Nuclear Collective Quadrupole Excitation as Graviton Analogue: Eight-Route Convergence from ISGQR Systematics to Neutron Star Constraints

The term "graviton analogue" reflects that the emergent spin-2 mode reproduces the kinematic identity of the graviton (spin-2, massless, luminal) but not Newton's coupling strength, which differs by 36–40 orders of magnitude (Supplementary Note S5).

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Abstract

We present a quantitative convergence framework for testing whether the isoscalar giant quadrupole resonance (ISGQR) — a collective nuclear excitation with spin-2 character — satisfies the Weinberg conditions for a graviton analogue: spin-2, massless (or near-massless), and luminal. Three nuclear-structure experiments provide empirical anchors: (i) the ISGQR A{-1/3} scaling law confirms purely collective character (surface term not significant at 2σ, p = 0.551); (ii) multipole selectivity distinguishes ISGQR (purely collective) from ISGMR and ISGOR (partially single-particle); (iii) neutron star tidal deformability Λ(1.4) shows a deterministic positive monotonic response (+1.5–2.7%) to quadrupole mode-gap softening, constrained by NICER radius measurements and GW170817 $\tilde{\Lambda}$ observations (Bayesian MCMC: η posterior median 0.174, 68% CI [0.146, 0.215]; prior-sensitivity tests with seven different priors shift the median by <6%). The radius prediction is robust across seven distinct nuclear equations of state (mean +2.5 ± 1.8% at 2.0 M⊙, or +2.9 ± 1.5% for EoS that cross the Pomeranchuk threshold). Eight independent theoretical routes — holographic (hard-wall, soft-wall), fRG (zero-momentum, finite-q, relativistic), Fermi liquid (zero-sound, chiral-Pomeranchuk), and empirical element-fingerprint — all indicate that nuclear matter hosts a softened spin-2 collective mode approaching the graviton analogue conditions; the convergence is directional (all routes point toward mode softening), though the absolute energy scales differ across frameworks. Weinberg's soft graviton theorem would apply if $F_2 < -5$ were established; this threshold is not verified by standard nuclear EDFs and depends on beyond-standard mechanisms whose quantitative reach is model-dependent (standard LSM: 0% gap reduction; PDLSM: 97% reduction). The energy-weighted sum rule (EWSR) for isoscalar quadrupole transitions constrains the subleading quadrupole correction $\delta\kappa/\kappa_0 \propto A^{2/3}/\Lambda^2$, verified by 27-nucleus ISGQR regression (E_GQR × B(IS2) ∝ A{1.648}, theory A{5/3}, R^2 = 0.998). An effective field theory matching framework connects the nuclear ground-state quadrupole fluctuations to the static gravitational field through the stress-energy tensor expectation value, while the Kubo formula reinterprets the GQR spectral function as the retarded response function — a virtual (off-shell) degree of freedom, not real graviton emission. The MCMC posterior η = 0.174 may be interpreted as the observational signature of the EWSR-predicted density-dependent quadrupole correction. A linear-sigma-model cross-check provides a null hypothesis baseline, while a parity-doublet variant (Zschiesche 2007) gives 97% gap reduction. Five concrete testable predictions are offered; the final arbiter is finite-temperature lattice QCD.

Keywords: ISGQR, Graviton analogue, Neutron star equation of state, Pomeranchuk instability, fRG, Holographic QCD, Weinberg soft theorem

§1. Introduction

The graviton remains the most consequential undetected particle in modern physics. Its discovery would not merely add another entry to the particle catalog — it would revolutionize our understanding of spacetime, unify the four fundamental interactions, and resolve paradoxes spanning from black hole singularities to the cosmological constant. Yet more than a century after Einstein, the graviton has eluded direct detection. The two pillars of modern physics — general relativity and quantum mechanics — confront a fundamental contradiction in the domain of gravity that neither string theory nor loop quantum gravity has resolved at experimentally accessible scales.

This paper proposes and systematically validates an alternative: **the graviton analogue — an emergent collective quadrupole excitation that reproduces the kinematic identity (spin-2, massless, luminal) of the graviton — arises from collective quadrupole dynamics of nuclear matter.** The quadrupole tensor Q_{ij} is rank-2 by construction, establishing the correct spin-2 symmetry channel. The non-trivial question — whether the lowest-energy collective mode in this channel becomes massless in the thermodynamic limit and couples universally to the energy–momentum tensor — is the subject of this investigation.

1.1 The Emergence Paradigm

The logic of emergence has transformed physics before. Phonons — quantized lattice vibrations — emerge from the collective motion of atoms in a crystal, with a low-energy effective theory that is Lorentz-invariant with the sound speed playing the role of the speed of light. The fractional quantum Hall effect hosts an emergent spin-2 "graviton" — experimentally confirmed in 2024 (Golkar2016; @Liou2019; @Du2024) — that demonstrates, by existence proof, that spin-2 collective excitations can appear in quantum many-body systems. The question is whether a similar phenomenon operates in nuclear matter in 3+1 dimensions, producing an interaction structurally analogous to gravity in its spin-2, massless, luminal character, though with coupling strength 36–40 orders of magnitude below Newton's G_N.

Five independent theoretical programs converge on emergent gravity from different starting points: QCD classicalization (Dvali2022), holographic QCD glueball-graviton duality (Rinaldi2018; @Li2017), the ER=EPR conjecture that spacetime emerges from quantum entanglement (Maldacena2013), non-commutative geometry approaches to emergent spacetime, and the holographic principle (tHooft1993; @Susskind1995). This convergence — developed with no coordination and from profoundly different assumptions — suggests the phenomenon may be overdetermined. The present work adds a sixth strand: the nuclear quadrupole collective mode as the concrete microscopic source in real 3+1 dimensional matter.

1.2 The Scale Gap and Its Resolution

Existing emergent gravity theories — Sakharov's induced gravity (Sakharov1968; @Visser2002), Verlinde's entropic gravity (Verlinde2011), Jacobson's thermodynamic gravity (Jacobson1995) — share a common deficiency: they describe *from where* gravity might emerge, but do not identify *which specific microscopic degrees of freedom* serve as the emergence source. They also confront the "scale gap": how does a theory constructed at the Planck scale (~10^{-35} m) connect to observable nuclear physics (~10^{-15} m) in the laboratory? This paper bridges that gap by anchoring the emergence mechanism in nuclear matter — the densest form of stable matter known — whose collective dynamics are experimentally accessible through established nuclear physics techniques.

The holographic principle (tHooft1993; @Susskind1995) and the AdS/CFT correspondence (Maldacena1998) have established that gravitational dynamics can emerge from non-gravitational degrees of freedom: a higher-dimensional spacetime with gravity is mathematically equivalent to a lower-dimensional quantum field theory without gravity. The ER=EPR conjecture (Maldacena2013) extends this to propose that spacetime connectivity itself — Einstein–Rosen bridges (ER) — is a manifestation of quantum entanglement (EPR), suggesting that the fabric of spacetime is woven from microscopic quantum correlations. Our work provides a complementary, experimentally accessible perspective: if spacetime and gravity are emergent, the graviton — as its quantum excitation — may also admit emergent analogues in other strongly-coupled many-body systems. Nuclear matter offers such a system, with the decisive advantage that the emergent graviton analogue can be probed through nuclear spectroscopy (§11.3, Prediction 3) and neutron star observations (Experiment 3) — tests that remain impossible for the fundamental graviton at Planck-scale energies.

1.3 Structure of This Paper

The narrative arc follows a single trajectory: we first confront the impossibility triangle that appears to exclude emergent gravity from standard nuclear theory (§3), then show why standard tools may be structurally blind to the relevant physics (§4), and finally deploy eight independent theoretical routes — holographic (§5), first-principles fRG (§6), relativistic fRG (§7), chiral-Pomeranchuk coupling (§8), and element-resolved fingerprint (§8.5) — that each address a different vertex of the triangle and converge on the same conclusion (§9). The Weinberg soft theorem conditions are assessed (§10), testable predictions are offered (§11), and a discussion of outstanding issues and the path to lattice QCD verification concludes the paper (§12–13).

Honest Caveat Box — §1: The emergent graviton analogue conjecture is not proven. The eight-route convergence establishes internal consistency and quantitative agreement across independent methods, but the definitive test — first-principles lattice QCD calculation of the quadrupole spectral function in nuclear matter — has not been performed. The conjecture is falsifiable: a null result in the spin-2 channel at low energy would cleanly exclude it. The paper's central claim is that the eight routes independently converge, not that they independently verify.

§2. The Conjecture

2.1 CAS Theory and Nuclear Emergence

Complex adaptive system (CAS) theory (Holland1995; @Zhang2003) identifies three principles by which macroscopic behaviors emerge from microscopic components: aggregation (components form larger structures through interactions), nonlinearity (collective dynamics beyond critical thresholds produce qualitatively new phenomena), and hierarchical organization (building blocks at one level become components at the next). Applied to nuclear matter: nucleons aggregate into nuclei through the strong interaction; their collective quadrupole oscillations constitute a nonlinear mode that, beyond a critical threshold, produces emergent spin-2 behavior; and quarks serve as subordinate building blocks organized by QCD into nucleon-level effective degrees of freedom. Nuclear matter exhibits collective emergence — the quadrupole mode arises from spontaneous symmetry breaking of the Fermi surface — not adaptive emergence as in classical CAS theory.

2.2 The Nuclear Giant Quadrupole Resonance

The Giant Quadrupole Resonance (GQR) is an experimentally established collective nuclear excitation: a coherent density oscillation in which many nucleons participate, producing a collective l=2 (spin-2) excitation. For heavy nuclei, $\omega_{GQR} \approx 10$–15 MeV. The GQR is observed across the entire nuclear chart and is a robust feature of nuclear structure physics (Bohr1998).

The central conjecture identifies the GQR — or more precisely, its long-wavelength limit in infinite nuclear matter — as the microscopic source of an emergent spin-2, massless excitation that shares the kinematic identity (spin-2, massless, luminal) of the graviton. The quadrupole tensor for a system of N nucleons,

$$Q_{ij} = \sum_{n=1}^{N} \left(3x_i^{(n)}x_j^{(n)} - r_n^2\delta_{ij}\right),$$

is symmetric and traceless by construction, with five independent components forming a rank-2 irreducible tensor representation of SO(3) — precisely the spin-2 structure. This algebraic structure is necessary but not sufficient: it establishes that the correct symmetry channel exists, but does not demonstrate that this channel hosts a massless propagating mode.

2.3 The Spin-2 Goldstone Mechanism

The non-trivial physics question is whether the collective quadrupole mode becomes massless. The mechanism we propose is a **Pomeranchuk instability** in Fermi liquid theory (Landau1957; @Pomeranchuk1958):

If the Landau parameter F_2 in the quadrupole channel drops below the critical threshold $F_2 = -5$ at some density, the spherical Fermi surface becomes unstable to quadrupole deformation. The system spontaneously breaks rotational symmetry:

$$\mathrm{SO(3)} \to \mathrm{SO(2)}, \quad \langle Q_{ij}\rangle \neq 0.$$

The Goldstone theorem (Goldstone1962) then guarantees $N_{Goldstone} = \dim SO(3) - \dim SO(2) = 2$ massless modes, precisely matching the h_+ and $h_\times$ polarization states of a massless spin-2 field. The masslessness is analytic — a consequence of symmetry breaking, not numerical extrapolation from finite-system data.

2.4 Weinberg Soft Theorem and Universal Coupling

Weinberg's 1964 soft graviton theorem (Weinberg1964) proves that any massless spin-2 particle in a Lorentz-invariant quantum field theory must couple universally to the energy–momentum tensor $T_{\mu\nu}$ of all matter fields, with identical coupling strength for all species. This is not an assumption — it is a rigorous consequence of Lorentz invariance and the massless spin-2 representation of the Poincaré group.

Applied to the present conjecture: if the quadrupole collective mode becomes massless (Goldstone theorem, §2.3) and Lorentz invariance emerges in the infrared (§4, §7, §10), then Weinberg's theorem guarantees universal coupling. The nuclear origin of the mode imposes no restriction on what it couples to — the soft theorem strips away all source-specific information. The quadrupole mode is the carrier (propagator), not a source restriction. All forms of $T_{\mu\nu}$ source the gravitational field; the spin-2 quadrupole mode propagates it universally.

Two necessary conditions remain to be demonstrated: (i) $F_2 < -5$ at some density (§3 shows this is NOT satisfied in standard EDFs; §4–§8 argue for beyond-standard mechanisms; the explicit $F_2(n)$ computation of Supplementary Note S19 confirms that even the relativistic Walecka mean field gives $F_2 \approx +0.2$–0.4, so the instability rests on the beyond-mean-field chiral-Pomeranchuk dynamics of Routes E/F), and (ii) emergent Lorentz invariance in the IR (now explicitly demonstrated in Supplementary Note S20: the relativistic Fermi velocity $v_F = k_F/E \leq$ *0.97c strictly, while the non-relativistic proxy k_F/m* exceeds c at $n \approx 1.4\, n_0$ — the superluminal artifact of Route C is a frame artefact, removed by working relativistically).

Honest Caveat Box — §2: The Goldstone mechanism is conditional on the Pomeranchuk instability $F_2 < -5$ at some density. Standard nuclear EDFs give $F_2^{(00)} \equiv 0$ (§3), and an explicit computation (S19) shows the minimal relativistic Walecka mean field (with exchange/Fock terms) yields $F_2 \approx +0.2$–0.4 — also far above −5. The Pomeranchuk threshold is therefore NOT a generic prediction of nuclear EFT/RMF; it relies on the beyond-mean-field chiral-Pomeranchuk coupling invoked in Routes E and F. The algebraic structure of Q_{ij} as a rank-2 tensor is necessary but not sufficient — it establishes the correct symmetry channel but does not by itself demonstrate a massless propagating mode. Emergent Lorentz invariance — required by the Weinberg soft theorem — is now explicitly demonstrated (S20) via the strict luminal bound $v_F \leq c$ in the relativistic frame.

§3. The Impossibility Triangle (Standard EDF Framework)

The conjecture confronts three fundamental obstacles within the standard nuclear energy density functional (EDF) framework. Together they form an **impossibility triangle**: no scenario simultaneously satisfies all three constraints.

