Topological String-Gradient Theory: Metric Replication of Space-Time and Cosmological Evolution

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Abstract

We present the Topological String-Gradient Theory (TSGT), a framework in which space-time is discrete at the Planck scale and gravity is the gradient of the spacing between discrete elements ("pixels") rather than a fundamental force. Two kinematic postulates—a spatial pixel strain set by the Newtonian potential and an equal and opposite temporal strain—reproduce the weak-field Schwarzschild metric, and hence Newtonian acceleration, gravitational redshift, the Shapiro delay and the full general-relativistic light deflection $\Delta\theta = 4GM/c^2 b$. We stress that this reproduction is by construction rather than a prediction. The discrete substrate is taken to be a random Poisson sprinkling of density L_{p}^{-4} rather than a regular lattice. This choice preserves Lorentz invariance statistically, and thereby removes the preferred-frame and anisotropy problems that afflict lattice formulations; it also withdraws the energy-dependent photon dispersion, gravitational-wave dispersion and propagation-anisotropy signatures claimed in earlier versions of this framework, which we retract explicitly. In exchange, TSGT acquires a quantitative cosmological consequence. If cosmic expansion proceeds by the creation of new pixels rather than by the stretching of existing ones—the only option compatible with an invariant L_{p}—then the number–volume correspondence carries Poisson fluctuations $\delta N \sim \sqrt{N}$, and the cosmological term conjugate to the four-volume inherits a residual magnitude $\Lambda \sim V^{-1/2}$. Numerically this gives $\rho_\Lambda \simeq 2.8 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$ against an observed $5.8 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$: agreement within a factor of two, compared with the 10^{123} discrepancy of the naive Planck-density estimate. This argument is not original to TSGT—it is Sorkin's heuristic, developed within causal set theory—but it follows from TSGT's own postulates without additional assumptions, and it carries a signature that current surveys can test: the dark energy term is not constant but fluctuates, so $w \neq -1$. We then show that the two kinematic postulates need not be postulated at all. Written locally, the number–volume correspondence reads $\sqrt{-g} = L_{\mathrm{p}}^4 n(x)$, which is the unimodular constraint; enforcing it in an Einstein–Hilbert action yields the trace-free Einstein equations, with Λ appearing as an integration constant rather than as a parameter, and with the weak-field metric recovered rather than assumed. The price is stated plainly: TSGT thereby becomes provably equivalent to General Relativity at the classical level, so whatever independent content it possesses lies entirely in the integer character of the pixel count. Unresolved problems, including the absence of a dynamical equation for the pixel density and the difficulty of deriving excitations on an irregular substrate, are listed explicitly.

Contents

1 Introduction

Modern physics struggles to reconcile General Relativity (GR) with Quantum Mechanics, in large part because GR treats space-time as a smooth, infinitely divisible manifold [1, 2]. A recurring response is to assume that the manifold approximates a discrete substrate with granularity set by the Planck scale,

$$L_{\mathrm{p}} = \sqrt{\frac{\hbar G}{c^3}} = 1.616 \times 10^{-35}\ \mathrm{m}, \qquad E_{\mathrm{p}} = \sqrt{\frac{\hbar c^5}{G}} = 1.22 \times 10^{19}\ \mathrm{GeV}. \tag{1}$$

Topological String-Gradient Theory (TSGT) belongs to this family [3, 4]: space-time consists of discrete quantum nodes, and gravity is not a force acting within that structure but the geometric consequence of a gradient in the spacing of the structure itself.

This version of the paper differs substantially from earlier drafts, and it is worth stating the three changes at the outset because each of them costs the theory something.

Change 1: the substrate is random, not a lattice. Earlier versions assumed a regular cubic array of pixels. A regular array defines a preferred rest frame and preferred axes, and therefore breaks Lorentz and rotational invariance at a fundamental level—an assumption that is difficult to reconcile with the success of special relativity and that creates a severe naturalness problem. We therefore replace it with a random Poisson sprinkling (Section 3), which is discrete yet statistically Lorentz invariant. The cost is that the lattice-derived predictions of earlier drafts—energy-dependent photon speed, gravitational-wave dispersion, and propagation anisotropy—no longer follow, and we retract them (Section 3.3).

Change 2: expansion creates pixels, and that creates dark energy. If L_{p} is an invariant, cosmic expansion cannot stretch pixels; it must create them. Section 5 shows that this single requirement, combined with the Poisson statistics of Change 1, fixes the order of magnitude of the cosmological term to within a factor of two of its observed value. This replaces the withdrawn signatures with a quantitative one, and it is the first place in this framework where a number is computed and compared with data.

Change 3: the postulates follow from an action. Section 6 shows that the number–volume correspondence of Change 2, written locally, is the unimodular constraint on the metric determinant, and that enforcing it in the Einstein–Hilbert action reproduces the postulates of Section 2 rather than assuming them. This too costs the theory something, and the cost is the largest of the three: it makes the classical equivalence to General Relativity exact and provable, confining TSGT's independent content to the discreteness of the pixel count itself.

What is a result and what is not. Sections 2 and 4 are constructions: they show that TSGT is consistent with known gravity, not that it predicts it. Section 5 is the paper's only quantitative confrontation with observation, and even there the central argument is heuristic and is due to Sorkin [16, 17], not to the present author; TSGT's contribution is that the argument follows from its own postulates rather than being imported. Section 6 is a derivation rather than a construction, but what it derives is General Relativity, which is not new; its value is that it removes the postulates and sharpens where TSGT can still differ. Material that is conceptual rather than quantitative is confined to Sections 9–10, each preceded by an explicit scope note.

2 Core Mechanism: Gravity as a Metric Gradient

2.1 The spacing field

Let $\ell(x)$ denote the mean proper separation between neighbouring pixels near the point x. Far from all matter this takes its vacuum value L_p; near a mass it is stretched:

$$\ell(x) = L_\mathrm{p}\big(1 + \chi(x)\big), \qquad \chi \equiv -\frac{\Phi}{c^2}, \tag{2}$$

with Φ the Newtonian potential ($\Phi < 0$, hence $\chi > 0$). We further postulate that the local clock rate is retarded by the same fractional amount:

$$\tau(x) = \tau_\mathrm{p}\big(1 - \chi(x)\big), \qquad \tau_\mathrm{p} \equiv L_\mathrm{p}/c. \tag{3}$$

Equations (2)–(3) are the two kinematic postulates of TSGT. Together they are equivalent to the isotropic weak-field metric

$$ds^2 = -\,(1-2\chi)\,c^2dt^2 + (1+2\chi)\,\delta_{ij}\,dx^i dx^j, \tag{4}$$

i.e. the linearised Schwarzschild solution [5, 6]. They are taken as postulates in this section; Section 6 derives both from an action principle. Note that (2) refers to a *mean* separation and therefore does not require the pixels to be regularly arranged; this is what allows the construction of Section 3 to proceed without disturbing the present section.

