannot be maintained. A regular array specifies a rest frame (the frame in which it is at rest) and a set of axes (its principal directions). Both are observable in principle, and both conflict with the empirical success of local Lorentz invariance. The conflict is not merely aesthetic: Lorentz-violating operators generated at the Planck scale propagate to low energies through radiative corrections with only loop suppression rather than L_{p} suppression, so a Planck-scale violation generically implies laboratory-scale violations many orders of magnitude larger than observed [10].

One may attempt to rescue the array by making it dynamical, so that the preferred frame becomes a field rather than a fixed background. This is the Einstein-aether programme [11]. It is a viable class of theories, but it retains a preferred frame at every point and is now tightly constrained by the coincident detection of GW170817 and GRB 170817A [12, 13]. We do not pursue it here.

3.2 The sprinkling postulate

TSGT instead takes the pixels to be distributed by a **Poisson sprinkling** of the space-time manifold at density $\rho = L_{\mathrm{p}}^{-4}$. The probability of finding exactly N pixels in a four-volume V is

$$P(N) = \frac{(\rho V)^N e^{-\rho V}}{N!}, \qquad \langle N \rangle = \frac{V}{L_{\mathrm{p}}^4}, \qquad \delta N = \sqrt{\langle N \rangle}. \tag{12}$$

The pixels are discrete and have a well-defined mean separation L_{p}, but they form no regular pattern, no rows and no axes.

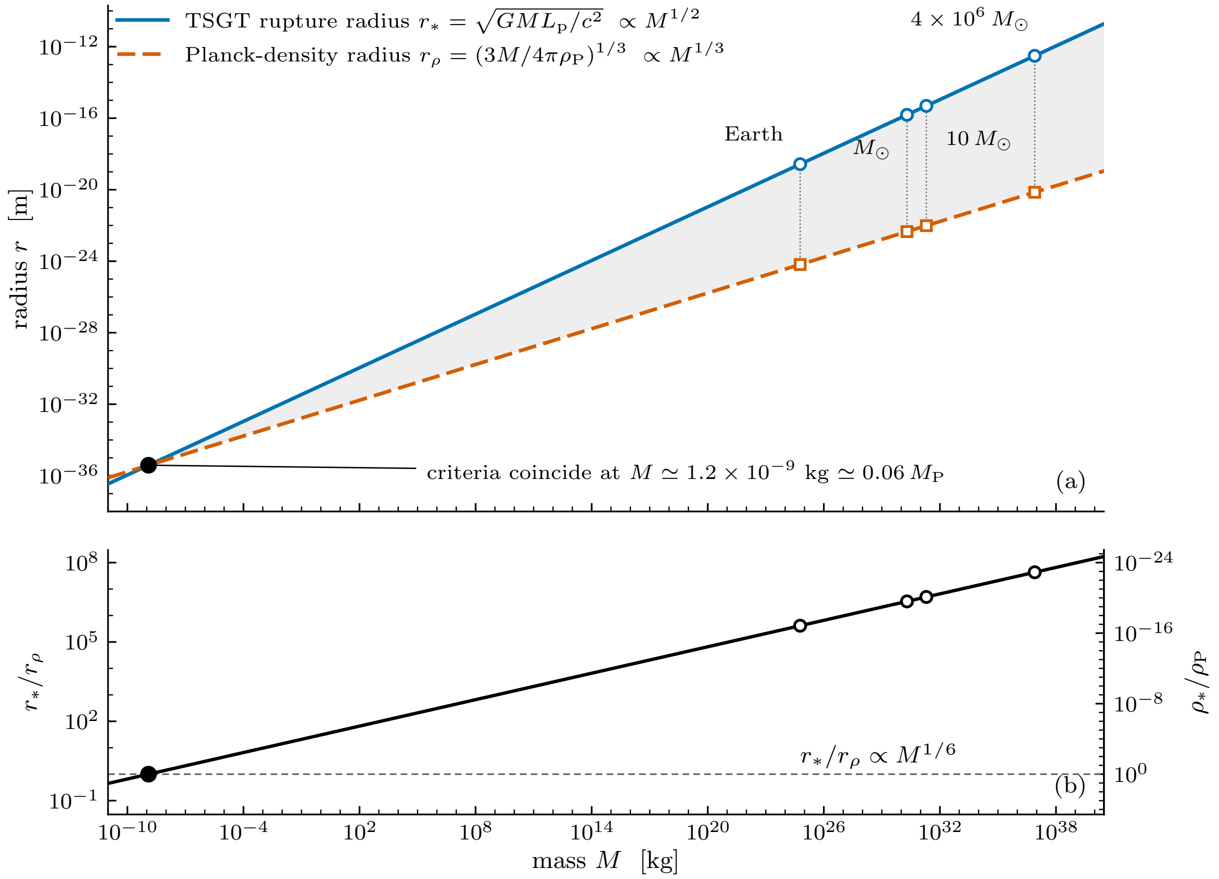

Figure 1: TSGT's rupture threshold $r_* = \sqrt{GML_p/c^2}$ compared with the radius $r_\rho = (3M/4\pi\rho_P)^{1/3}$ at which a body of mass M reaches the Planck density. (a) The two radii against mass; open circles and open squares mark the entries of Table 1 and the filled circle the mass at which the two criteria coincide. (b) Their ratio, a power law of index 1/6 crossing unity at that mass; the right-hand axis carries the equivalent density ratio $\rho_*/\rho_P = (r_*/r_\rho)^{-3}$, the last column of Table 1. The two criteria agree only near the Planck mass and separate as $M^{1/6}$ above it: for a solar mass TSGT places the transition some twenty orders of magnitude below the Planck density.

The essential property is that this construction is **statistically Lorentz invariant**: a Poisson sprinkling of Minkowski space looks the same, in distribution, to every inertial observer (Fig. 2). This is a theorem, not an approximation [15]. Discreteness therefore does not require a preferred frame, and the naturalness problem of the previous subsection does not arise. The apparent paradox—that a boosted observer should see contracted spacing—is resolved because there is no fixed spacing to contract: only a Lorentz-invariant density, which every observer agrees on. This is also why the spacing field $\ell(x)$ of Section 2 does not conflict with the present postulate: ℓ varies as a coordinate quantity, while the proper density that Eq. (12) fixes does not vary at all.

