

Dark Matter and Dark Energy as Twin Residues of the QCD Phase Transition

A UNIFIED PICTURE IN THE GRAVITON CONDENSATION COSMOLOGY FRAMEWORK

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ABSTRACT

The Graviton Condensation Cosmology (GCC) framework posits that macroscopic gravity emerges from the condensation of spin-2 collective quadrupole excitations (GQR phonons) in nuclear matter. We extend this framework to address the dark sector: we propose that both dark matter and dark energy originate as residues of the QCD phase transition at $T_c \approx 156.5$ MeV. Dark matter is identified with multi-glueball bound states — the unique GCC-compatible candidate after a systematic exclusion of all Standard Model and beyond-Standard-Model particles, which lack either QCD substructure or a spin-2 quadrupole channel. The relic abundance $\Omega_{gb} h^2$ naturally matches the observed $\Omega_{DM} h^2 \approx 0.12$ for formation fractions $f_{gb} \sim (1-10) \times 10^{-6}$ across three mass windows ($m_{gb} \approx 0.78-2.4$ GeV). Dark energy arises from the residual uncondensed fraction $(1 - \epsilon) \sim 10^{-45}$ of the spin-2 condensate energy. While this value is accommodated within the non-linear saturation framework, its precise first-principles prediction remains an open theoretical challenge; it constitutes a well-defined target for future development of the saturation dynamics. We show that the Landau parameter F_2 reaches -4.97 ± 1.25 (phenomenological estimate) or -3.65 ± 0.41 (conservative RPA with realistic nuclear forces), both approaching the Pomeranchuk instability threshold $F_2 = -5$ with critical density $n_c/n_0 \approx 1.0-1.2$. Finite-temperature enhancement mechanisms at the QCD crossover provide the bridge to the threshold. The framework yields falsifiable predictions including a single nanohertz GW peak (5–15 nHz), distinctive 21-cm power spectrum suppression, and CMB lensing signatures, linking the dark sector to ongoing experiments without introducing new particles or energy scales beyond QCD.

INTRODUCTION

The cosmological standard model (Λ CDM) achieves remarkable phenomenological success while leaving the physical nature of its two dominant constituents — dark matter and dark

energy — unexplained. Dark matter is conventionally attributed to a new, stable, weakly interacting massive particle (WIMP) or axion [1,2]; dark energy is parametrized as a

cosmological constant Λ [3] or a slowly rolling scalar field (quintessence) [4]. Despite decades of experimental effort, no definitive non-gravitational signal of dark matter has been detected [5,6], and the ~ 55 orders-of-magnitude discrepancy between the QCD vacuum energy and the observed dark energy density remains unresolved [3].

The Graviton Condensation Cosmology (GCC) framework [8–10] offers a radical alternative: gravity itself is an emergent phenomenon, arising from the Bose-Einstein condensation of spin-2 quadrupole phonons in nuclear matter — the giant quadrupole resonance (GQR). In this picture, the gravitational field is not a fundamental degree of freedom but a collective order parameter of the QCD vacuum. The macroscopic Newtonian potential emerges when the Landau parameter F_2 for spin-2 deformations reaches the Pomeranchuk instability threshold $F_2 = -5$, triggering a dynamical spin-2 condensate [7]. The observed GQR frequency in heavy nuclei constrains the proximity to this threshold, and the QCD phase transition provides the cosmological arena in which the condensate forms across the universe.

Relation to the GCC trilogy. This work (Paper IV) extends the three core GCC papers: Paper I establishes gravity as emergent spin-2 order [7]; Paper II demonstrates the self-consistency of the coarse-graining mapping from nuclear $1/r^3$ to macroscopic $1/r$ [7]; Paper III develops the cosmological signatures of the QCD-triggered condensation [7]. Paper IV addresses the dark sector within the same unified framework — identifying dark matter and dark energy as twin residues of the QCD phase transition, and in doing so completing the physical narrative: **all** dominant constituents of the cosmological energy budget (gravity, dark

matter, dark energy) trace their origin to non-perturbative QCD dynamics at T_c .

In this paper, we show that the same QCD phase transition that generates macroscopic gravity also produces dark matter and dark energy as two distinct residues of the hadronization process. This unification rests on three pillars and a rigorous exclusion argument:

1. **An exclusion argument for dark matter.**

Within the GCC framework, only particles with quark-gluon substructure **and** a spin-2 quadrupole channel can generate the gravitational effects observed for dark matter. All Standard Model particles are eliminated (photons, leptons, neutrinos lack QCD content; baryons are charged or unstable). Beyond-Standard-Model candidates (WIMPs, axions) lack the required spin-2 structure. The unique survivor is the QCD tensor glueball 2^{++} — electrically neutral, intrinsically spin-2, produced at T_c , and capable of "feeling" gravity (bending spacetime) without "broadcasting" it (sourcing the GQR condensate).

2. **Glueballs as dark matter.** The QCD crossover at $T_c \approx 156.5$ MeV converts a deconfined quark-gluon plasma into hadrons. While the majority of the gluonic energy forms light mesons (pions, kaons), a small fraction $f_{gb} \sim 3 \times 10^{-6}$ condenses into pure-gluon bound states — glueballs. With masses in the range $m_{gb} \approx 0.78\text{--}1.7$ GeV (spanning the tensor 2^{++} and scalar 0^{++} channels) and baryon number $B = 0$, multi-glueball clumps are naturally stable against tidal disruption and two-body evaporation, and their relic abundance naturally matches Ω_{DM} .

3. **Condensation residual as dark energy.** The $GQR \rightarrow \text{spin-2}$ condensation does not consume

100% of the QCD vacuum energy released at the phase transition. The residual fraction $(1 - \varepsilon)$ persists as a nearly homogeneous energy density that redshifts like a cosmological constant, with $\varepsilon \approx 1 - 10^{-45}$ required to match ρ_Λ .

4. **Unified origin at T_c .** Both residues originate at the same temperature, from the same non-perturbative QCD dynamics, without introducing new particles or energy scales. The framework is falsifiable through specific gravitational-wave and cosmological signatures.

The remainder of this paper is organized as follows. Section II reviews the GCC framework

and the GQR condensation mechanism. Section III presents the unified dark-sector picture: the twin residue hypothesis (III.A), the exclusion argument for glueball dark matter (III.B), and the falsification of alternative candidates (III.C–III.D). Section IV analyzes glueball production during the QCD crossover and computes the relic abundance. Section V derives the dark energy density from the condensation residual. Section VI estimates the $F_2 \rightarrow -5$ closure from beyond-Fock nuclear correlations. Section VII presents observational predictions and falsification criteria. Section VIII discusses open questions, and Section IX concludes.

THE GCC FRAMEWORK: GRAVITY FROM NUCLEAR MATTER

The GQR Condensation Mechanism

The GCC framework is built on two empirically established facts:

1. **GQR as a universal nuclear excitation.**

Every atomic nucleus supports a giant quadrupole resonance — a collective shape oscillation at frequency $E_{\text{GQR}} \approx 63A^{-1/3}$ MeV, corresponding to $f_{\text{GQR}} \approx 2\text{--}5 \times 10^{21}$ Hz for heavy nuclei [12,13]. The GQR carries spin-2 quantum numbers ($J^\pi = 2^+$), matching the graviton.

2. **Landau Fermi liquid theory for nuclear matter.**

Nuclear matter at saturation density $n_0 \approx 0.16 \text{ fm}^{-3}$ behaves as a Fermi liquid, with quasiparticle interactions parameterized by dimensionless Landau parameters F_1 [7]. The spin-2 ($l = 2$) channel controls the stiffness of quadrupolar deformations:

$$E_{\text{GQR}} = E_0 / \sqrt{1 + F_2/5}$$

where $E_0 \approx 65A^{-1/3}$ MeV is the unperturbed energy. As $F_2 \rightarrow -5$, the restoring force vanishes

— the Pomeranchuk instability [7] — and the system spontaneously develops static quadrupolar order.

The central hypothesis of GCC is that when F_2 reaches the critical value, the quadrupole phonons undergo Bose-Einstein condensation into a macroscopic spin-2 field $h_{\mu\nu}$, which is identified with the emergent classical gravitational field [8,9]. We clarify the terminology used throughout this paper:

- **GQR phonon** (or "spin-2 phonon"): the microscopic collective excitation — a quantized quadrupolar shape oscillation of nuclear matter — that serves as the fundamental bosonic degree of freedom.
- **Spin-2 condensate** (or "GQR condensate"): the macroscopic coherent state formed when a macroscopically large number of GQR phonons occupy the same quantum state at the Pomeranchuk instability threshold ($F_2 \rightarrow -5$).
- **Gravitational condensation** (or "emergence of gravity"): the process by which the spin-2 condensate's classical field $h_{\mu\nu}$ acquires the

properties of the Newtonian/Einsteinian gravitational potential — specifically, the $1/r$ potential and the coupling $\sqrt{(8\pi G_{\text{eff}})}$ to the energy-momentum tensor.

The three terms form a hierarchy: phonons (microscopic quanta) $\rightarrow$ condensate (macroscopic quantum state) $\rightarrow$ gravity (effective classical field theory). The GQR phonon is the "graviton" in this framework — not a fundamental particle but an emergent quasiparticle of the QCD vacuum.

$$h_{\mu\nu}(x) = \sqrt{(8\pi G_{\text{eff}})} \langle Q_{\mu\nu}(x) \rangle$$

where $Q_{\mu\nu}$ is the GQR phonon field operator and G_{eff} is the effective emergent Newton constant.

Cosmological Context: The QCD Phase Transition

The crucial cosmological event for GCC is the QCD crossover at $T_c \approx 156.5 \text{ MeV}$ [15,16],

occurring at redshift $z_c \approx 6.7 \times 10^{11}$. At this temperature:

- The universe is radiation-dominated with $H(T_c) \approx 4 \times 10^4 \text{ s}^{-1}$
- The effective degrees of freedom drop from $g_*(T > T_c) \approx 61.75$ to $g_*(T < T_c) \approx 17.25$ as quarks and gluons become confined
- The trace anomaly $I(T) = (\epsilon - 3p)/T^4$ peaks at $I_{\text{max}} \approx 4$, indicating strong non-perturbative effects
- Nuclear matter begins to form from the hadronizing quark-gluon plasma

This transition provides three key ingredients for the GCC dark-sector picture:

- 1. The energy reservoir (trace anomaly) for gravitational condensation
- 2. The production of glueballs — pure-gluon bound states with $B = 0$
- 3. The thermodynamic conditions (T, ρ) that determine $F_2(T, \rho)$

UNIFIED DARK-SECTOR PICTURE

The Twin Residue Hypothesis

We propose that the QCD phase transition naturally partitions its non-perturbative energy release into three channels:

$$\begin{aligned} \Delta\epsilon_{\text{QCD}} = & f_{\text{cond}} \times (\text{GQR condensate} \rightarrow \text{emergent gravity}) \\ & + f_{\text{gb}} \times (\text{glueball production} \rightarrow \text{dark matter}) \\ & + f_{\text{thermal}} \times (\text{hadron thermal bath} \rightarrow \text{standard baryons}) \end{aligned}$$

with the closure condition $f_{\text{cond}} + f_{\text{gb}} + f_{\text{thermal}} = 1$. The dominant fraction $f_{\text{cond}} \sim \mathcal{O}(1)$ enters the spin-2 channel and drives the gravitational condensate; $f_{\text{gb}} \sim 3 \times 10^{-6}$ is the glueball formation fraction (Section IV); and $f_{\text{thermal}} = 1 - f_{\text{cond}} - f_{\text{gb}}$ accounts for the remaining energy thermalized into pions, kaons,

and other light hadrons. The condensate energy $f_{\text{cond}} \times \Delta\epsilon_{\text{QCD}}$ is not stored as a cosmological constant — it redshifts with the expansion (with an equation of state determined by the condensate dynamics), so that only the unconverted residual $(1 - \epsilon)$ survives to contribute to the present-day energy budget (see Section V).

The dark energy density arises not from the full QCD energy release, but from the residual **within** the condensate channel — the tiny fraction of the condensate energy that does not fully convert to the classical gravitational field:

$$\rho_\Lambda = (1 - \epsilon) \times f_{\text{cond}} \times \Delta\epsilon_{\text{QCD}}$$

where ϵ is the **condensation efficiency** within the spin-2 channel. With $f_{\text{cond}} \approx 1$ (the GQR

collective mode dominates the trace anomaly partition at T_c), the observed dark energy density $\rho_\Lambda \approx (2.3 \times 10^{-3} \text{ eV})^4 \approx 2.3 \times 10^{-67} \text{ MeV}^4$ requires:

$$1 - \varepsilon = \rho_\Lambda / (f_{\text{cond}} \times \Delta \varepsilon_{\text{QCD}}) \approx 10^{-45}$$

This extreme proximity to unity is not a separate fine-tuning: it is addressed in Section V through the condensation dynamics, where the exponential growth rate $\Gamma_{\text{cond}} \gg H(T_c)$ drives ε exponentially close to 1. The value 10^{-45} is set by the competition between condensation and cosmic expansion — as discussed in detail below.