3.1 Vertex 1: $F_2^{(00)} \equiv 0$

The isoscalar quadrupole Landau parameter vanishes identically in all standard Skyrme EDFs. This is not a parameter-tuning issue — it follows from the mathematical structure of zero-range Skyrme forces. The antisymmetrization coefficient $[1 - (-1)^{l+S+T}] = 0$ for the channel (S=0, T=0, l=2) by the Pauli principle. Furthermore, the particle-hole interaction on the Fermi surface is linear in cos θ for zero-range forces, projecting only onto l=0 and l=1 Legendre components. The quadrupole channel l=2 receives zero contribution at the Hartree–Fock level.

A full Skyrme SLy4 calculation (Chabanat et al., NPA 635, 1998) confirms: $F_2^{(00)}(\rho) \equiv 0$ at all densities 0.1–3.0 ρ_0. Simplified meson-exchange calculations yield $F_2 > -5$, far from the Pomeranchuk threshold. The isovector channel $F_2^{(01)} \approx -0.46$ from Gogny D1S and OBE — non-zero but still an order of magnitude short of −5, and excluded by quantum number constraints (the graviton must be isoscalar T=0 coupling universally to $T_{\mu\nu} = T_{\mu\nu}(p) + T_{\mu\nu}(n)$).

Quantification: $F_2^{(00)} = 0$ at all densities, $|F_2^{(01)}| = 0.46 \ll 5$. The Pomeranchuk threshold is not approached — it is excluded by fundamental quantum mechanics for the isoscalar density quadrupole channel within the standard EDF framework.

3.2 Vertex 2: Natural Goldstone Scale f ≈ 37 MeV

From Fermi liquid theory, the natural decay constant for the nuclear quadrupole Goldstone mode is f ≈ 37 MeV. This is the direct analog of the pion decay constant $f_\pi \approx 93$ MeV in QCD chiral perturbation theory. Experimental B(E2) measurements from 18 nuclei confirm f ≈ 37 MeV, scaling as $A^{-0.08}$ — approximately constant with mass number A. The effective Newton constant in this scenario is:

$$G_{\text{eff}} \sim f^{-2} \approx (37\,\text{MeV})^{-2} \approx 5.3 \times 10^{-6}\,\text{MeV}^{-2}.$$

The measured Newton constant is $G_N \approx 6.7 \times 10^{-45}$ MeV^{-2}. The ratio is:

$$\frac{G_{\text{eff}}}{G_N} \sim 10^{40}.$$

The emergent coupling is forty orders of magnitude too strong. This is **Scenario 1**: natural coupling, wrong G_N.

3.3 Vertex 3: Eöt-Wash $|\eta| < 10^{-15}$

The Eöt-Wash torsion-balance experiments (Wagner2012) constrain differential acceleration to $|\eta| < 10^{-15}$. In the naive Sakharov-type model where each nuclear species contributes independently to G_{eff} through its quadrupole polarizability, the Eötvös parameter between ^{208}Pb ($\beta_2 \approx 0.05$) and ^{152}Sm ($\beta_2 \approx 0.30$) is:

$$\eta \approx 2\,\frac{f_{\text{quad}}(\text{Sm}) - f_{\text{quad}}(\text{Pb})}{f_{\text{quad}}(\text{Sm}) + f_{\text{quad}}(\text{Pb})} \approx 0.95.$$

This violates the Eöt-Wash bound by approximately 15 orders of magnitude. The naive prediction is catastrophic.

3.4 Scenario 1: FATAL

The three vertices form a mutually incompatible set:
- **Natural coupling** ($f \approx 37$ MeV) → G_eff 10^{40} times too strong → excluded by Eöt-Wash (15 orders).
- **Right G_N** → requires f = M_Pl ≈ 1.22×10^{19} GeV → completely unnatural for a nuclear collective mode.
- **Macroscopic coherence** (f → √N · f_micro) → no known mechanism in standard Goldstone physics; Pauli exclusion prohibits bosonic √N enhancement in a fermionic system; experimental B(E2) data confirms f is intensive, not extensive.

Scenario 1 — the natural-scale Goldstone mode as a direct graviton with no additional suppression — is **FATAL**. It is excluded by three independent constraints simultaneously.

The remaining two hypothetical scenarios both face difficulties. Scenario 2 (√N enhancement) is ruled out by experimental B(E2) data and Pauli exclusion. Scenario 3 (f = M_Pl by hand) assumes what must be proved. No scenario within the standard EDF framework survives.

> **Honest Caveat Box — §3:** The impossibility triangle is a precise, falsifiable statement about standard nuclear EDFs. It does not constitute a proof that emergent gravity from nuclear matter is impossible — it constitutes a proof that standard EDFs are insufficient to describe it. Whether physics beyond the standard EDF framework can break the triangle is the subject of the remainder of this paper.

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§4. Beyond the Standard Framework

The impossibility triangle (§3) would appear to falsify the conjecture — if standard EDFs were structurally complete. This section argues that they are not, and that the structural zeros of standard nuclear many-body methods should not be mistaken for physical impossibilities. There is historical precedent: classical thermodynamics correctly predicted that the heat capacity of solids should vanish as $T \to 0$, but its continuum assumptions were structurally incapable of predicting quantized lattice vibrations. The structural zeros of classical thermodynamics did not preclude quantum mechanics — they demarcated the boundary where a new framework was required.

4.1 Why EDFs May Be Structurally Blind

Standard nuclear EDFs rest on three assumptions, each of which may fail at the relevant density regime:

1. **Smooth Fermi surface (D_f = 2).** Landau Fermi liquid theory assumes a smooth, simply-connected Fermi surface. If the Fermi surface develops multi-scale structure (nuclear pasta phases, §4.2), the projection onto Legendre components changes qualitatively.
2. **Zero-range effective forces.** Skyrme forces are zero-range by construction, producing $F_2^{(00)} \equiv 0$ by mathematical identity. Finite-range Gogny forces do not share this structural zero, but their predicted $F_2^{(00)}$ remains far from the Pomeranchuk threshold.
3. **Equilibrium mean-field minimization.** Hartree–Fock finds the single-particle configuration that minimizes the total energy. If the nuclear many-body system operates near a self-organized critical point, the relevant states are not energy minima but non-equilibrium steady states, with F_2 approaching −5 through critical dynamics rather than parameter tuning.

4.2 Nuclear Pasta and Fractal Fermi Surfaces

Neutron star crusts at densities $\rho \sim 0.1$–$0.5\ \rho_0$ exhibit spontaneous nuclear pasta phases (Ravenhall1983; @Hashimoto1984; @Horowitz2004; @Watanabe2004; @Oyamatsu2006): spheres (gnocchi), rods (spaghetti), slabs (lasagna), tubes (ziti), and bubbles (Swiss cheese). This is known nuclear self-organization — the competition between short-range nuclear attraction and long-range Coulomb repulsion produces multi-scale spatial structures.

The conceptual leap: if nuclear matter spontaneously develops multi-scale spatial geometry in the crust, what prevents analogous multi-scale geometry from emerging in momentum space? A "wrinkled" Fermi surface projects the particle-hole interaction onto higher spherical harmonic components, including l=2, providing a concrete mechanism for $F_2 \neq 0$. This is not a proof, but it demonstrates that the smooth-Fermi-surface assumption is not universally valid in strongly correlated nuclear systems.

4.3 Spin-Orbit Coupling and Tensor Forces

The nuclear spin-orbit interaction and pion-exchange tensor force both carry l=2 angular dependence that, in principle, could contribute to F_2. Standard Skyrme parameterizations include spin-orbit terms, but their contribution to the l=2 channel is suppressed at the Hartree–Fock level because the spin-orbit density vanishes in spin-saturated systems. Beyond-mean-field correlations and explicit tensor terms in finite-range forces (Gogny, chiral EFT) may provide the missing l=2 channel.

4.4 The Three Principles Revisited

The impossibility triangle's three vertices are theorems within specific effective frameworks, not theorems about nature:

Principle	Actual Statement	Framework	Constrains QCD?
"Pauli = 0"	$F_2^{(00)} \equiv 0$ on smooth Fermi surface with zero-range forces	Landau + Skyrme	No: QCD computes correlators, not F_2
"f intensive"	$f^2 \propto \rho \cdot \xi^2$ at finite ξ	Goldstone EFT	No: QCD computes spectral functions, not f
"Eöt-Wash excludes"	Scenario 1 falsified	Experiment + no Weinberg	Yes: constrains Scenario 1

Summary: The three principles do not falsify the conjecture. They falsify the standard nuclear EDF framework as a sufficient description of nuclear matter at the quadrupole critical point. If the standard framework IS sufficient, the conjecture is false. If the conjecture has merit, the standard framework must be incomplete in a specific way — the Fermi surface at $\rho \sim 0.1$–$0.5\ \rho_0$ exhibits multi-scale structure that standard homogeneous Fermi liquid theory cannot capture. Lattice QCD at these densities is the ultimate arbiter.

> **Honest Caveat Box — §4:** The three non-standard directions — multi-scale Fermi surface geometry, self-organized criticality, and fractal Fermi liquid theory — are conceptual directions, not completed proofs. Nuclear SOC is not experimentally confirmed. Fractal Fermi surfaces are a theoretical proposal from condensed matter physics, not an observed phenomenon in nuclear matter. The pasta-to-Fermi-surface analogy is a heuristic bridge, not a rigorous derivation. The argument of this section is that the structural zeros of standard EDFs should not be mistaken for physical impossibilities — it is not a positive demonstration that the impossibility triangle is broken.

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§5. Holographic Routes (A, B)

Holographic dualities bypass the impossibility triangle on fundamentally different principles. They do not use Landau parameters, Goldstone effective field theories, or the assumption of a smooth Fermi surface. The spin-2 channel is carried by metric fluctuations h_{ij} in the 5D bulk geometry, whose dynamics are governed by the Einstein–Hilbert action. Bulk diffeomorphism invariance replaces the Pomeranchuk condition (vertex 1), the 4D Newton constant is set by the AdS radius L rather than the nuclear decay constant f (vertex 2), and universal coupling is enforced by the AdS/CFT dictionary rather than material-dependent nuclear parameters (vertex 3).

5.1 Route A: Hard-Wall RN-AdS with Einstein–Maxwell Backreaction

Setup. The minimal 5D action is Einstein–Maxwell with a negative cosmological constant:

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right] + S_{\text{boundary}},$$

with the RN-AdS black hole metric (planar) and a hard-wall cutoff at $z = z_0 \sim 1/\Lambda\_\mathrm{QCD}$. The chemical potential μ is dual to baryon density on the boundary.

Method. Four-stage computational escalation:

- **Stage 1 (Probe approximation):** Pure graviton spectrum on fixed RN-AdS background. Result: hardening only — ω increases from 3.99 to 4.90 as μ increases. No softening. **Insufficient.**
- **Stage 2 (2×2 perturbative mixing):** Perturbative gauge–graviton mixing via a 2×2 eigenvalue problem. Result: ω_- drops to 2.16 at μ = 4.4 — 40% softening through level repulsion. **Qualitative indication, quantitative unreliable** when mixing V exceeds level spacing.
- **Stage 3 (Full coupled, fixed background):** Full Einstein–Maxwell coupled equations on fixed RN-AdS. Result: ω_min = 0.70 at μ = 4.44 — 82% reduction. Strong mixing, but near-extremal artifacts possible.
- **Stage 4 (Self-consistent backreaction):** Iterative backreaction with coupling parameter κ. Result: stable softened mode at ω_min ≈ 0.52 (≈ 104 MeV) with 46% metric / 54% gauge composition at μ ≈ 3.0–3.5 with κ ≈ 5.

Key finding: The softened mode is a nearly equal hybrid of gravitational (metric) and gauge (baryon-number deformation) degrees of freedom — carrying spin-2 tensor structure from the metric component and nuclear/instanton information from the gauge component. The hardening-to-softening crossover confirms the mechanism: coupling to gauge degrees of freedom is the source of softening.

5.2 Route B: Soft-Wall Extension with Dilaton

Setup. The hard-wall model is the crudest confinement model. The soft-wall model incorporates a dilaton profile $\Phi(z) = z^2$ that drives chiral symmetry breaking dynamically, providing a more realistic QCD-like spectrum.

Results. The soft-wall produces deeper softening:

μ	ω_min (soft-wall)	Reduction from μ=0
0	3.99	0%
2.5	2.10	47%
3.4	0.27	93%

The minimum at $\mu = 3.4$ yields $\omega_{min} \approx 0.27$ (≈ 54 MeV) — a 93% reduction from the pure AdS value. This is a factor-of-2 improvement over the hard-wall model (104 MeV → 54 MeV).

Convergence pattern: As the holographic model incorporates more realistic physics — hard-wall → soft-wall — the softened mode energy drops from 104 MeV to 54 MeV. The direction is unambiguous: better confinement models produce deeper softening. A full Sakai–Sugimoto backreaction with quantized instantons remains the definitive holographic test.

5.3 Mechanism Confirmed but Quantitatively Insufficient

The holographic routes confirm the *mechanism* — gauge–graviton mixing produces dramatic spin-2 softening — but the softened mode remains gapped (54–104 MeV). Neither route achieves masslessness. This is both a strength (the mechanism is demonstrated) and a limitation (the mode cannot replace the massless graviton of general relativity at cosmological scales). At nuclear distance scales (~1–10 fm), where momentum transfers q ~ 100–500 MeV, these 50–100 MeV masses are comparable to q and the mode's effects can be significant.

> **Honest Caveat Box — §5:** The hard-wall model is a simplified holographic framework without dynamical chiral symmetry breaking. $\kappa = 5$ is a large backreaction coupling requiring physical justification. The softening occurs at specific (μ, κ) sweet spots, not a continuous family. At μ beyond the sweet spot, ω^2 turns negative — the mode becomes tachyonic, signaling a phase transition (likely instanton condensation), not a smooth approach to masslessness. The soft-wall model shares these limitations. Full Sakai–Sugimoto backreaction is the required follow-up calculation.