3.3 What this costs: retracted predictions

Honesty requires an explicit retraction. The following results appeared in earlier versions of this framework and do not survive the replacement of the regular array by a sprinkling:

(i) **Energy-dependent photon velocity.** The prediction $v(E) \simeq c\,[1 - \xi(E/E_p)]$ arises from a substrate with a preferred frame. A statistically Lorentz-invariant substrate does

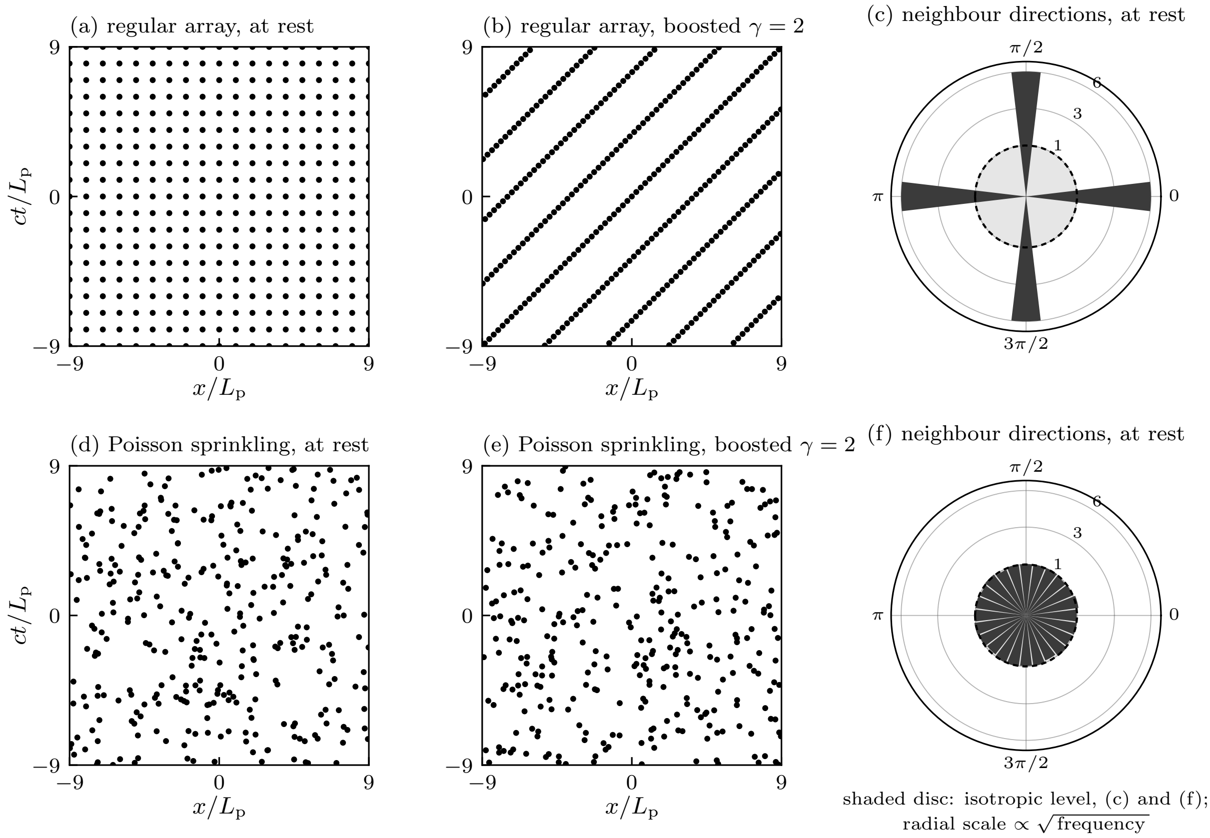

Figure 2: Discreteness without a preferred frame or preferred axes. Panels (a)–(c) show a regular array of pixels, panels (d)–(f) a Poisson sprinkling of the same mean density, both drawn in the (x, ct) plane in units of L_p. Panels (b) and (e) show the same configurations as seen by an observer boosted with $\gamma = 2$; the Lorentz transformation has unit determinant, so the mean density is identical in every panel. The boosted array differs visibly from the array at rest—this is what it means for a regular array to define a rest frame—whereas the sprinkling is statistically indistinguishable from itself, in accordance with the theorem of Ref. [15]. Panels (c) and (f) give the distribution of directions from each element to its four nearest neighbours, measured on the rest-frame configurations of (a) and (d); the radial scale is $\sqrt{\text{frequency}}$, so that petal area rather than petal radius is proportional to the number of neighbours in an angular bin, and the shaded disc of unit radius marks the isotropic level. The array confines the directions to four rays that overshoot that level sixfold, whereas the sprinkling's petals sit on it. Discreteness therefore requires neither a preferred frame nor preferred axes.

not generate leading-order vacuum dispersion, since there is no frame in which to define the energy dependence. The prediction is withdrawn.

(ii) **Gravitational-wave dispersion.** The relation $\omega^2 = c^2k^2\left(1 - \alpha k^2 L_\mathrm{p}^2/2\right)$ was derived from a nearest-neighbour cubic array via its Brillouin zone. No Brillouin zone exists here. Withdrawn.

(iii) **Propagation anisotropy.** The predicted cubic angular pattern $\alpha(\hat{n}) = \frac{1}{6}\sum_i \hat{n}_i^4$ was a direct consequence of the cubic axes. There are no axes. Withdrawn.

We note that (i)–(iii) were in any case unobservable by margins of between sixteen and sixty orders of magnitude, so little empirical content is lost. What is gained is the removal of the framework's most serious structural objection.

3.4 Relation to causal set theory

The sprinkling construction of Eq. (12) is not original here: it is the kinematic foundation of causal set theory, introduced by Bombelli, Lee, Meyer and Sorkin [14]. Intellectual honesty requires stating this plainly, and equally requires stating what TSGT does differently.

