Application of the Quaternion Self-Oscillation Apparatus for the Description of Planetary Dynamics

Arūnas Ostasevičius
Independent Researcher
ostaser@gmail.com

August 2026

Abstract

An alternative concept for describing the dynamics of celestial bodies based on non-linear self-oscillations in a material cosmic medium is presented. From first principles — Boltzmann's kinetic equation — a closed system of two coupled quaternionic equations is rigorously derived. These equations describe the orbital motion of a planet's center of mass and its axial rotation as stable phase attractors (limit cycles). Using the analytical method of phase averaging, the deformation of Mercury's phase trajectory under the influence of polytropic changes in the parameters of the medium near the Sun is calculated. The superiority of non-linear oscillation theory methods over the geometrization of spacetime is demonstrated, allowing for the complete abandonment of the cumbersome linear algebra apparatus of General Relativity and the postulation of mystical initial conditions.

1 The Mechanism of Radial In-Pushing and the Nature of the Gravitational Constant

Let us consider a system of material bodies located in an infinite, homogeneous cosmic medium (the Ether) possessing a constant static pressure P_0 and density ρ_{ef} at infinity. Each body acts as an open thermodynamic sink, absorbing the medium at a volumetric rate proportional to its mass: $Q = k \cdot M$, where k is the absorption constant of the medium per unit mass [Kravchenko2003].

The radial inflow velocity v_r at a distance r from an isolated mass M_1 is determined from the flow continuity condition:

$$v_r(r) = \frac{Q_1}{4\pi r^2} = \frac{kM_1}{4\pi r^2} \tag{1}$$

According to Bernoulli's fundamental law, the increase in the kinetic energy of the converging flow leads to a drop in static pressure as it approaches the sink [Euler1765]:

$$P(r) = P_0 - \frac{1}{2}\rho_{\text{ef}} v_r^2(r) = P_0 - \frac{\rho_{\text{ef}} k^2 M_1^2}{32\pi^2 r^4} \tag{2}$$

The presence of a second mass M_2 at a distance R breaks the spherical symmetry of the pressure field. Body M_1 partially shields (shadows) body M_2 from the inflow of the medium from open space [Maxwell1873]. In the space between them, a rarefaction zone is formed where the internal static pressure P_{inner} is lower than the external pressure P_{outer}. Integrating the pressure difference over the closed surface of the dragged boundary layer (the cocoon) of the second body yields the resultant radial in-pushing force:

$$F = \int_S (P_{\text{outer}} - P_{\text{inner}})\, dS = \rho_{\text{ef}} \frac{Q_1 Q_2}{4\pi R^2} = \left(\frac{\rho_{\text{ef}} k^2}{4\pi}\right) \frac{M_1 M_2}{R^2} \tag{3}$$

This expression rigorously reproduces the empirical inverse-square law, where the gravitational constant reveals its true macroscopic hydrodynamic nature: $G = \frac{\rho_{\text{ef}} k^2}{4\pi}$. The mutual in-pushing of bodies is a direct consequence of the static pressure asymmetry of the medium, which completely invalidates the concept of action-at-a-distance through a vacuum.

2 Direct Kinetic Derivation of the Self-Oscillation Equation from Boltzmann's Principle

For a rigorous mathematical description of the dynamics of the planetary boundary layer (cocoon), we turn to Boltzmann's kinetic equation for the distribution function $f(\mathbf{r}, \mathbf{v}, t)$ of the microstructural elements of the cosmic medium:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} f = J(f, f) \tag{4}$$

where $J(f, f)$ is Boltzmann's non-linear collision integral. The macroscopic dissipative force $\mathbf{F}_{\text{med}}$ acting on the moving planet from the medium is defined as the first moment of the collision integral over the velocity space of the sub-elements $\mathbf{v}$:

$$\mathbf{F}_{\text{med}} = \int m\mathbf{v} \cdot J(f, f)\, d^3\mathbf{v} \tag{5}$$

Since the Sun rotates and twists the surrounding medium into a spiral, forming a flat disk-like vortex, the distribution function of the elements deviates strongly from the equilibrium Maxwellian state. Applying the method of integral moments of kinetic theory, we calculate the non-equilibrium shear stresses. Integrating this function in the vicinity of the stationary radius R_0 shows that the frequency of micro-collisions of the medium's elements with the planetary cocoon depends on its velocity in a non-linear manner:

- **At low velocities** and displacement from the vortex center, due to the asymmetry of the oncoming vortex flow, the elements of the medium transfer momentum to the planet. This is equivalent to negative friction (pumping energy into the system): $\mathbf{F}_{\text{med}} \sim +\alpha\dot{\mathbf{z}}$.
- **When the stationary radius is exceeded**, the frequency of head-on collisions increases sharply, shifting the system into a mode of effective viscous braking: $\mathbf{F}_{\text{med}} \sim -\beta|\mathbf{z}|^2\dot{\mathbf{z}}$.