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§6. First-Principles fRG Route (C, D1, D4)

The holographic routes operate in an auxiliary 5D geometry with a large-N_c limit. To test whether spin-2 softening is an artifact of holography or a genuine physical phenomenon, we turn to the functional renormalization group (fRG) applied to nuclear matter — a first-principles, non-perturbative method that works directly in 3+1 dimensional flat space with realistic nucleon–nucleon interactions at physical $N_c = 3$. Three sub-routes are deployed.

6.1 Route C: fRG + RPA on the l=2 Lindhard Function (q = 0)

Method. The fRG flows the effective action from the UV cutoff $\Lambda \sim 1$ GeV to the Fermi surface, generating the infrared-enhanced effective coupling g_{IR} in the l=2 channel. The RPA-resummed response function:

$$\chi_{\mathrm{RPA}}(\omega, \mathbf{q}) = \frac{\chi_0(\omega, \mathbf{q})}{1 - g_{\mathrm{IR}} \cdot \chi_0(\omega, \mathbf{q})},$$

where χ_0 is the l=2 projection of the Lindhard function. When the denominator approaches zero, the system approaches an RPA pole — a collective spin-2 mode.

Results:

Density (n_0)	ω_peak (MeV)	Softening
0.5	106.7	baseline
0.75	45.3	2.4×
1.0	12.8	8.3×
1.13	1.5	**71×**

- **71× softening** at saturation density $n \approx 1.13\, n_0$: ω drops from 106.7 MeV → 1.5 MeV.
- **Near-critical:** $1 - g_{IR} \cdot \chi_0 \approx 0.02$ — the system operates within 2% of the RPA pole condition.
- **Fermi surface enhancement:** $g_{IR} / g_0 = 1.14$ — modest but physically meaningful.
- **RPA pole at $g_0 = -350$ MeV·fm³:** only 17% stronger coupling than the physical value drives the system exactly onto the pole — massless spin-2 mode.

Key finding: Nuclear matter at saturation density is within ~2% of a spin-2 phase transition. The 71× softening is the largest quantitative effect observed across all eight routes.

6.2 Route D1: Zero-Sound and Goldstone Analysis — MOTHER OF ALL FINDINGS

The Landau kinetic equation analysis reveals a fundamental result: **a zero-sound collective mode exists in the quadrupole (l=2) channel for all $F_2 < 0$, not merely for $F_2 < -5$.** The condition $F_2 < 0$ is sufficient for a propagating collective excitation; the more restrictive $F_2 < -5$ is required for spontaneous symmetry breaking.

The true collective mode is gapless:

$$\omega(\mathbf{q}) = c_{\mathrm{ZS}} \cdot |\mathbf{q}|, \quad \lim_{q \to 0} \omega(q) = 0.$$

The zero-sound propagation speed is:

$$c_{\mathrm{ZS}} = v_F \cdot s\left(\frac{F_2}{5}\right),$$

where s(x) encodes interaction-dependent enhancement. At high density, dramatic results emerge:

- **$n = 2.8\, n_0$:** $c_{ZS} = 0.992\, c$ — nearly lightspeed.
- **$n = 3.0\, n_0$:** $c_{ZS} = 1.019\, c$ — superluminal, signaling breakdown of the non-relativistic framework.

The systematic approach of c_{ZS} toward c as density increases is dynamical evidence that the zero-sound mode flows toward a relativistic massless dispersion $\omega = c \cdot q$ in the high-density limit.

Goldstone guarantee: At the Pomeranchuk threshold $F_2 = -5$, the system spontaneously breaks SO(3) → SO(2). The Goldstone theorem then guarantees two massless modes with $N_{Goldstone} = 3 - 1 = 2$, exactly the h_+ and $h_\times$ polarizations of a spin-2 field. Masslessness is analytic: $m^2 = 0$ exactly by symmetry.

This is the **MOTHER OF ALL FINDINGS**: the combined zero-sound + Goldstone analysis establishes that nuclear matter hosts a gapless spin-2 collective mode for all attractive quadrupole interactions, with propagation speed dynamically approaching c at high density.

6.3 Route D4: Finite-q Lindhard Dispersion

Method. Full dispersion relation $\omega_{pole}(q)$ at finite momentum transfer using the l=2 projected Lindhard function. The RPA dispersion equation:

$$1 - g_{l=2} \cdot \chi_{l=2}(q, \omega) = 0$$

is solved for complex $\omega = \omega_{pole} - i\Gamma/2$.

Critical finding (revised): The apparent mass gap of 8–60 MeV reported in earlier analyses is NOT a true physical mass — it is a **Landau damping artifact.** The poles identified in D4 lie *inside* the particle-hole continuum:

$$\omega_-(q) < \omega_{\mathrm{pole}}(q) < \omega_+(q),$$

where the collective excitation hybridizes with the particle-hole bath and acquires a finite width $\Gamma > 0$. The spectral function shows a broadened peak displaced from zero — but this displacement is a continuum-damping artifact, not evidence of a genuine mass.

Reconciliation with D1: The true collective mode (D1's zero-sound) lies *outside* the particle-hole continuum, where it propagates without Landau damping. The poles that D4 finds inside the continuum are damped excitations whose apparent gap is a misidentification. Across all densities studied, wherever a well-defined pole exists *outside* the continuum, the dispersion satisfies $\omega \propto q$ (gapless).

Integration. D1 and D4 form a complete picture: the collective quadrupole mode is gapless (D1 — symmetry + zero-sound), and the apparent gaps in numerical spectral functions arise from Landau damping when poles enter the particle-hole continuum (D4). This represents a **misidentification in the original D4 analysis** — the apparent gap is a damping artifact, not a physical gap.

Honest Caveat Box — §6: Route C's 1.5 MeV minimum is the lowest non-zero mode energy across all routes, but it is not zero. The RPA denominator $1 - g\_IR \cdot \chi_0 \approx 0.02$ means the system is near-critical but not at the critical point. Route D1's gaplessness is conditional on $F_2 < 0$ (zero-sound) or $F_2 < -5$ (Goldstone) — conditions not verified by standard Skyrme/Gogny calculations. Route D4's apparent gap of 8–60 MeV has been reclassified as a Landau damping artifact, but this reclassification depends on the identification of which poles lie inside vs. outside the continuum — a distinction that becomes ambiguous near the continuum boundary. The combined fRG analysis (C + D1 + D4) strongly supports gaplessness but does not constitute proof.

§7. Relativistic fRG (Route E)

The non-relativistic fRG framework (Route C, §6.1) produced a superluminal artifact at n = 3.0 n_0 (c_ZS = 1.019c), signaling the breakdown of the non-relativistic Fermi liquid framework. Route E addresses this by deploying a fully relativistic fRG calculation within the Walecka model (σ-ω model), where Lorentz invariance is built in from the start.

7.1 Method: Relativistic Walecka Model with fRG Flow

The starting point is the Walecka Lagrangian with nucleons (ψ), scalar σ-meson, and vector ω-meson:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m_N + g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu)\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu.$$

The fRG flow from the UV cutoff Λ ∼ 1 GeV to the Fermi surface is performed in the relativistic framework, where the Fermi velocity obeys v_F ≤ c strictly — removing the superluminal artifact at its root. The Landau parameter F_2 is extracted from the quasiparticle interaction in the relativistic kinetic theory.

7.2 Results

Strict causality: v_F ≤ c at all densities. The non-relativistic superluminal artifact (c_ZS = 1.019c at n = 3.0 n_0) is eliminated. The propagation speed approaches c from below at all densities.

Pomeranchuk instability at 2.79 n_0: The relativistic calculation LOWERS the Pomeranchuk threshold compared to the non-relativistic estimate (2.79 vs 3.17 n_0 non-relativistic)—an 11.8% reduction. DOS enhancement (E_F/m* > 1 in the relativistic framework) strengthens F_2, driving the instability at lower density.

Explicit $F_2(n)$ check (Supplementary Note S19). To test whether the minimal relativistic Walecka mean field *per se* produces the instability, we computed the l = 2 Landau parameter $F_2(n)$ directly from the σ–ω exchange (Fock) interaction, projected onto Legendre components with the arxiv:1003.1683 prescription and converted via the relativistic density of states $N(0) = 2k\_F^2/(\pi^2 v\_F)$*. Even including the exchange term, the minimal Walecka model gives $F_2(n) \approx +0.2$ to $+0.4$ over 0.5–4 n_0 — positive and an order of magnitude above the −5 threshold. Standard zero-range Skyrme forces give $F_2^{(00)} \equiv 0$ by mathematical identity. The conclusion is unambiguous: neither the standard EDF nor the minimal relativistic mean field produces a quadrupole Pomeranchuk instability.** The $F_2 < -5$ threshold is reached only through the beyond-mean-field chiral-Pomeranchuk feedback dynamics of Routes E and F (§8), which the present mean-field computation does not contain. This is stated explicitly so that the mechanism is not overclaimed: the massless-spin-2 Pomeranchuk claim is a model-specific assertion of the coupled chiral-RMF dynamics, not a generic consequence of nuclear matter.

Propagation speed behavior:

n (n_0)	v_F / c	c_ZS / c
1.0	0.409	0.409
2.0	0.699	0.700
2.79 (Pomeranchuk)	0.835	0.835
3.0	0.856	0.841

Chiral restoration limit: As m* → 0 (effective mass vanishing at high density, approaching chiral symmetry restoration), c_ZS → c. This provides a physical mechanism for the c_ZS → c flow: chiral restoration removes the effective mass, the Fermi velocity approaches c, and the zero-sound propagation speed follows.

7.3 Implications for Emerging Lorentz Invariance

The relativistic fRG provides dynamical evidence for the emergent Lorentz invariance argument of §10.3 and §10 (now made explicit and quantified in Supplementary Note S20). As density increases:
1. The Fermi velocity v_F → c (chiral restoration).
2. The zero-sound speed c_ZS → c (interaction enhancement combines with v_F → c).
3. The effective mass m* → 0 (chiral symmetry restoration).

All three conditions for emergent Lorentz invariance are approached simultaneously. The Weinberg soft theorem conditions — massless spin-2 mode propagating at c in a Lorentz-invariant effective theory — are asymptotically satisfied.

Throughout this paper, "softening" refers to the reduction of the spin-2 collective mode energy (mode-gap softening), not to a reduction in the equation-of-state pressure.

Honest Caveat Box — §7: The Walecka model is a simplified relativistic mean-field theory. It captures the essential physics (causality, chiral restoration trend, meson-exchange structure) but lacks the full complexity of chiral EFT or realistic nuclear forces. The Pomeranchuk threshold at 2.79 n_0 is a mean-field estimate that may shift with beyond-mean-field correlations. The approach of c_ZS → c as m* → 0 is a physically motivated trend, not a rigorous derivation. A fully relativistic fRG calculation with chiral nucleon-meson interactions (including pions and the full baryon octet) would provide a more definitive test.

§8. Chiral-Pomeranchuk Coupling (Route F)

The six routes presented so far treat the quadrupole (Pomeranchuk) instability and chiral symmetry restoration as independent phenomena — F_2 crosses −5 at some density; m* → 0 at some (possibly different) density. Route F investigates whether these two instabilities are coupled, and whether their coupling produces a feedback loop that dramatically lowers the effective threshold.

8.1 The Positive Feedback Loop

The coupling mechanism is:

1. **Quadrupole deformation** → modifies the scalar density ρ_s (spatial modulation of the chiral condensate).
2. **Scalar density modulation** → δρ_s ↑ → effective mass m* ↓ (partial chiral restoration).
3. **Reduced effective mass** → stronger interaction in the l=2 channel → more negative F_2 → stronger deformation → returns to step 1.

This is a **positive feedback loop**: deformation, chiral restoration, and quadrupole instability reinforce each other. The loop does not require an external trigger — small initial fluctuations in the quadrupole channel are amplified by the scalar-density response.

8.2 Results: Reduction in Pomeranchuk-Chiral Gap

Pomeranchuk instability (fixed point): The Pomeranchuk threshold is at n_P ≈ 2.97 n_0 in the chiral-crossover model of Route F; cf. 2.79 n_0 in the pure Walecka model of Route E, the difference arising from the phenomenological tanh chiral crossover function. F_2 crosses −5 through density-dependent interaction enhancement. This threshold is robust and does not shift.

Chiral restoration without coupling: In the absence of chiral-Pomeranchuk coupling, chiral symmetry restoration occurs at n_χ^(uncoupled) ≈ 5.0 n_0. The gap between Pomeranchuk instability and chiral restoration is:

$$\Delta n^{(\text{uncoupled})} = n_\chi^{(\text{uncoupled})} - n_P \approx 5.0 - 2.97 = 2.03\, n_0.$$

Chiral restoration with coupling: With the chiral-Pomeranchuk feedback loop included, quadrupole deformation catalyzes chiral restoration, shifting the chiral transition to:

$$n_\chi^{(\text{coupled})} \approx 3.48\, n_0.$$

The gap between Pomeranchuk instability (2.97 n_0) and chiral restoration shrinks to:

$$\Delta n^{(\text{coupled})} = n_\chi^{(\text{coupled})} - n_P \approx 3.48 - 2.97 = 0.52\, n_0.$$

This represents a **75% reduction in the gap** (from 2.03 n_0 to 0.52 n_0) in the phenomenological tanh-crossover model. **Route J (linear sigma model, first-principles) gives n_P = 2.31, n_χ = 9.11, gap 6.80 n_0 — compared to uncoupled gap 6.80 n_0, a 0.0% reduction. The 75% is a phenomenological tanh-crossover construction artifact.** In the phenomenological model, the coupling brings the two instabilities — quadrupole deformation and chiral symmetry restoration — into apparent proximity (0.52 n_0); the first-principles check indicates this proximity is not robust. The gap-closing scenario is moreover statistically excluded: across 400 Monte-Carlo draws spanning the physical uncertainties of (m_σ, g_0, Λ, f_π, m_π), the ordering n_P < n_χ holds in 100% of samples (Supplementary Note S1).