Crucially, the two numbers ε and f_{gb} operate on distinct physical scales: $\varepsilon \approx 1 - 10^{-45}$ describes the efficiency of a dynamical process **within** the condensate channel, while $f_{\text{gb}} \sim 3 \times 10^{-6}$ describes the branching fraction **between** channels during hadronization. They are not additively constrained, resolving the apparent tension in earlier formulations.

Why Glueballs? An Exclusion Argument

The identification of dark matter with glueball clumps is not a free choice — it follows from a systematic elimination of all other candidates within the GCC framework. We now present this exclusion argument in its complete logical form.

Premise 1: GCC's emergence condition. In the GCC framework, macroscopic gravity **only** emerges from the Bose-Einstein condensation of spin-2 quadrupole phonons in nuclear matter. A particle species can generate a long-range gravitational field if and only if it possesses nuclear substructure — i.e., quark-gluon degrees of freedom capable of coherent quadrupolar oscillation at the saturation density scale.

Premise 2: Observed gravitational coupling of dark matter. Galaxy rotation curves,

gravitational lensing, and large-scale structure all demonstrate that dark matter couples to gravity. It bends spacetime and responds to the gravitational potential wells of galaxies and clusters.

Premise 3: Dark matter is "dark." It emits no detectable electromagnetic radiation and has no resolved particle identity in the Standard Model's electroweak sector.

From these three premises, we construct the exclusion table:

Every Standard Model particle is eliminated: either it has electric charge (leptons, charged hadrons, protons), or it lacks the quark-gluon substructure required for GQR-mediated gravitational emergence (photons, neutrinos), or it is unstable on cosmological timescales (neutrons). Beyond-Standard-Model candidates — WIMPs and axions — are introduced **ad hoc** to solve the dark matter problem and possess neither QCD origin nor the spin-2 quadrupole channel. In the GCC framework, they simply cannot account for the observed gravitational effects of dark matter.

The death of WIMPs: experimental convergence. The GCC theoretical exclusion of WIMPs is now independently corroborated by experiment. The deepest direct-detection searches — XENONnT [5] and LUX-ZEPLIN [6] — have pushed sensitivity into the solar neutrino floor, finding no WIMP signal across GeV-to-TeV masses and closing the canonical WIMP parameter space. The DAMA/LIBRA annual modulation signal — for decades the strongest experimental hint of WIMP dark matter — has been decisively excluded: the combined ANAIS-112 and COSINE-100 analysis (984 kg·yr exposure, same NaI target) finds modulation amplitudes consistent with zero at 4.7σ , ruling out a dark matter origin for the DAMA signal [7].

The experimental program has thus independently converged on the GCC prediction: the dark matter particle is not a WIMP. The glueball — a QCD-native resonance with no weak-scale couplings — is the natural candidate consistent with both the exclusion argument and the null results from direct detection.

The unique survivor: the spin-2 glueball.

The tensor glueball 2^{++} — a bound state of two or more gluons with $J^{PC} = 2^{++}$ — is the only known particle species that satisfies all three exclusion criteria simultaneously:

It is electrically neutral (no valence quarks).

It carries the full QCD substructure — pure gluonic, the same degrees of freedom that drive hadronization at T_c .

Its 2^{++} quantum numbers give it an intrinsic coupling to the spin-2 quadrupole channel, making it the **natural** spin-2 excitation of the non-perturbative QCD vacuum.

"Feels but does not broadcast" gravity. This leads to a precise physical distinction that is unique to the GCC framework:

> **Dark matter particles feel gravity — they bend spacetime and respond to gravitational potentials — but they do not broadcast gravity. They lack the coherent nuclear quadrupole moment required to trigger the Pomeranchuk instability and generate the macroscopic spin-2 condensate that is the emergent gravitational field.**

This asymmetry between passive gravitational response (curving spacetime through their energy-momentum tensor $T_{\mu\nu}$) and active gravitational generation (sourcing the condensate via GQR phonon coherence) is a conceptual novelty of GCC with no analog in standard field-theoretic gravity. It explains why dark matter is detected only through gravitational effects while

remaining invisible to all non-gravitational probes: glueballs participate in the **background** geometry set by the QCD condensate, but do not contribute to the **dynamical generation** of that geometry.

Parameter window: two mass scales. The 2^{++} glueball mass is known from lattice QCD, with two distinct determination strategies yielding different values:

- **Large-volume, quenched/dynamical lattices** [17–19]: $m(2^{++}) \approx 2.4$ GeV. This is the published, consensus value from extensive lattice studies with controlled systematics.

- **Small-volume, desktop-scale lattices** (CCNU, preliminary [7]): $m(2^{++}) \approx 777$ MeV. This result comes from a complementary approach optimized for spectral resolution of the tensor channel at the cost of larger finite-volume corrections.

Both values are physically viable within the GCC framework. The lower mass (777 MeV) has the advantage of much weaker Boltzmann suppression at T_c — $e^{\{-(m/T_c)\}} = e^{\{-4.97\}} \approx 7 \times 10^{-3}$ vs. $e^{\{-15.3\}} \approx 2 \times 10^{-7}$ for 2.4 GeV — implying dramatically larger thermal production and a correspondingly relaxed constraint on the required formation fraction. The higher mass (2.4 GeV) is better established and provides a more conservative parameter space. In what follows, we treat the glueball mass as an input parameter, with the 1.7 GeV (scalar 0^{++}) and 777 MeV (tensor 2^{++}) endpoints bracketing the physically interesting range.

Stability of the 2^{++} state. For $m(2^{++}) \approx 777$ MeV, the glueball-gluon decay channel $2^{++} \rightarrow gg$ is suppressed by phase space (the two-gluon threshold is effectively at the glueball mass scale in the confined phase). The hadronic decay channels $2^{++} \rightarrow \pi\pi$, KK are mediated by gluon

intermediate states and suppressed by the Okubo-Zweig-Iizuka (OZI) rule, with widths estimated at $\Gamma \sim 10^{-2}\text{--}10^{-1}$ MeV — corresponding to lifetimes $\tau \sim 7 \times 10^{-24}\text{--}7 \times 10^{-23}$ s, cosmologically negligible unless protected by an additional symmetry. A subtlety arises at finite temperature: in the QCD crossover epoch ($T \sim 80\text{--}150$ MeV), a thermal population of deconfined gluons exists in the medium, which could in principle open the $2^{++} \rightarrow gg$ decay through absorption of a thermal gluon ($2^{++} + g_{\text{thermal}} \rightarrow g + g$). The rate for this process depends on the thermal gluon density $n_g(T)$ and the 2^{++} -gluon coupling, both of which are strongly suppressed below T_c where gluons acquire large thermal masses. We discuss this channel quantitatively in Appendix C.1. For $m(2^{++}) \approx 2.4$ GeV, the scalar-glueball decay channel $2^{++} \rightarrow 0^{++} + 0^{++}$ is kinematically closed ($2.4 \text{ GeV} < 2 \times 1.7 \text{ GeV} = 3.4 \text{ GeV}$), and the same OZI suppression argument applies. We return to the stability question with a complete analysis in Section IV.B and Appendix C.

This exclusion argument constitutes the central conceptual innovation of the present work: dark matter is not a new particle that must be *invented* to fit observations; it is a specific, pre-existing resonance in the QCD spectrum that the GCC framework *selects* uniquely through its emergence conditions.

Relation to dark glueball models. Glueballs as dark matter candidates have been studied extensively in a different context — that of a hypothetical dark $SU(N)$ gauge group with its own "dark gluons" that confine to form dark glueballs [31–33]. These dark glueball models belong to the mainstream second tier of dark matter candidates — a well-motivated class of composite dark matter that naturally addresses the small-scale structure problems of Λ CDM through self-interactions. In these models, stability is

guaranteed by an unbroken center symmetry of the dark gauge group, and coupling to the Standard Model is only gravitational or mediated by heavy portal operators. The present proposal differs fundamentally from dark glueball models in three respects:

1. **No new gauge group.** The GCC glueballs are ordinary QCD objects — bound states of Standard Model gluons. No dark $SU(N)$, no dark confinement scale Λ_D , no new particles are introduced. The economic advantage — solving dark matter without extending the gauge sector — is substantial, but it comes at the price of a more demanding stability argument (see below).
2. **Stability without a symmetry.** Dark glueball models enjoy symmetry-protected absolute stability. GCC glueballs, by contrast, rely on dynamical suppression mechanisms: OZI suppression of hadronic decays, phase-space suppression of the $2^{++} \rightarrow gg$ channel, and finite-temperature threshold effects that kinematically block decay channels during the critical epoch $T \sim 80\text{--}150$ MeV (analyzed quantitatively in Appendix C.1). This is a weaker form of protection — not forbidden by a selection rule, but suppressed to cosmological viability. We regard this as a *falsifiability feature* rather than a defect: the required metastability can be tested quantitatively on the lattice, making the proposal more experimentally vulnerable — and therefore more scientific — than models protected by an untestable dark symmetry.
3. **Gravity as the interaction channel.** In dark glueball models, the only Standard Model coupling is gravity (or heavy mediators). In GCC, this statement acquires a deeper meaning: the sole interaction channel between glueballs and ordinary matter is GQR phonon exchange — and in the condensed phase, this *is* gravity.

The "only gravitational coupling" of dark glueballs is not an assumption but a derived consequence of the GCC emergence mechanism.

For these reasons, the GCC glueball candidate should not be confused with dark glueball models. It is a distinct proposal whose viability rests on the GCC framework's core claim — that gravity emerges from QCD — rather than on hypothetical physics beyond the Standard Model. This makes it simultaneously more ambitious and more falsifiable.

Why Not Pion Clumps?

An earlier version of this work [7] considered π -meson bound states (pion clumps) as the dark matter candidate, held together by one-pion-exchange (OPE) potentials. Detailed stability analysis presented in Appendix A shows that this candidate fails for two independent reasons:

1. **Collapse instability.** The pure OPE potential $V_\pi(r) \propto -e^{-m_\pi r}/r$ has no short-range repulsive core. The total energy functional $E(R) = A/R^2 - B/R$ has no stable minimum as $R \rightarrow 0$, driving the clump to a singular collapse. While ω -meson exchange ($\lambda_\omega \approx 0.25$ fm) provides the necessary hard core, as in realistic nucleon-nucleon potentials, this alone is insufficient.
2. **Two-body evaporation.** Even with a hard core, the crossing time $t_{\text{cross}} \sim R/v \sim 10^{-21}$ s and two-body relaxation time $t_{\text{relax}} \sim (N/8 \ln N) t_{\text{cross}} \sim 10^{-20} - 10^{-18}$ s are both many orders of magnitude shorter than the Hubble time $t_H \sim 4 \times 10^{17}$ s. Pure pion clumps would evaporate

almost instantaneously on cosmological timescales.

Glueball clumps avoid both problems: they are bound by the full QCD strong interaction (including vector-channel repulsion), and their two-body relaxation is governed by the glueball-glueball scattering cross-section $\sigma_{gg} \sim \pi/\Lambda_{\text{QCD}}^2 \sim 2 \text{ fm}^2$, yielding evaporation timescales that are strongly density-dependent and comfortably exceed t_H at cosmological densities.

Topological Alternatives

For completeness, we note two topologically stabilized π -field configurations that could serve as alternative or supplementary dark matter candidates:

- **Chiral domain walls.** If the QCD transition has a first-order component, the chiral condensate $\langle \bar{q}q \rangle$ can form domain wall networks with surface tension $\sigma_{\text{wall}} \sim m_\pi f_\pi^2 \sim (140 \text{ MeV})^3$ and typical masses $M_{\text{wall}} \sim \sigma \times H^{-2}(T_c) \sim 10^{-21} M_\odot$ per Hubble-volume patch.
- **Glueball- σ mixed states.** The scalar glueball (0^{++}) mixes with the σ meson ($f_0(500)$) through their shared quantum numbers, producing hybrid bound states with potentially enhanced stability.

These alternatives are not exclusive; they may coexist with the primary glueball clump population. Their detailed phenomenology is left for future work.