Causal set theory takes the fundamental data to be the discrete elements together with their *causal order*, from which metric information is recovered ("order plus number equals geometry"). TSGT takes the fundamental data to be the discrete elements together with their *spacing*, and supplies an explicit relation—Eq. (6)—between the gradient of that spacing and gravitational acceleration, expressed in Planck units. Causal set theory has no such relation; deriving Einstein's equations from causal order remains one of its central open problems.

Whether TSGT's spacing-gradient relation can be derived from causal order, or whether the two descriptions are genuinely independent, is not settled here. It is the most interesting technical question the present framework raises, and we record it as open problem OP4.

4 Pixel Excitations and the Emergent Spin-2 Sector

A discrete framework that removes gravity as a fundamental force must still account for the massless spin-2 quantum of linearised GR. TSGT's answer is that it is a collective excitation of the substrate, in the same sense that a phonon is a collective excitation of a crystal. The argument below is stated in the continuum effective description, which is where it is valid; the corresponding derivation directly on a sprinkling is not available and is recorded as an open problem.

4.1 The fundamental variable must be a tensor

Suppose one starts, as in ordinary elasticity, from a displacement vector field u_i and builds the strain $h_{ij} = \partial_i u_j + \partial_j u_i$. In Fourier space $h_{ij} = i(k_i u_j + k_j u_i)$. The transverse-traceless (TT) projector is built from $P_{ij} = \delta_{ij} - \hat{n}_i \hat{n}_j$ with $\hat{n} = \mathbf{k}/|\mathbf{k}|$ and satisfies $P_{ij} k_j = 0$. Every term in h_{ij} carries a free index contracted with $\mathbf{k}$, hence

$$h_{ij}^{TT} = \mathcal{P}_{ij,kl}\, h_{kl} \equiv 0 \quad \text{identically, for any } u_i. \tag{13}$$

A substrate whose only degrees of freedom are element displacements therefore has no spin-2 sector at all: its excitations are one longitudinal and two transverse sound modes, none transverse-traceless. This is the standard obstruction to obtaining gravitons from elasticity.

TSGT evades it because its primitive object is a *spacing*, and spacing is direction dependent. This motivates promoting the fundamental variable to a symmetric rank-2 field,

$$g_{ij}(x) = \delta_{ij} + h_{ij}(x,t), \qquad h_{ij} = h_{ji}, \tag{14}$$

with six independent components. Its isotropic part $h_{ij} = 2\chi\delta_{ij}$ recovers the static configuration of Section 2, while its traceless part describes shear of the pixel distribution.

This h_{ij} is the spatial part of the full four-dimensional field $h_{\mu\nu}$ whose dynamics is fixed in Section 6, and whose temporal component carries the clock postulate of Eq. (3). The present section works with the spatial sector alone because the helicity classification is a statement about spatial rotations.

4.2 Irreducible decomposition and the TT projection

Under spatial rotations $h'_{ij} = R_{im} R_{jn} h_{mn}$, and $3 \otimes 3 = 1 \oplus 3 \oplus 5$; since h_{ij} is symmetric the antisymmetric 3 is absent. For a given wavevector the relevant classification is by helicity under the little group SO(2). With $P_{ij} = \delta_{ij} - \hat{n}_i \hat{n}_j$, the TT projector is (written $\mathcal{P}$ rather than Λ, which is reserved throughout for the cosmological term)

$$\mathcal{P}_{ij,kl}(\hat{n}) = P_{ik}P_{jl} - \tfrac{1}{2}P_{ij}P_{kl}, \qquad h_{ij}^{TT} = \mathcal{P}_{ij,kl}h_{kl}, \tag{15}$$

which is idempotent and satisfies

$$\hat{n}_i h_{ij}^{TT} = 0, \qquad \delta_{ij} h_{ij}^{TT} = 0. \tag{16}$$

The six components split into two transverse-traceless modes (helicity ± 2), two vector modes (helicity ± 1) and two scalar modes (helicity 0). The conditions (16) are exactly those satisfied by the linearised graviton, and the helicity-± 2 pair corresponds to the $+$ and $\times$ polarisations.

Note. Equation (15) replaces the subtraction $h_{ij} - \frac{1}{3}\delta_{ij}h - (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)\nabla^{-2}h$ used in earlier presentations. That expression removes the trace and one scalar piece but leaves the transverse-vector part, so the result does not satisfy $\partial_i h_{ij} = 0$. Only the full projector isolates the helicity-2 sector.

4.3 Quantisation

Quantising the TT sector gives

$$h_{ij}^{TT}(\mathbf{x}, t) = \sum_{\mathbf{k}} \sum_{s=\pm 2} \sqrt{\frac{16\pi G\hbar}{2\omega_k \mathcal{V} c^2}} \left[\epsilon_{ij}^{(s)}(\mathbf{k})\, \hat{a}_{\mathbf{k},s} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_k t)} + \text{h.c.} \right], \tag{17}$$

with $[\hat{a}_{\mathbf{k},s}, \hat{a}^\dagger_{\mathbf{k}',s'}] = \delta_{\mathbf{k}\mathbf{k}'}\delta_{ss'}$, and $\mathcal{V}$ a normalisation volume, not to be confused with the four-volume V of Section 5. The prefactor $16\pi G$ is the conventional graviton normalisation, and the explicit c^2 is required for h_{ij}^{TT} to be dimensionless. In a lattice formulation the momentum sum would be cut off by a Brillouin zone; on a sprinkling no such sharp cutoff exists, and the effective cutoff is statistical—modes with wavelength approaching L_p lose meaning because the sprinkling does not resolve them. Making this precise is open problem OP3.

4.4 The extra-polarisation problem

Nothing above gaps the scalar and vector sectors, so they propagate alongside the two TT modes. GR has only the TT pair, and this is constrained observationally: a propagating scalar mode coupling to the trace of the stress tensor shifts the PPN parameter γ, measured to $|\gamma - 1| < 2.3 \times 10^{-5}$ [8, 9], and searches for non-tensorial polarisations in gravitational-wave data find none [23]. TSGT must therefore gap or decouple these sectors. Earlier versions of this paper recorded that failure as an open problem; it is resolved in Section 6, where the adoption of an action principle is shown to remove the scalar and vector sectors from the physical spectrum, leaving exactly the two transverse-traceless modes. The resolution comes from having any consistent dynamics for a massless spin-2 field, not from the unimodular constraint specifically.