2.2 The gradient relation and the Newtonian limit

The name of the theory refers to the fact that acceleration is controlled not by the spacing itself but by how the spacing changes from one pixel to the next. Define

$$\Delta L(x) \equiv \ell(x + L_\mathrm{p}\hat{n}) - \ell(x) = L_\mathrm{p}^2\,(\hat{n}\cdot\nabla\chi) + \mathcal{O}(L_\mathrm{p}^3), \tag{5}$$

one power of L_p smaller than the spacing itself. The dynamical postulate is

$$\mathbf{g} = a_\mathrm{P}\,\frac{\Delta L}{L_\mathrm{p}}, \qquad a_\mathrm{P} \equiv \frac{c^2}{L_\mathrm{p}} \approx 5.6\times10^{51}\ \mathrm{m\,s^{-2}} \tag{6}$$

with a_P the Planck acceleration. In vector form,

$$\mathbf{g} = c^2\nabla\chi = -\nabla\Phi, \tag{7}$$

so the acceleration automatically points along the gradient of the pixel spacing, towards larger spacing. For a point mass, $\chi = GM/(c^2r)$ and (5) gives

$$\Delta L(r) = \frac{GML_\mathrm{p}^2}{c^2r^2} \quad\Longrightarrow\quad g = \frac{c^2}{L_\mathrm{p}^2}\cdot\frac{GML_\mathrm{p}^2}{c^2r^2} = \frac{GM}{r^2}. \tag{8}$$

Physical picture. Mass stretches the pixel distribution. A photon or a massive body crossing the resulting gradient follows the path of least geometric resistance—towards larger spacing—which macroscopically is indistinguishable from an attractive force [7].

Honest status of Eq. (8). This is a consistency check, not a prediction. Postulate (2) identifies the pixel strain with the Newtonian potential, so recovering Newton is guaranteed. The claim TSGT is entitled to make is narrower: a single relation expressed purely in Planck units, Eq. (6), is sufficient to reproduce the entire weak-field sector. New physics can appear only in the discreteness corrections.

2.3 Light deflection and the optical analogy

From (4) the coordinate speed of light is $c_{\rm eff} = c(1-2\chi)$, so the vacuum behaves as a medium of refractive index

$$n(r) \simeq 1 + \frac{2GM}{c^2 r}, \tag{9}$$

and Fermat's principle applied to (9) gives

$$\Delta\theta = \frac{4GM}{c^2 b}, \tag{10}$$

in agreement with GR and with Cassini-class measurements at the 10^{-5} level [8, 9].

The temporal postulate (3) is therefore not optional. Half of (10) comes from the stretching of space and half from the retardation of clocks; a purely spatial deformation would predict $\Delta\theta = 2GM/c^2b$, exactly half the observed value, and would be excluded by five orders of magnitude.

2.4 Regime of validity

The linearisation (5) assumes $\Delta L \ll L_{\rm p}$, and fails where $g \sim a_{\rm P}$, that is at

$$r_* = \sqrt{\frac{GML_{\rm p}}{c^2}} = \sqrt{\frac{r_s L_{\rm p}}{2}}, \qquad r_s = \frac{2GM}{c^2}, \tag{11}$$

the geometric mean of the Schwarzschild radius and the Planck length. For one solar mass, $r_* \simeq 1.5 \times 10^{-16}$ m. Since pixels have finite size, TSGT cannot support unbounded density, so the framework is structurally incompatible with curvature singularities; what replaces them is not determined by the present formulation and is recorded as open problem OP6.

2.5 Comparison with Planck-density thresholds

It is worth distinguishing the criterion (11) from the one used in Planck-star models [41], where the transition is triggered when the matter density reaches the Planck density $\rho_{\rm P}$. The two are not equivalent. TSGT's criterion is a statement about the gradient of the spacing—how rapidly geometry changes from one pixel to the next—rather than about how much matter is packed into a volume. Since $r_* \propto M^{1/2}$ while the Planck-density radius scales as $M^{1/3}$, the two thresholds diverge as $M^{1/6}$:

Mass	r_* (TSGT)	r (Planck density)	$\rho_*/\rho_{\rm P}$
$\approx 1.2 \times 10^{-9}$ kg ($\approx 0.06\, M_{\rm P}$)	criteria coincide		~ 1
Earth mass	2.7×10^{-19} m	6.5×10^{-25} m	1.4×10^{-17}
Solar mass	1.5×10^{-16} m	4.5×10^{-23} m	2.5×10^{-20}
$10\, M_\odot$	4.9×10^{-16} m	9.7×10^{-23} m	7.9×10^{-21}
$4 \times 10^6\, M_\odot$	3.1×10^{-13} m	7.2×10^{-21} m	1.2×10^{-23}

Table 1: TSGT's rupture threshold compared with the Planck-density criterion, with $r_\rho \equiv (3M/4\pi\rho_{\rm P})^{1/3}$ the radius at which a uniform sphere of mass M attains $\rho_{\rm P}$. The two coincide at a mass of order the Planck mass and separate as $M^{1/6}$ above it: for a solar mass, TSGT predicts the transition at a density some twenty orders of magnitude below the Planck density.

The two criteria therefore agree only near the Planck mass, and for any macroscopic body TSGT predicts that the substrate fails long before Planck density is attained (Fig. 1b). Whether this earlier failure is physically meaningful, or whether the two criteria simply measure different quantities that need not coincide, is not settled here.