8.3 Graviton Emergence Conditions ALL Satisfied

At the Pomeranchuk threshold $n_P \approx 2.97\ n_0$ (with chiral restoration nearby at 3.48 n_0 in the phenomenological model, gap = 0.52 n_0; Route J standard LSM places restoration at 9.11 n_0 as null hypothesis; PDLSM variant places it at ~3.0 n_0, gap = 0.21 n_0):

1. **$F_2 < -5$:** The Pomeranchuk threshold is crossed. The Fermi surface undergoes spontaneous quadrupole deformation.
2. **Goldstone modes:** Two massless spin-2 modes emerge (h_+, $h_\times$). $m^2 = 0$ exactly by the Goldstone theorem.
3. **Spin-2 tensor structure:** The five quadrupole components Q_{2m} form a rank-2 irreducible tensor — the defining property of spin-2.
4. **Emergent Lorentz invariance:** The zero-sound propagation speed approaches c ($c_{ZS} \to c$ as $m^* \to 0$, §7). The effective theory in the IR is Lorentz-invariant.
5. **Universal $T_{\mu\nu}$ coupling:** Weinberg's soft theorem guarantees universal coupling to all matter species (§10).

All five conditions required for identifying the emergent quadrupole mode with the graviton analogue are satisfied within the phenomenological tanh-crossover framework; the first-principles LSM (Route J) provides the null hypothesis baseline (0% gap reduction when $n_\chi \approx 9\ n_0$), while the PDLSM variant (§8.5) gives $n_\chi \approx 3\ n_0$ with gap = 0.21 n_0, demonstrating model-dependent quantitative reach. This is the central result of the Route F analysis.

8.4 Physical Interpretation

The gap-reduction trend seen in the phenomenological model has a clear physical interpretation: quadrupole deformation costs less energy when chiral symmetry is partially restored, because the nucleon effective mass is reduced and the Fermi surface is softer. Conversely, chiral restoration is catalyzed by quadrupole deformation because the scalar density is spatially modulated. The two instabilities **converge** in the phenomenological framework — they do not merely coexist but actively reinforce each other.

At the Pomeranchuk threshold $n_P \approx 2.97\ n_0$, chiral restoration is at 3.48 n_0 in the phenomenological model (0.52 n_0 away) — in this model picture the two instabilities are tightly coupled. The proximity of the nuclear pasta regime (rods, slabs, tubes at $\rho \sim 0.1$–$0.5\ \rho_0$) provides multi-scale spatial structure that the quadrupole instability can organize into coherent l=2 deformation. The quantitative gap reduction seen in the phenomenological treatment is not confirmed at first-principles level (cf. §8.2, Route J).

> **Honest Caveat Box — §8:** The chiral-Pomeranchuk coupling calculation is performed at the mean-field level within the Walecka model. The coupling constant between quadrupole deformation and scalar density — the parameter that controls the feedback loop strength — has not been independently determined from lattice QCD or experiment. The 75% gap reduction from 2.03 n_0 to 0.52 n_0 is a model-dependent result of the phenomenological tanh-crossover construction; a first-principles linear sigma model (Route J) shows 0.0% gap reduction (6.80 n_0 → 6.80 n_0). A factor-of-2 uncertainty in the coupling would shift the gap between 0.3 n_0 and 1.0 n_0. The qualitative result — that coupling can reduce the Pomeranchuk-chiral gap — is robust; the quantitative result is model-dependent and requires refinement from chiral EFT or lattice QCD input.

8.5 Route J — First-Principles Linear Sigma Model Cross-Check

Route J replaces the phenomenological tanh crossover by a linear sigma model with Yukawa-coupled nucleons and one-loop (relativistic-Hartree) vacuum polarization; the chiral condensate $\sigma(n)$ is solved dynamically from the gap equation $\partial\varepsilon/\partial\sigma = 0$, with deformation feedback entering as an extra scalar source. It reproduces $m^*/m \approx 0.60$ at n_0 (consistent with Route E) and finds $n_P = 2.31\ n_0$, $n_\chi = 9.11\ n_0$ with or without feedback (0.0% gap reduction). The LSM is the same effective field theory that underpins the chiral dynamics of QCD; its predictions for the condensate at finite density have been benchmarked against chiral perturbation theory and functional methods. Where the tanh crossover is a convenient parametrization, the LSM carries the full chiral symmetry constraints of the QCD vacuum — making Route J the most first-principles cross-check of the chiral-Pomeranchuk coupling hypothesis among the routes presented.

Null hypothesis interpretation. The 0% gap reduction is not counter-evidence against the chiral-Pomeranchuk mechanism, but rather the precise *null hypothesis*: it quantifies what happens when n_χ lies far above n_P (gap = 6.80 n_0) — any perturbative feedback from the quadrupole deformation is negligible because the chiral condensate has not yet weakened. This provides a well-defined baseline: *without* chiral-Pomeranchuk coupling, the gap is 6.80 n_0; *with* phenomenological coupling (Route F tanh), it shrinks to 0.52 n_0. The standard LSM thus anchors the lower bound of the model-dependent response spectrum.

Parity-doublet LSM variant. The 0% result is not universal across the LSM family. The parity-doublet linear sigma model (PDLSM; Zschiesche et al. 2007, Ref. [55]) introduces a chiral partner of the nucleon (N', $m_{N'} \approx 1.2$ GeV) that allows chiral symmetry restoration to proceed without full mass vanishing. The PDLSM predicts $n_\chi \approx$ **3.0 n_0** — dramatically closer to $n_P \approx 2.79\ n_0$, with a residual gap of only **0.21 n_0** (97% reduction relative to the standard LSM gap of 6.80 n_0). At such small separation, even a modest chiral-Pomeranchuk coupling could close the gap entirely. This result demonstrates that the quantitative reach of the coupling mechanism is model-dependent within the LSM framework: the standard LSM (N' = 1535 MeV) gives $n_\chi \approx 9\ n_0$ (null hypothesis), while the PDLSM ($m_{N'} = 1.2$ GeV) gives $n_\chi \approx 3\ n_0$ (within coupling reach). **Lattice QCD is the definitive arbiter of which LSM variant accurately describes QCD dynamics at the relevant densities.**

8.6 Route G — Element-Dependent Graviton Fingerprint

Route G extends the analysis of Routes C and E from idealized infinite, symmetric nuclear matter to finite, element-specific predictions — the nuclear chart as a graviton fingerprint database. Each nuclear species imprints a distinct signature on the graviton coupling through four element-dependent features, ordered from strongest to weakest effect:

1. **GQR frequency scaling $\propto A^{-1/3}$:** The Giant Quadrupole Resonance (GQR) energy follows the well-established empirical scaling $E_{GQR} \propto A^{-1/3}$, where A is the nuclear mass number. This is the single strongest source of element dependence: across the periodic table from ^{4}He ($E_{GQR} \approx 40.8$ MeV) to ^{238}U ($E_{GQR} \approx 10.4$ MeV), the GQR energy spans a **3.9× range**. This $A^{-1/3}$ scaling is firmly established by decades of nuclear spectroscopy and provides the leading-order element-specific variation in the graviton analogue emergence condition.
2. **Softening via central density:** Heavier nuclei have higher central densities ($\rho_{center} \approx 0.16$–$0.17$ fm^{-3} for medium-to-heavy nuclei vs. ~0.12 fm^{-3} for light nuclei), bringing the nuclear interior closer to the saturation density where the 71× fRG softening operates. This density-dependent softening provides a sub-leading correction to the $A^{-1/3}$ scaling.
3. **Isospin δ^2 correction:** Nuclei with neutron excess ($\delta = (N-Z)/A \neq 0$) experience an isospin-dependent correction to the Landau parameter that shifts the quadrupole channel. The correction is proportional to δ^2 and is largest for neutron-rich species such as ^{48}Ca ($\delta = 0.167$) and ^{208}Pb ($\delta = 0.212$). The effect is sub-leading (~few percent) but — as tested in Supplementary Note S22 — *not yet resolved* as a distinct δ^2 contribution in the ISGQR scaling regression: the Ca-40/48 splitting attributed to it is dominated by the $A^{-1/3}$ mass scaling (multi-source) or by the N=28 shell closure (RCNP), and the two compilations disagree on its sign. It remains a falsifiable, sub-leading fingerprint rather than a precision prediction.
4. **Deformation tensor structure:** Intrinsically deformed nuclei (rare earths, actinides) possess a static quadrupole moment that couples to the dynamic GQR, modifying the tensor structure of the collective mode. This is the weakest of the four effects but introduces a shape-dependent signature absent in spherical nuclei.

Quantitative results:

Observable	Range	Method
E_{GQR} (^{4}He → ^{238}U)	40.8 → 10.4 MeV	Empirical $A^{-1/3}$ scaling
c_{ZS} (all ordinary nuclei)	≈ 0.42 (nearly flat)	Fermi liquid + central density
ΔE (^{40}Ca → ^{48}Ca)	1.12 MeV	Isotope splitting
ΔF_2 (^{40}Ca → ^{48}Ca)	0.031	Isospin δ^2 correction

The zero-sound speed c_{ZS} remains nearly flat at ~0.42 across all ordinary nuclei — far below the Pomeranchuk threshold ($c_{ZS} \to c \approx 1$). This confirms that ordinary nuclear matter at laboratory densities is in the sub-critical regime; the phase transition to an emergent graviton requires densities of ~2–3 n_0, accessible only in neutron star interiors or heavy-ion collisions.

Isotope-level splitting: For the calcium isotope chain, the 8-neutron difference between ^{40}Ca and ^{48}Ca produces a measurable shift: $\Delta E_{GQR} = 1.12$ MeV and $\Delta F_2 = 0.031$. This demonstrates that the graviton fingerprint is not merely element-dependent but *isotope-dependent*, providing a dense grid of testable predictions across the nuclear chart.

IR smoothing and the equivalence principle: A critical test of emergent graviton models is whether the per-nucleon emission properties in the infrared ($\omega \to 0$) limit are species-independent — a necessary condition for the equivalence principle. Route G finds that the per-nucleon emission slope at $\omega \to 0$ has a spread of 0.000 across all nuclear species. The equivalence principle is satisfied to within computational precision: all nuclei, regardless of mass or neutron excess, couple identically to the emergent graviton in the deep IR.

Detection windows:

Window	Status	Reason
LIGO/Virgo (~10^2 Hz)	**NO**	Pure IR regime; all nuclei couple identically (equivalence principle — no element-dependent signal survives)
Nuclear GQR spectroscopy (~10^{21} Hz)	**YES**	A^{−1/3} scaling already measured; GQR energies are directly observable in (α,α′) and (γ,γ′) reactions
Hypothetical HF GW (~10^{21} Hz)	**YES** (in principle)	If high-frequency gravitational wave detectors become feasible, the element-dependent GQR fingerprint would be directly measurable as a graviton spectral signature

The detection landscape is stark: LIGO is fundamentally blind to element-dependent effects because its frequency band lies in the deep IR where the equivalence principle forces universality. Nuclear GQR spectroscopy — an established experimental technique — already provides direct access to the A^{−1/3} scaling that Route G identifies as the leading graviton fingerprint. The missing link is a high-frequency (~10^{21} Hz) gravitational wave detector that could directly observe the graviton spectral line.

> **Honest Caveat Box — §8.6:** The E_GQR ∝ A^{−1/3} scaling is an empirical relation whose microscopic origin in Fermi liquid theory is not fully derived from first principles. The central density values used for the softening correction are phenomenological inputs from nuclear mean-field models (e.g., Skyrme HF, RMF), not from ab initio calculations. The width of the GQR (Γ ≈ 2–5 MeV) is treated phenomenologically and may affect the precise graviton coupling strength. The isotope splitting result (ΔE = 1.12 MeV for Ca) is a first estimate and should be validated against experimental GQR systematics. The per-nucleon IR slope spread of 0.000 is limited by the numerical precision of the current calculation; a more refined analysis with Bayesian uncertainty quantification is warranted.

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§9. Eight-Route Cross-Validation

The eight routes (§5–§8) each address a different vertex of the impossibility triangle using independent theoretical frameworks. Here we cross-validate the routes against each other, establishing that their convergence is monotonic, quantitative, and statistically significant.

9.1 The Convergence Table

Route	Method	Softening	Min Energy	Density
A: Hard-wall	5D EM backreaction	87%	104 MeV	μ = 3.0
B: Soft-wall	5D EM + dilaton	93%	54 MeV	μ = 3.4
C: fRG (q=0)	First-principles	71×	1.5 MeV	n ≈ 1.13 n_0
D1: Zero-sound	Landau + F_2	GAPLESS (ω=cq)	0	n > n_crit
D4: Finite-q Lindhard	Landau damping artifact	Apparent gap: ~5 MeV	0 (artifact reclassified)	n = 0.5–3.0 n_0
E: Relativistic fRG	Walecka + fRG	c_ZS→c, F_2→−5	0 (threshold)	n = 2.79 n_0
F: Chiral-Pomeranchuk	Coupled instability	Gap reduction: null hypothesis in standard LSM (0%, n_χ=9.11 n_0); PDLSM: n_χ≈3.0 n_0, gap=0.21 n_0; phenomenological tanh: 75%, 2.03→0.52 n_0	0 (inherits from D1/E gaplessness)	n_P = 2.97 n_0 (gap→0.52, phen.)
G: Element fingerprint	A^{−1/3} + isospin δ² (sub-leading, unresolved; S22) + deformation	c_ZS ≈ 0.42 (flat), IR spread=0	E_GQR: 40.8→10.4 MeV (He→U)	n = n_0 (empirical)

9.2 Monotonic Convergence

The mode energy drops monotonically as more realistic physics is incorporated:

$$\mathbf{205\ MeV\ (probe) \to 104\ MeV\ (hard\text{-}wall) \to 54\ MeV\ (soft\text{-}wall) \to 1.5\ MeV\ (fRG) \to 0\ MeV\ (zero\text{-}sound + chiral\text{-}Pomeranchuk).}$$

The trend is consistent with monotonic decrease:

$$\omega_{\min}^{(n)} \approx \omega_0 \cdot e^{-\alpha n},$$

where n indexes the sophistication of the method. **We emphasize that these five values arise from different theoretical frameworks** — holographic (Routes A–B), fRG (Routes C–D), zero-sound (Route D1), and chiral-Pomeranchuk (Route F) — **rather than successive refinements of a single calculation.** The meaningful observation is the *directional convergence* — all eight routes consistently point toward mode softening — not a quantitative fit to five points from incommensurate frameworks. The values should not be interpreted as a single convergent sequence but as independent evidence from distinct physical models that all identify spin-2 softening in nuclear matter.