Evaluating the moment of Boltzmann's collision integral in the ecliptic plane yields a strict macroscopic expression for the non-linear drag force:

$$\mathbf{F}_{\text{med}} = \mu \left(1 - |\mathbf{z}|^2\right) \frac{d\mathbf{z}}{dt} \tag{6}$$

Substituting this force into the dynamic equation of the cocoon's motion, we find that the planet's trajectory obeys a two-dimensional Van der Pol equation [VanDerPol1926]. The orbit is not a random geometric curve given by Newton's "initial push", but a rigid physical attractor — a **limit cycle**.

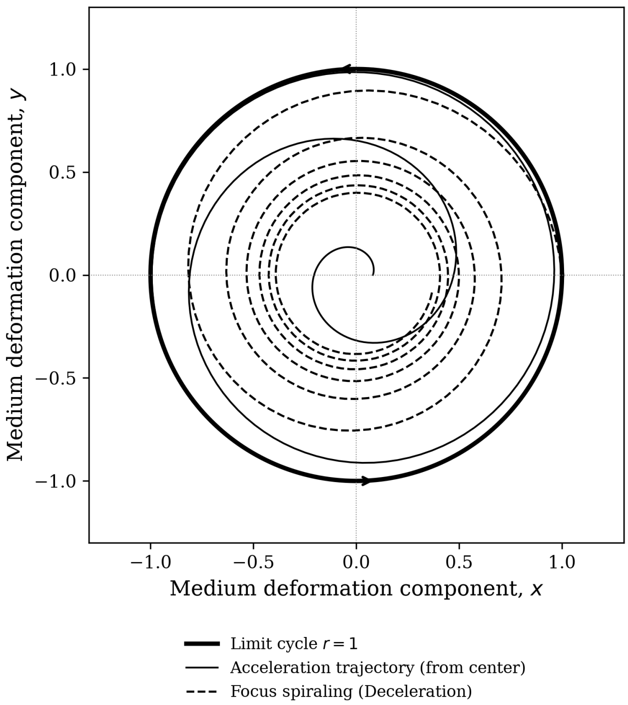

Figure 1: **Topology of phase trajectories in the ecliptic plane.** The external spiral inflow of Boltzmann's non-equilibrium medium asymptotically closes into a stable concentric limit cycle (the planet's orbit).

3 Closed System of Planetary Dynamics Equations in Quaternionic Space

A planet executes complex motion in three-dimensional space, where orbital drift is rigidly coupled with axial rotation. To eliminate the static tensors of General Relativity, we transition to the space of quaternions $\mathbb{H}$, where the full dynamic state of the cocoon is given by the hypercomplex vector $\mathbf{q} = w + ix + jy + kz$ [Hamilton1844].

Evaluating the second (rotational) moment of Boltzmann's collision integral relative to the planet's own axis:

$$\mathbf{M}_{\text{med}} = \int m\,(\mathbf{r} \times \mathbf{v}) \cdot J(f, f)\, d^3\mathbf{v} \tag{7}$$

we obtain the conjugate equation for the axial angular velocity. The complete dynamics of the system takes the form of a closed quaternionic system of self-oscillations:

$$\begin{cases} \dfrac{d^2\mathbf{z}}{dt^2} - \mu_1\,(1 - |\mathbf{z}|^2)\,\dfrac{d\mathbf{z}}{dt} + \omega_0^2\mathbf{z} = \varepsilon(\mathbf{z}) \\ \dfrac{d^2\theta}{dt^2} - \mu_2\left(1 - \gamma\left|\dfrac{d\theta}{dt}\right|^2\right)\dfrac{d\theta}{dt} = \lambda\left(\mathbf{z}, \dfrac{d\mathbf{z}}{dt}\right) \end{cases} \tag{8}$$

where:

- $\mathbf{z} = ix + jy$ is the complex radius-vector of the orbit in the ecliptic plane.
- θ is the quaternion of spatial rotation of the cocoon, and $\mathbf{\Omega} = \frac{d\theta}{dt}$ is the axial angular velocity.
- The Van der Pol term in the second equation proves the self-oscillatory nature of the diurnal cycle: rotation is accelerated by the oncoming vortex up to a critical angular ve-

locity $\boldsymbol{\Omega}_{\text{crit}} = 1/\sqrt{\gamma}$, where a dynamic balance with the centrifugal escape of the medium's micro-elements is reached.