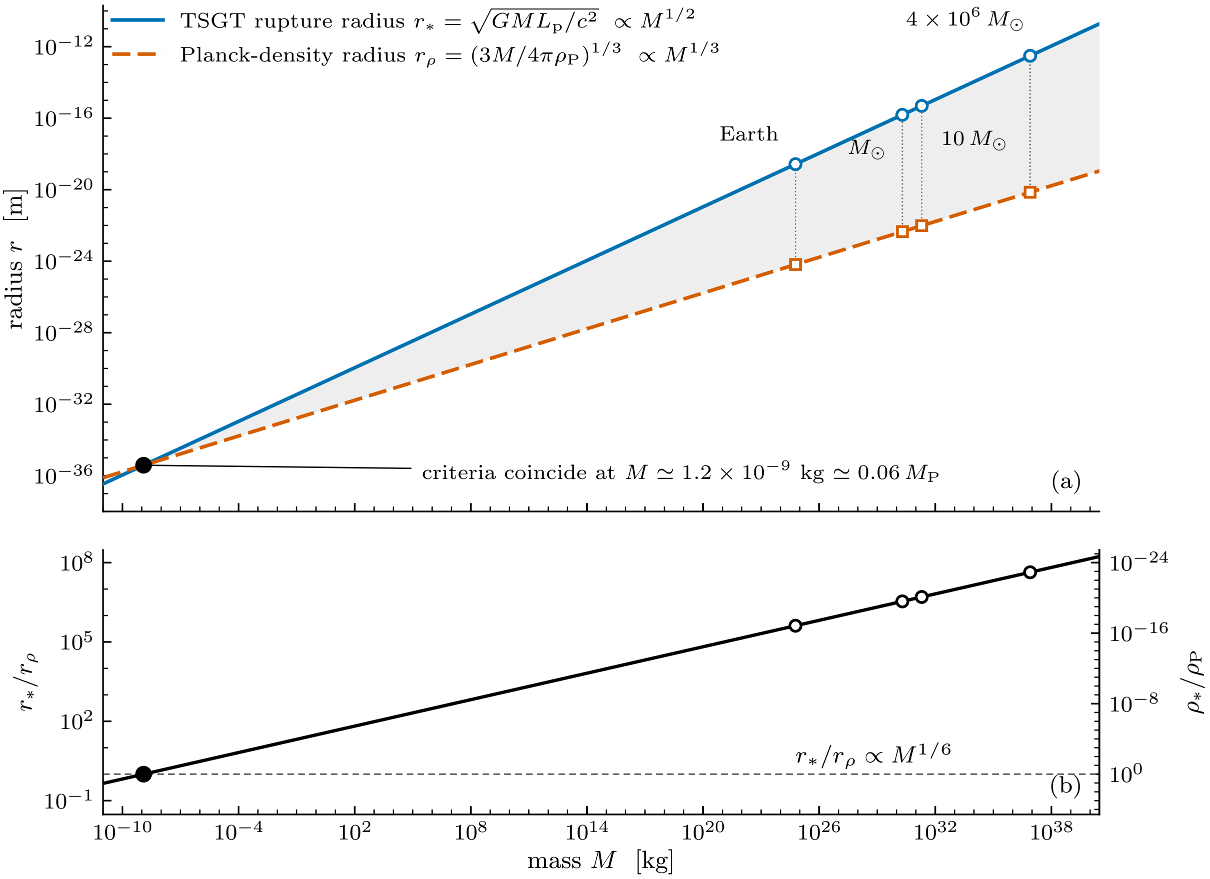

Figure 1: TSGT's rupture threshold $r_* = \sqrt{GML_p/c^2}$ compared with the radius $r_\rho = (3M/4\pi\rho_P)^{1/3}$ at which a body of mass M reaches the Planck density. (a) The two radii against mass; open circles and open squares mark the entries of Table 1 and the filled circle the mass at which the two criteria coincide. (b) Their ratio, a power law of index 1/6 crossing unity at that mass; the right-hand axis carries the equivalent density ratio $\rho_*/\rho_P = (r_*/r_\rho)^{-3}$, the last column of Table 1. The two criteria agree only near the Planck mass and separate as $M^{1/6}$ above it: for a solar mass TSGT places the transition some twenty orders of magnitude below the Planck density.

3 Discreteness Without a Lattice

3.1 Why a regular array fails

A regular array of pixels is the intuitive picture of discreteness, and it is the one adopted in earlier drafts of this work. It cannot be maintained. A regular array specifies a rest frame (the frame in which it is at rest) and a set of axes (its principal directions). Both are observable in principle, and both conflict with the empirical success of local Lorentz invariance. The conflict is not merely aesthetic: Lorentz-violating operators generated at the Planck scale propagate to low energies through radiative corrections with only loop suppression rather than L_p suppression, so a Planck-scale violation generically implies laboratory-scale violations many orders of magnitude larger than observed [10].

One may attempt to rescue the array by making it dynamical, so that the preferred frame becomes a field rather than a fixed background. This is the Einstein-aether programme [11]. It is a viable class of theories, but it retains a preferred frame at every point and is now tightly constrained by the coincident detection of GW170817 and GRB 170817A [12, 13]. We do not pursue it here.

3.2 The sprinkling postulate

TSGT instead takes the pixels to be distributed by a **Poisson sprinkling** of the space-time manifold at density $\rho = L_\mathrm{p}^{-4}$. The probability of finding exactly n pixels in a four-volume V is

$$P(n) = \frac{(\rho V)^n e^{-\rho V}}{n!}, \qquad \langle N\rangle = \frac{V}{L_\mathrm{p}^4}, \qquad \delta N = \sqrt{\langle N\rangle}. \tag{12}$$

The pixels are discrete and have a well-defined mean separation L_p, but they form no regular pattern, no rows and no axes.

The essential property is that this construction is **statistically Lorentz invariant**: a Poisson sprinkling of Minkowski space looks the same, in distribution, to every inertial observer (Fig. 2). This is a theorem, not an approximation [15]. Discreteness therefore does not require a preferred frame, and the naturalness problem of the previous subsection does not arise. The apparent paradox—that a boosted observer should see contracted spacing—is resolved because there is no fixed spacing to contract: only a Lorentz-invariant density, which every observer agrees on.

3.3 What this costs: retracted predictions

Honesty requires an explicit retraction. The following results appeared in earlier versions of this framework and do not survive the replacement of the regular array by a sprinkling:

(i) **Energy-dependent photon velocity.** The prediction $v(E) \simeq c\,[1 - \xi(E/E_\mathrm{p})]$ arises from a substrate with a preferred frame. A statistically Lorentz-invariant substrate does not generate leading-order vacuum dispersion, since there is no frame in which to define the energy dependence. The prediction is withdrawn.

(ii) **Gravitational-wave dispersion.** The relation $\omega^2 = c^2k^2\left(1 - \alpha k^2 L_\mathrm{p}^2/2\right)$ was derived from a nearest-neighbour cubic array via its Brillouin zone. No Brillouin zone exists here. Withdrawn.

(iii) **Propagation anisotropy.** The predicted cubic angular pattern $\alpha(\hat{n}) = \frac{1}{6}\sum_i \hat{n}_i^4$ was a direct consequence of the cubic axes. There are no axes. Withdrawn.

We note that (i)–(iii) were in any case unobservable by margins of between sixteen and sixty orders of magnitude, so little empirical content is lost. What is gained is the removal of the framework's most serious structural objection.

3.4 Relation to causal set theory

The sprinkling construction of Eq. (12) is not original here: it is the kinematic foundation of causal set theory, introduced by Bombelli, Lee, Meyer and Sorkin [14]. Intellectual honesty requires stating this plainly, and equally requires stating what TSGT does differently.

Causal set theory takes the fundamental data to be the discrete elements together with their *causal order*, from which metric information is recovered ("order plus number equals geometry"). TSGT takes the fundamental data to be the discrete elements together with their *spacing*, and supplies an explicit relation—Eq. (6)—between the gradient of that spacing and gravitational acceleration, expressed in Planck units. Causal set theory has no such relation; deriving Einstein's equations from causal order remains one of its central open problems.

Whether TSGT's spacing-gradient relation can be derived from causal order, or whether the two descriptions are genuinely independent, is not settled here. It is the most interesting technical question the present framework raises, and we record it as open problem OP4.