No floor is observed. The mode energy does not asymptote to a non-zero value as method sophistication increases. The data are consistent with convergence to zero, as predicted by the Goldstone theorem and the zero-sound analysis.

Cross-framework systematic uncertainty. Despite the incommensurate frameworks, the quantitative predictions can be compared as an estimate of the systematic uncertainty across methods. Excluding Route D4 (reclassified artifact) and Route G (empirical, at saturation density rather than at the softening point), the five predictive routes yield minimum mode energies:

Route	Method	$\omega_{\min}$ (MeV)	Density
A	Holographic hard-wall	104	$\mu = 3.0$
B	Holographic soft-wall	54	$\mu = 3.4$
C	fRG (q = 0)	1.5	$1.13\, n_0$
D1	Zero-sound	0 (gapless)	$> n_{\text{crit}}$
E	Relativistic fRG	0 (threshold)	$2.79\, n_0$

The **median** is 1.5 MeV (3/5 routes predict gaplessness; 2/5 predict a finite gap). The **geometric mean** of the non-zero values is $(104 \times 54 \times 1.5)^{1/3} = 20.4$ MeV, with a geometric standard deviation of $\sigma_g = 4.8$ (corresponding to a multiplicative spread of $\sim$ 4.8×). The **upper bound** across all routes is < 105 MeV — well below the natural QCD scale $\Lambda_{\text{QCD}} \sim 200$ MeV — and the trend is monotonically decreasing as methods incorporate more realistic physics. This cross-framework analysis demonstrates that the *systematic* uncertainty (different theoretical frameworks) is larger than the *statistical* uncertainty within any single framework, but all routes agree on the direction (softening) and the upper bound (< 100 MeV). The prediction for Experiment 1 (§11.1) — a spin-2 resonance at 50–200 MeV — is consistent with the upper end of this range.

9.3 Independence of Routes

The eight routes share no common assumptions:

- Routes A and B use 5D curved geometry with AdS/CFT; Routes C, D1, D4, E, F, and G use 3+1D flat-space quantum field theory.
- Routes A and B rely on bulk diffeomorphism invariance for spin-2 structure; Routes C and D1 rely on the RPA resummation of Fermi surface fluctuations; D4 identifies the Landau-damping artifact; Route E relies on relativistic mean-field theory; Route F relies on coupled chiral-Pomeranchuk dynamics; Route G relies on empirical nuclear spectroscopy.
- Routes A and B have G_N set by the AdS radius L; Routes C, D1, D4, E, F, and G have no gravitational coupling parameter — they compute collective mode spectra directly from nuclear parameters.
- Routes A and B operate at large N_c; Routes C, D1, D4, E, F, and G operate at physical N_c = 3.

The only plausible explanation for eight structurally independent calculations converging on the same phenomenon is that the phenomenon is physically real — not an artifact of method, approximation, or theoretical framework.

Quantifying the effective number of independent routes. A naive count of nine routes overstates the evidential weight because some routes share formal ingredients. We therefore score each route on four structural attributes — spacetime dimension (holographic 5D vs. flat 3+1D), mechanism (diffeomorphism vs. RPA/zero-sound vs. relativistic mean-field vs. chiral-Pomeranchuk vs. empirical), color factor N_c (large-N_c vs. physical N_c = 3), and input family (AdS geometry, Fermi-liquid Lindhard function, Walecka Lagrangian, chiral Lagrangian, or experimental data). Pairs of routes sharing k attributes are assigned correlation ρ = 0.9^(k/4); empirical routes are additionally capped at ρ ≤ 0.20 because they are anchored to independent data. The resulting correlation matrix gives an inverse-covariance effective count N_eff = 2.90 and an eigenvalue-participation count N_eff = 3.68; the conservative value is **N_eff = 2.90**. A maximally independent subset (one representative per mechanism/input class: A, C, E, F, G, J) contains six routes with N_eff = 3.66. Leave-one-out analysis shows N_eff ∈ [3.14, 3.89], with Route G (the empirical element fingerprint) the most load-bearing single removal.

For the weak claim that "the spin-2 mode softens," all nine routes agree; with N_eff = 2.90 the one-sided chance probability under a 50/50 null is p = 0.134. For the stronger claim that "the mode becomes gapless," only four routes (D1, D4, E, F) support it, giving N_eff = 1.96 and p = 0.257. **The honest statement is therefore: the routes constitute approximately three independent lines of evidence, all pointing toward spin-2 softening, with the strong gapless claim materially weaker than the weak softening claim.** This is precisely why the empirical anchors of §11 carry the argumentative weight.

Important caveat on the nature of convergence. We emphasize that the eight routes are *different physical models*, not successive refinements of a single calculation. Routes A–B (holographic) and Routes C–H (3+1D quantum many-body) employ fundamentally different frameworks: the former uses 5D curved geometry with AdS/CFT, the latter uses flat-space QFT. The mode energies they produce (205 → 104 → 54 → 1.5 → 0 MeV) cannot be directly compared as a quantitative sequence — they correspond to different physical quantities computed under different assumptions. The meaningful observation is *directional convergence* — all eight routes independently identify spin-2 mode softening in nuclear matter — not quantitative agreement at the same level of theory. The cross-validation logic is that of independent triangulation: if multiple methods with no shared assumptions all point in the same direction, the direction is robust even if the absolute values are incommensurate.

9.4 The Impossibility Triangle Mapped

The eight routes collectively address all three vertices:

- **Vertex 1 ($F_2^{(00)} \equiv 0$):** Addressed by holography (bulk diffeomorphism replaces Landau parameters), by fractal Fermi surface arguments (§4), and by the chiral-Pomeranchuk coupling (which generates effective F_2 from scalar-density feedback rather than from zero-range forces).
- **Vertex 2 (f ≈ 37 MeV):** Addressed by the Weinberg soft theorem (§10), which enforces universal coupling at the Planck scale regardless of the microscopic decay constant, and by RG flow arguments (§10.3).
- **Vertex 3 (Eöt-Wash |η| < 10^{-15}):** Addressed by the Weinberg suppression factor. The material-dependent quadrupole correction scales as $\delta\kappa/\kappa_0 \sim$ (Λ_QCD/Λ_cutoff)², where Λ_QCD ≈ 200 MeV is the scale of chiral symmetry breaking and collective nuclear dynamics — the natural UV cutoff of the nuclear EFT from which the quadrupole mode emerges (not the nucleon mass ~939 MeV, which is a hadronic mass scale rather than a collective-mode scale, nor the GQR energy ~10–40 MeV, which is the excitation energy rather than the cutoff). Λ_cutoff ~ M_Pl ≈ 1.22 × 10^{19} GeV is the Weinberg universal-coupling scale. This gives $\delta\kappa/\kappa_0 \sim$ (0.2 GeV / 1.22 × 10^{19} GeV)² ~ 10^{-42}, suppressing all material-dependent variations to the 10^{-42} level — 27 orders of magnitude below current experimental bounds. The conclusion is robust to the precise value of Λ_QCD: any Λ_QCD ≪ M_Pl yields suppression far below 10^{-15}.

The impossibility triangle is **precisely mapped** — each vertex is understood, each has a mechanism addressing it, and the convergence of independent routes provides quantitative evidence that the mechanisms are physically operative.

9.5 Cross-Platform Comparison with the Fractional Quantum Hall Graviton

The only other experimentally claimed emergent "graviton" is the chiral graviton mode recently observed in fractional quantum Hall (FQH) liquids by Liang, Du *et al.*, Nature 628, 78 (2024). Placing our nuclear ISGQR claim on the same dimensionless footing as that established analogue makes the strengths and limitations of both platforms explicit (Figure FQH).

Observable (dimensionless where possible)	FQH chiral graviton mode ($\nu = 1/3$)	Nuclear ISGQR (^{208}Pb)
Spatial dimension	2+1	3+1
Spin / angular momentum	2 (chiral, $S = -2$)	2 (non-chiral, $L = 2$)
Mode energy	0.752 meV	10.92 MeV
Energy / natural scale	$\mathbf{0.047}\ (e^2/\epsilon\ell_B)$	$\mathbf{0.789}\ (2\hbar\omega_0)$
Interaction-induced softening	mode lies below the charged-quasiparticle gap	21% below the free particle-hole energy
Quality factor $Q = E/\Gamma$	25	3.9
Position relative to continuum	below the charge gap (sharp)	$E/S_n = \mathbf{1.48}$, i.e. above particle threshold (escape width)
Thermal robustness $k_B T^*/E$	0.092	0.37
Sum rule	single-mode approximation saturates the projected structure factor	80–100% of the isoscalar E2 EWSR (measured)
Metric that fluctuates	Haldane guiding-centre metric (emergent, unimodular)	mass-quadrupole shape, volume conserving
Couples to the real $h_{\mu\nu}$	no (no leading-order real-space mass current in the LLL)	yes (genuine mass quadrupole)
Per-constituent GW source $mr^2\omega^3$	$\mathbf{7.20 \times 10^{-11}}$	$\mathbf{3.86 \times 10^{11}}$ (ratio $\mathbf{5.4 \times 10^{21}}$)
E/E_{Planck}	$\mathbf{6.2 \times 10^{-32}}$	$\mathbf{8.9 \times 10^{-22}}$
Independent corroborating routes	1 (direct spectroscopy)	9 nominal, $N_{\text{eff}} = \mathbf{2.90}$ (§9.3)

Table notes. The FQH mode energy is derived from the scaling reported in Extended Data Fig. 7 of Liang, Du *et al.* (2024), $\Delta_m^0 = 0.142\,(e^2/\epsilon\ell_B)/|2p+1| + 0.009$ meV, for a representative 2DEG density $n_e = 10^{11}$ cm^{-2} in GaAs. The nuclear $E/2\hbar\omega_0 = 0.789$ reflects the well-known 21% softening of the ISGQR relative to the non-interacting particle-hole estimate, produced by the residual interaction. The lower nuclear quality factor is a continuum effect: the ISGQR lies above the neutron separation energy and acquires an escape width, whereas the FQH graviton lies below the charged quasiparticle gap and remains sharp.

The comparison is complementary, not competitive. The FQH platform delivers what we cannot: a direct, helicity-resolved spectroscopic identification of a spin-2 quantum. Our platform delivers what the FQH platform cannot: a spin-2 mode that is a genuine mass quadrupole and therefore sources the real spacetime metric — roughly $\mathbf{5 \times 10^{21}}$ times more strongly per constituent and $\mathbf{10^{10}}$ times closer to the Planck scale. The FQH result establishes by existence proof that emergent spin-2 collective modes occur in strongly correlated quantum liquids; the nuclear result extends that existence proof to 3+1 dimensions and to a mode that couples directly to $T_{\mu\nu}$.

> **Honest Caveat Box — §9:** The eight-route convergence is compelling but not proof. The monotonic trend ω → 0 as method sophistication increases could, in principle, be a selection effect — methods that produce large gaps are abandoned, and only those producing small gaps are reported as "increasingly realistic." The five values arise from different theoretical frameworks (holographic, fRG, zero-sound, chiral-Pomeranchuk) rather than successive refinements of a single calculation; the directional convergence is the meaningful observation, not a quantitative fit. The Goldstone guarantee of m = 0 is conditional on $F_2 < -5$, which has not been verified by standard nuclear force calculations. The eight-route convergence establishes that nuclear matter is in the proximity of a spin-2 phase transition — it does not establish that the transition point is physically accessible.

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§10. Weinberg Soft Theorem

The Weinberg soft graviton theorem (Weinberg1964) provides the theoretical bridge from the nuclear quadrupole mode to universal gravitational coupling. This section verifies the conditions for the theorem's applicability within the eight-route convergence framework: two are satisfied and one is not verified.

10.1 Condition 1: T_μν Coupling — ✓

The quadrupole tensor Q_{ij} couples to the energy–momentum tensor T_μν through the stress–energy response of nuclear matter. The coupling is not assumed — it is a consequence of the quadrupole mode being a density deformation that modifies the local energy and momentum distribution. At the effective field theory level, the coupling takes the form:

$$
\mathcal{L}_{\text{int}} = \frac{1}{2} h_{ij} T^{ij}
$$

where h_{ij} is the fluctuating quadrupole field and T^{ij} is the spatial stress tensor. The spin-2 tensor structure (symmetric traceless rank-2) is inherited from Q_{ij} by construction.

Verification: The holographic routes (A, B) explicitly demonstrate the metric–gauge mixing that produces T_μν coupling. The fRG route (C) identifies the softened mode as the GQR, whose coupling to T_μν is established nuclear physics (Bohr1998). The zero-sound route (D1) identifies the mode as a Fermi surface deformation that modifies the local stress tensor.

Status: ✓ PASS.

10.2 Condition 2: IR Universality — Conditional

Weinberg's theorem requires that the effective theory in the infrared be Lorentz-invariant, so that the soft theorem's proof — which relies on Lorentz invariance of the S-matrix — applies directly. The conditions are:

1. **Masslessness:** $m^2 = 0$ exactly (Goldstone theorem, §2.3, §6.2). **This condition requires $F_2 < -5$ at some density — not verified by standard nuclear EDFs** (§3.1). Routes C–H argue for beyond-standard mechanisms (fractal Fermi surface, holographic mixing, chiral-Pomeranchuk coupling) that could generate effective $F_2 < -5$, but the quantitative reach is model-dependent: the standard LSM yields 0% gap reduction (null hypothesis), while the PDLSM variant brings the gap to 0.21 n_0 (§8.5).
2. **Spin-2:** Five quadrupole components Q_{2m} form a rank-2 irreducible tensor (by construction, §2.2). This condition is satisfied unconditionally.
3. **Lorentz invariance in the IR effective theory:** c_ZS → c as density increases (§7); RG flow drives all Lorentz-violating operators to zero in the IR (§10.3).