5 Cosmology: Pixel Creation and the Cosmological Term

This section contains the framework's only quantitative confrontation with data.

5.1 Expansion must create pixels

Consider a homogeneous expanding universe with scale factor $a(t)$. Two options exist for the pixels:

(A) **Stretching.** The number of pixels is fixed and their separation grows as $a(t)$. Then L_p is time dependent. But L_p is the unit against which ΔL is measured in Eq. (6), and it is also the sprinkling density that guarantees Lorentz invariance in Eq. (12). Option (A) therefore destroys both foundations of the framework and is excluded internally.

(B) **Creation.** The separation is fixed at L_p and new pixels are created, so that the number in a comoving region grows as $N \propto a^3$ spatially, or equivalently $N = V/L_\mathrm{p}^4$ in four-volume terms.

TSGT is committed to (B). This is not a free choice; it is forced by the invariance of L_p.

5.2 Number, volume and their fluctuation

Option (B) makes the pixel count the primitive measure of space-time extent, with the four-volume a derived quantity:

$$V = NL_\mathrm{p}^4. \tag{18}$$

Because the sprinkling is Poisson, Eq. (18) is not exact but carries the fluctuation of Eq. (12):

$$N = \frac{V}{L_\mathrm{p}^4} \pm \sqrt{\frac{V}{L_\mathrm{p}^4}}. \tag{19}$$

The relative uncertainty in the four-volume is therefore $1/\sqrt{N}$—small, but not zero, and this residual is the origin of what follows.

5.3 The magnitude of the cosmological term

In unimodular formulations of gravity, the cosmological term Λ and the four-volume V are conjugate variables [28, 29], obeying an uncertainty relation of the form $\delta\Lambda\,\delta V \sim \hbar$; this is assumed here and derived from the action in Section 6. Combining it with Eq. (19) and working in Planck units gives

$$\delta\Lambda \sim \frac{1}{\delta V} \sim \frac{1}{\sqrt{V}} = \frac{1}{\sqrt{N}}\,. \tag{20}$$

Suppose further that whatever mechanism governs Λ drives its mean value to zero, so that the observed value is entirely this residual fluctuation. Then Λ is not a constant of nature at all: it is Poisson noise in the pixel count, and its magnitude is fixed by the size of the universe.

Motivation for $\langle\Lambda\rangle = 0$. The assumption that the mean vanishes is the strongest unproven ingredient of the argument, and it deserves more than a passing mention. In the unimodular formulation of Section 6, Λ is an integration constant rather than a parameter of the action, so no value is preferred *a priori*. Zero is nevertheless special: it is the value that corresponds to exact energy conservation in the matter sector (Case 1 of Section 6), and it is the only value compatible with the symmetry $\Lambda \to -\Lambda$ that the trace-free field equations (30) possess before the integration constant is introduced. Neither observation constitutes a derivation, and the mechanism that would enforce $\langle\Lambda\rangle = 0$ dynamically remains the central missing ingredient of the cosmological argument.

What the estimate is an estimate of. The status of (20) deserves to be stated explicitly, because the distinction it rests on governs the two subsections that follow and was elided in earlier versions of this paper. Once $\langle\Lambda\rangle = 0$ is assumed, the quantity on the right-hand side is the root-mean-square of a random variable, not the value of a function of time:

$$\sigma_\Lambda(V) \equiv \left\langle\Lambda^2\right\rangle^{1/2} \sim V^{-1/2}. \tag{21}$$

What TSGT computes is σ_Λ. The realised Λ at any epoch is a single draw from a distribution of that width, and (21) says nothing about its sign: a zero-mean fluctuation is as likely to be negative as positive. Statements about σ_Λ are statements about an envelope, and they do not transfer to the realisation inside it. In particular $\dot{\sigma}_\Lambda < 0$ does not imply $\dot{\Lambda} < 0$; an envelope may shrink monotonically while the quantity it bounds changes sign repeatedly. This distinction does real work in Sections 5.4 and 6.

Choice of four-volume. The identification of V with the Hubble four-volume $V \sim (c/H_0)^4$ requires comment. Sorkin's original formulation and its development in Ref. [17] use the past causal diamond—the intersection of the past light cone of a future event with the future light cone of the initial singularity—which differs from the Hubble four-volume by an $\mathcal{O}(1)$ numerical factor depending on the expansion history. The Hubble four-volume is used here for simplicity; the resulting difference is absorbed into the factor-of-two discrepancy between σ_Λ and the observed realisation, and cannot be meaningfully resolved without a derivation that selects one volume over another.

Evaluating (20) for the present epoch, with the four-volume taken to be the Hubble four-volume $V \sim (c/H_0)^3 \cdot (c/H_0)$:

$$H_0 \tau_{\mathrm{P}} = 1.18 \times 10^{-61}, \tag{22}$$

$$N = \left(\frac{1}{H_0 \tau_{\mathrm{P}}} \right)^4 = 5.2 \times 10^{243}, \qquad \sqrt{N} = 7.2 \times 10^{121}, \tag{23}$$

$$\sigma_\Lambda^{\mathrm{TSGT}} \sim N^{-1/2} = 1.4 \times 10^{-122} \quad \text{(Planck units)}. \tag{24}$$

The observed value [20] is $\Lambda_{\mathrm{obs}} = 2.8 \times 10^{-122}$ in the same units, larger by a factor of 2.1. The comparison is between a predicted width and a single measured realisation drawn from it, so agreement to a factor of a few is the most the argument can deliver, and it would be a mistake to read the residual factor of 2.1 as a discrepancy waiting to be improved. In terms of energy density,

$$\rho_\Lambda^{\mathrm{TSGT}} \simeq 2.8 \times 10^{-27}\ \mathrm{kg\,m^{-3}}, \qquad \rho_\Lambda^{\mathrm{obs}} \simeq 5.8 \times 10^{-27}\ \mathrm{kg\,m^{-3}}. \tag{25}$$

For comparison, the naive estimate that assigns each pixel one Planck energy gives $\rho_\Lambda \sim \rho_{\mathrm{P}} = 5.2 \times 10^{96}\ \mathrm{kg\,m^{-3}}$, too large by a factor of 8.8×10^{122}. The fluctuation argument replaces a discrepancy of 123 orders of magnitude with a discrepancy of a factor of two (Fig. 3).