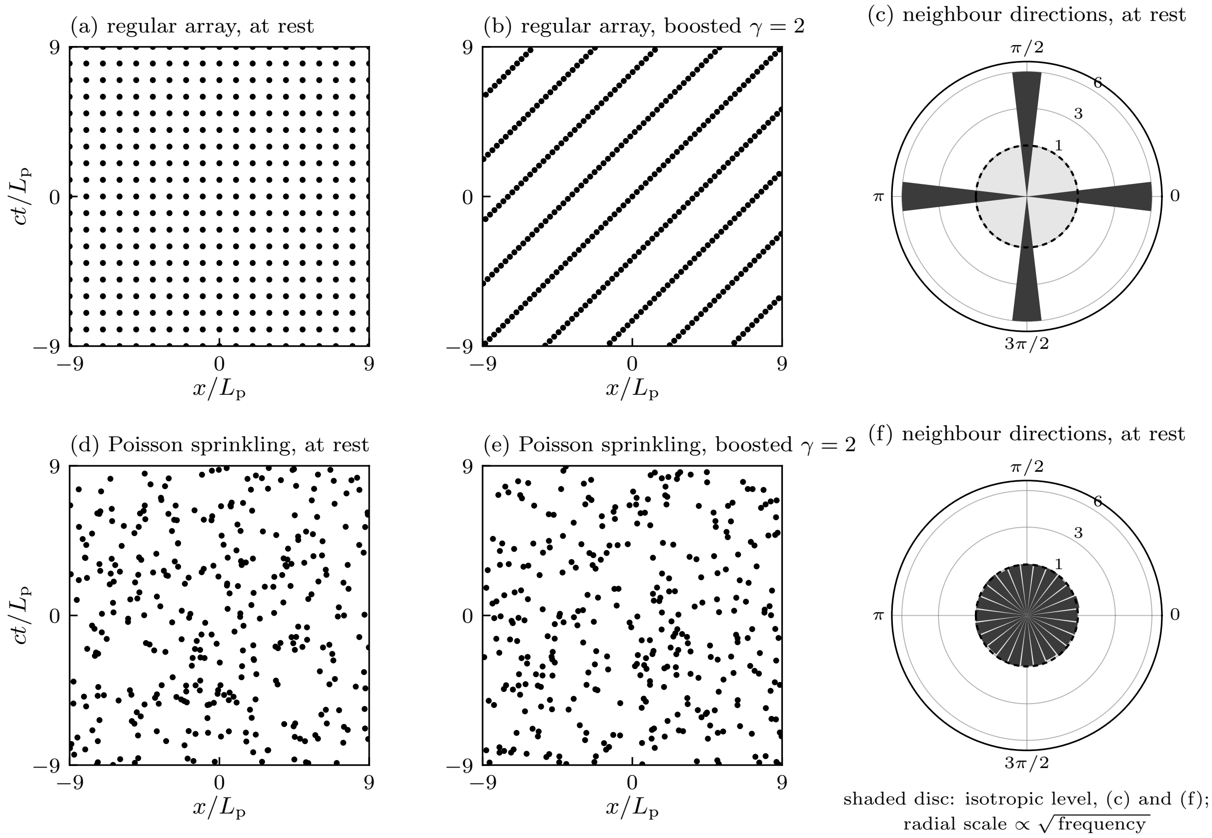

Figure 2: Discreteness without a preferred frame or preferred axes. Panels (a)–(c) show a regular array of pixels, panels (d)–(f) a Poisson sprinkling of the same mean density, both drawn in the (x, ct) plane in units of L_p. Panels (b) and (e) show the same configurations as seen by an observer boosted with $\gamma = 2$; the Lorentz transformation has unit determinant, so the mean density is identical in every panel. The boosted array differs visibly from the array at rest—this is what it means for a regular array to define a rest frame—whereas the sprinkling is statistically indistinguishable from itself, in accordance with the theorem of Ref. [15]. Panels (c) and (f) give the distribution of directions from each element to its four nearest neighbours, measured on the rest-frame configurations of (a) and (d); the radial scale is $\sqrt{\text{frequency}}$, so that petal area rather than petal radius is proportional to the number of neighbours in an angular bin, and the shaded disc of unit radius marks the isotropic level. The array confines the directions to four rays that overshoot that level sixfold, whereas the sprinkling's petals sit on it. Discreteness therefore requires neither a preferred frame nor preferred axes.

4 Pixel Excitations and the Emergent Spin-2 Sector

A discrete framework that removes gravity as a fundamental force must still account for the massless spin-2 quantum of linearised GR. TSGT's answer is that it is a collective excitation of the substrate, in the same sense that a phonon is a collective excitation of a crystal. The argument below is stated in the continuum effective description, which is where it is valid; the corresponding derivation directly on a sprinkling is not available and is recorded as an open problem.

4.1 The fundamental variable must be a tensor

Suppose one starts, as in ordinary elasticity, from a displacement vector field u_i and builds the strain $h_{ij} = \partial_i u_j + \partial_j u_i$. In Fourier space $h_{ij} = i(k_i u_j + k_j u_i)$. The transverse-traceless (TT) projector is built from $P_{ij} = \delta_{ij} - \hat{n}_i \hat{n}_j$ with $\hat{n} = \mathbf{k}/|\mathbf{k}|$ and satisfies $P_{ij} k_j = 0$. Every term in h_{ij} carries a free index contracted with $\mathbf{k}$, hence

$$h_{ij}^{TT} = \Lambda_{ij,kl}\, h_{kl} \equiv 0 \quad \text{identically, for any } u_i. \tag{13}$$

A substrate whose only degrees of freedom are element displacements therefore has no spin-2 sector at all: its excitations are one longitudinal and two transverse sound modes, none transverse-traceless. This is the standard obstruction to obtaining gravitons from elasticity.

TSGT evades it because its primitive object is a *spacing*, and spacing is direction dependent. The natural variable is therefore a symmetric rank-2 field,

$$g_{ij}(x) = \delta_{ij} + h_{ij}(x,t), \qquad h_{ij} = h_{ji}, \tag{14}$$

with six independent components. Its isotropic part $h_{ij} = 2\chi\delta_{ij}$ recovers the static configuration of Section 2, while its traceless part describes shear of the pixel distribution.

This h_{ij} is the spatial part of the full four-dimensional field $h_{\mu\nu}$ whose dynamics is fixed in Section 6, and whose temporal component carries the clock postulate of Eq. (3). The present section works with the spatial sector alone because the helicity classification is a statement about spatial rotations.

4.2 Irreducible decomposition and the TT projection

Under spatial rotations $h'_{ij} = R_{im} R_{jn} h_{mn}$, and $3 \otimes 3 = 1 \oplus 3 \oplus 5$; since h_{ij} is symmetric the antisymmetric 3 is absent. For a given wavevector the relevant classification is by helicity under the little group SO(2). With $P_{ij} = \delta_{ij} - \hat{n}_i \hat{n}_j$, the TT projector is

$$\Lambda_{ij,kl}(\hat{n}) = P_{ik} P_{jl} - \tfrac{1}{2} P_{ij} P_{kl}, \qquad h_{ij}^{TT} = \Lambda_{ij,kl} h_{kl}, \tag{15}$$

which is idempotent and satisfies

$$\hat{n}_i h_{ij}^{TT} = 0, \qquad \delta_{ij} h_{ij}^{TT} = 0. \tag{16}$$

The six components split into two transverse-traceless modes (helicity ± 2), two vector modes (helicity ± 1) and two scalar modes (helicity 0). The conditions (16) are exactly those satisfied by the linearised graviton, and the helicity-± 2 pair corresponds to the $+$ and $\times$ polarisations.

Note. Equation (15) replaces the subtraction $h_{ij} - \frac{1}{3}\delta_{ij} h - (\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2)\nabla^{-2} h$ used in earlier presentations. That expression removes the trace and one scalar piece but leaves the transverse-vector part, so the result does not satisfy $\partial_i h_{ij} = 0$. Only the full projector isolates the helicity-2 sector.