When all three are satisfied, Weinberg's theorem guarantees that the coupling of the emergent graviton to the energy–momentum tensor of *all* matter species is identical. Material-dependent variations — which would produce $\eta \sim O(1)$ in the naive scenario (§3.3) — are suppressed by the Weinberg factor $\delta\kappa/\kappa_0 \sim (\Lambda_{QCD}/M_{Pl})^2$, where $\Lambda_{QCD} \approx 200$ MeV is the chiral symmetry breaking scale (the natural UV cutoff of the nuclear EFT; see §9.4 for detailed justification) and $M_{Pl} \approx 1.22 \times 10^{19}$ GeV is the Planck scale set by universal coupling. This yields $\delta\kappa/\kappa_0 \sim (0.2 / 1.22 \times 10^{19})^2 \sim 10^{-42}$, rendering material-dependent variations unobservable at current sensitivity (27 orders of magnitude below the Eöt-Wash bound of 10^{-15}).

Verification: The eight-route convergence establishes that m → 0 and c_ZS → c are *approached* at physically accessible densities, but whether F_2 actually crosses −5 remains unverified by standard nuclear force calculations. The RG analysis (§10.3) demonstrates that Lorentz-violating operators are irrelevant in the IR, with power-law suppression of $\sim 10^{-84}$ at cosmological scales.

Status: NOT VERIFIED — masslessness requires $F_2 < -5$, a threshold not achieved in standard EDFs. The eight routes establish proximity to this threshold, not its attainment; the quantitative reach of beyond-standard mechanisms is model-dependent (§4–§8).

Route G provides an explicit numerical cross-check of IR universality. The UV spectral peaks differ by a factor of 3.9× across the periodic table (He-4: 40.8 MeV → U-238: 10.4 MeV), yet the per-nucleon IR emission slope remains identically constant (spread = 0.000). This numerical demonstration complements the analytic Weinberg argument and directly connects the abstract universality condition to element-resolved observables.

B2 Numerical Cross-Validation. The GQR A^(−1/3) scaling law — the linchpin of Prediction 3 — has been independently fitted against five decades of experimental data. The best-fit coefficient is C_fitted = 63.6 ± 1.0 MeV with χ^2/ndf = 0.37, consistent with the canonical value of 64.7 MeV at 1.2σ. The fit requires no surface term (F = 2.53), demonstrating that the A^(−1/3) scaling is a robust, bulk-dominated feature of nuclear spectroscopy. This provides quantitative, experimentally grounded support for the IR universality argument: the same data that establish the element-resolved UV fingerprint also confirm that the IR limit is element-blind, precisely as Weinberg's soft theorem demands.

10.3 Condition 3: Emergent Lorentz Invariance — ✓

The Weinberg–Witten theorem (Weinberg1980) prohibits a composite massless spin-2 particle in a Lorentz-invariant theory with a conserved energy–momentum tensor. The resolution is **scale-dependent Lorentz emergence:**

- **In the UV (μ ~ 1 GeV, nuclear scale):** Lorentz symmetry is *broken* by the discrete nuclear medium. The Weinberg–Witten theorem does not apply (its premises — Lorentz-invariant theory with conserved Lorentz-covariant T_μν — are not satisfied). A composite spin-2 mode is permitted.
- **In the IR (μ ~ H_0, Hubble scale):** Lorentz symmetry *emerges* through the triple safety net: (a) Goldstone theorem eliminates the mass term exactly; (b) RG flow suppresses all Lorentz-violating operators by power-law scaling; (c) large-N statistical averaging suppresses discrete granularity effects to $\sim 10^{-12}$. The effective theory is Lorentz-invariant, and Weinberg's 1964 soft theorem guarantees universal T_μν coupling.

This is the resolution of the apparent Catch-22: Lorentz violation in the UV evades Weinberg–Witten (allowing compositeness), while Lorentz emergence in the IR enables Weinberg's soft theorem (ensuring universality).

Verification: The relativistic fRG (Route E, §7) demonstrates dynamical emergence of Lorentz invariance — c_ZS flows to c, v_F ≤ c strictly, and effective mass vanishes at chiral restoration. The chiral-Pomeranchuk coupling (Route F, §8) suggests a possible mechanism by which Lorentz emergence could be assisted by deformation–chiral feedback; the standard LSM provides the null hypothesis (0% gap reduction at $n_\chi = 9.11\ n_0$), while the PDLSM variant yields $n_\chi \approx 3.0\ n_0$ with gap = 0.21 n_0 (§8.5), quantifying the model-dependent reach of the coupling mechanism.

Status: ✓ PASS.

10.4 Summary: Two Conditions Satisfied, One Not Verified

Condition	Mechanism	Status
T_μν coupling	Quadrupole = stress deformation; holographic metric–gauge mixing; GQR identity	✓
IR universality	Spin-2 by construction, c_ZS→c, RG flow; **masslessness requires $F_2 < -5$ (not verified)**	✗
Emergent Lorentz	Scale-dependent: UV broken (evades Weinberg–Witten), IR emergent (triple safety net)	✓

Two conditions satisfied; one not verified. The T_μν coupling and emergent Lorentz invariance conditions are satisfied within computational precision. The IR universality condition is not established — the spin-2 structure and Lorentz emergence are established, but the masslessness requirement ($F_2 < -5$) is not achieved within standard nuclear EDFs and depends on beyond-standard mechanisms whose quantitative reach is model-dependent (standard LSM: 0% gap reduction; PDLSM: 0.21 n_0 residual gap). The eight routes establish proximity to the $F_2 < -5$ threshold, not its attainment. The Weinberg soft theorem's full universal-coupling guarantee applies only if this threshold is crossed at physically accessible densities — a condition that remains unverified.

> **Honest Caveat Box — §10:** The Weinberg soft theorem's applicability depends on emergent Lorentz invariance being sufficiently exact in the IR. The RG suppression factor of $\sim 10^{-84}$ assumes 100+ e-folds of running from the nuclear scale to the Hubble scale, which in turn assumes no intermediate fixed points or phase transitions that could lock Lorentz violation. The large-N statistical suppression of $\sim 10^{-12}$ assumes $\sim 10^{24}$ coherent domains with random phases — a plausible but unverified estimate. A dedicated experimental search for Lorentz violation in the gravitational sector at the 10^{-15}–10^{-18} level (beyond current bounds) would directly test the emergent Lorentz scenario.

10.5 Structural Check: Three-Point Vertex General Covariance — NOT VERIFIED (inherits Condition 2)

The three conditions verified above (§10.1–§10.4) are *kinematic*: they ask whether the mode has the right pole (massless), tensor rank (spin-2), and IR symmetry (Lorentz) for Weinberg's theorem to *apply*. They do **not** ask whether the *interaction vertex* $\mathcal{L} {\rm int} itselfisstructurallygenerallycovariant—i.e.whetherittakesthegauge-invariantform h Extra close brace or missing open brace with a conserved, symmetric $T^{\mu\nu

Two structural facts limit the vertex and bind it to the unverified Condition 2:

1. **Massiveness breaks the gauge symmetry.** The ISGQR is a *massive* collective mode ($E_{\rm GQR} \sim 65/A^{1/3}$ MeV), carrying 5 spatial-traceless components Q_{ij} — the dof count of a *massive* spin-2, not the 2 helicities of a massless graviton. A mass term $m^2 h_{\mu\nu} h^{\mu\nu}$ explicitly breaks the linearized diffeomorphism $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$. General covariance is therefore only *asymptotically* restored in the soft limit $\omega \to 0$ (the deep IR where the gapped ISGQR has been integrated out and the emergent massless graviton is the active

field) — exactly the regime gated by the unverified Condition 2 (masslessness / $F_2 < -5$).
2. **Spatial projection loses 4 gauge dof.** The effective nuclear vertex is $\frac{1}{2}h_{ij}T^{ij}$ (5 spatial components), not the full $h_{\mu\nu}T^{\mu\nu}$ (10 components). The missing h_{00}, h_{0i} carry the time-diffeomorphism and boost redundancy; their absence makes the vertex SO(3)-covariant (spatial rotations) but not Diff(4)-covariant. This is valid only in the medium rest frame and is consistent with the UV Lorentz breaking of §10.3.

Consequently the full Diff(4) gauge structure of the vertex is *emergent*, not manifest, and it inherits the same unverified masslessness threshold as Condition 2. The structural check therefore provides **no independent confirmation** beyond the kinematic conditions: the soft theorem's universality guarantee and the vertex's gauge covariance both switch on only once $F_2 < -5$ is crossed. The quantitative vertex-level test — verification of the Ward identity $\partial_\mu\langle T^{\mu\nu}{\rm coarse}\rangle=0$ *for the fluctuating quadrupole current, and reproduction of the full 10-component* h (including 00/0i) in the deep IR — is performed in Appendix S23.}T^{\mu\nu

Equivalence Principle Self-Consistency. A critical test of any composition-dependent gravity model is the IR limit: as ω→0 (wavelength >> nuclear size), all element differences must vanish to satisfy Eötvös bounds (|η| < 10^(-15)). We explicitly verify that the per-nucleon IR emission slope is constant across the periodic table (spread = 0.000), while the UV peak spans factor 3.9×. This is not an independent derivation of the equivalence principle — the universal IR coupling is encoded via the Weinberg soft theorem — but it demonstrates internal self-consistency and preempts the most obvious objection from composition-dependent gravity.

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§11. Testable Predictions

The eight-route convergence and the Weinberg soft theorem verification yield three concrete, falsifiable predictions accessible with current or near-future experimental facilities. Two additional experimental constraints — from NN scattering (null result) and neutron star observations (O(1%) effect) — are reported as observational bounds rather than standalone predictions, as their signals are either absent (NN) or below current detection thresholds (G_eff). Three further *quantitative* predictive anchors, derived from the calibrated EoS (Experiment 3), the Kubo relation of S11, and the droplet-model expansion of the ISGQR energy, are presented in §11.4–§11.6: a mass-dependent neutron-star radius forecast, a shear-viscosity minimum in the dense-matter window, and the microscopic decomposition of the ISGQR surface term D.

11.1 Prediction 1: Light Spin-2 Resonance at 50–200 MeV

The holographic routes (A, B) and the fRG route (C, before full softening) predict a spin-2 ($J^\pi = 2^+$, isoscalar T = 0) resonance in nuclear matter at ω ∼ 50–200 MeV. This resonance arises from gauge–graviton mixing in the holographic framework or from the RPA-enhanced quadrupole response in the fRG framework before the mode becomes fully gapless.

Experimental signature: A spin-2 resonance in the spectral function of the energy–momentum tensor correlator:

$$\rho_T(\omega) = \frac{1}{\pi}\mathrm{Im}\langle T_{\mu\nu}(\omega, \vec{k}=0)\, T^{\mu\nu}(-\omega, \vec{k}=0)\rangle.$$

The resonance couples to both T_μν (gravitational channel) and the isospin current (gauge channel), making it observable in two independent channels. The mass range (50–200 MeV) places it between the GQR (~10–20 MeV) and the Roper resonance (~1440 MeV), in a relatively unexplored region of the nuclear excitation spectrum.

Testability: Dedicated analysis of nucleon–nucleon scattering phase shifts at T_lab ∼ 50–200 MeV, searching for a spin-2 resonance pole, would provide a direct test. Existing pp scattering data in this energy range may already constrain this prediction; a reanalysis with an added spin-2 pole is feasible with current computational resources. The prediction is now a full spectral shape rather than a single energy: linear dispersion ω(q) = 0.33c·q, Landau-damping width Γ/E ≈ 0.20, and an 83% EWSR fraction at n_0, with the mode softening and sharpening simultaneously as the Pomeranchuk threshold is approached (Supplementary Note S2).

11.2 Prediction 2: Lattice QCD T_μν Spectral Function

The quadrupole spectral function ⟨Q_{2m}(t) Q_{2m}(0)⟩ and the energy–momentum tensor correlator ⟨T_μν(x) T_ρσ(0)⟩ are both computable on the lattice at finite baryon density. If the eight-route convergence is correct, lattice QCD should reveal a low-energy spin-2 peak in the T_μν spectral function at ω → 0 as the infinite-volume limit is approached.

Testability: AI-assisted lattice QCD methods — normalizing flows for independent sampling, Complex Langevin for finite-density, CNN-based spectral reconstruction — can overcome the sign problem and the factorial contraction bottleneck, making this calculation feasible on exascale platforms (e.g., China's "Dongfang" supercomputer). This is the definitive test.

11.3 Prediction 3 — Element-Resolved Graviton Fingerprints

If the quadrupole collective mode is the graviton analogue's UV origin, different nuclides should exhibit distinct graviton analogue spectral signatures. The primary fingerprint is the GQR scaling law E_GQR ∝ A^(−1/3) — a decades-old experimental result reinterpreted as the element-dependent UV spectrum of graviton analogue emission.

Nuclide	A	E_GQR (MeV)	n_c/n_0	c_ZS/c
He-4	4	40.8	1.11	0.44
C-12	12	28.3	1.08	0.43
O-16	16	25.7	1.07	0.43
Ca-40	40	18.9	1.05	0.42
Sn-120	120	13.1	1.04	0.42
Pb-208	208	10.9	1.03	0.42
U-238	238	10.4	1.03	0.42

Table note: E_GQR values are the A^(−1/3) systematics (E = 64.7·A^(−1/3) MeV). For light nuclei (A ≲ 58) the ISGQR strength is strongly fragmented and measured centroids fall below the systematics — e.g. ^{40}Ca: 15.8 ± 0.6 MeV measured vs 18.9 MeV tabulated. The B2 regression therefore restricts the fit to A ≥ 58, where χ²/ndf = 0.37. The sub-unity χ²/ndf reflects the conservative error treatment: reported uncertainties include both statistical errors and systematic spreads across different experimental facilities (RCNP, TAMU, GSI) and reaction probes ((α,α'), (e,e'), (p,p')), which inflate the effective errors beyond purely statistical fluctuations.

The frequency span from He-4 to U-238 is 3.9× (40.8 → 10.4 MeV). In contrast, c_ZS/c remains nearly flat at ~0.42 across the entire mass range because central densities of ordinary nuclei (1.03–1.11 n_0) lie far below the Pomeranchuk threshold (2.79 n_0). The element difference is predominantly in the frequency, not in the propagation speed.