Attribution. This argument is not original to TSGT. It is Sorkin's heuristic, presented in 1987–1990 within causal set theory [16] and developed quantitatively by Ahmed, Dodelson, Greene and Sorkin [17]. It was advanced roughly a decade before the supernova observations of 1998 [18, 19], and it therefore constitutes a genuine prediction in advance rather than a retrofit. What TSGT adds is that the two ingredients the argument requires—a Poisson-distributed substrate and a number-volume correspondence generated by expansion—are not imported assumptions here but consequences of Sections 3 and 5.

What the argument does not establish. The relation (20) is heuristic. It assumes without proof that the mean of Λ vanishes, it does not specify the mechanism that enforces this, the identification of V with the Hubble four-volume is a choice rather than a derivation, and the final comparison sets an rms against a single realisation. Order-of-magnitude agreement obtained from an argument with these gaps should be treated as encouraging, not as confirmation.

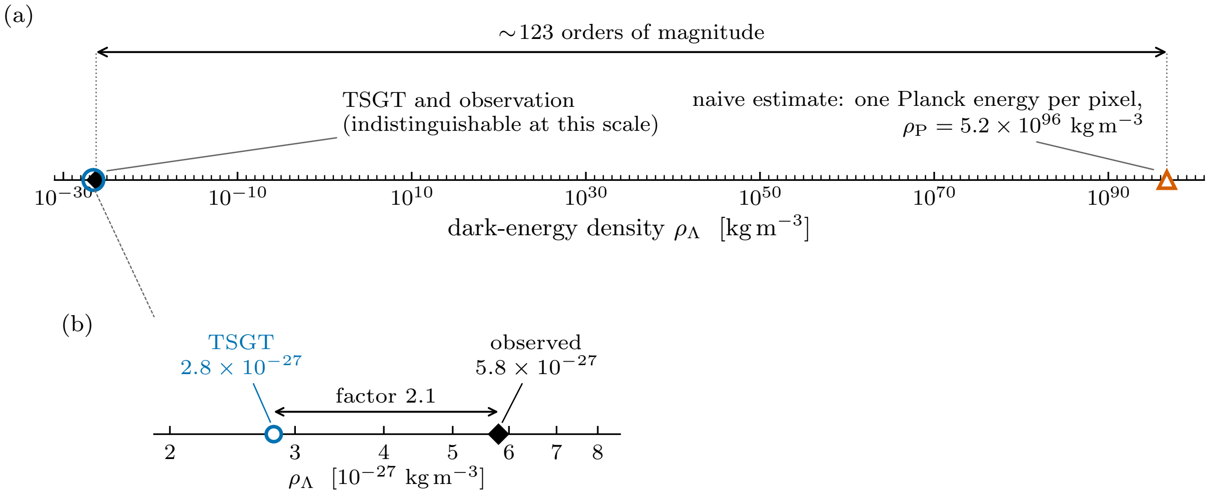

Figure 3: The cosmological term of Section 5.3 on a logarithmic density axis. (a) The naive estimate that assigns one Planck energy to every pixel, $\rho_P = 5.2 \times 10^{96}$ kg m^{-3}, exceeds the observed dark-energy density by some 123 orders of magnitude; on this scale the TSGT estimate $\Lambda \sim N^{-1/2}$ and the observation are separated by 0.3% of the axis and are drawn as one symbol, a black diamond inside a blue ring. (b) The same two values resolved: 2.8×10^{-27} against 5.8×10^{-27} kg m^{-3}, a factor of 2.1. A bar chart is deliberately avoided: on a logarithmic scale the base of a bar is arbitrary, and only the positions of the three points on the axis carry meaning.

5.4 A testable consequence: Λ is not constant

Equation (21) makes the width of the distribution depend on V, which grows with time. Taking the Hubble four-volume $V \sim (c/H)^4$ at each epoch gives

$$\sigma_\Lambda \sim H^2, \tag{26}$$

so the scale on which Λ fluctuates falls over cosmic history. The realised Λ therefore neither sits at a fixed value nor follows a smooth law: it wanders inside a shrinking envelope. This is a sharp qualitative difference from the standard ΛCDM picture, in which $w = -1$ exactly, and it is testable with current surveys. If the correlation time of the fluctuation is of order the Hubble time H^{-1}, as is natural for a Poisson process whose rate is set by expansion, then Λ changes appreciably on cosmological timescales, and the effective equation-of-state parameter w varies by $\mathcal{O}(1)$ over each Hubble epoch. This is qualitatively distinct from the slow-roll w_0–w_a parametrisation used in survey analyses, and it suggests that standard CPL fits may not capture the TSGT signal faithfully.

Why the envelope must not be mistaken for the term itself. Equation (26) constrains σ_Λ, and reading it instead as a smooth law $\Lambda(t) \propto H^2(t)$ produces a model that is already excluded—though not for the reason recorded in earlier versions of this paper. Write $\varepsilon \equiv \rho_\Lambda/\rho$ for the ratio of the cosmological to the matter density. In a spatially flat universe a Λ that tracks H^2 exactly holds ε fixed at all epochs, and the balance equation (35) of Section 6 then integrates to

$$\rho \propto a^{-3/(1+\varepsilon)}, \qquad a \propto t^{\,2(1+\varepsilon)/3}, \qquad w_{\text{eff}} = -\frac{\varepsilon}{1+\varepsilon} = -\Omega_\Lambda. \tag{27}$$

Such a universe does accelerate: the exponent exceeds unity whenever $\Omega_\Lambda > 1/3$, and for the observed $\Omega_\Lambda = 0.69$ one finds $a \propto t^{2.14}$. The objection is instead that ε becomes a constant of nature. The same ratio then holds during nucleosynthesis and at recombination as holds today, spoiling the light-element abundances and shifting the acoustic scale, and w_{eff} is pinned at $-\Omega_\Lambda \simeq -0.69$, many standard deviations from the measured value. The smooth branch is therefore not merely uninteresting but observationally dead, and TSGT is committed to reading (26) as an envelope.