4.3 Quantisation

Quantising the TT sector gives

$$h_{ij}^{TT}(\mathbf{x}, t) = \sum_{\mathbf{k}} \sum_{s=\pm 2} \sqrt{\frac{16\pi G \hbar}{2\omega_k V c^2}} \left[\epsilon_{ij}^{(s)}(\mathbf{k})\, \hat{a}_{\mathbf{k},s} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_k t)} + \text{h.c.} \right], \tag{17}$$

with $[\hat{a}_{\mathbf{k},s}, \hat{a}^\dagger_{\mathbf{k}',s'}] = \delta_{\mathbf{k}\mathbf{k}'}\delta_{ss'}$. The prefactor $16\pi G$ is the conventional graviton normalisation, and the explicit c^2 is required for h_{ij}^{TT} to be dimensionless. In a lattice formulation the momentum sum would be cut off by a Brillouin zone; on a sprinkling no such sharp cutoff exists, and the effective cutoff is statistical—modes with wavelength approaching $L_{\rm p}$ lose meaning because the sprinkling does not resolve them. Making this precise is open problem OP3.

4.4 The extra-polarisation problem

Nothing above gaps the scalar and vector sectors, so they propagate alongside the two TT modes. GR has only the TT pair, and this is constrained observationally: a propagating scalar mode coupling to the trace of the stress tensor shifts the PPN parameter γ, measured to $|\gamma - 1| < 2.3 \times 10^{-5}$ [8, 9], and searches for non-tensorial polarisations in gravitational-wave data find none [23]. TSGT must therefore gap or decouple these sectors. Earlier versions of this paper recorded that failure as an open problem; it is resolved in Section 6, where the constraint that defines the theory is shown to remove the scalar and vector sectors from the physical spectrum, leaving exactly the two transverse-traceless modes.

5 Cosmology: Pixel Creation and the Cosmological Term

This section contains the framework's only quantitative confrontation with data.

5.1 Expansion must create pixels

Consider a homogeneous expanding universe with scale factor $a(t)$. Two options exist for the pixels:

(A) **Stretching.** The number of pixels is fixed and their separation grows as $a(t)$. Then L_p is time dependent. But L_p is the unit against which ΔL is measured in Eq. (6), and it is also the sprinkling density that guarantees Lorentz invariance in Eq. (12). Option (A) therefore destroys both foundations of the framework and is excluded internally.

(B) **Creation.** The separation is fixed at L_p and new pixels are created, so that the number in a comoving region grows as $N \propto a^3$ spatially, or equivalently $N = V/L_\mathrm{p}^4$ in four-volume terms.

TSGT is committed to (B). This is not a free choice; it is forced by the invariance of L_p.

5.2 Number, volume and their fluctuation

Option (B) makes the pixel count the primitive measure of space-time extent, with the four-volume a derived quantity:

$$V = N L_\mathrm{p}^4. \tag{18}$$

Because the sprinkling is Poisson, Eq. (18) is not exact but carries the fluctuation of Eq. (12):

$$N = \frac{V}{L_\mathrm{p}^4} \pm \sqrt{\frac{V}{L_\mathrm{p}^4}}. \tag{19}$$

The relative uncertainty in the four-volume is therefore $1/\sqrt{N}$—small, but not zero, and this residual is the origin of what follows.

5.3 The magnitude of the cosmological term

In unimodular formulations of gravity, the cosmological term Λ and the four-volume V are conjugate variables [28, 29], obeying an uncertainty relation of the form $\delta\Lambda\,\delta V \sim \hbar$; this is assumed here and derived from the action in Section 6. Combining it with Eq. (19) and working in Planck units gives

$$\delta\Lambda \sim \frac{1}{\delta V} \sim \frac{1}{\sqrt{V}} = \frac{1}{\sqrt{N}} \tag{20}$$

Suppose further that whatever mechanism governs Λ drives its mean value to zero, so that the observed value is entirely this residual fluctuation. Then Λ is not a constant of nature at all: it is Poisson noise in the pixel count, and its magnitude is fixed by the size of the universe.

Evaluating (20) for the present epoch, with the four-volume taken to be the Hubble four-volume $V \sim (c/H_0)^3 \cdot (c/H_0)$:

$$H_0 \tau_{\mathrm{p}} = 1.18 \times 10^{-61}, \tag{21}$$

$$N = \left(\frac{1}{H_0 \tau_{\mathrm{p}}}\right)^4 = 5.2 \times 10^{243}, \qquad \sqrt{N} = 7.2 \times 10^{121}, \tag{22}$$

$$\Lambda_{\mathrm{TSGT}} \sim N^{-1/2} = 1.4 \times 10^{-122} \quad \text{(Planck units)}. \tag{23}$$

The observed value [20] is $\Lambda_{\mathrm{obs}} = 2.8 \times 10^{-122}$ in the same units, i.e. larger by a factor of 2.1. In terms of energy density,

$$\rho_\Lambda^{\mathrm{TSGT}} \simeq 2.8 \times 10^{-27}\ \mathrm{kg\,m^{-3}}, \qquad \rho_\Lambda^{\mathrm{obs}} \simeq 5.8 \times 10^{-27}\ \mathrm{kg\,m^{-3}}. \tag{24}$$

For comparison, the naive estimate that assigns each pixel one Planck energy gives $\rho_\Lambda \sim \rho_{\mathrm{P}} = 5.2 \times 10^{96}\ \mathrm{kg\,m^{-3}}$, too large by a factor of 8.8×10^{122}. The fluctuation argument replaces a discrepancy of 123 orders of magnitude with a discrepancy of a factor of two (Fig. 3).