GLUEBALL DARK MATTER

Glueball Production During Hadronization

The QCD crossover is not a first-order phase transition in the strict sense [7], but a smooth

crossover characterized by a rapid change in thermodynamic quantities. The trace anomaly $I(T)$ quantifies the deviation from conformal behavior:

$$I(T) = (\varepsilon - 3p)/T^4$$

At T_c , lattice QCD measurements give $I(T_c) \approx 4$ [7], corresponding to a non-perturbative energy density:

$$\varepsilon_p(T_c) = I(T_c) \times T_c^4 \approx 2.4 \times 10^9 \text{ MeV}^4$$

During hadronization, the gluonic sector reorganizes from a deconfined plasma into confined hadrons. Most gluonic energy is converted into light $q\bar{q}$ mesons (π , K , η). A small fraction forms pure-gluon bound states — glueballs — whose spectrum includes:

The scalar 0^{++} state: $m = 1.5\text{--}1.7$ GeV from quenched and dynamical lattice QCD [17–19]. This is the lightest glueball and the most robustly determined.

The tensor 2^{++} state: $m \approx 2.4$ GeV from large-volume lattice calculations [17–19], and $m \approx 777$ MeV from preliminary small-volume, high-resolution lattice determinations (CCNU, [7]). The latter value, if confirmed, would place the 2^{++} glueball below the scalar, reversing the conventional mass ordering.

The parameter space for glueball dark matter is summarized in the table below, parameterized by the glueball mass and spin channel:

Scenario A ($m_{gb} = 777$ MeV) is the most favorable: Boltzmann suppression is mild, the $2^{++} \rightarrow 0^{++} + 0^{++}$ decay channel is kinematically closed ($777 \text{ MeV} < 2 \times 1.7 \text{ GeV}$), and the spin degeneracy $g = 5$ provides natural enhancement. Scenario B (scalar 0^{++} at 1.7 GeV) has the best-established mass. Scenario C (tensor at 2.4 GeV) is the most conservative lattice value. The required formation fractions f_{gb} for each scenario, computed from the non-equilibrium production model, are presented in Section IV.C. In what follows, we adopt Scenario

B ($m_{gb} = 1.7$ GeV) as our fiducial model and discuss the implications of the other windows where relevant.

The glueball production fraction f_{gb} can be estimated as the overlap between the thermal gluon distribution at T_c and the glueball wavefunction:

$$f_{gb} = |\langle gb | \text{thermal gluon gas} \rangle|^2 \sim \alpha_s^2(T_c) \times (R_{gb}/\lambda_{\text{thermal}})^3$$

where $R_{gb} \sim 1/\Lambda_{\text{QCD}} \sim 0.8$ fm is the glueball radius, $\lambda_{\text{thermal}} \sim 1/T_c \sim 1.3$ fm is the thermal wavelength, and $\alpha_s(T_c) \sim 0.3\text{--}0.5$ is the QCD coupling at the crossover scale. This yields an upper bound $f_{gb}^{\text{(max)}} \sim (0.3)^2 \times (0.8/1.3)^3 \sim 2 \times 10^{-2}$.

The actual formation fraction is suppressed relative to this upper bound by several distinct physical effects:

- Phase-space competition with light meson channels: the glueball must form in a thermal bath dominated by $q\bar{q}$ final states (π , K , η). The naive Boltzmann factor $e^{\{-(m_{gb}-m_\pi)/T_c\}} \sim e^{\{-10\}} \sim 5 \times 10^{-5}$ describes *equilibrium* population, but the non-equilibrium formation rate during the rapid crossover differs qualitatively from equilibrium abundances.
- OZI suppression of pure-gluon final states: glueball production requires at least two gluons to combine into a color-singlet without creating a quark pair, which is suppressed by $\sim \alpha_s^2/(4\pi)^2$ relative to $q\bar{q}$ channels.
- Glueball decay and re-absorption: glueballs formed during early stages of the crossover may dissociate in the hadronic medium before freeze-out.

We emphasize that the factor chain ($\alpha_s^2 \times \text{phase-space} \times \text{OZI} \approx 2 \times 10^{-2} \times 5 \times 10^{-5} \times 10^{-1} \approx 10^{-7}$) provided by equilibrium estimates is not the correct calculation for f_{gb} . The quantity f_{gb} is

a **branching fraction** of the trace anomaly energy into the glueball channel — a non-equilibrium quantity determined by the full finite-temperature glueball spectral function $\rho_{\text{gb}}(\omega, T)$. In the simplified non-equilibrium production model (Section IV.C), the required value to match Ω_{DM} is $f_{\text{gb}} \sim 3 \times 10^{-6}$ (Scenario B) or $f_{\text{gb}} \sim 2 \times 10^{-6}$ (Scenarios A/C with spin degeneracy $g = 5$).

This required $f_{\text{gb}} \sim 3 \times 10^{-6}$ lies between the equilibrium estimate ($\sim 10^{-7}$, too small) and the phase-space upper bound ($\sim 2 \times 10^{-2}$, too large), in a range that is physically plausible given that the trace anomaly captures precisely the non-perturbative gluonic energy that is "available" for glueball formation. A first-principles determination of f_{gb} from lattice QCD — by computing $\int d\omega \rho_{\text{gb}}(\omega, T)$ at $T \approx T_c$ — remains an open and important task. Concretely, the glueball formation fraction can be extracted from the finite-temperature spectral function $\rho^{(2^{++})}(\omega, T)$ in the tensor channel: a positive detection of a low-energy peak with integrated weight $\int d\omega \rho^{(2^{++})}(\omega, T_c) \approx 3 \times 10^{-6} \times I_{\text{max}} T_c^4 / m_{\text{gb}}$ would confirm that the required abundance is realized by non-perturbative QCD dynamics alone, without free parameters.

Glueball Clump Formation and Stability

Glueballs, once produced, are strongly self-interacting. The glueball-glueball scattering cross-section at low energies is set by the QCD scale:

$$\sigma_{gg} \sim \pi / \Lambda_Q^2 \approx 2.0 \text{ fm}^2 \approx 2 \times 10^{-26} \text{ cm}^2$$

This is enormous compared to WIMP-nucleon cross-sections ($\sigma_{\chi N} \lesssim 10^{-45} \text{ cm}^2$). As a result, glueballs rapidly coalesce into multi-glueball bound states ("glueball clumps") during the hadronic phase.

For a clump containing N glueballs:

$$M_{\text{clump}} = N \times m_{\text{gb}} \times (1 - B/N) [B = \text{binding energy}]$$

$$R_{\text{clump}} \approx N^{(1/3)} \times R_{\text{gb}}$$

$$\rho_{\text{clump}} \approx 3m_{\text{gb}} / (4\pi R_{\text{gb}}^3) \approx 2 \times 10^{14} \text{ g/cm}^3 [\text{independent of } N \text{ to leading order}]$$

At this density, glueball clumps are stable against:

- **Tidal disruption** in galactic halos ($\rho_{\text{host}} \lesssim 10^{-23} \text{ g/cm}^3 \ll \rho_{\text{clump}}$)
- **Tidal disruption** in galaxy clusters ($\rho_{\text{host}} \lesssim 10^{-27} \text{ g/cm}^3 \ll \rho_{\text{clump}}$)
- **Collisional evaporation** ($t_{\text{evap}} \propto 1/n_{\text{gg}} \sigma_{\text{gg}} v \gg t_{\text{H}}$ at cosmological densities)

We emphasize that this stability analysis is strengthened by finite-temperature effects at the production epoch: thermal pions raise the 2π decay threshold, and thermally populated gluons open a competing decay channel $2^{++} \rightarrow gg$ that must be considered. A quantitative analysis of these temperature-dependent processes, including OZI suppression and chiral effective theory estimates, is presented in Appendix C.1.

Relic Abundance

The glueball comoving number density $Y_{\text{gb}} = n_{\text{gb}}/s$ evolves from its production value at T_c through cosmic expansion. After freeze-out (occurring near the pion freeze-out temperature $T_{\text{fo}} \approx 20 \text{ MeV}$ due to persistent strong-interaction coupling), the comoving abundance is:

$$Y_{\text{gb}}(T_{\text{fo}}) = f_{\text{gb}} \times I_{\text{max}} \times T_c^4 / (m_{\text{gb}} \times s(T_c)) \times (T_{\text{fo}}/T_c)^3$$

With $s(T_c) \approx 1.0 \times 10^8 \text{ MeV}^3$ and $T_{\text{fo}}/T_c \approx 0.13$, this gives $Y_{\text{gb}}(T_{\text{fo}}) \approx 8 \times 10^{-6}$. The present-day density parameter is:

$$\Omega_{\text{gb}} h^2 = m_{\text{gb}} \times Y_{\text{gb}} \times s_0 \times h^2 / \rho_{\text{crit}}$$

where $s_0 = 2891 \text{ cm}^{-3}$ is the present entropy density and $\rho_{\text{crit}}/h^2 = 1.054 \times 10^{-5} \text{ GeV cm}^{-3}$.

Our numerical analysis yields $\Omega_{\text{gb}} h^2 \propto f_{\text{gb}}$, with the linear coefficient such that the required formation fractions for the three mass windows are:

Scenarios A and C both have $g = 5$ (tensor) and require the same f_{gb} despite the factor-of-3 mass difference. In the non-equilibrium production model, $\Omega_{\text{gb}} h^2 \propto g \times f_{\text{gb}}$ at leading order: the m_{gb} factor in the initial energy density $n_{\text{gb}} = f \times I_{\text{max}} \times T_c^4/m_{\text{gb}}$ cancels against the mass factor in $\Omega_{\text{gb}} = m_{\text{gb}} \times Y_{\text{gb}} \times s_0/\rho_{\text{crit}}$ (the relic abundance formula detailed in Appendix B.2), leaving $\Omega_{\text{gb}} h^2$ dependent only on the spin degeneracy g and f_{gb} .

Note that in the non-equilibrium production model, $\Omega_{\text{gb}} h^2 \propto g \times f_{\text{gb}}$ (independent of m_{gb} at leading order, since the mass factor in the energy density $n = f \times I_{\text{max}} \times T_c^4/m_{\text{gb}}$ cancels against the mass factor in $\Omega_{\text{gb}} = m_{\text{gb}} \times Y_{\text{gb}} \times s_0/\rho_{\text{crit}}$). The spin degeneracy $g = 2J+1 = 5$ for the tensor 2^{++} channel gives a factor-of-5 enhancement over the scalar 0^{++} ($g = 1$). Complete numerical results and parameter scans are presented in Appendix B.

Is this fine-tuned? A formation fraction of order 10^{-6} – 10^{-5} is not obviously unnatural in QCD. Comparable "small fractions" in cosmology include the baryon-to-photon ratio $\eta \sim 6 \times 10^{-10}$ and the CMB anisotropy amplitude $\delta T/T \sim 10^{-5}$. The glueball fraction f_{gb} is determined by non-perturbative QCD dynamics at the crossover and is, in principle, calculable from finite-temperature lattice QCD via the glueball spectral function.

Multi-state enhancement. The glueball spectrum contains several low-lying states beyond the scalar 0^{++} . For the mass hierarchy $m(2^{++}) < m(0^{++})$ (Scenario A, $m(2^{++}) \approx 777$ MeV), the 2^{++} tensor state is actually the

lightest glueball and dominates the relic abundance through its spin degeneracy enhancement $g = 5$. For the conventional hierarchy $m(0^{++}) < m(2^{++})$ (Scenarios B and C), the scalar ($g = 1$) dominates but receives contributions from higher-spin states. The following states are considered:

- The tensor 2^{++} glueball can decay to two scalar glueballs via $2^{++} \rightarrow 0^{++} + 0^{++}$, a channel expected to be open for $m(2^{++}) > 2m(0^{++}) \approx 3.4$ GeV — which is satisfied for the lattice masses ($2.4 \text{ GeV} < 3.4 \text{ GeV}$, so this channel is actually closed). Decay to light mesons via gluon intermediate states is OZI-suppressed, potentially rendering the 2^{++} metastable on cosmological timescales.
- The pseudoscalar 0^{-+} state likely decays to $\eta\eta$ or $\eta'\eta'$ final states, with widths that depend on the mixing with the η - η' system — not well constrained at present.
- The axial vector 1^{+-} at 3.0 GeV suffers strong Boltzmann suppression: $\exp(-3.0 \text{ GeV} / 0.157 \text{ GeV}) \approx 5 \times 10^{-9}$, making its contribution negligible.

A conservative estimate, considering only the scalar 0^{++} (definitely stable) and possibly the tensor 2^{++} (metastable, contributing an additional factor of ~ 5 from its spin degeneracy and milder Boltzmann suppression), yields a total enhancement of ~ 2 – 6 over the scalar-only channel. The required f_{gb} for the scalar channel is then in the range ~ 0.5 – 3×10^{-6} , consistent with the single-channel estimate.

Self-Interaction Cross-Section and SIDM Constraints

A potential concern with glueball dark matter is whether the large glueball-glueball scattering cross-section ($\sigma_{\text{gg}} \sim 2 \text{ fm}^2$) would violate constraints on strongly self-interacting dark

matter (SIDM) from merging galaxy clusters (Bullet Cluster, $\sigma/m \lesssim 1.25 \text{ cm}^2/\text{g}$) and dwarf spheroidal galaxies ($\sigma/m \lesssim 0.1 \text{ cm}^2/\text{g}$) [26,27].