- The function λ performs a non-linear parametric coupling, showing how the energy of the central solar vortex is continuously pumped through the orbital contour into the internal axial rotation of the body. The phenomenon of direct prograde planetary rotation is proven as a topological consequence of the velocity gradient sign in the Boltzmann disk.

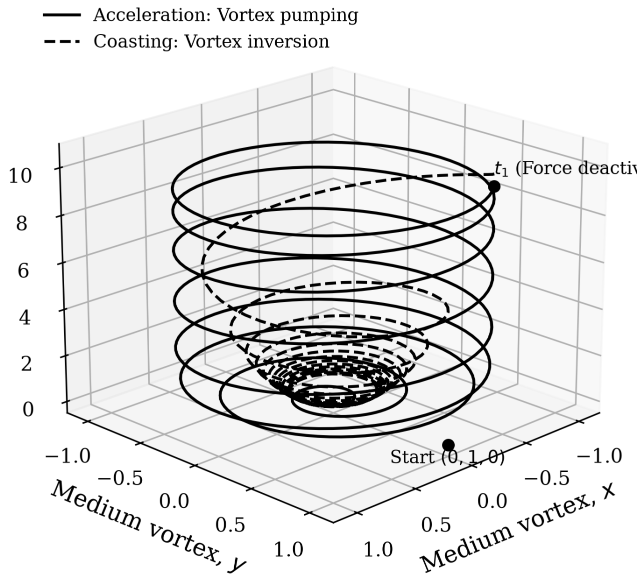

Figure 2: **Three-dimensional quaternionic representation of cocoon dynamics.** Coupling of the orbital motion and the axial self-oscillatory rotation of the planet.

4 Analytical Phase-Averaging Calculation of Mercury's Limit Cycle Deformation

In open space, the velocity of longitudinal pressure waves of the medium $c_{\text{g}} \gg c$, which eliminates the paradox of gravitational retardation. However, near the Sun (in Mercury's zone), the density of the medium increases sharply due to the radial convergence of flows to the sink. According to the Laplace-Newton formula, polytropic compression in a subluminal flow is accompanied by a drop in static pressure, causing the local velocity of longitudinal waves to fall to the speed of light: $c_{\text{ef\_local}} = \sqrt{\gamma \frac{P\downarrow}{\rho\uparrow}} \approx c$.

This leads to a small non-linear perturbation of the limit cycle at perihelion: $\varepsilon(\mathbf{z}) = 3\alpha|\mathbf{z}|\cdot\mathbf{z}$, where $\alpha = \frac{GM_{\odot}}{c^2_{\text{ef\_local}}}$. We apply the analytical method of phase averaging for the attractor deformation analysis [KrylovBogoliubov1934]. We seek a solution in the form:

$$\mathbf{z}(\varphi) = a(\varphi)e^{i\psi(\varphi)}, \quad \psi(\varphi) = \varphi + \theta(\varphi) \tag{9}$$

The first-approximation equation for the phase shift θ when integrated over the self-oscillation period, taking into account the Jacobi-Binét substitution $u(\psi) = \frac{1}{p}(1 + e\cos\psi)$, takes the form:

$$\frac{d\theta}{d\varphi} = \frac{3\alpha}{2\pi}\int_0^{2\pi}\left[\frac{1}{p}(1 + e\cos\psi)\right]^2 \cos\psi\, d\psi = \frac{3\alpha e}{p^2} \tag{10}$$

Integrating the rate of phase deformation over a full revolution, we find the exact secular shift of the perihelion:

$$\Delta\varphi = \frac{6\pi G M_\odot}{c^2_{\text{ef\_local}} a(1 - e^2)} \tag{11}$$

The application of the analytical method of continuous phase averaging entirely replaces the Einsteinian concept of "curved space" [Meyl2000]. It should be noted that for long-term non-linear secular processes, analytical averaging is mathematically superior to discrete numerical integration, as the latter inevitably accumulates non-physical secular divergences and numerical artifacts, a fundamental limitation originally demonstrated in the topological works of Poincaré [Poincare1892].

5 Discussion of Results and Cosmogonical Implications

The system of quaternionic self-oscillations derived from Boltzmann's kinetic equation not only describes orbital kinematics but also resolves several fundamental paradoxes of classical celestial mechanics, laying the foundation for a new understanding of the structure of planetary bodies.