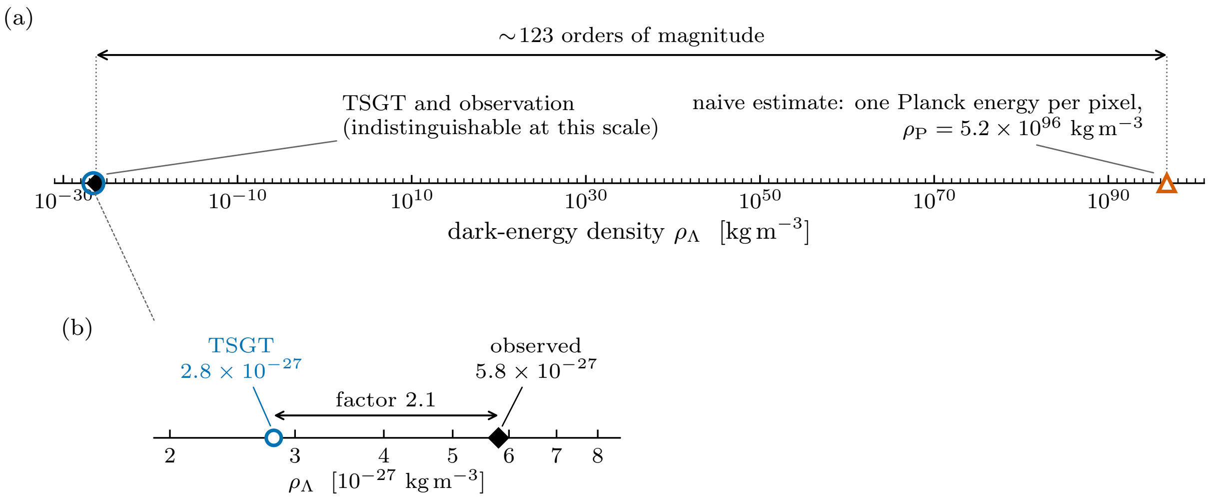

Figure 3: The cosmological term of Section 5.3 on a logarithmic density axis. (a) The naive estimate that assigns one Planck energy to every pixel, $\rho_{\mathrm{P}} = 5.2 \times 10^{96}\ \mathrm{kg\,m^{-3}}$, exceeds the observed dark-energy density by some 123 orders of magnitude; on this scale the TSGT estimate $\Lambda \sim N^{-1/2}$ and the observation are separated by 0.3% of the axis and are drawn as one symbol, a black diamond inside a blue ring. (b) The same two values resolved: 2.8×10^{-27} against $5.8 \times 10^{-27}\ \mathrm{kg\,m^{-3}}$, a factor of 2.1. A bar chart is deliberately avoided: on a logarithmic scale the base of a bar is arbitrary, and only the positions of the three points on the axis carry meaning.

Attribution. This argument is not original to TSGT. It is Sorkin's heuristic, presented in 1987–1990 within causal set theory [16] and developed quantitatively by Ahmed, Dodelson, Greene and Sorkin [17]. It was advanced roughly a decade before the supernova observations of 1998 [18, 19], and it therefore constitutes a genuine prediction in advance rather than a retrofit. What TSGT adds is that the two ingredients the argument requires—a Poisson-distributed substrate and a number-volume correspondence generated by expansion—are not imported assumptions here but consequences of Sections 3 and 5.

What the argument does not establish. The relation (20) is heuristic. It assumes without proof that the mean of Λ vanishes, it does not specify the mechanism that enforces this, and the identification of V with the Hubble four-volume is a choice rather than a derivation. Order-of-magnitude agreement obtained from an argument with these gaps should be treated as encouraging, not as confirmation.

5.4 A testable consequence: Λ is not constant

Equation (20) makes Λ depend on V, which grows with time. The cosmological term must therefore decrease over cosmic history, tracking $\Lambda \sim H^2$ at each epoch, and it must fluctuate rather than sit at a fixed value. This is a sharp qualitative difference from the standard ΛCDM picture, in which $w = -1$ exactly, and it is testable with current surveys.

The relation $\Lambda \sim H^2$ should be read as a statement about the magnitude of the fluctuation at each epoch, not as a smooth tracking law: a Λ that follows H^2 exactly rescales the matter era without producing acceleration, and it is the fluctuating character of the term, not its envelope, that distinguishes it from a rescaled matter component.

The observational situation is genuinely unsettled and should be reported as such. DESI DR2 reports a preference for an evolving dark-energy equation of state over a cosmological constant, reaching up to 4.2σ for particular data combinations [21]. Independent Bayesian reanalyses find the evidence substantially weaker, and find that it largely disappears when a recalibrated supernova sample is used [22]. TSGT sides with the evolving interpretation, and a firm confirmation that $w = -1$ exactly would count against this section.

A second consequence deserves emphasis because it is a liability rather than an asset: a Λ that tracks H^2 at all epochs is present during nucleosynthesis and structure formation, where it can spoil light-element abundances and suppress galaxy formation. Existing treatments address this by letting Λ fluctuate about zero in sign rather than remaining positive [17]. Whether TSGT's pixel-creation picture permits a sign-changing Λ is not established here (open problem OP5).

6 The Action Principle

Sections 2–5 proceed from postulates imposed by hand: a spacing relation, a clock relation, and a gradient relation. This section removes that defect by exhibiting a single action from which all of them follow, and states plainly what the construction achieves and what it costs.

6.1 Why the action cannot be freely invented

It is tempting to write a new action for the field h_{ij} of Section 4 and hope that something other than General Relativity emerges. This route is closed. For a massless spin-2 field, Lorentz invariance together with the absence of negative-norm states fixes the quadratic action uniquely to the Fierz–Pauli form [24], and consistency of the self-coupling at higher orders forces the full Einstein–Hilbert action [25, 26]. Any attempt to deform this either reproduces linearised General Relativity or introduces a ghost.

The consequence for TSGT is decisive: *the theory's new content cannot live in the propagation of $h_{\mu\nu}$*. It must live in the structure that TSGT actually adds, which is the statement that the space-time volume element is not free but is fixed by a discrete pixel count. That is a constraint on the measure, not a modification of the graviton, and it evades the Fierz–Pauli–Deser uniqueness theorems because it does not alter the tensor sector at all.

6.2 The TSGT action

Let $n(x)$ denote the coordinate-space number density of pixels, so that the number of pixels in a coordinate cell is $dN = n(x)\, d^4x$. The correspondence $V = NL_\mathrm{p}^4$ of Eq. (18), written locally, becomes a condition on the metric determinant:

$$\sqrt{-g} = L_{\mathrm{p}}^4\, n(x). \tag{25}$$

This is the statement, in covariant form, that pixel number *determines* volume rather than merely measuring it. We take it as the defining structural postulate of TSGT and enforce it with a Lagrange multiplier $\lambda(x)$:

$$\boxed{S[g_{\mu\nu}, \lambda, \psi] = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,R \;-\; \frac{1}{8\pi G}\int d^4x\;\lambda(x)\Big[\sqrt{-g} - L_{\mathrm{p}}^4 n(x)\Big] \;+\; S_{\mathrm{m}}[g_{\mu\nu}, \psi]} \tag{26}$$

where ψ denotes matter fields. The fundamental variable is the full four-dimensional metric $g_{\mu\nu}$, whose ten components—six spatial, three mixed and one temporal—are reduced to nine independent ones by the constraint (25). The temporal component is what carries the clock postulate of Eq. (3), which is therefore no longer an independent assumption but part of the same field.

Equation (26) is the unimodular form of gravity [27, 28, 29]. TSGT does not claim to have invented it. What TSGT supplies is the physical identification (25): in unimodular gravity the fixed volume element is a background structure of unexplained origin, whereas here it is the pixel count, and the count is an integer.

6.3 Field equations

Varying (26) with respect to λ returns the constraint (25). Because that constraint holds, the variation of $g_{\mu\nu}$ is restricted to be volume preserving, and only the trace-free part of the metric variation is free. The resulting field equations are the trace-free Einstein equations,

$$R_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}R = 8\pi G\left(T_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}T\right). \tag{27}$$

Taking the divergence of (27) and using the contracted Bianchi identity $\nabla^\mu R_{\mu\nu} = \frac{1}{2}\nabla_\nu R$ gives

$$\nabla_\nu(R + 8\pi G\,T) = 32\pi G\,J_\nu, \qquad J_\nu \equiv \nabla^\mu T_{\mu\nu}. \tag{28}$$

Case 1: conserved matter. If $J_\nu = 0$, then $R + 8\pi G\,T$ is constant. Writing that constant as 4Λ and substituting back into (27) recovers the full Einstein equations,

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G\,T_{\mu\nu}, \tag{29}$$

in which Λ appears as an *integration constant* rather than as a parameter of the action [29, 30]. Its weak-field limit is the metric of Eq. (4), so postulates (2) and (3) are now derived rather than assumed, and Eq. (6) becomes a rewriting of the Newtonian limit in Planck units rather than an independent dynamical law.