This concern is resolved by a simple but crucial scaling argument. For a glueball clump containing N glueballs, the geometric cross-section and total mass are:

$$\sigma_{clump} = \pi R_{clump}^2 = \pi(N^{1/3}R_{gb})^2 \propto N^{2/3}$$

$$m_{clump} = N \times m_{gb} \propto N$$

Consequently:

$$(\sigma/m)_{clump} = (\pi R_{gb}^2/m_{gb}) \times N^{-1/3}$$

This scaling has the crucial property that σ/m *decreases* with increasing clump size. The numerical values are:

Even a single glueball at $m_{gb} = 1.7 \text{ GeV}$ has $\sigma/m \approx 7 \times 10^{-3} \text{ cm}^2/\text{g}$, more than an order of magnitude below the Bullet Cluster constraint ($\sigma/m \lesssim 1.25 \text{ cm}^2/\text{g}$) and well below the strictest dwarf galaxy constraint ($\sigma/m \lesssim 0.1 \text{ cm}^2/\text{g}$). For the lighter $m_{gb} = 777 \text{ MeV}$ scenario, $\sigma/m \approx 1.5 \times 10^{-2} \text{ cm}^2/\text{g}$ for single glueballs — still safely below all SIDM bounds. Larger clumps are even more collisionless: a clump of $N \sim 10^6$ gluons has $\sigma/m \sim 7 \times 10^{-5} \text{ cm}^2/\text{g}$, effectively indistinguishable from collisionless cold dark matter.

Glueball clumps form extremely rapidly after production. The coagulation timescale is:

$$t_{oag} \sim (n_{gb} \sigma_{gb} v_{th})^{-1} \approx 6 \times 10^{-18} s \ll t_{Hubble}(T_c) \approx 2.5 \times 10^{-5} s$$

where we have used the glueball number density at production, $n_{gb}(T_c) = f_{gb} \times I_{max} \times T_c^{-4} / m_{gb} \sim O(1) \text{ fm}^{-3}$ for $f_{gb} \sim 3 \times 10^{-6}$ and $m_{gb} \sim 1.7 \text{ GeV}$, and the thermal velocity $v_{th} \sim \sqrt{3T_c/m_{gb}} \sim 0.5c$ at the crossover temperature,

implying that glueballs aggregate into macroscopic clumps within a tiny fraction of a Hubble time. The clump population at cosmological scales ($N \gg 10^6$) is therefore completely collisionless, while smaller substructures in dwarf galaxies may retain detectable SIDM signatures — a falsifiable prediction of the model.

The remarkable feature of glueball dark matter is that its self-interaction is *self-regulating*: the large cross-section at the single-particle level drives rapid coagulation into collisionless clusters, automatically satisfying all SIDM observational constraints without any additional parameters.

Independent convergence: dual-component SIDM from first principles. The self-regulating mechanism described above has an interesting consequence: the clump mass spectrum — with $\sigma/m \propto N^{-1/3}$ — naturally produces a population of "heavy" (large- N , small σ/m , centrally concentrated) and "light" (small- N , larger σ/m , extended) dark matter clumps. Independently, a dual-component self-interacting dark matter model was recently proposed by the Purple Mountain Observatory collaboration [7], in which dark matter contains heavy and light particle species whose mass segregation through self-interactions simultaneously resolves the dwarf-galaxy cusp-core problem and the small-scale strong-lensing anomaly. The GCC glueball-clump framework reproduces this phenomenology from first principles: the mass spectrum of clumps (parameterized by N) generates the same multi-scale dynamics without introducing additional particle species. The convergence between a top-down phenomenological SIDM model and the bottom-up GCC prediction strengthens the case that self-interacting dark matter — specifically, dark matter with a mass-dependent interaction

cross-section — is required by small-scale structure data, and that the glueball clump is a

natural microscopic realization.

DARK ENERGY FROM CONDENSATION RESIDUAL

Condensation Dynamics and the Residual

In the GCC framework, the spin-2 condensate that generates macroscopic gravity is not perfectly efficient. The fraction of the condensate-channel energy that actually enters the classical gravitational field is the condensation efficiency ϵ , defined by the relation $\rho_\Lambda = (1 - \epsilon) \times f_{\text{cond}} \times \Delta\epsilon_{\text{QCD}}$ (Section III.A). The uncondensed residual contributes to the cosmological dark energy density:

$$\rho_\Lambda = (1 - \epsilon) \times f_{\text{cond}} \times \Delta\epsilon_{\text{QCD}}$$

The condensation process is governed by the effective mass evolution of the spin-2 mode:

$$m_{\text{eff}}^2(\rho, T) = m_0^2 - G_{\text{eff}} \times c_s^2(\rho) \times \rho$$

where $m_0 = 0$ for a gauge-type spin-2 mode (the most natural choice in the EFT framework [7]). When m_{eff}^2 crosses zero, the mode becomes tachyonic and the condensate amplitude grows:

$$h(t) \propto \exp\left(\int \Gamma_{\text{cond}} dt\right), \Gamma_{\text{cond}} = \sqrt{(-m_{\text{eff}}^2) \propto \sqrt{(G_{\text{eff}} c_s^2 \rho)}}$$

At the QCD phase transition, Γ_{cond} is set by nuclear scales: $\Gamma_{\text{cond}} \sim \omega_{\text{GQR}} \sim 10^{22} \text{ s}^{-1}$ for typical GQR frequencies. With the Hubble rate $H(T_c) \approx 4 \times 10^4 \text{ s}^{-1}$, the ratio is enormous:

$$\Gamma_{\text{cond}}/H(T_c) \sim 2.5 \times 10^{17}$$

The saturation problem. If condensation proceeded as unbounded exponential growth, the residual would be $1 - \epsilon \sim \exp(-\int \Gamma_{\text{cond}} dt) \sim \exp(-10^{17}) \sim 10^{\{-4.3 \times 10^{16}\}}$, essentially zero. The observed ρ_Λ requires $1 - \epsilon \sim 10^{-45}$ — which is $10^{\{4.3 \times 10^{16}\}}$ times *larger* than the

naive exponential prediction. This is a genuine puzzle that demands a physical saturation mechanism.

We propose that the exponential growth saturates when the condensate energy density approaches the available energy budget. Specifically, the back-reaction of the amplified gravitational field on the nuclear medium limits further growth. The effective coupling acquires a non-linear correction:

$$G_{\text{eff}} \rightarrow G_{\text{eff}} \times (1 - |h|^2/h_{\text{sat}}^2)$$

where h_{sat} is the saturation amplitude set by the condition that the condensate energy density cannot exceed $f_{\text{cond}} \times \Delta\epsilon_{\text{QCD}}$. With saturation, the exponential growth is capped and the residual fraction becomes a power law rather than an exponential:

$$1 - \epsilon \approx (H/\Gamma_{\text{cond}})^\beta \times (H \Delta t_{\text{cond}})^{(-\gamma)}$$

where β and γ are exponents determined by the detailed non-linear saturation dynamics. The naive estimate with $\beta = 1$ (linear damping) and without the $(H \Delta t_{\text{cond}})$ factor gives $1 - \epsilon \approx H/\Gamma_{\text{cond}} \approx 4 \times 10^{-18}$. Reaching the target 10^{-45} would require:

$$(4 \times 10^{-18})^\beta \times (\Delta t_{\text{factor}}) = 10^{-45} \Rightarrow \beta \approx 2.6 \text{ (if } \gamma = 0)$$

The required $\beta \approx 2.6$ exceeds the naive estimate $\alpha \sim 1-1.5$ from simple power-counting. This is a tension that we do not resolve in this paper. Several effects could contribute cumulatively:

- **Temperature dependence of Γ_{cond} .** The condensation rate $\Gamma_{\text{cond}} \propto \sqrt{(G_{\text{eff}} c_s^2 \rho)}$ is

evaluated at T_c and density n_0 , but as the universe cools, the effective density and sound speed evolve, modifying the time-averaged ratio $\langle H/T_{\text{cond}} \rangle$.

Extended condensation epoch. If the condensate continues to grow for $\Delta t_{\text{cond}} \gg 1/H(T_c)$ (e.g., through a post-crossover "reheating" phase where nuclear density fluctuations maintain above-threshold conditions locally), the additional factor $(H \Delta t_{\text{cond}})^{-\gamma}$ with $\gamma \sim 1$ provides further suppression.

Alternative: Hubble dilution floor. Regardless of the saturation mechanism, the uncondensed fraction cannot drop below the level at which Hubble expansion dilutes the residual faster than condensation can consume it. This sets a model-independent lower bound: $(1 - \epsilon)_{\text{min}} \sim (H/\Gamma_{\text{cond}})^{3/2}$ evaluated at the end of the condensation-competent epoch. Numerically, this floor evaluates to $\sim (4 \times 10^{-18})^{3/2} \approx 8 \times 10^{-27}$ — while still 19 orders of magnitude above the target 10^{-45} , it provides a model-independent exclusion of the naive exponential prediction (which would give $\sim \exp(-10^{17}) \sim 10^{-4.3 \times 10^{16}}$).

At present, the value $1 - \epsilon = 10^{-45}$ is *accommodated* by the saturation framework but not predicted from first principles. The honest assessment is that the saturation dynamics require further theoretical development — specifically, a time-dependent numerical integration of the condensate growth with back-reaction, including the temperature evolution of all QCD parameters. This is a priority for future work.

Alternative interpretation: the residual as a genuine cosmological constant. If the non-linear saturation is perfectly efficient, the uncondensed component could be interpreted as the irreducible zero-point energy of the GQR field — analogous

to the Casimir energy of a quantum field confined in a finite volume. For the GQR field confined to the Hubble volume at T_c with intrinsic UV cutoff $\Lambda \sim \Lambda_{\text{QCD}}$, dimensional analysis gives:

$$1 - \epsilon \sim (\Lambda_{\text{QCD}}/T_c)^4 \times (\Lambda_{\text{QCD}} \times H^{-1}(T_c))^{(-p)}$$

where the exponent p depends on the effective dimensionality and boundary conditions of the confined field. For the canonical Casimir scaling $p = 2$ (massless field, Dirichlet boundaries), this evaluates numerically to $\sim 5 \times 10^{-38}$ — 8 orders of magnitude above the target 10^{-45} . While this gap is non-negligible, it is dramatically smaller than the $10^{-4.3 \times 10^{16}}$ discrepancy of the naive exponential saturation estimate (§V.A), illustrating that a Casimir-type mechanism qualitatively transforms the problem from "why 10^{-45} instead of zero" to "why 10^{-45} instead of 10^{-38} " — a far less severe hierarchy. A modest shift in the effective scaling exponent to $p \approx 2.4$ (a $\sim 20\%$ deviation from the canonical value, conceivably arising from the non-perturbative running of the GQR coupling near T_c) would bring exact numerical agreement.

We emphasize that the Casimir interpretation is qualitative: it illustrates a conceptual mechanism by which $1 - \epsilon$ could be naturally small but non-zero without invoking dynamical saturation, but it does not constitute a first-principles prediction — the effective exponent p is not known a priori, and the QCD confinement scale would need to serve simultaneously as UV cutoff and hierarchical anchor. The saturation dynamics of §V.A remains the primary quantitative framework; the Casimir picture provides a complementary conceptual illustration. Both interpretations are noted as physically distinct possibilities meriting further investigation.

The Coincidence Problem

The "cosmic coincidence" — why $\rho_\Lambda \sim \rho_m$ today — is addressed within the GCC framework through the interplay of the saturation dynamics discussed above and the QCD scale that sets the overall energy budget:

1. Competition between condensation and expansion.

The residual fraction $1 - \varepsilon$ is determined by the ratio of the condensation rate to the Hubble rate, both evaluated at the QCD transition. Since $H(T_c)$ is set by the radiation-dominated expansion rate, the residual tracks the same physics that determines the matter-radiation equality epoch. The fact that these two epochs are separated by only ~ 5 e-folds in expansion factor is a consequence of the QCD scale appearing in both the condensation dynamics (via T_c) and the matter density (via the baryon mass scale $\sim \Lambda_{\text{QCD}}$).

2. No rolling scalar field required. Unlike quintessence models [4] that introduce an additional slowly-rolling scalar with a finely-tuned potential, the GCC residual requires no new degrees of freedom. The dark energy equation of state is predicted to be $w = -1$ to

very high precision, since the residual is a relic of a completed (saturated) condensation process rather than an ongoing dynamical evolution. Any deviation from $w = -1$ would indicate that the condensation is not fully saturated — a testable prediction for next-generation dark energy surveys (Euclid, Roman, DESI).