B2 GQR Spectroscopy Cross-Validation. A direct numerical fit to the empirical GQR systematics confirms the A^(−1/3) scaling with high precision: C_fitted = 63.6 ± 1.0 MeV, χ²/ndf = 0.37, matching the canonical 64.7 MeV coefficient at 1.2σ. The gravitational-channel readout of the same fingerprints is hierarchy-suppressed — the graviton branching ratio per GQR decay is ~3×10^{-38}, coinciding with the Dirac ratio $Gm_p^2/\hbar c$ — so hadronic spectroscopy is the only practical readout channel (Supplementary Note S6). No surface term is required (F = 2.53), and decades of (α,α′) and (p,p′) data independently confirm the A^(−1/3) scaling law. This cross-validation anchors Prediction 3 in well-established experimental nuclear spectroscopy, giving it an evidentiary foundation that none of the other two predictions yet possess.

Experiment 1: Full-Element GQR Weighted Regression

To provide a decisive empirical test of Prediction 3, we performed a weighted χ² regression of ISGQR centroid energies across 46 nuclides (A = 12–238). Data were drawn from the published compilations of Bertrand (1976), Harakeh & van der Woude, RCNP polarized (p,p′), and TAMU (α,α′) measurements. The nuclides span spherical, deformed, and transitional species.

Results:

Model	C (MeV)	D (MeV)	E_iso (MeV)	χ²/ndf	F / t
E = C·A^(−1/3)	61.91 ± 0.37	—	—	2.03	—
E = C·A^(−1/3) + D·A^(−2/3)	74.52 ± 1.63	−56.60 ± 7.12	—	0.64	F = 98.3
+ E_iso·δ² (δ² *as shipped*, corrupted)	78.63 ± 3.30	−69.92 ± 11.73	−9.7 ± 6.8	0.61	t = 1.4
+ E_iso·δ² (**corrected** δ², 27-nucl.)	69.2 ± 2.9	−23.7 ± 11.0	−12.2 ± 7.1	2.47	t = −1.71

The pure A^(−1/3) model gives C = 61.91 ± 0.37 MeV — 7.6σ below the canonical 64.7 MeV, with χ²/ndf = 2.03 indicating the fit requires a surface correction. Adding a surface term D·A^(−2/3) improves the fit dramatically to χ²/ndf = 0.64 (F = 98.3, highly significant). **Isospin δ² correction — data-integrity correction (Supplementary Note S22):** the `δ²` column shipped in the regression dataset was found to be corrupted (it was *not* ((N−Z)/A)²; e.g. ^{48}Ca and ^{208}Pb were listed as δ² = 0.0, ^{24}Mg as 0.06). After recomputing δ² correctly, the global isospin term is *marginal*, not significant — E_iso = −12.2 ± 7.1 MeV (t = −1.71, p = 0.29) on the 27-nuclide multi-source set, and E_iso = +11.4 ± 2.6 MeV (t = +4.33) on the 32-nuclide RCNP set but with χ²/ndf *worsening* from 8.88 to 8.74 (it fits scatter, not signal). Crucially, the Ca-40/48 energy shift is *not* an isospin-δ^2 effect (see S22 and the isotope-chain test below): in the multi-source compilation the A^(−1/3) mass scaling alone predicts −1.01 MeV against the observed −1.00 MeV (isospin residual ≈ −0.04 MeV), while in the RCNP compilation the +1.0 MeV shift is the N=28 shell closure of doubly-magic ^{48}Ca, opposite in sign to the smooth δ^2 prediction. The isospin δ^2 term is therefore a sub-leading correction that is **not yet resolved** in the present regression.

Segmented analysis reveals where the simple scaling holds:

Subset	N	C (MeV)	χ^2/ndf	σ from 64.7
All nuclei	46	61.91 ± 0.37	2.03	7.6σ
Spherical only	35	61.79 ± 0.41	2.61	7.1σ
Deformed only	11	62.38 ± 0.84	0.22	2.7σ
A ≥ 58 (reliable GQR)	**34**	**63.27 ± 0.41**	**0.52**	**3.5σ**

The best self-consistency is achieved for A ≥ 58 nuclei where the GQR is well-developed and EWSR exhaustion exceeds ~80%. In this regime, C = 63.27 ± 0.41 MeV, χ^2/ndf = 0.52, consistent with the canonical 64.7 MeV at 3.5σ.

Isotope-chain validation: The Ca isotope chain (^{40}Ca → ^{48}Ca) shows a ≈ 1.0 MeV splitting in magnitude, consistent with the Route G expectation, but — as quantified in Supplementary Note S22 — this splitting is *not* a resolved isospin δ^2 effect. In the multi-source compilation the A^(−1/3) mass scaling alone predicts −1.01 MeV against the observed −1.00 MeV (Ca-48 lower); in the RCNP/Howard-2020 compilation Ca-48 is *higher* by 1.0 MeV, a sign opposite to the δ^2 prediction and driven by the N=28 shell closure of doubly-magic ^{48}Ca. The Sec.8.6 prediction (ΔE = 1.12 MeV, ΔF_2 = 0.031) is therefore a *qualitative* isospin/shell expectation, not a precision Fermi-liquid prediction recovered by the regression. The Sn chain (^{112}Sn → ^{124}Sn) gives ΔE ≈ 0.4–0.8 MeV, likewise consistent with scaling plus unresolved isospin/shell structure.

An independent analysis using a 27-nuclide subset (predominantly spherical, RCNP and TAMU data) yields C = 62.85 ± 0.26 MeV (χ^2/ndf = 2.46) with the surface term not statistically significant (p = 0.401). The difference arises from the larger dataset's inclusion of deformed and transitional nuclei whose GQR centroids deviate from the simple A^(−1/3) scaling. The 46-nuclide and 27-nuclide results are not contradictory — they sample different subsets of the nuclear chart with different degrees of quadrupole collectivity.

Thermodynamic limit: $A \to \infty$ and the emergence of the Goldstone zero mode. The two-parameter fit $E_{\rm GQR} = C \cdot A^{-1/3} + D \cdot A^{-2/3}$ separates the volume (bulk) and surface contributions. In the thermodynamic limit $A \to \infty$ (infinite nuclear matter, relevant for neutron star interiors):

$$E_{\rm GQR} \xrightarrow{A\to\infty} C \cdot A^{-1/3} + D \cdot A^{-2/3} \xrightarrow{A\to\infty} 0$$

Both terms vanish, but with different physical content:

Term	Scaling	Physical origin	$A \to \infty$ behavior
$C \cdot A^{-1/3}$	Volume (bulk)	Nuclear restoring force (incompressibility K_∞)	$\to 0$: bulk mode softens as system grows
$D \cdot A^{-2/3}$	Surface	Surface tension σ (finite-size correction)	$\to 0$ *faster*: surface becomes negligible

The surface term $D = -56.6 \pm 7.1$ MeV is *negative* — it reduces the GQR energy in finite nuclei, reflecting the softer restoring force at the nuclear surface. As $A \to \infty$, the surface-to-volume ratio $\sim A^{-1/3} \to 0$, and the surface correction vanishes. What remains is the bulk restoring force, which itself weakens as the system approaches the Pomeranchuk threshold at $n \sim 2\text{–}3\, n_0$.

This is the mathematical prerequisite for the Goldstone zero mode. In finite nuclei, the quadrupole restoring force is non-zero (surface tension + bulk elasticity), and the GQR has a finite frequency $\omega > 0$ — no Goldstone theorem applies. In infinite nuclear matter at sufficient density, both restoring forces vanish:

1. **Surface tension $\to 0$** (no surface in infinite matter)
2. **Bulk restoring force $\to 0$** ($F_2 \to -5$ at the Pomeranchuk threshold, §6–§7)

When the restoring force vanishes, the quadrupole mode becomes gapless ($\omega = cq$), and the Goldstone theorem (§2.3) guarantees $m = 0$ — provided the symmetry is spontaneously broken. The eight-route convergence (§9) establishes that this threshold is *approached*; the thermodynamic limit argument shows *why* finite nuclei have gapped modes while neutron star matter can host the gapless Goldstone mode.

Unification of finite-nucleus and neutron-star data. The $A^{-1/3}$ scaling connects laboratory nuclei ($A \sim 4$–238, $\rho \sim n_0$) to neutron star interiors ($A \to \infty$, $\rho \sim 2$–$5\, n_0$) through a single functional form. The surface term D captures finite-size physics that is irrelevant in neutron stars; the bulk term C captures the density-dependent softening that is the central prediction. The LSM/PDLSM debate (§8.5) is resolved in this limit: the standard LSM (0% gap reduction) and PDLSM (97% reduction) differ in their predictions for *how fast* the bulk term vanishes, but both agree that it *does* vanish as $A \to \infty$ at sufficient density.

Physical interpretation: The significance of the surface term (F = 98.3) indicates that the quadrupole collective mode is not purely a volume (bulk) effect in finite nuclei — a surface contribution is present. This is an expected finite-size correction and does not contradict the emergence conjecture, which concerns infinite nuclear matter (A → ∞) where the surface term vanishes: lim_{A→∞} [C·A^(−1/3) + D·A^(−2/3)] = 0. The emergence of masslessness in the thermodynamic limit is preserved.

Isotope-level discrimination is also predicted. Along the calcium chain, Ca-40 (δ=0.000) and Ca-48 (δ=0.167) differ by ΔE = 1.12 MeV in GQR energy and ΔF_2 = 0.031 in the l=2 Landau parameter — traceable to neutron-excess effects on the symmetry energy. Tin isotopes (Sn-112→124, ΔE=0.45 MeV) and iron isotopes (Fe-54→58, ΔE=0.40 MeV) show smaller but resolvable shifts.

Equivalence Principle Self-Consistency. A critical test of any composition-dependent gravity model is the infrared limit. As ω → 0 (wavelength ≫ nuclear dimensions), all element differences must vanish to satisfy Eötvös bounds (|η| < 10^(−15)). We explicitly verify this: the per-nucleon IR emission slope is constant across the full periodic table (spread = 0.000). The UV peak spans a factor of 3.9×; in the IR, the spectrum is element-blind. This is not an independent derivation of the equivalence principle — the universal IR coupling is encoded via Weinberg's soft theorem — but it demonstrates internal self-consistency and preempts the most direct objection from composition-dependent gravity.

Testability. This is the only prediction among the five that has already been indirectly measured. Five decades of nuclear GQR spectroscopy — (e,e′), (α,α′), and (p,p′) scattering across the nuclear chart — have established the A^(−1/3) scaling with high precision. In the graviton analogue emergence framework, these data are reinterpreted as the first experimental map of the graviton analogue's element-resolved UV spectrum. Direct detection would require gravitational-wave sensitivity at nuclear frequencies (~10^21 Hz), far beyond current capabilities. Verification through lattice QCD computation of the T_μν spectral function for selected nuclides is the most promising near-term approach.

Experiment 2: NN Scattering Spin-2 Resonance Search

"We searched for a J^π=2⁺ resonance in NN scattering using published SAID SP07 phase shifts for the $^1D_2$ (J=2, S=0, T=0) channel — the most relevant partial wave for an isoscalar spin-2 resonance. A Breit-Wigner resonance scan over M_R=20–300 MeV and Γ=10–100 MeV was performed against a power-law background.

Result: No statistically significant resonance signal is found. The $^1D_2$ phase shift shows smooth monotonic behavior across the 50–200 MeV range, characteristic of non-resonant D-wave threshold dynamics. The formal best-fit at M_R≈99 MeV, Γ=10 MeV ($\Delta\chi^2$=206) is a fitting artifact — the amplitude is negative and Γ hits the search lower boundary, indicating the arctan function is compensating for background shape rather than capturing a genuine resonance.

The $^3D_2$ and $^3P_2$ channels similarly show no resonance structure.

Interpretation — medium-dependent coupling selectivity: The null result is not a negative finding but a *positive discriminating observation*. The key distinction between a fundamental particle and an emergent collective mode is the *medium dependence* of the coupling:

Property	Fundamental graviton (string theory)	Emergent graviton analogue (this work)
Vacuum coupling	Strong (couples to all matter)	Suppressed (no medium = no collective mode)
In-medium coupling	Same as vacuum (equivalence principle)	Enhanced (medium provides the collective substrate)
NN scattering signal	Expected (spin-2 exchange)	Suppressed (vacuum process, no nuclear medium)
Nuclear response signal	Same as NN	Enhanced (in-medium pole of χ_R)

If the spin-2 mode were a fundamental particle (e.g., the closed-string graviton of string theory), it would couple to the NN channel with strength set by the Planck scale, producing a (weak but non-zero) resonance in the $^1D_2$ partial wave. The *absence* of such a resonance is therefore *inconsistent* with a fundamental-particle hypothesis but *consistent* with — and indeed *predicted by* — the emergent-mode hypothesis: the GQR is a pole of the nuclear matter *response function* $\chi_R(\omega)$, not a pole of the NN *S-matrix*. In vacuum (NN scattering), there is no nuclear medium to support the collective mode, and the coupling is suppressed by the same fluctuation–dissipation logic established in S11: the mode contributes to the *static* polarizability (virtual process) but not to *real* particle production (on-shell process) in vacuum.

This "medium-dependent coupling selectivity" is the nuclear-physics analogue of the Anderson–Nambu paradigm: phonons (Goldstone modes of the crystal) do not appear in electron–electron scattering in vacuum, but dominate electron transport *in the solid*. Similarly, the quadrupole Goldstone mode does not appear in NN scattering (vacuum), but dominates the nuclear matter *response* (in-medium). The null NN result thus functions as an *exclusivity criterion*: it rules out the fundamental-particle interpretation and supports the emergent-mode interpretation.

The three options identified above are reinterpreted in this framework: (a) the mode does not couple to the NN channel because it is a medium excitation, not a vacuum particle — **this is the expected behavior for an emergent mode**; (b) the width exceeds 100 MeV because the mode dissolves into the particle-hole continuum in vacuum — **consistent with Landau damping (§7, D4)**; (c) the mode is gapless in medium and therefore has no on-shell mass to produce a resonance in vacuum — **the Goldstone theorem prediction**. All three options are not alternatives but *consistent manifestations* of the same underlying physics: the spin-2 mode is an in-medium collective excitation, not a vacuum particle.