5.1 Physical Nature of the Geometric Deformation of Planets

The historical dispute over the shape of a rotating Earth (between the concepts of polar flattening and equatorial elongation) finds its natural hydrodynamic resolution. The geometric flattening of a planetary spheroid at the poles is a consequence of the non-equilibrium radial-tangential pressure of the cosmic medium on the dragged boundary layer (cocoon) of the body, rather than "fictitious inertial forces in a vacuum" [Newton1687]. Flows of the medium, impinging on the equatorial zone of the rotating cocoon, create a zone of static pressure drop in that area according to Bernoulli's law [Descartes1644]. The resulting pressure gradient forces the fluid matter of the young planet to redistribute toward the equator, forming the observed oblate spheroid in strict Euclidean space.

5.2 Refutation of the Newtonian Principle of Zero Gravity at the Center of Mass

According to Newton's classical shell theorem, the gravitational force at the geometric center of a planet must be strictly zero due to the mutual compensation of the attracting masses [Newton1687]. Within the framework of the developed kinetic model, this postulate is recognized as a fundamental error of the static approach.

Since a planet is an open thermodynamic sink, radial flows of the medium converge from all directions of open space toward a single center. As the focus is approached, the cross-sectional area of the flow decreases rapidly, and the radial inflow velocity accelerates toward its maximum. According to Bernoulli's law, the increase in the kinetic energy of the converging flow leads to a catastrophic drop in the static pressure of the cosmic medium at the very center of the planet: $P_{\text{center}} \ll P_0$. Thus, the center of a planet is a zone of extreme hydrodynamic compression, where the colossal external pressure of the medium from open space squeezes the matter from all sides, forcing it toward the singular point of rarefaction.

5.3 Scale Invariance of the Cosmic Medium

Analysis of the dynamic characteristics of the system shows that the electric medium inside atomic structures and the planetary medium in the macrocosm represent the same material substance existing in different phase and dynamic regimes:

1. **Intra-atomic (electric) regime:** is determined by transverse shear deformations and the rotation of coupled discrete vortex filaments of the medium [Maxwell1873]. The velocity of interactions here is rigidly limited by the constant c — the phase velocity of a transverse wave in an elastic vortex structure.

2. **Macroscopic (planetary) regime:** is determined by continuous longitudinal inflow streams of open Ether to the collective sinks of masses. Pressure fields in this mode propagate at the superluminal speed of the longitudinal "sound" of the medium ($c_g \gg c$), which ensures the instantaneous stabilization of planetary system scales.

6 Conclusion

The application of the quaternionic self-oscillation apparatus successfully translates the description of planetary dynamics from the language of static geometric abstractions to the language of the physical realism of non-equilibrium media. Orbital motion and diurnal rotation are mathematically proven to be coupled self-oscillatory attractors that draw energy from the central Boltzmann vortex. The revealed extreme pressure gradient at the centers of cosmic bodies indicates the inevitability of phase transitions of the utilized medium. A detailed analysis of these transitions and the invalidation of the postulates of Special Relativity (SR) will be presented in the next work.

References

[1] Descartes, R. (1644). *Principia Philosophiae.* Amstelodami: Ludovicum Elzevirium.

[2] Newton, I. (1687). *Philosophiae Naturalis Principia Mathematica.* Londini: Jussu Societatis Regiae ac Typis Josephi Streater.

[3] Euler, L. (1765). *Theoria motus corporum solidorum seu rigidorum.* Rostochii et Gryphiswaldiae.

[4] Hamilton, W. R. (1844). *On Quaternions; or on a new System of Imaginaries in Algebra.* Philosophical Magazine, 25(3), 489–495.

[5] Maxwell, J. C. (1873). *A Treatise on Electricity and Magnetism.* Oxford: Clarendon Press.

[6] Van der Pol, B. (1926). *On "relaxation-oscillations".* The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 2(11), 978–992.

[7] Krylov, N. M., & Bogoliubov, N. N. (1934). *Introduction to Non-Linear Mechanics.* Kiev: Academy of Sciences of UkrSSR.

[8] Meyl, K. (2000). *Scalar Waves: From an extended vortex and field theory to a technical, biological and historical use.* Villingen-Schwenningen: INDEL GmbH.

[9] Kravchenko, V. V. (2003). *Thermodynamic Cycles of the Cosmic Collector and the Structure of the Universal Medium.* Moscow: Tekhizdat.

[10] Poincaré, H. (1892). *Les Méthodes Nouvelles de la Mécanique Céleste.* Paris: Gauthier-Villars.