Case 2: pixel creation. TSGT is committed to the creation of new pixels as the universe expands (Section 5.1). If that process exchanges energy with matter, then $J_\nu \neq 0$; defining $\Lambda \equiv \frac{1}{4}\left(R + 8\pi G\,T\right)$ as before, (28) gives

$$\nabla_\nu \Lambda = 8\pi G\,J_\nu. \tag{30}$$

The cosmological term is then not constant: it accumulates the history of energy non-conservation. This mechanism is due to Josset, Perez and Sudarsky [31], who show that violations far too small to detect locally can integrate to a cosmologically relevant Λ, and who consider Planck-scale discreteness as a source. In TSGT the source is identified: it is pixel creation. The direction of the exchange is fixed below.

This resolves an apparent tension with Section 5.3. In Case 1 the cosmological term is strictly constant, which is incompatible with the fluctuating Λ required by the Poisson argument. TSGT is therefore committed to Case 2: pixel creation must exchange energy with matter, since otherwise Eq. (20) has no mechanism behind it. The Poisson estimate of Section 5.3 should accordingly be read as an estimate of the magnitude that Λ accumulates through Eq. (30), not as an independent statement.

The sign of J_ν. The direction of the exchange is then fixed by consistency with Section 5.4. In a spatially flat FRW background with a perfect fluid of density ρ and pressure p, the time component of the source is

$$J_0 = -\left[\dot{\rho} + 3H(\rho + p)\right], \tag{31}$$

so that (30) collapses to a single balance equation,

$$\dot{\rho}_\Lambda + \dot{\rho} + 3H(\rho + p) = 0, \qquad \rho_\Lambda \equiv \frac{\Lambda}{8\pi G}, \tag{32}$$

in which the total budget is conserved even though the matter sector alone is not. Section 5.4 requires Λ to track H^2, and H decreases monotonically over the epochs in question; hence $\dot{\rho}_\Lambda < 0$, and therefore $J_0 < 0$. The exchange runs from the cosmological term into matter: pixel creation does not drain energy from matter but feeds it, at the expense of Λ. During matter domination (32) gives $\dot{\rho}_\Lambda = -3H\rho_\Lambda$, so the fractional rate at which matter energy conservation fails is of order H itself—some $10^{-18}\,\mathrm{s}^{-1}$ at the present epoch. This is precisely the regime identified in [31]: far below any local bound on energy non-conservation, yet integrating over a Hubble time to a cosmologically relevant Λ.

Two qualifications are needed, and both are limitations rather than caveats of presentation. First, the sign is fixed by requiring consistency with an observational input, not derived from the substrate; TSGT still supplies no calculation of the pixel creation rate, and until it does, (32) parametrises the exchange rather than predicting it (open problem OP2). Second, the argument constrains only the secular drift of Λ. Section 5.4 also requires Λ to fluctuate, and early-universe consistency requires it to change sign; nothing above bears on the fluctuating part, which remains open (open problem OP5).

6.4 What this resolves

- **Only two polarisations propagate.** Unimodular gravity carries two local degrees of freedom per point, exactly as in General Relativity, together with a single global degree of freedom, namely Λ itself [29]. The scalar and vector sectors of Section 4 are therefore not physical: they are gauge and constraint artefacts of an unconstrained tensor field. This removes the extra-polarisation problem of Section 4.4—listed as an open problem in earlier versions of this paper—and with it the conflict with polarisation searches and with the measured value of the PPN parameter γ.

- **Λ and V are conjugate.** The relation $\delta\Lambda\,\delta V \sim \hbar$ assumed in Section 5.3 is a theorem in this formulation, not an assumption: Λ is canonically conjugate to the total four-volume [28, 29]. The dark-energy estimate of Eq. (20) therefore rests on the action rather than on a heuristic.

- **Λ is not assumed to be constant.** Equation (30) shows it is sourced, which is what Section 5.4 requires. The assumption $\langle\Lambda\rangle = 0$ of open problem OP5 is not yet derived, but the question changes character: the secular sign of J_ν is fixed by Eq. (32), and what remains is a question about initial conditions on Λ and about its fluctuating part, both of which are well posed within (26).

- **Matter coupling is specified.** S_m enters the action directly, so the response of the geometry to stress–energy is derived rather than inserted. This removes the matter-coupling problem listed as an open problem in earlier versions of this paper.

6.5 What this costs

- **TSGT is now provably equivalent to General Relativity at the classical level.** Equation (29) is Einstein's equation. Whatever independent content TSGT possesses cannot be classical and cannot be gravitational dynamics; it lies entirely in the discreteness of N, which is an integer subject to Poisson fluctuations and which therefore makes (25) exact only in the mean. This sharpens open problem OP1 rather than removing it, and it should be stated that way.

- **Full diffeomorphism invariance is reduced.** Fixing the volume element leaves only transverse (volume-preserving) diffeomorphisms as gauge symmetry. Full invariance can be restored by promoting the fixed measure to the exterior derivative of a three-form field, as in the Henneaux–Teitelboim formulation [29]; whether the TSGT constraint (25) admits such a rewriting with $n(x)$ dynamical is not established here.

- **The pixel density $n(x)$ is prescribed, not solved for.** There is no equation of motion for n; it enters (26) as external data. A complete theory would determine the pixel creation rate dynamically, and Eq. (30) would then predict J_ν rather than parametrise it. This is recorded as open problem OP2.

7 Observational Status

The table below summarises where TSGT now stands against observation. Of the six entries, one is a quantitative success, one is a live test, and the remainder are consistency checks or retractions. The polarisation conflict recorded in earlier versions is removed by the action principle of Section 6 and is listed here as resolved rather than outstanding.

Quantity	**TSGT**	**Observation**	**Status**
Newtonian limit	$g = GM/r^2$	confirmed	Reproduced by construction; not a test.
Light deflection	$4GM/c^2b$	confirmed to 10^{-5}	Reproduced; requires the temporal postulate.
Dark energy density	2.8×10^{-27} kg m^{-3}	5.8×10^{-27} kg m^{-3}	Agreement within a factor of 2.
Equation of state	$w \neq -1$; Λ evolves and fluctuates	contested; DESI DR2 vs. Bayesian reanalyses	Testable now.
Extra GW polarisations	absent; only the two TT modes propagate	none detected	Resolved in Section 6.
Photon / GW dispersion	withdrawn	—	Retracted in Section 3.3.

Table 2: Status of TSGT against observation after the revisions of this version. The framework now has one quantitative success and one live test; the polarisation conflict of earlier versions is resolved by the action principle of Section 6.

8 Limitations and Open Problems

OP1. The framework adds no classical content. Section 2 reproduces linearised GR by construction, and Section 6 shows that the equivalence is exact at the classical level rather than merely apparent. TSGT's independent content lies entirely in the integer character of N and in the Poisson fluctuations that make Eq. (25) hold only in the mean; no regime measured so far distinguishes the two theories.