The observational situation is genuinely unsettled and should be reported as such. DESI DR2 reports a preference for an evolving dark-energy equation of state over a cosmological constant, reaching up to 4.2σ for particular data combinations [21]. Independent Bayesian reanalyses find the evidence substantially weaker, and find that it largely disappears when a recalibrated supernova sample is used [22]. TSGT sides with the evolving interpretation, and a firm confirmation that $w = -1$ exactly would count against this section.

The envelope reading does not by itself escape the early universe. Equation (26) makes σ_Λ comparable to the dominant energy density at every epoch—this is what "everpresent" means—so a Λ that kept one sign would still be present during nucleosynthesis and structure formation, where it would spoil light-element abundances and suppress galaxy formation. Existing treatments address this by letting Λ fluctuate about zero in sign rather than remaining positive [17], which is consistent with $\langle\Lambda\rangle = 0$ but is not derived from it. Whether TSGT's pixel-creation picture supplies such a sign-changing fluctuation is not established here (open problem OP5).

6 The Action Principle

Sections 2–5 proceed from postulates imposed by hand: a spacing relation, a clock relation, and a gradient relation. This section removes that defect by exhibiting a single action from which all of them follow, and states plainly what the construction achieves and what it costs.

6.1 Why the action cannot be freely invented

It is tempting to write a new action for the field h_{ij} of Section 4 and hope that something other than General Relativity emerges. This route is closed. For a massless spin-2 field, Lorentz invariance together with the absence of negative-norm states fixes the quadratic action uniquely to the Fierz–Pauli form [24], and consistency of the self-coupling at higher orders forces the full Einstein–Hilbert action [25, 26]. Any attempt to deform this either reproduces linearised General Relativity or introduces a ghost.

The consequence for TSGT is decisive: *the theory's new content cannot live in the propagation of $h_{\mu\nu}$*. It must live in the structure that TSGT actually adds, which is the statement that the space-time volume element is not free but is fixed by a discrete pixel count. That is a constraint on the measure, not a modification of the graviton, and it evades the Fierz–Pauli–Deser uniqueness theorems because it does not alter the tensor sector at all.

6.2 The TSGT action

Let $n(x)$ denote the coordinate-space number density of pixels, so that the number of pixels in a coordinate cell is $dN = n(x)\, d^4x$. The correspondence $V = NL_{\mathrm{p}}^4$ of Eq. (18), written locally, becomes a condition on the metric determinant:

$$\sqrt{-g} = L_{\mathrm{p}}^4\, n(x). \tag{28}$$

This is the statement, in covariant form, that pixel number *determines* volume rather than merely measuring it. We take it as the defining structural postulate of TSGT and enforce it with a Lagrange multiplier $\lambda(x)$:

$$\boxed{S[g_{\mu\nu}, \lambda, \psi] = \frac{1}{16\pi G}\int d^4x\, \sqrt{-g}\, R \;-\; \frac{1}{8\pi G}\int d^4x\; \lambda(x)\Big[\sqrt{-g} - L_{\mathrm{p}}^4 n(x)\Big] \;+\; S_{\mathrm{m}}[g_{\mu\nu}, \psi]} \tag{29}$$

where ψ denotes matter fields. The fundamental variable is the full four-dimensional metric $g_{\mu\nu}$, whose ten components—six spatial, three mixed and one temporal—are reduced to nine independent ones by the constraint (28). The temporal component is what carries the clock postulate of Eq. (3), which is therefore no longer an independent assumption but part of the same field.

Equation (29) is the unimodular form of gravity [27, 28, 29]. TSGT does not claim to have invented it. What TSGT supplies is the physical identification (28): in unimodular gravity the fixed volume element is a background structure of unexplained origin, whereas here it is the pixel count, and the count is an integer.

6.3 Field equations

Varying (29) with respect to λ returns the constraint (28). Because that constraint holds, the variation of $g_{\mu\nu}$ is restricted to be volume preserving, and only the trace-free part of the metric variation is free. The resulting field equations are the trace-free Einstein equations,

$$R_{\mu\nu} - \tfrac{1}{4} g_{\mu\nu} R = 8\pi G \left(T_{\mu\nu} - \tfrac{1}{4} g_{\mu\nu} T \right) . \tag{30}$$

Taking the divergence of (30) and using the contracted Bianchi identity $\nabla^\mu R_{\mu\nu} = \frac{1}{2}\nabla_\nu R$ gives

$$\nabla_\nu (R + 8\pi G\, T) = 32\pi G\, J_\nu, \qquad J_\nu \equiv \nabla^\mu T_{\mu\nu}. \tag{31}$$

Case 1: conserved matter. If $J_\nu = 0$, then $R + 8\pi G T$ is constant. Writing that constant as 4Λ and substituting back into (30) recovers the full Einstein equations,

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \tag{32}$$

in which Λ appears as an *integration constant* rather than as a parameter of the action [29, 30]. Its weak-field limit is the metric of Eq. (4), so postulates (2) and (3) are now derived rather than assumed, and Eq. (6) becomes a rewriting of the Newtonian limit in Planck units rather than an independent dynamical law.

Case 2: pixel creation. TSGT is committed to the creation of new pixels as the universe expands (Section 5.1). If that process exchanges energy with matter, then $J_\nu \neq 0$; defining $\Lambda \equiv \frac{1}{4}\left(R + 8\pi G\, T\right)$ as before, (31) gives

$$\nabla_\nu \Lambda = 8\pi G\, J_\nu. \tag{33}$$

The cosmological term is then not constant: it accumulates the history of energy non-conservation. This mechanism is due to Josset, Perez and Sudarsky [31], who show that violations far too small to detect locally can integrate to a cosmologically relevant Λ, and who consider Planck-scale discreteness as a source. In TSGT the source is identified: it is pixel creation. Its magnitude is estimated below; its direction is not fixed.