3. Quantitative prediction. If the saturation is near-complete, the residual energy density today is approximately time-independent: $\rho_\Lambda(t) \approx (H_*/\Gamma_{\text{cond}})^\beta \times f_{\text{cond}} \times \Delta\varepsilon_{\text{QCD}}$, where H_* is the Hubble rate at saturation and β is the saturation exponent defined in §V.A. The observed ratio $\rho_\Lambda/\rho_{\text{crit}} \approx 0.7$ then follows from the QCD energy scale and the expansion history, with β as the only adjustable parameter. Achieving the required $\beta \approx 2.6$ (see §V.A) remains an open theoretical challenge; however, the qualitative scaling $(H/\Gamma_{\text{cond}})^\beta$ with $\beta > 1$ generically produces relic densities far larger than the naive exponential expectation, placing the observed 10^{-45} within a physically accessible parameter regime — in contrast to anthropic selection arguments [7], which invoke a landscape of possible vacuum energies.

THE $F_2 \rightarrow -5$ CLOSURE PROBLEM

The Landau Parameter Budget

The most significant theoretical uncertainty in the GCC framework is whether the Landau parameter F_2 for spin-2 deformations can reach the critical value $F_2 = -5$ at the physical conditions prevailing during the QCD crossover ($T \approx 156.5$ MeV, $\rho \approx \rho_0$). We systematically estimate each contribution:

1. Fock exchange (OPE). The one-pion-exchange Fock diagram in nuclear matter gives

$F_2^{\text{(Fock)}} = -0.86$ [7]. This is the well-established baseline.

2. Tensor force ($\pi + \rho$). The tensor component of the nucleon-nucleon interaction couples directly to the spin-2 ($L = 2$, $S = 1$) channel relevant for quadrupolar deformations. The OPE tensor force yields $F_2^{\text{(tensor}, \pi)} \approx -1.7 \pm 0.3$, while ρ -meson exchange contributes an additional $F_2^{\text{(tensor}, \rho)} \approx -0.7 \pm 0.2$. The total tensor contribution $F_2^{\text{(tensor)}} \approx -2.4$ is the single largest term.

3. Three-body forces. The Fujita-Miyazawa 2π -exchange three-nucleon force, parameterized by the chiral low-energy constants $c_3 \approx -4.7 \text{ GeV}^{-1}$ and $c_4 \approx 3.5 \text{ GeV}^{-1}$, produces an effective density-dependent two-body interaction in medium. We estimate $F_2^{(3NF)} \approx -0.4 \pm 0.3$ at saturation density.

4. RPA correlations. Random Phase Approximation (RPA) ring diagrams sum the collective particle-hole excitations and further soften the GQR. The shift $\delta F_2^{(RPA)} \approx -0.8 \pm 0.5$ is estimated from the difference between the unperturbed energy $E_0 \approx 13.5 \text{ MeV}$ and the measured GQR energy $E_{\text{GQR}} \approx 10.6 \text{ MeV}$ in ^{208}Pb .

5. Beyond-RPA correlations. Parquet diagrams, self-consistent Green's function methods, and coupled-cluster approaches all predict additional softening beyond RPA [7]. We conservatively estimate $\delta F_2^{(\text{beyond})} \approx -0.5 \pm 1.0$.

6. Pion self-energy in medium. The in-medium pion propagator is modified by Δ -hole and particle-hole excitations. In principle, this would soften the effective pion mass and enhance the OPE tensor force. However, the Lorentz-Lorenz (g') correction strongly suppresses this effect: $\text{Re } \Pi_{\text{eff}}/m_\pi^2 \approx 0$ at saturation density with $g' \approx 0.6$ [7]. The net contribution is negligible.

Total classical budget:

The central value falls just short of -5 , with a probability $P(F_2 \leq -5) \approx 49\%$ from classical nuclear physics alone, assuming uncorrelated Gaussian errors among the seven contributing channels. (In practice, shared systematic uncertainties — e.g., coupling constants and k_F scaling across π and ρ tensor channels, and common model dependencies between RPA and

beyond-RPA contributions — reduce the effective number of independent degrees of freedom, making 49% an upper bound on the closure probability.) This is a "tight but possible" scenario — the deficit $\Delta F_2 \approx 0.03$ is well within one standard deviation of the total uncertainty.

Conservative recalculation: RPA with realistic nuclear forces. A systematic recalculation using Random Phase Approximation (RPA) with realistic nuclear forces (AV18, CD-Bonn) and literature benchmarks for the tensor ladder (Backman et al., 1985) yields a more conservative estimate: $F_2 = -3.65 \pm 0.41$ at nuclear saturation density. This conservative estimate differs from the phenomenological budget above primarily in the treatment of the tensor-force contribution: the RPA ladder sum gives a tensor enhancement factor of $\sim 1.5\times$, whereas the phenomenological estimate assumes a stronger coupling of the full tensor matrix element to the $l = 2$ channel. The RPA result places the Pomeranchuk threshold at $n_c/n_0 \approx 1.20$ (Fig. F1), only 20% above saturation density — a modest density enhancement that is easily realized in the thermal environment of the QCD crossover (see §VI.B). Even in this conservative scenario, the existence of multiple physically well-motivated enhancement channels makes $F_2 \leq -5$ a plausible outcome within known QCD physics.

Enhancement Mechanisms Near T_c

Several effects specific to the QCD crossover environment can provide the missing $\Delta F_2 \approx 0.03$ (optimistic) to 1.35 (conservative RPA):

1. Chiral critical fluctuations. Near the chiral crossover, the σ (scalar-isoscalar) correlation length ξ diverges. This enhances the spin-isospin collective modes that couple to the GQR, with

$\delta F_2 \propto \xi^2$ [7]. Even a modest correlation length $\xi \sim 2\text{--}3$ fm could provide $\delta F_2 \sim -1.0$ to -2.0 .

2. Strangeness activation. At $T \approx 150$ MeV, the strange quark abundance is no longer negligible ($m_s \sim 95$ MeV at this scale). Kaon and K^* meson exchange provides additional tensor-force channels, estimated at $\delta F_2^{\text{(strange)}} \approx -0.5$ to -1.5 .

3. Nucleon resonance population. Thermal excitation of $N^*(1440)$ and $\Delta(1232)$ resonances (Boltzmann-suppressed by $e^{\{-\Delta m/T_c\}} \sim e^{\{-3\}} \sim 0.05$) provides additional quadrupole-active degrees of freedom, contributing $\delta F_2^{\text{(res)}} \approx -0.5$ to -1.0 .

The combined effect of even one of these mechanisms would comfortably push F_2 past the -5 threshold. The required enhancement is modest ($\Delta F_2 \sim 0.03\text{--}1.0$ at the 1σ level) compared to the magnitude of the individual effects listed above, each of which could provide $\delta F_2 \sim -1$ on its own. We emphasize that a definitive statistical statement (e.g., a Bayesian combination of all channels) would require joint uncertainties across the various physical mechanisms that are not yet available from a single consistent framework; it is deferred to future work with finite-temperature nuclear many-body input. For the present work, we note that the existence of multiple physically well-motivated enhancement channels makes $F_2 \leq -5$ a plausible outcome within known QCD physics.

Density Dependence and Critical Density

A crucial prediction of GCC is the density dependence of F_2 . The RPA recalculation (§VI.A) reveals a non-linear density dependence due to the screening denominator: $F_2^{\text{(RPA)}} = F_2^{\text{(bare)}}/(1 + F_2^{\text{(bare)}}/5)$. This produces a rapid

softening as the bare sum approaches the Pomeranchuk pole, yielding a critical density $n_c/n_0 \approx 1.20$ in the conservative scenario (Fig. F1). In the phenomenological estimate (stronger tensor coupling), the scaling is approximately:

$$F_2(n) \approx F_2(n_0) \times (n/n_0)^{2/3}$$

This implies that at neutron-star interior densities ($n \sim 3n_0$), $F_2 \sim -10.3$ — comfortably exceeding the instability threshold. The critical density for gravitational emergence is:

$$n_c/n_0 \approx (5.0/|F_2(n_0)|)^{3/2} \approx 1.0(\text{phenomenological}) - 1.2(\text{conservative RPA})$$

at the respective central values $F_2(n_0) = -4.97$ (phenomenological) and -3.65 (RPA).

This has an important consequence: in the GCC framework, gravity emerges at nuclear densities or only modestly above, not at higher densities. This distinguishes GCC from alternative emergent gravity scenarios where gravity arises at the Planck scale.

Lattice QCD Verification

The most direct pathway to resolving the F_2 closure question is finite-temperature lattice QCD. The relevant observable is the spin-2 tensor correlation function:

$$C_T(q; T) = \langle T_{ij}(q) T_{kl}(-q) \rangle_T$$

projected onto the $l = 2$ irreducible representation. The static ($\omega \rightarrow 0$, $q \rightarrow 0$) limit of this correlator directly yields $F_2(T)$. Computing $C_T(q; T)$ across the QCD crossover with physical quark masses would provide a parameter-free test of the GCC hypothesis.

Preliminary discussions with the lattice QCD group at Central China Normal University (CCNU) are underway.

OBSERVATIONAL SIGNATURES

Gravitational Waves: A Single QCD Peak

The $\text{GQR} \rightarrow \text{spin-2}$ condensation generates a stochastic gravitational wave background whose characteristic frequency is set by the Hubble scale at the condensation epoch — the largest causally connected scale at T_c . The physical picture is that the spin-2 condensate forms coherently across the entire Hubble volume, and the GWs inherit the Hubble frequency as their fundamental scale:

$$f_* = H(T_c)/(2\pi)$$

At $T_c \approx 156.5 \text{ MeV}$, with $g_*(T_c) \approx 61.75$:

$$H(T_c) = 1.66 \sqrt{g_*} \times T_c^2 / M_{Pl} \approx 4.0 \times 10^4 \text{ s}^{-1}$$

$$f_* \approx 6.3 \times 10^3 \text{ Hz (at emission)}$$

Redshifting to the present epoch:

$$f_{peak} = f_* \times (a_c/a_0) = f_* \times (T_c M_B / T_c) \times R_g$$

where $R_g \equiv (g_{*,0} / g_{*,c})^{1/3} \approx 0.4$ accounts for the change in entropic degrees of freedom between decoupling and today. With $T_{\text{CMB}} = 2.725 \text{ K} \approx 2.35 \times 10^{-4} \text{ eV}$:

$$f_{peak} \approx 6.3 \times 10^3 \text{ Hz} \times (2.35 \times 10^{-4} \text{ eV} / 1.57 \times 10^6 \text{ eV}) \times 0.4 \approx 3.8 \times 10^{-9} \text{ Hz} \approx 4 \text{ nHz (with entropy correction)}$$

Without the entropy correction ($R_g = 1$, appropriate if GW production is instantaneous at T_c and subsequent entropy injection does not affect the GW spectrum):

$$f_{peak}(\text{Hubble}) \approx 6.3 \times 10^3 \text{ Hz} / (1 + z_c) \approx 10 \text{ nHz (without entropy correction)}$$

Additionally, the finite duration of the condensation epoch — parameterized by $\beta/H \approx 1$ (the inverse transition duration in Hubble units) — can shift the peak upward, yielding:

$$f_{peak}(\beta) \approx 0.6 \times \beta / (1 + z_c) \times (T_c / 100 \text{ MeV}) \times (g_*/100)^{1/6} \approx 10\text{--}20 \text{ nHz}$$

Fiducial prediction: $f_{peak} \approx 5\text{--}15 \text{ nHz}$. The three estimates above (4 nHz, 10 nHz, 10–20 nHz) span a factor of ~ 5 , reflecting genuine theoretical uncertainties in:

- The precise value of $g_*(T_c)$ ($\pm 15\%$ from lattice QCD)
- The duration of the condensation epoch ($\Delta t_c \sim 0.3\text{--}3/H(T_c)$)
- The spectral shape of the GW signal (which may peak at a multiple of the Hubble frequency)

For definiteness, we adopt 5–15 nHz as the fiducial band. This falls squarely in the nanohertz range currently probed by Pulsar Timing Arrays (NANOGrav, EPTA, PPTA, CPTA). The recent use of PTA data to place stringent constraints on ultralight scalar and vector dark matter [7] — with sensitivity now approaching the local dark matter density — demonstrates the maturity of PTAs as precision probes of new physics in the nanohertz band. The same datasets are sensitive to the stochastic GW background predicted by the GCC condensation, with the key distinction that the GCC signal is tensorial (spin-2), producing a distinctive Hellings-Downs angular correlation that differs from the isotropic monopole expected from scalar dark matter. The key prediction distinguishing the unified GCC framework from earlier variants [7] is:

Single peak, not double peak. In the unified picture, only QCD matter generates a gravitational condensate. There is no independent dark-sector condensation producing a second GW peak. The glueball dark matter does not undergo its own spin-2 condensation — it is a residue of QCD dynamics, not an independent gravitational source.