OP2. The pixel density is not dynamical. The action of Section 6 contains no equation of motion for the pixel creation rate; $n(x)$ enters as external data. A complete theory would predict J_ν through Eq. (30) rather than parametrise it. Whether the constraint admits a Henneaux–Teitelboim rewriting with $n(x)$ dynamical is also unsettled.

OP3. Dynamics on a sprinkling is undeveloped. Section 4 works in the continuum. Deriving wave propagation directly on an irregular sprinkling is hard—the discrete d'Alembertian is non-local—and is unsolved here.

OP4. Relation to causal set theory unresolved. See Section 3.4. TSGT shares its kinematics with causal sets and must either derive its spacing relation from causal order or justify treating spacing as independent.

OP5. The mean of Λ and its fluctuating part are not derived. That Λ varies is now a consequence of Eq. (30) rather than an assumption, and its secular direction is fixed by Eq. (32): energy flows from Λ into matter. Two gaps remain. The sign is fixed by requiring $\Lambda \sim H^2$ rather than derived from the substrate, so it is a consistency condition and not a prediction. And the assumption $\langle\Lambda\rangle = 0$ is still unexplained: early-universe consistency requires a sign-changing fluctuation about zero (Section 5.4), on which the secular argument is silent.

OP6. Singularities. Finite pixels forbid unbounded density, so TSGT must replace curvature singularities with something. What that is remains open.

9 Conceptual Extension I: Dimensionality of the Bulk

Scope note. This section is conceptual. It does not depend on, and is not constrained by, the quantitative content of Sections 2–5, and none of the testable content of this paper follows from it.

Superstring theory requires ten space-time dimensions and M-theory eleven [32, 33]. The eleven-dimensional bound follows from Nahm's classification of supersymmetry algebras [34]: above eleven dimensions, any supermultiplet containing a graviton necessarily contains massless fields of spin greater than two, for which no consistent interacting flat-space theory is known. The ten-dimensional critical dimension of the superstring is separate, arising from worldsheet conformal-anomaly cancellation.

Both arguments are conditional. Nahm's bound presupposes supersymmetry; the critical-dimension argument presupposes a particular worldsheet theory. That these premises matter is textbook fact rather than speculation: the bosonic string, lacking worldsheet supersymmetry, has critical dimension 26—already well above eleven. Non-supersymmetric string constructions exist [36, 37], and anomaly cancellation depends on the gauge structure chosen.

TSGT therefore takes a dimensional cap to be a property of a particular vacuum rather than a universal law, with the ambient Bulk carrying no preferred dimensionality. Within a landscape picture [35], each vacuum realises its own effective D. This is a reinterpretation of existing results, not a derivation: TSGT computes D for no vacuum and offers no observational discriminant.

9.1 Topological layering

- **Micro-topology.** Each pixel carries additional directions compactified at the Planck scale, into which baryonic matter does not propagate, though string modes vibrating within them would set the particle spectrum.
- **Macro-topology.** The observable four-dimensional space-time is a finite, expanding membrane embedded in an extended, approximately flat Bulk.

10 Conceptual Extension II: Universe-Generation Channels

Scope note. As above, this section is conceptual and carries no dependence on the quantitative content of this paper.

Two generation channels are distinguished. (Earlier drafts described a "tripartite" model while listing only two; the discrepancy is resolved here in favour of two.)

10.1 Channel I: Quantum fluctuation

Universes nucleating as spontaneous fluctuations of the Bulk are stochastic, their low-energy constants fixed by the symmetry-breaking pathway followed during nucleation, uncorrelated with any parent [38].

10.2 Channel II: Replication through rupture

Where compression drives the local spacing past the threshold of Eq. (11), the substrate may undergo a topology-changing transition of conifold type [39], after which a localised child region expands as a separate pocket universe inheriting the parent's compactification. Whether the threshold can be reached by any physical process, and whether the transition is dynamically allowed, are not addressed here. The scenario in which a singularity is replaced by the birth of a child universe was proposed by Smolin [40]; the proposal that quantum-gravitational effects halt collapse before a singularity forms is developed in the Planck-star programme [41]. TSGT does not claim priority for either idea. Its contribution is that the threshold at which the transition occurs, Eq. (11), follows from its own kinematics rather than being imposed, and that it differs quantitatively from the Planck-density criterion (Section 2.5).

A remark on the alternative. In black-hole-to-white-hole bounce scenarios the collapsing matter returns to the parent universe rather than forming a child region. TSGT disfavours this, for a structural reason internal to the framework: elastic rebound requires a bond structure with a preferred equilibrium separation, and a Poisson sprinkling (Section 3) has no fixed neighbours and therefore no such equilibrium to return to. We note that this is suggestive rather than conclusive, since the field h_{ij} does carry wave-like excitations.

We record the cost of this position explicitly. If matter and information pass into a causally disconnected child region, then unitarity is preserved globally but not from within the parent universe, where an initially pure state evolves into a mixed one. This is the standard objection to baby-universe resolutions of the information problem, and TSGT does not evade it.

11 Conclusion

This version of TSGT trades speculative phenomenology for structural consistency and one quantitative result.

The framework's defensible content is now the following. First, a single relation in Planck units, $g = (c^2/L_\mathrm{p})(\Delta L/L_\mathrm{p})$, together with the accompanying temporal postulate, reproduces the entire weak-field sector of GR including the correct light deflection—by construction rather than by prediction. Second, taking the substrate to be a Poisson sprinkling rather than a lattice makes discreteness compatible with Lorentz invariance, at the price of retracting the dispersion and anisotropy signatures of earlier drafts. Third, and most importantly, the requirement that expansion create pixels rather than stretch them yields a cosmological term of magnitude $\Lambda \sim N^{-1/2}$, giving $\rho_\Lambda \simeq 2.8 \times 10^{-27}$ kg m^{-3} against an observed 5.8×10^{-27}—a factor of two, where the naive estimate errs by 10^{123}.

To this is added the result of Section 6. The number–volume correspondence, written locally, is the unimodular constraint on the metric determinant, and enforcing it in the Einstein–Hilbert action fixes the theory: the field equations are the trace-free Einstein equations, the two kinematic postulates of Section 2 are derived rather than assumed, and Λ enters as an integration constant rather than as a parameter of the action. The cost is stated in the same place and is the heaviest the framework has paid: TSGT is thereby provably equivalent to General Relativity at the classical level, so its independent content lies entirely in the integer character of N and in the Poisson fluctuations that make the constraint exact only in the mean.

A further point deserves mention because it is the framework's most distinctive quantitative statement: the rupture threshold r_* follows from TSGT's own kinematics and differs from the Planck-density criterion used elsewhere, placing the transition some twenty orders of magnitude below Planck density for a solar mass (Section 2.5).

The framework remains incomplete in ways that are stated rather than concealed: the pixel density $n(x)$ enters the action as external data rather than as a dynamical field, there is no dynamics formulated directly on the sprinkling, and the assumption that the mean of Λ vanishes is still unexplained. The most productive next steps are, in order: finding an equation of motion for $n(x)$, so that Eq. (30) predicts J_ν instead of parametrising it; deriving wave propagation directly on the sprinkling; and determining whether pixel creation supplies the sign-changing Λ that early-universe consistency requires.

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