This resolves an apparent tension with Section 5.3. In Case 1 the cosmological term is strictly constant, which is incompatible with the fluctuating Λ required by the Poisson argument. TSGT is therefore committed to Case 2: pixel creation must exchange energy with matter, since otherwise Eq. (20) has no mechanism behind it. The Poisson estimate of Section 5.3 should accordingly be read as an estimate of the width σ_Λ that Λ accumulates through Eq. (33), not as an independent statement.

The magnitude of J_ν. The size of the exchange can be estimated, even though its direction cannot. In a spatially flat FRW background with a perfect fluid of density ρ and pressure p, the time component of the source is

$$J_0 = -\left[\dot\rho + 3H(\rho + p)\right], \tag{34}$$

so that (33) collapses to a single balance equation,

$$\dot{\rho}_\Lambda + \dot{\rho} + 3H(\rho + p) = 0, \qquad \rho_\Lambda \equiv \frac{\Lambda}{8\pi G}, \tag{35}$$

in which the total budget is conserved even though the matter sector alone is not. Section 5.4 fixes the scale on which Λ varies: it wanders inside an envelope $\sigma_\Lambda \sim H^2$ which itself changes appreciably only over a Hubble time, so that $|\dot{\rho}_\Lambda| \sim H\rho_\Lambda$ and hence

$$|J_0| \ \sim \ H\rho_\Lambda. \tag{36}$$

The fractional rate at which matter energy conservation fails is therefore of order H itself—some $10^{-18}\,\mathrm{s}^{-1}$ at the present epoch. This is precisely the regime identified in [31]: far below any local bound on energy non-conservation, yet integrating over a Hubble time to a cosmologically relevant Λ. Since TSGT supplies no calculation of the pixel creation rate, (35) parametrises the exchange rather than predicting it (open problem OP2).

Why the sign is not determined. It is tempting to go further and fix the direction of the exchange, and earlier versions of this paper did so: σ_Λ falls monotonically, therefore $\dot{\rho}_\Lambda < 0$ and $J_0 < 0$, therefore pixel creation feeds matter at the expense of Λ. That argument does not survive the distinction drawn in Section 5.3. What falls monotonically is the envelope σ_Λ, not the realised Λ, and with $\langle\Lambda\rangle = 0$ the realisation changes sign on a timescale of order the Hubble time. Then $\dot{\rho}_\Lambda$ changes sign with it, J_0 acquires no definite secular direction, and the exchange runs from Λ into matter during some epochs and from matter into Λ during others. Only the magnitude (36) is available.

The same conclusion follows from the balance equation directly, which is worth recording because the smooth reading also gets the coefficient wrong. Setting $p = 0$ and $\rho_\Lambda = \varepsilon\rho$ with ε constant, (35) gives $(1+\varepsilon)\dot{\rho} = -3H\rho$ and therefore

$$\dot{\rho}_\Lambda = -\frac{3H\rho_\Lambda}{1+\varepsilon}, \tag{37}$$

not $-3H\rho_\Lambda$ as stated in earlier versions; the two differ by a factor $1+\varepsilon \simeq 3.2$ at the present epoch. The order of magnitude in (36) is unaffected, but (37) holds only on the smooth branch that Section 5.4 excludes, and it is quoted here only to make the comparison. Fixing the sign would require a mechanism governing the mean of Λ and the correlation structure of its fluctuation, which TSGT does not supply (open problem OP5).

We note that this failure is not a new defect but an old one seen from a new angle. The missing ingredient—a dynamical account of how pixel creation generates Λ, and on what timescale—is the same one that leaves $\langle\Lambda\rangle = 0$ unexplained in Section 5.3 and leaves the early-universe sign problem open in Section 5.4. What appeared in earlier versions as three separate gaps is one gap.

6.4 What this resolves

- **Only two polarisations propagate.** Section 4 counted six components of h_{ij} and found nothing to gap the scalar and vector sectors. That count was kinematic: it classified a field for which no dynamics had yet been specified, and a field without an action has no notion of which of its components propagate. Supplying an action settles the question. In the Einstein–Hilbert action—and, by the uniqueness argument of Section 6.1, in any consistent action for a massless spin-2 field—the scalar and vector parts of the metric perturbation are not propagating modes at all: they are fixed by the constraint equations or removed by gauge freedom, leaving exactly the two transverse-traceless polarisations. It is therefore the passage from kinematics to an action principle, and not the unimodular constraint, that removes the extra polarisations. The constraint neither adds nor subtracts local degrees of freedom: unimodular gravity carries the same two per point as General Relativity, together with a single global degree of freedom, namely Λ itself [29]. This resolves the extra-polarisation problem of Section 4.4—listed as an open problem in earlier versions of this paper—and with it the conflict with polarisation searches and with the measured value of the PPN parameter γ.

- **Λ and V are conjugate.** The relation $\delta\Lambda\,\delta V \sim \hbar$ assumed in Section 5.3 is a theorem in this formulation, not an assumption: Λ is canonically conjugate to the total four-volume [28, 29]. The dark-energy estimate of Eq. (20) therefore rests on the action rather than on a heuristic.
- **Λ is not assumed to be constant.** Equation (33) shows it is sourced, which is what Section 5.4 requires, and (36) fixes the scale of the source. What remains undetermined is the sign of Λ and the correlation structure of its fluctuation, and with them the assumption $\langle\Lambda\rangle = 0$ used in Section 5.3. The question nevertheless changes character. It is no longer whether Λ can vary at all, which (29) settles, but what the statistics of that variation are—and that is a well-posed question within (29) as soon as an equation of motion for $n(x)$ is supplied.
- **Matter coupling is specified.** S_{m} enters the action directly, so the response of the geometry to stress–energy is derived rather than inserted. This removes the matter-coupling problem listed as an open problem in earlier versions of this paper.