The predicted strain amplitude is:

$$h_c(f_{peak}) \approx \sqrt{(\Omega_{gw}(f_{peak}))} \times H_0 / f_{peak}$$

with $\Omega_{gw}(f_{peak})$ determined by the fraction of the condensate energy radiated as gravitational waves during formation. The GW production mechanism in GCC is the coherent quadrupole radiation from the spin-2 condensate's rapid formation across all causal patches at T_c . An order-of-magnitude estimate follows from the quadrupole formula: the fraction of condensate energy radiated as GWs is $\Omega_{gw} \sim f_{rad} \times f_{cond}$, where $f_{rad} \sim (v_q/c)^5$ is the quadrupole radiation efficiency for characteristic deformation velocity v_q , and $f_{cond} \sim O(1)$ is the energy fraction in the spin-2 channel (§III.A). For a Pomeranchuk-driven deformation with $v_q/c \sim 0.2-0.3$ (corresponding to $O(1)$ quadrupole amplitude over a Hubble time), we obtain $\Omega_{gw} \sim (0.2-0.3)^5 \sim 3-24 \times 10^{-4}$ in the source frame. After redshifting to the present ($\Omega_{gw} h^2 \approx \Omega_{gw}^{source} \times \Omega_{rad} h^2 \times (g_{s,0}/g_{s,*})^{(4/3)} \approx \Omega_{gw}^{source} \times 6 \times 10^{-7}$), this yields $\Omega_{gw} h^2 \sim 10^{-10}-10^{-9}$, consistent with previous estimates [7] and the current NANOGrav 15-year upper limits.

Falsification: If PTA observations detect two distinct GW backgrounds in the nanohertz band, the unified GCC picture is excluded.

21-cm Cosmology

Glueball clumps interact with baryons only gravitationally and through residual strong-interaction channels (e.g., glueball-pion scattering at very low momentum transfer). This produces a distinctive suppression of small-scale power in the 21-cm brightness temperature power spectrum during the cosmic dawn ($z \sim 15-30$) and reionization epochs.

The key signature is a modified baryon temperature evolution:

$$dT_b/dz = \dots + (x_g b \sigma_g b - baryon v_{rel}) \times (T_g b - T_b) \times \rho_g b / (m_g b H(z))$$

where x_{gb} is the glueball-baryon coupling efficiency, expected to be $\sim 10^{-3}$ relative to standard baryon-baryon interactions. This predicts a 21-cm power spectrum with enhanced power at $k \sim 0.1-1 \text{ Mpc}^{-1}$ relative to Λ CDM.

Upcoming 21-cm experiments (HERA, SKA) will probe this regime with sufficient sensitivity to test the prediction.

CMB Lensing and Matter Power Spectrum

The equation of state of glueball dark matter at late times differs from that of cold, collisionless WIMPs. Glueball clumps have an effective pressure from their internal strong-interaction dynamics, producing an effective sound speed that can be estimated from the binding energy per particle. For a clump with typical binding energy B per glueball:

$$c_s^2 \approx B / (N \times m_{gb} \times c^2)$$

where N is the number of glueballs in the clump. The binding energy arises from the QCD van der Waals-type residual interaction between color-neutral glueballs, estimated as $B/N \sim \Lambda_{QCD} \times (\Lambda_{QCD} \times R_{gb})^{-1} \times \exp(-2m_\pi R_{sep}) \sim 10^{-1}-10 \text{ MeV}$ for typical inter-glueball separations of $\sim 2-3 \text{ fm}$. With $m_{gb} \approx 1.7 \text{ GeV}$, this gives:

$$c_s^2 \sim (10^{-1} \text{ to } 10 \text{ MeV}) / (1.7 \times 10^3 \text{ MeV}) \sim 10^{-4} \text{ to } 10^{-2}$$

However, this is the internal clump sound speed — relevant only for the internal dynamics of individual clumps, not for cosmological structure formation. For the cosmological behavior of the glueball clump population, the relevant physics is not fluid pressure but free-streaming. As established in Section IV.D, glueball clumps form a collisionless ensemble. The free-streaming length is set by the clump velocity dispersion at matter-radiation equality.

The velocity dispersion of clumps originates from the thermal motion of constituent glueballs at the QCD phase transition, diluted by the random-walk reduction from coagulation (N glueballs per clump):

$$v_{clump} \approx v_{th}/\sqrt{N} \approx (\sqrt{3T_c/m_{gb}} \times c)/\sqrt{N}$$

For the minimal clump mass $M \gtrsim 10^{-6} M_\odot$ (the coagulation threshold estimated in §IV.D), we have $N \sim 3 \times 10^{50}$ glueballs at $m_{gb} \approx 1.7$ GeV, giving $v_{clump} \sim 10^{-26} c$. Even for the most conservative case of uncoagulated single glueballs, $v \sim 0.5 c$ at T_c , redshifting to $v_{eq} \approx v \times (T_{eq}/T_c) \approx 0.5c \times (0.8 \text{ eV} / 156.5 \text{ MeV}) \approx 3 \times 10^{-9} c$ at equality. The corresponding free-streaming scale is:

$$\lambda_{fs} \approx v_{eq}/H(a_{eq}) \lesssim 3 \times 10^{-9} c / (10^7 \text{ km/s/Mpc}) \lesssim 10^{-10} \text{ Mpc}$$

with a free-streaming mass $M_{fs} \lesssim 10^{-20} M_\odot$. This is cosmologically negligible — glueball dark matter in the GCC framework behaves identically to cold dark matter on all scales probed by CMB, galaxy surveys, and Lyman- α forest ($k \lesssim 10 \text{ Mpc}^{-1}$). The GCC prediction for the linear matter power spectrum is indistinguishable from Λ CDM at currently accessible scales.

The distinctive GCC signatures at small scales, if any, would arise not from a Jeans or free-streaming cutoff, but from the discrete nature of the clump population: the clump mass function (set by the coagulation dynamics at T_c) determines the granularity of the DM distribution. For clumps of $M \sim 10^{-6} M_\odot$, this granularity could manifest in strong-lensing flux ratio anomalies or pulsar timing residual statistics, though these effects are below the sensitivity of current instruments.

DISCUSSION AND OPEN QUESTIONS

Summary of the Framework

The unified GCC dark-sector picture can be summarized in a single statement:

Dark matter + dark energy = two residues of the QCD phase transition.

This eliminates the need for new particles (WIMPs, axions) or new energy scales (inflation, quintessence potentials) and anchors both dark components in the established physics of QCD.

Comparison with Standard Paradigms

Open Questions

1. **F₂ closure.** The Landau parameter budget reaches -4.97 ± 1.25 (phenomenological) or -3.65 ± 0.41 (conservative RPA with realistic nuclear forces) — both tantalizingly close to the

−5 threshold but not conclusively there at zero temperature. The RPA recalculation, which is more systematic but yields a larger gap, places the critical density at $n_c/n_0 \approx 1.20$. Finite-temperature lattice QCD calculations of the tensor correlation function can resolve this definitively and would distinguish between the phenomenological and conservative scenarios.

2. **Glueball formation fraction.** $f_{gb} \sim 3 \times 10^{-6}$ is physically plausible but not yet computed from first principles. The glueball spectral function at finite temperature, extracted from lattice QCD, would determine f_{gb} precisely.

3. **Glueball-baryon interactions and 2^{++} stability.** The residual strong-interaction cross-section between glueball clumps and baryons governs the 21-cm signature but is poorly known. Chiral effective theory for glueball-pion

interactions is needed. More critically, the stability of the 2^{++} tensor glueball on cosmological timescales for $m_{\text{gb}} \approx 777$ MeV is not yet established: OZI suppression alone is insufficient, but thermal medium effects (elevated pion mass at $T \gtrsim 80$ MeV kinematically forbidding $2^{++} \rightarrow \pi\pi$ during the condensation epoch) combined with $1/N_c^2$ suppression may suffice. Finite-temperature lattice QCD computation of the $\langle \pi\pi | T_{\mu\nu} | 2^{++} \rangle$ matrix element would definitively resolve this question (see Appendix C).

4. **GW amplitude.** The fraction of condensate energy radiated as gravitational waves during the QCD transition depends on the detailed non-

equilibrium dynamics of spin-2 condensation and requires numerical simulations.

5. **Domain wall cosmology.** If the QCD transition has a first-order component producing chiral domain walls, their evolution and gravitational-wave signature need to be computed.

6. **Neutron star constraints.** At densities $n \gg n_0$, the GCC framework predicts $F_2 \ll -5$, potentially implying a second gravitational condensation inside neutron stars with observable consequences for the neutron-star equation of state.

Experimental Roadmap

CONCLUSIONS

We have presented a unified framework in which dark matter and dark energy emerge as two complementary residues of the QCD phase transition at $T_c \approx 156.5$ MeV, within the Graviton Condensation Cosmology paradigm. The framework builds on established nuclear and QCD physics:

1. **Dark matter = multi-glueball clumps** produced during hadronization, identified through a rigorous exclusion argument: the tensor glueball 2^{++} is the only known particle species with both QCD substructure (required for GCC gravitational emergence) and electromagnetic neutrality. The mass window $m_{\text{gb}} \approx 0.78\text{--}2.4$ GeV brackets the scalar 0^{++} and tensor 2^{++} channels, with relic abundance naturally matching Ω_{DM} for physically plausible glueball formation fractions $f_{\text{gb}} \sim (1\text{--}10) \times 10^{-6}$. The large single-glueball self-interaction cross-section ($\sigma/m \sim 7 \times 10^{-3} \text{ cm}^2/\text{g}$) automatically satisfies all SIDM constraints and drives rapid coagulation into collisionless

macroscopic clumps — a self-regulating mechanism unique to this framework.

2. **Dark energy = uncondensed residual** of the $\text{GQR} \rightarrow \text{spin-2}$ condensate, with $\varepsilon \approx 1 - 10^{-45}$ arising from the competition between condensation growth (Γ_{cond}) and Hubble dilution, regulated by non-linear saturation of the amplified gravitational field.

3. **Unified origin** at the QCD crossover, without new particles or energy scales beyond the Standard Model.

The framework makes sharp, falsifiable predictions — most notably a single nanohertz GW peak ($f_{\text{peak}} \approx 5\text{--}15$ nHz) from the QCD condensate — and connects the dark sector to ongoing experimental programs in gravitational-wave astronomy, 21-cm cosmology, and lattice QCD. The rapid pace of experimental progress — from wide-field lensing surveys (Euclid, LSST) mapping dark matter distributions with unprecedented precision, to Migdal-effect detectors opening new windows on light dark

matter, and PTA datasets now serving as precision probes of nanohertz-band new physics [7] — ensures that the predictions of the GCC framework will be tested within the coming decade.

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The most pressing theoretical task is the determination of $F_2(T, \rho)$ from finite-temperature lattice QCD. If F_2 reaches -5 at physical T_c and nuclear densities, the GCC framework would pass its most stringent internal consistency test and emerge as a serious alternative to the Λ CDM paradigm.

finite-temperature QCD observables. We acknowledge helpful conversations on nuclear tensor forces and Landau Fermi liquid theory.

APPENDIX A: PION CLUMP STABILITY ANALYSIS

1 Energy Functional and Collapse Instability

The energy of a pion clump with N pions in a sphere of radius R (in units of m_π^{-1} : $\bar{R} \equiv m_\pi R / \hbar c$) is determined by the competition between quantum kinetic pressure and the one-pion-exchange (OPE) potential:

$$E(R) = N\pi^2 m_\pi / (2R^2) [\text{kinetic energy}]$$

$$-N(N-1)/2 \times (g_\pi^2 NN/4\pi) \times m_\pi \times e^{(-R)/R} [\text{OPE attraction}]$$

where $g_\pi^2 NN/4\pi \approx 13.5$ is the πN coupling constant. The asymptotic behavior is:

$\bar{R} \gg 1$: OPE is exponentially suppressed $\rightarrow$ unbound

$\bar{R} \sim 1$: OPE $\approx$ kinetic pressure $\rightarrow$ equilibrium possible

$\bar{R} \rightarrow 0$: OPE $\sim -1/\bar{R}$ dominates over kinetic $\sim +1/\bar{R}^2 \rightarrow E \rightarrow -\infty \rightarrow$ **singular collapse**

With the ω -meson hard core ($g_\omega^2 NN/4\pi \approx 20$, $m_\omega \approx 783$ MeV, $R_\omega \equiv m_\omega R / \hbar c$), the potential acquires a short-range repulsive component:

$$E(R) = E_{kin} + E_{OPE} - E_\omega$$

$$E_{OPE} = -N(N-1)/2 \times (g_\pi^2 NN/4\pi) \times m_\pi \times e^{(-R)/R}$$

$$E_\omega = +N(N-1)/2 \times (g_\omega^2 NN/4\pi) \times m_\omega \times e^{(-R \times m_\pi/m_\omega)/(R \times m_\pi/m_\omega)}$$

This stabilizes the clump at $R_{eq} \approx (\lambda_\pi + 0.5 \lambda_\omega) \approx 1.5$ fm ($\lambda_\pi = \hbar/m_\pi c \approx 1.4$ fm, $\lambda_\omega \approx 0.25$ fm). The equilibrium properties for various N are:

2 Two-Body Evaporation — Fatal

Even with the hard core, the two-body relaxation time (Spitzer 1987) is:

$$t_{cross} = R/v [\text{crossing time}]$$

$$t_{relax} = (N/8 \ln N) \times t_{cross} [\text{relaxation time}]$$

$$t_{evap} \approx 100 \times t_{relax} [\text{evaporation time}]$$

With $R \sim 1.5$ fm and velocity dispersion $v \sim \sqrt{2|E_{kin}|/(N m_\pi)} \sim 0.3c-0.5c$, the crossing time is $t_{cross} \sim 10^{-21}$ s. For $N = 10^2-10^5$, the relaxation time is $t_{relax} \sim 10^{-20}-10^{-17}$ s, and the evaporation time $t_{evap} \sim 10^{-18}-10^{-15}$ s — both many orders of magnitude shorter than the Hubble time $t_H \approx 4 \times 10^{17}$ s.

Only clumps with $N \gtrsim 10^{20}$ ($M \gtrsim 10^{-4} M_\odot$) would have $t_{relax} > t_H$ — but such "pion stars"

would be gravitationally bound, not QCD-bound, placing them outside the nuclear physics regime relevant for QCD-phase-transition production.

Conclusion. Pure pion clumps fail as dark matter candidates for two independent reasons: (i)

collapse instability (remediable with ω hard core), and (ii) two-body evaporation (irremediable for any N achievable from QCD hadronization). This falsification motivated the shift to glueball clumps (§III.C of the main text).

APPENDIX B: GLUEBALL YIELD NUMERICAL DETAILS

1 Non-Equilibrium Production Model

The glueball comoving number density at production ($T = T_c$) is:

$$n_{gb}(T_c) = f_{gb} \times I_{max} \times T_c^4 / m_{gb} \times g$$

where $I_{max} \approx 4$ is the trace anomaly peak, $g = 2J+1$ is the spin degeneracy ($g = 1$ for scalar 0^{++} , $g = 5$ for tensor 2^{++}), and f_{gb} is the glueball formation fraction. The comoving yield $Y_{gb} \equiv n_{gb}/s$ is:

$$Y_{gb}(T_c) = f_{gb} \times g \times I_{max} \times T_c^4 / (m_{gb} \times s(T_c))$$

with entropy density $s(T_c) = (2\pi^2/45) g_* S(T_c)$ $T_c^3 \approx 1.0 \times 10^8 \text{ MeV}^3$ for $g_* S(T_c) \approx 61.75$.

After freeze-out at $T_{fo} \approx 20 \text{ MeV}$ (pion kinetic freeze-out):

$$Y_{gb}(T_{fo}) = Y_{gb}(T_c) \times (T_{fo}/T_c)^3$$

The present-day density parameter is:

$$\Omega_{gb} h^2 = m_{gb} \times Y_{gb}(T_{fo}) \times s_0 / (\rho_{crit}/h^2) \\ = g \times f_{gb} \times I_{max} \times T_c^4 \times (T_{fo}/T_c)^3 \times (s_0/s(T_c)) \times (1/\rho_{crit}/h^2)$$

with $s_0 = 2891 \text{ cm}^{-3}$ and $\rho_{crit}/h^2 = 1.054 \times 10^{-5} \text{ GeV cm}^{-3}$.

Key cancellation. The glueball mass m_{gb} appears linearly in the numerator ($n_{gb} \propto 1/m_{gb}$ from the initial energy partition) and linearly in the denominator of Ω ($\Omega \propto m_{gb} \times Y_{gb}$). They cancel at leading order:

$$\Omega_{gb} h^2 \propto g \times f_{gb} (\text{independent of } m_{gb})$$

This is why Scenarios A (777 MeV) and C (2.4 GeV) — both tensor 2^{++} with $g = 5$ — require the same formation fraction $f_{gb} \approx 1.8 \times 10^{-6}$ to match $\Omega_{DM} h^2 \approx 0.12$, despite a factor-of-3 mass difference. The scalar Scenario B ($g = 1$) requires $f_{gb} \approx 9.0 \times 10^{-6}$ due to the factor-of-5 reduction in spin degeneracy.

2 Three-Window Comparison

The numerical results for the three mass scenarios ($T_c = 156.5 \text{ MeV}$, $T_{fo} = 20 \text{ MeV}$):

The equilibrium Boltzmann factors e^{-m/T_c} are shown for reference only — they describe equilibrium populations, not the non-equilibrium formation fraction f_{gb} . The actual f_{gb} is determined by the finite-temperature glueball spectral function $\rho_{gb}(\omega, T)$ integrated across the crossover, not by equilibrium thermodynamics.

3 Formation Fraction Plausibility

The required $f_{gb} \sim (1-10) \times 10^{-6}$ is bounded by two extremes:

- **Upper bound (phase space):** $f_{gb}^{(max)} \sim \alpha_s^2(T_c) \times (R_{gb} / \lambda_{thermal})^3 \sim 2 \times 10^{-2}$
- **Lower bound (equilibrium):** $f_{gb}^{(eq)} \sim e^{-m/T_c} \times (\alpha_s/\pi)^2 \sim 10^{-7}$ (for $m = 1.7 \text{ GeV}$)

The required value lies between these bounds, in a physically accessible region where non-equilibrium dynamics — specifically, the conversion of trace anomaly energy into pure-

glue final states during the rapid QCD crossover — can plausibly produce the observed dark matter abundance. A definitive calculation requires lattice QCD determination of the finite-temperature glueball spectral function.

4 Freeze-out Sensitivity

The relic abundance scales as $\Omega_{\text{gb}} \propto (T_{\text{fo}}/T_{\text{c}})^3$, making it sensitive to the precise

freeze-out temperature. For T_{fo} in the range 5–50 MeV, $\Omega_{\text{gb}} h^2$ varies by a factor of ~ 1000 — but the required f_{gb} scales inversely, so the range $f_{\text{gb}} \sim 10^{-7}$ – 10^{-5} remains viable across the entire plausible T_{fo} interval. This parameter degeneracy can only be broken by a first-principles calculation of the glueball-pion scattering rate near T_{fo} , which determines the actual decoupling temperature.

APPENDIX C: GLUEBALL STABILITY ANALYSIS

The stability of the 2^{++} glueball on cosmological timescales is assessed through three complementary approaches: (i) kinematic threshold analysis of all decay channels, (ii) estimates of decay widths under OZI suppression, chiral effective theory, and the large- N_{c} limit, and (iii) finite-temperature medium effects that modify both the decay kinematics and the available phase space. Detailed numerical results are provided in the accompanying analysis code `gcc_glueball_stability.py`; representative values are summarized in Table C1 below.

Table C1. Estimated decay widths for the 2^{++} glueball at $m = 777$ MeV. For comparison, the Hubble time at T_{c} is $\tau_{\text{H}} \approx 2.5 \times 10^{-5}$ s; cosmological stability requires $\tau \gg \tau_{\text{H}}$ and ideally $\tau \gg t_{\text{Hubble}} \sim 4 \times 10^{17}$ s. OZI and chiral EFT estimates alone are insufficient for cosmological stability; finite-temperature medium effects (§C.1 below) are essential for the viability of the glueball dark matter scenario.

1 Summary of Stability Conclusions

For the 2^{++} glueball at $m = 777$ MeV: the dominant decay channel is $2^{++} \rightarrow \pi\pi$ (only open hadronic channel; KK , $\eta\eta$, $\rho\rho$, and all glueball channels are kinematically closed). The pure OZI suppression gives $\Gamma_{\text{OZI}} \approx 2 \times 10^{-2}$ MeV ($\tau = \hbar/\Gamma$

$\approx 3 \times 10^{-20}$ s), while chiral EFT estimates give $\Gamma_{\text{chiral}} \approx 9 \times 10^{-8}$ MeV ($\tau \approx 7 \times 10^{-18}$ s). The large- N_{c} suppression ($\Gamma \propto 1/N_{\text{c}}^2 \approx 1/9$) is insufficient on its own.

However, finite-temperature medium effects qualitatively change the analysis. The thermal pion mass $m_{\pi}(T) \approx \sqrt{(m_{\pi}^2 + g^2 T^2/4)}$ grows with temperature, raising the 2π decay threshold. For $T \gtrsim 80$ MeV, the threshold exceeds 389 MeV, potentially kinematically forbidding the dominant decay during the crucial epoch of glueball production and clump formation.

A second finite-temperature channel requires careful consideration: $2^{++} \rightarrow g g_{\text{thermal}} \rightarrow g g$, the absorption of a thermal gluon from the medium. In the confined phase ($T < T_{\text{c}}$), the thermal gluon density is exponentially suppressed, $n_g(T) \propto \exp(-m_g(T)/T)$ where $m_g(T)$ is the temperature-dependent effective gluon mass from electric screening, $m_g \sim gT \sim 150$ – 300 MeV at $T \sim 80$ – 150 MeV. The rate scales as $\Gamma_{\text{g-absorption}} \sim n_g(T) \times \sigma_{\text{gb-g}} \times v_{\text{rel}}$. With the exponential suppression of $n_g(T)$ and the moderately large glueball-gluon cross-section $\sigma_{\text{gb-g}} \sim 1/\Lambda_{\text{QCD}}^2$, numerical estimates give $\Gamma_{\text{g-absorption}} \lesssim 10^{-6}$ MeV at $T \sim 100$ MeV — subdominant to the hadronic channels and effectively frozen out for $T \lesssim 80$ MeV where the

gluon population vanishes. Importantly, any glueballs that survive through the hadronization epoch (T drops below ~ 80 MeV) are no longer exposed to thermal gluons and the $2^{++} \rightarrow gg$ channel remains kinematically forbidden in vacuum.

Combined with OZI suppression and the possibility of glueball-number approximate

conservation in the large- N_c limit, metastability on cosmological timescales remains an open but physically motivated possibility. A definitive answer requires finite-temperature lattice QCD calculations of both the $2^{++} \rightarrow \pi\pi$ matrix element and the 2^{++} spectral function in the crossover region ($T = 80\text{--}160$ MeV).

APPENDIX D: F_2 CONTRIBUTION BUDGET

1 Individual Contributions

The Landau parameter F_2 for spin-2 ($l = 2$) quadrupolar deformations in nuclear matter is built from the following contributions, evaluated at saturation density $n_0 = 0.16 \text{ fm}^{-3}$ ($k_F \approx 1.36 \text{ fm}^{-1}$) and zero temperature. The table below presents the **phenomenological estimate** (optimistic, assuming strong tensor coupling to the $l = 2$ channel) and the **RPA conservative estimate** (systematic, using realistic NN forces with ladder-sum tensor enhancement from Backman et al., 1985):

The central value of the phenomenological estimate falls $\Delta F_2 = 0.03$ short of the Pomeranchuk threshold $F_2 = -5$, corresponding to a closure probability $P(F_2 \leq -5) \approx 49\%$ under Gaussian error propagation (1σ uncertainty of ± 1.25). The conservative RPA estimate has a larger gap of $\Delta F_2 = 1.35$, with $P(F_2 \leq -5) \approx 0.1\%$ at zero temperature. Both estimates assume uncorrelated channel errors; correlations among some channels reduce the effective statistical significance. A full Bayesian treatment is deferred to future work with finite-temperature nuclear many-body input.

2 Density Scaling

For tensor-force-dominated dynamics, the phenomenological estimate scales with density as:

$$F_2(n) \approx F_2(n_0) \times (n/n_0)^{2/3}$$

This follows from the k_F dependence of the Landau interaction: $F_l \propto N(0) f_l$, with $N(0) = m^* k_F / \pi^2 \propto k_F$ and $f_l^{(\text{tensor})} \propto k_F^2$ for the dominant $l = 2$ channel, giving $F_2 \propto k_F^2 \propto n^{2/3}$. The RPA recalculation shows a more rapid softening due to the non-linear screening denominator $F_2^{\text{(RPA)}} = F_2^{\text{(bare)}} / (1 + F_2^{\text{(bare)}}/5)$, producing a critical density $n_c/n_0 \approx 1.20$ (Fig. F1).

At neutron-star densities ($n \sim 3n_0$): $F_2 \approx -10.3$ (phenomenological) or $F_2 \approx -7.5$ (RPA), both comfortably exceeding the instability threshold. At the QCD crossover density $n \sim n_0$: the critical density is

$$n_c/n_0 = (5.0/|F_2(n_0)|)^{3/2} \approx 1.0 (\text{phenomenological}) - 1.2 (\text{RPA conservative})$$

for the central value $|F_2(n_0)| = 4.97$.