6.5 What this costs

- **TSGT is now provably equivalent to General Relativity at the classical level.** Equation (32) is Einstein's equation. Whatever independent content TSGT possesses cannot be classical and cannot be gravitational dynamics; it lies entirely in the discreteness of N, which is an integer subject to Poisson fluctuations and which therefore makes (28) exact only in the mean. This sharpens open problem OP1 rather than removing it, and it should be stated that way.
- **Full diffeomorphism invariance is reduced.** Fixing the volume element leaves only transverse (volume-preserving) diffeomorphisms as gauge symmetry. Full invariance can be restored by promoting the fixed measure to the exterior derivative of a three-form field, as in the Henneaux–Teitelboim formulation [29]; whether the TSGT constraint (28) admits such a rewriting with $n(x)$ dynamical is not established here.
- **The pixel density $n(x)$ is prescribed, not solved for.** There is no equation of motion for n; it enters (29) as external data. A complete theory would determine the pixel creation rate dynamically, and Eq. (33) would then predict J_ν rather than parametrise it. This is recorded as open problem OP2.

7 Observational Status

The table below summarises where TSGT now stands against observation. Of the six entries, one is a quantitative success, one is a live test, and the remainder are consistency checks or retractions. The polarisation conflict recorded in earlier versions is removed by the action principle of Section 6 and is listed here as resolved rather than outstanding.

8 Limitations and Open Problems

OP1. The framework adds no classical content. Section 2 reproduces linearised GR by construction, and Section 6 shows that the equivalence is exact at the classical level rather than merely apparent. TSGT's independent content lies entirely in the integer

Quantity	TSGT	Observation	Status
Newtonian limit	$g = GM/r^2$	confirmed	Reproduced by construction; not a test.
Light deflection	$4GM/c^2b$	confirmed to 10^{-5}	Reproduced; requires the temporal postulate.
Dark energy density	2.8×10^{-27} kg m^{-3} (rms)	5.8×10^{-27} kg m^{-3}	Predicted rms against one realisation; agreement within a factor of 2.
Equation of state	$w \neq -1$; Λ fluctuates inside a falling envelope	contested; DESI DR2 vs. Bayesian reanalyses	Testable now, but no $w(z)$ curve is predicted (OPOP5).
Extra GW polarisations	absent; only the two TT modes propagate	none detected	Resolved in Section 6.
Photon / GW dispersion	withdrawn	—	Retracted in Section 3.3.

Table 2: Status of TSGT against observation after the revisions of this version. The framework now has one quantitative success and one live test; the polarisation conflict of earlier versions is resolved by the action principle of Section 6.

character of N and in the Poisson fluctuations that make Eq. (28) hold only in the mean; no regime measured so far distinguishes the two theories.

OP2. **The pixel density is not dynamical.** The action of Section 6 contains no equation of motion for the pixel creation rate; $n(x)$ enters as external data. A complete theory would predict J_ν through Eq. (33) rather than parametrise it. Whether the constraint admits a Henneaux–Teitelboim rewriting with $n(x)$ dynamical is also unsettled.

OP3. **Dynamics on a sprinkling is undeveloped.** Section 4 works in the continuum. Deriving wave propagation directly on an irregular sprinkling is hard—the discrete d'Alembertian is non-local—and is unsolved here.

OP4. **Relation to causal set theory unresolved.** See Section 3.4. TSGT shares its kinematics with causal sets and must either derive its spacing relation from causal order or justify treating spacing as independent.

OP5. **The statistics of Λ are not derived.** That Λ varies is a consequence of Eq. (33) rather than an assumption, and Eq. (36) fixes the scale of the variation. Its statistics are not fixed, and three things are missing that are really one thing: the mean, assumed to vanish in Section 5.3 without a mechanism; the sign at any given epoch, which Section 6 shows cannot be inferred from the falling envelope; and the correlation time of the fluctuation, which is what would turn $w \neq -1$ into a definite prediction for $w(z)$ rather than a qualitative expectation. All three would follow from a dynamical account of pixel creation, i.e. from open problem OP2.

OP6. **Singularities.** Finite pixels forbid unbounded density, so TSGT must replace curvature singularities with something. What that is remains open.

9 Conclusion

This version of TSGT trades speculative phenomenology for structural consistency and one quantitative result.

The framework's defensible content is now the following. First, a single relation in Planck units, $g = (c^2/L_\mathrm{p})(\Delta L/L_\mathrm{p})$, together with the accompanying temporal postulate, reproduces the entire weak-field sector of GR including the correct light deflection—by construction rather than by prediction. Second, taking the substrate to be a Poisson sprinkling rather than a lattice makes discreteness compatible with Lorentz invariance, at the price of retracting the dispersion and anisotropy signatures of earlier drafts. Third, and most importantly, the requirement that expansion create pixels rather than stretch them yields a cosmological term of magnitude $\Lambda \sim N^{-1/2}$, giving $\rho_\Lambda \simeq 2.8 \times 10^{-27}$ kg m^{-3} against an observed 5.8×10^{-27}—a factor of two, where the naive estimate errs by 10^{123}.

To this is added the result of Section 6. The number–volume correspondence, written locally, is the unimodular constraint on the metric determinant, and enforcing it in the Einstein–Hilbert action fixes the theory: the field equations are the trace-free Einstein equations, the two kinematic postulates of Section 2 are derived rather than assumed, and Λ enters as an integration constant rather than as a parameter of the action. The cost is stated in the same place and is the heaviest the framework has paid: TSGT is thereby provably equivalent to General Relativity at the classical level, so its independent content lies entirely in the integer character of N and in the Poisson fluctuations that make the constraint exact only in the mean.

A further point deserves mention because it is the framework's most distinctive quantitative statement: the rupture threshold r_* follows from TSGT's own kinematics and differs from the Planck-density criterion used elsewhere, placing the transition some twenty orders of magnitude below Planck density for a solar mass (Section 2.5).

The framework remains incomplete in ways that are stated rather than concealed: the pixel density $n(x)$ enters the action as external data rather than as a dynamical field, there is no dynamics formulated directly on the sprinkling, and the assumption that the mean of Λ vanishes is still unexplained. Possible conceptual extensions—including higher-dimensional embeddings and universe-generation scenarios—are left to future work. The most productive next steps are, in order: finding an equation of motion for $n(x)$, so that Eq. (33) predicts J_ν instead of parametrising it; deriving wave propagation directly on the sprinkling; and determining whether pixel creation supplies the sign-changing Λ that early-universe consistency requires.

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