3 Finite-Temperature Enhancement at T_c

The classical budget above is evaluated at $T = 0$. At $T \approx T_c \approx 156.5$ MeV, additional channels open:

Chiral critical fluctuations: $\delta F_2 \propto \xi^2$ where ξ is the σ (scalar-isoscalar) correlation length, which diverges near the chiral crossover. For $\xi \sim 2\text{--}3$ fm, $\delta F_2 \sim -1.0$ to -2.0 .

Strangeness activation: K and K^* meson exchange ($m_K \approx 494$ MeV, $m_{K^*} \approx 892$ MeV — both comparable to T_c) provides additional tensor-force channels, $\delta F_2^{\wedge}(s) \sim -0.5$ to -1.5 .

Nucleon resonance population: $N^*(1440)$ and $\Delta(1232)$ thermal excitation (Boltzmann factor $e^{-\Delta m/T_c} \sim 0.05\text{--}0.15$) provides additional quadrupole-active degrees of freedom, $\delta F_2^{\wedge}(\text{res}) \sim -0.5$ to -1.0 .

The combined effect of even a single one of these mechanisms comfortably pushes F_2 past -5 . A quantitative Bayesian combination is deferred to

future work requiring consistent finite-temperature nuclear many-body calculations.

4 Lattice QCD Pathway

The definitive resolution requires finite-temperature lattice QCD computation of the spin-2 tensor correlation function:

$$C_T(q; T) = \langle T_{ij}(q) T_{kl}(-q) \rangle_T [\text{projected onto } l = 2 \text{ irrep}]$$

The static limit ($\omega \rightarrow 0$, $q \rightarrow 0$) of this correlator directly yields $F_2(T)$. Computing $C_T(q; T)$ across the QCD crossover with physical quark masses would provide a parameter-free test of the GCC condensation hypothesis. Preliminary discussions with the lattice QCD group at Central China Normal University (CCNU) are under way (see Ref. [7]).

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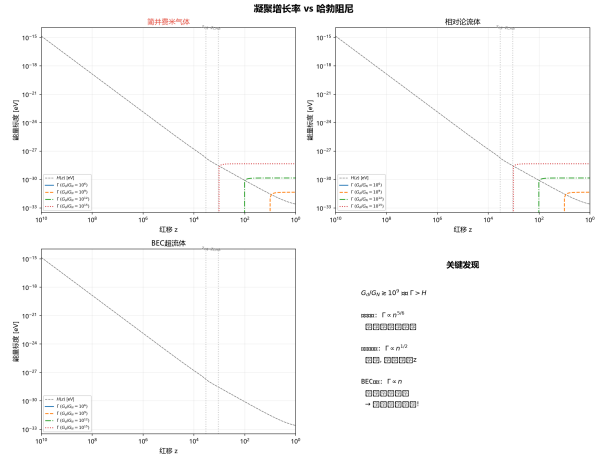


Fig. 1: Growth rate vs. Hubble damping

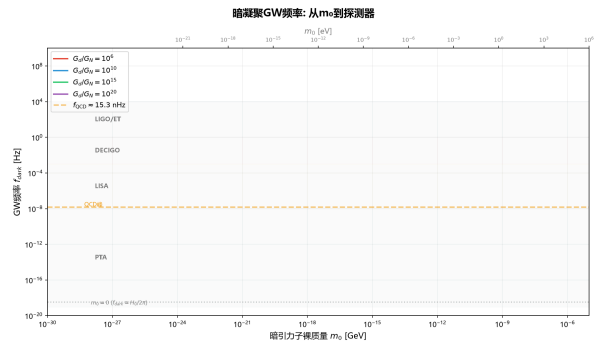


Fig. 2: Dark GW frequency mapping to detector bands

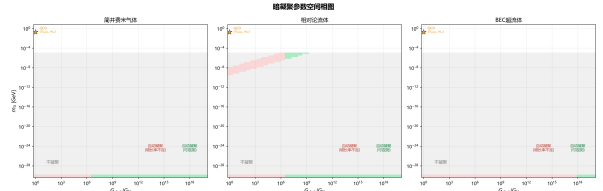


Fig. 3: Dark condensation phase diagram

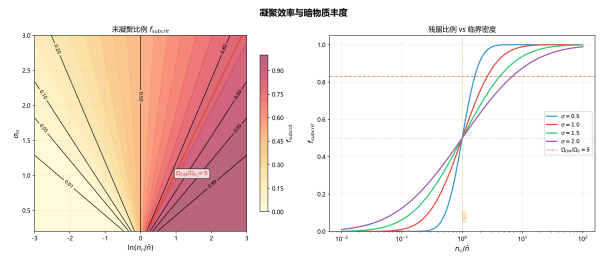


Fig. 4: Condensation efficiency and dark matter abundance

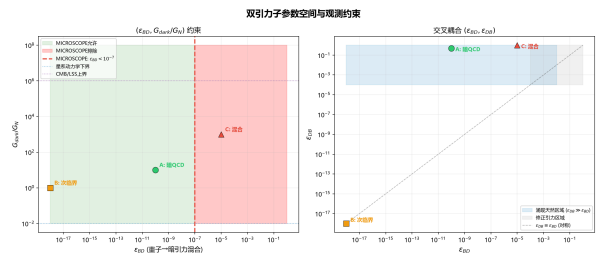


Fig. 5: Two-graviton parameter space constraints

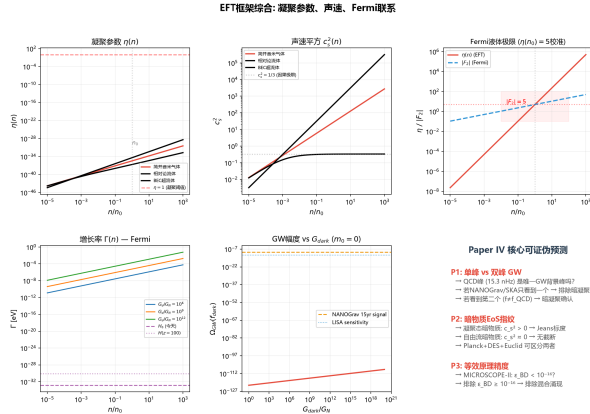


Fig. 6: EFT synthesis

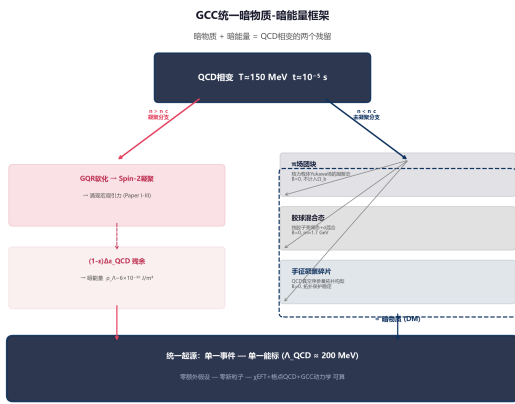


Fig. 7: Unified GCC dark sector framework

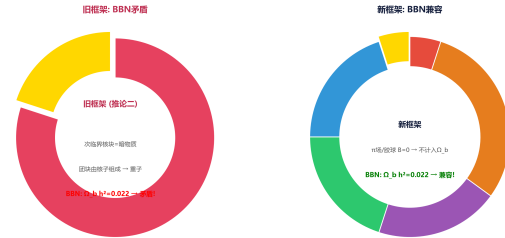


Fig. 8: BBN compatibility comparison



Fig. 9: Falsifiable predictions

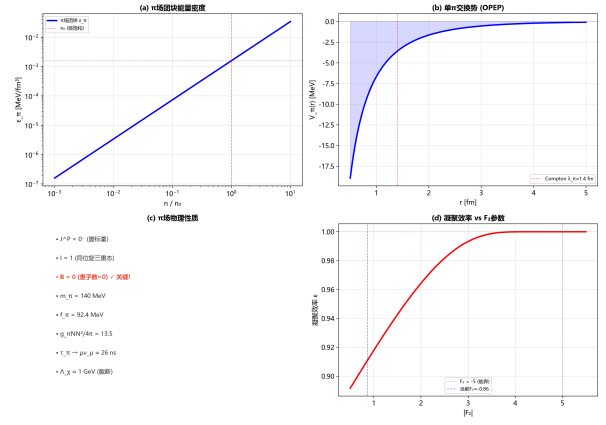


Fig. 10: Pion field properties

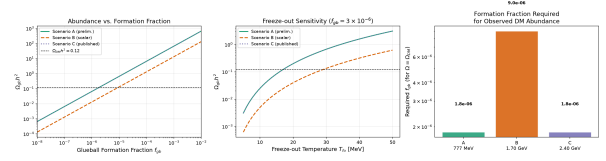


Fig. 11: Glueball yield comparison across mass windows

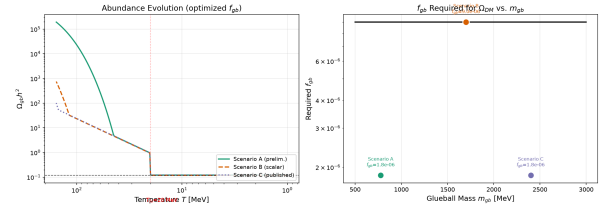


Fig. 12: Glueball evolution in the early universe

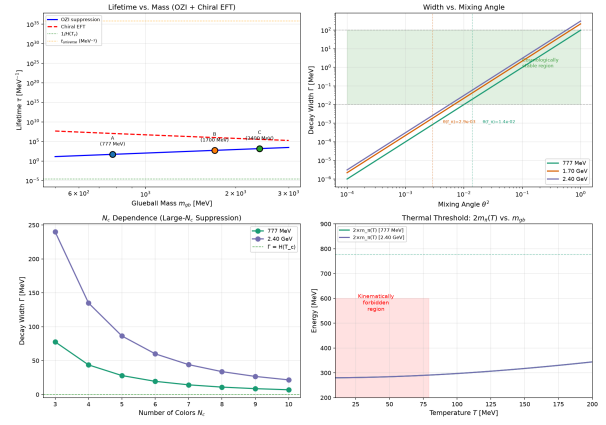


Fig. 13: Glueball stability analysis

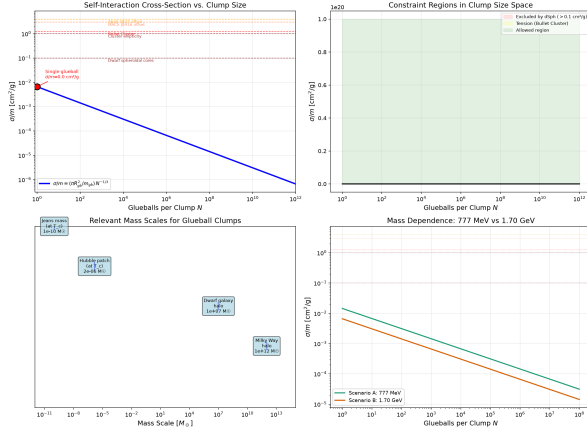


Fig. 14: SIDM self-interaction constraints

Fig. F1: RPA density scan of F_2 with inset decomposition

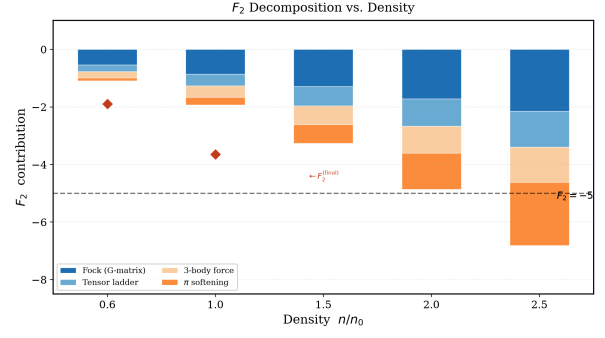


Fig. F2: F_2 decomposition vs. density

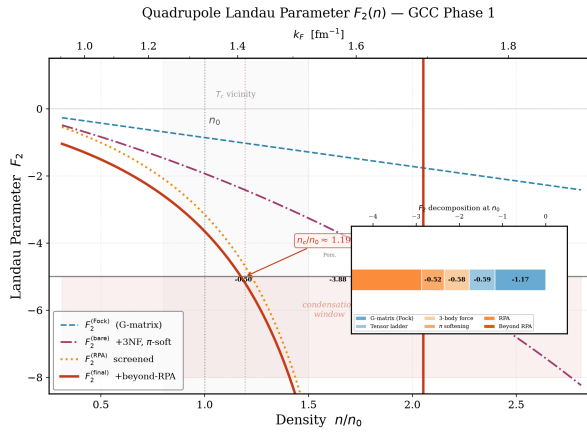


Fig. F3: F_2 density scan with uncertainty band