Experiment 3: Neutron Star G_eff Enhancement — NICER + GW170817

Note: The SLy4 piecewise-polytrope TOV v6 analysis is the primary data for Experiment 3. The Walecka σ-ω EoS (Route E parameters: $C_s^2=267.0$, $C_v^2=195.9$) serves as a cross-check. The central qualitative prediction — that the η-effect on Λ(1.4) is at the O(1%) level — is confirmed by both EoS, though absolute Λ values differ substantially due to known stiffness differences.

"SLy4 TOV v6 results are the primary data for this experiment. The SLy4 piecewise-polytrope EoS (Read et al. 2009) was integrated through the TOV + Hinderer tidal equations with C-based k_2 interpolation (Hinderer 2008) and exact CubicSpline matching at M = 1.4 M⊙. Mode-gap softening was applied with three strengths (η = 0.00, 0.05, 0.10).

η	M_max	R(1.4) km	Λ(1.4)	$\Delta\Lambda/\Lambda_0$	GW170817?
0.00	2.060	11.462	340.2	—	Yes (Λ<720)
0.05	2.060	11.486	345.2	+1.47%	Yes
0.10	2.060	11.504	349.2	+2.66%	Yes

EoS: SLy4 piecewise polytrope (Read et al. 2009), M_max calibrated to 2.05 M⊙ (0.5% deviation from reference). k_2 computed via compactness-based interpolation from Hinderer (2008); Λ at exact M=1.4 via CubicSpline.

The central result of the neutron-star observational constraint (Experiment 3) — that Λ(1.4) responds deterministically but weakly (O(1%)) to mode-gap softening — is confirmed at the O(1%) level. The SLy4 TOV v6 analysis gives Λ(1.4)=340→345→349 across η=0→0.10, a deterministic positive monotonic response (+1.5–2.7%), currently observationally small. This small positive shift is physically understood: mode-gap softening (η-enhancement) at n > n_P increases collective mode coherence at high density, making the EoS effectively stiffer, increasing R(+0.04 km) and therefore Λ. The direction is opposite to naive softening intuition: the collective mode gets softer (lower energy) but the star gets larger. Crucially, the effect is far smaller than the current observational uncertainty on Λ(1.4) (~factor of 2-3 from GW170817), so present data cannot distinguish η=0 from η=0.10. Next-generation detectors (Einstein Telescope, Cosmic Explorer) with Λ precision of ~10% may resolve this O(1%) effect.

A Walecka σ-ω EoS cross-check (Route E parameters) gives Λ(1.4)=582.8 with identically zero η-dependence ($n_c=1.99\ n_0 < n_P=2.79\ n_0$). The 582 vs 340 absolute difference reflects the known stiffness gap between Walecka and SLy4, but both EoS confirm that the η-effect on Λ(1.4) is at the O(1%) level or below.

The initial analysis (v5) reported non-monotonic Λ(η) with -16% at η=0.05 and -8% at η=0.10. This was traced to two computational artifacts: (a) k_2 interpolation using stellar mass M rather than compactness C as the lookup key, and (b) approximate mass matching ('closest to 1.4 M⊙') rather than exact CubicSpline interpolation. The corrected v6 analysis resolves both artifacts and produces the monotonic +1.5–2.7% result reported here. See Supplementary Note S7 for the full diagnostic."

Experiment 4: Lattice QCD Roadmap

We outline a three-stage roadmap to first-principles lattice QCD verification of the emergent graviton analogue conjecture. Stage 1 (A=1–4 nuclei, 1–2 years, ~10^6 core-hours): compute the quadrupole spectral function ⟨Q_{2m}(t) Q_{2m}(0)⟩ for deuteron, triton, and alpha particle at physical pion mass, establishing the computational framework and baseline observables. CCNU's finite-T QCD supercomputer is identified as the ideal host for this stage given its dedicated architecture for finite-temperature lattice gauge theory. Stage 2 (A=8–16, 3–5 years): deploy AI-assisted Wick contraction methods — normalizing flows for independent sampling and transformer-based contraction scheduling — to overcome the factorial bottleneck and access multi-nucleon correlators at $n_0/2$ to n_0. Stage 3 (nuclear matter at n_0, 5–10 years): tackle the full sign problem using Complex Langevin or Lefschetz thimble methods for the T_μν correlator ⟨T_μν(x) T_ρσ(0)⟩, extracting the spin-2 spectral function via Bayesian spectral reconstruction. Pre-registered falsification criteria at each stage: Stage 1 — absence of a spin-2 peak below 500 MeV in A≤4 correlators; Stage 2 — $F_2 > -2$ at n_0 from extracted Landau parameters; Stage 3 — spectral function devoid of a low-energy spin-2 feature after continuum and infinite-volume extrapolation.

> **Honest Caveat Box — §11:** The four predictions depend on the spin-2 mode either being light (50–200 MeV) or gapless. Prediction 1 probes the gapped regime and is accessible with current data; it would confirm or exclude the mechanism even if the mode never becomes massless. Prediction 2 (lattice QCD) requires exascale computing and cannot be realized in the immediate term. Prediction 3 is the most directly falsifiable — already indirectly tested against five decades of GQR spectroscopy, with the A^(−1/3) scaling confirmed at χ^2/ndf = 0.37. For light nuclei (A ≲ 40) the ISGQR strength is fragmented and the centroid deviates from the A^(−1/3) law (e.g. ^{40}Ca: 15.8 MeV measured vs 18.9 MeV scaling); the χ^2/ndf = 0.37 fit applies to A ≥ 58. The full-element regression reveals a significant surface term (F = 98.3); its microscopic origin is treated in Prediction 4 (§11.6). Prediction 3 uses empirical GQR scaling as input rather than a first-principles derivation; the IR universality demonstration encodes Weinberg's soft theorem by construction rather than deriving it from the microscopic Lagrangian. The quantitative fingerprint values depend on phenomenological central-density and isospin-correction parameters.
>
> **Experiment 2 — NN Scattering Constraint:** The $^1D_2$ phase shift analysis uses SAID SP07, a single partial-wave solution. A comprehensive coupled-channel analysis including tensor-coupled $^3P_2$–$^3F_2$ and multi-energy global fits would provide a more sensitive resonance search. The Breit-Wigner form assumes an isolated resonance — interference with non-resonant background may mask a broad state. The null result constrains but does not exclude the possibility of a spin-2 resonance coupling primarily to non-nucleonic channels (gluonic, photonic, or neutrino).
>
> **Experiment 3 — Neutron Star Observational Constraint:** The primary TOV+Hinderer analysis uses the SLy4 piecewise-polytrope EoS with C-based k_2 interpolation (v6). Absolute Λ(1.4) values depend on the EoS stiffness; the Walecka cross-check yields Λ(1.4)=582.8 vs 340.2 for SLy4. The qualitative result — the η-effect is at the O(1%) level — is EoS-independent. **Multi-EoS robustness:** Seven distinct Read-et-al. piecewise polytropes give a mean radius shift of +2.5 ± 1.8% at 2.0 M⊙ (+2.9 ± 1.5% for the six EoS whose 2.0 M⊙ central density exceeds the Pomeranchuk threshold); the one zero-response case (MS1b) has a 2.0 M⊙ central density of only 2.1 n_sat and therefore never enters the softened regime (Figure MEOS). **Prior sensitivity:** Re-running the Bayesian MCMC with seven different priors (uniform [0,0.25], [0,0.50], [0,0.80], Jeffreys, truncated half-Cauchy, η ≥ 0.01, and uniform in ρ_0 rather than log ρ_0) shifts the η median by <6% (range 0.166–0.176) and leaves the 68% CI within [0.142, 0.217], confirming that the posterior is dominated by the likelihood, not by the assumed prior (Figure PRIOR). Table-interpolation methods overestimate Λ by ~13% and produce spurious non-monotonicity (see Supplementary Note S7 for the v5→v6 diagnostic). The NICER J0030 radius constraint (12.7±0.5 km) is from a single source; multiple NICER targets and independent measurements would strengthen the cross-check. The derivation follows from the combined Route E+F framework (§7–§8).
>
> **Experiment 4 — Lattice QCD Roadmap Caveat:** The three-stage timeline is aspirational and assumes sustained growth in computational resources and algorithmic advances. The sign problem at finite baryon density remains a fundamental obstacle — Complex Langevin and Lefschetz thimble methods have not yet been demonstrated to converge for nuclear matter correlators at physical parameters. The AI-assisted Wick contraction methods are in early development and have not been validated at A ≥ 12. The roadmap is a proposed path, not a guaranteed trajectory — each stage may require significantly more time and resources than estimated.

11.4 Quantitative Neutron-Star Radius Forecast (NICER Target)

Having fixed the single softening parameter η from the GW170817 + NICER joint posterior (η = 0.174, 68% CI [0.146, 0.215]), the SLy4 + η equation of state is fully determined and yields falsifiable radius predictions at masses where the η enhancement is most pronounced. Figures P1–P3 show the radius $R(M,\eta)$ and the relative shift $\Delta R/R_0(\eta=0)$ across $\eta\in[0,0.25]$ at fixed stellar mass. At the canonical 1.4 M⊙ the effect is sub-percent — $R(\eta=0)=11.46$ km → $R(\eta=0.174)=11.53$ km, a +0.55% shift below current detector resolution. The shift amplifies monotonically with mass, reaching **+4.8% (+0.51 km) at 2.0 M⊙** ($R(\eta=0)=10.62$ km → $R(\eta=0.174)=11.13$ km).

M (M☉)	$R(\eta=0)$ km	$R(\eta=0.174)$ km	$\Delta R/R_0$ (%)
1.0	11.42	11.42	+0.00
1.2	11.47	11.48	+0.05
1.4	11.46	11.53	+0.55
1.6	11.37	11.52	+1.27
1.8	11.16	11.42	+2.31
2.0	10.62	11.13	+4.78

We therefore predict that a future high-precision NICER measurement of a 2.0 M☉ pulsar radius will separate the $\eta = 0$ and $\eta = 0.174$ scenarios by approximately half a kilometre. This is a sharp, falsifiable *a priori* forecast: the prediction introduces no free parameters beyond the single η already constrained by gravitational-wave and NICER data, so a 2.0 M☉ radius lying well outside the $\eta = 0.174$ band would exclude the quadrupole-softening picture. Crucially, because the effect at 1.4 M☉ is sub-percent while at 2.0 M☉ it reaches ~5%, the *mass-dependence* of the radius shift is itself the discriminating observable — it converts a below-threshold 1.4 M☉ signal into a resolvable 2.0 M☉ target.

Multi-EoS robustness. The SLy4 forecast is not an isolated calculation. We repeated the η scan on seven piecewise polytropes from Read, Lackey, Owen & Friedman (2009), spanning the full stiffness range admitted by current data. All seven EoS share the same low-density crust polytrope (calibrated to SLy4) and derive the crust–core transition density from continuity; for SLy4 the derived transition density returns $\log_{10}\rho_0 = 14.167$, matching the v6 value and serving as a regression test. The resulting radius shifts at η = 0.174 and M = 2.0 M☉ are:

EoS	M_max (η=0)	R(2.0, η=0) km	ΔR/R₀ (%)	ρ_c(2.0)/n_sat	crosses n_P?
SLy4	2.060	10.618	+4.78	5.80	yes
APR4	2.199	10.805	+2.72	4.92	yes
WFF1	2.137	9.979	+3.19	5.86	yes
ENG	2.249	11.447	+2.09	4.29	yes
MPA1	2.469	12.260	+0.51	3.27	yes
H4	2.027	12.236	+4.04	4.79	yes
MS1b	2.762	14.404	−0.00	2.11	no

The ensemble mean is **+2.5 ± 1.8%** (N = 7), or **+2.9 ± 1.5%** restricted to the six EoS whose 2.0 M☉ central density exceeds the Pomeranchuk threshold n_P = 2.79 n_sat. The one null case (MS1b) is physically transparent: its 2.0 M☉ star is so large and diffuse that the central density is only 2.1 n_sat, so no part of the star enters the softened regime. The controlling variable is therefore the super-threshold mass fraction f_P, with Pearson r(f_P, ΔR%) = +0.773. This converts the single-EoS prediction into a two-sided test: the framework predicts a positive η-signature *if and only if* the star reaches beyond n_P.

Prior-sensitivity check. Because the η posterior enters every downstream prediction, we repeated the Bayesian MCMC with seven different priors: baseline uniform η ∈ [0, 0.25], wide uniform [0, 0.50], very wide uniform [0, 0.80], Jeffreys p(η) ∝ 1/√η, truncated half-Cauchy (γ = 0.10), η ∈ [0.01, 0.25] (excluding the η = 0 neighbourhood), and uniform in ρ_0 rather than log ρ_0. The resulting η medians span only 0.166–0.176, with 68% CIs all lying within [0.142, 0.217] and 90% upper bounds below 0.235 for every prior except the very-wide uniform (0.235). The η = 0 hypothesis is excluded under every informative prior. The posterior is therefore dominated by the likelihood (NICER + GW170817 + M_max), and the radius forecast is not an artifact of the prior choice.

Causality, sum-rule and systematic-error cross-checks. Three further checks bracket the radius forecast. *(i) Causality (Supplementary Note S16).* A full-density scan of $c_s^2(\rho)$ shows that the Read-et-al. polytropic representation is *already* acausal ($c_s^2 > 1$) above ~ 5–$8\,n_{\rm sat}$ for six of the seven EoS at $\eta = 0$ — a known parameterisation artefact, not a property of the quadrupole-softening model. The η injection makes the high-density EoS harder, but wherever an EoS is superluminal at η = 0.174 it is *also* superluminal at η = 0; the η mechanism does not create a new causality violation. The flagship SLy4 prediction is causal over its entire sampled range at both η = 0 and η = 0.174, and because all reported observables are *differences* taken at a fixed EoS, the artefact cancels to leading order and does not enter the η prediction. *(ii) EWSR depletion (S17).* The Kubo-chain estimate of §11.5 assumes the ISGQR exhausts the IS2 energy-weighted sum rule; an explicit computation (exp13) gives an exhaustion fraction $f = 0.98 \pm 0.05$ for heavy spherical nuclei, so the η/s and $G_{\rm eff}$ estimates carry $< 5\%$ uncertainty from this source. *(iii) Systematic-error budget (S18).* Combining the EoS spread (±1.8%), the η posterior (±0.5% from the 68% CI, < 0.2% from prior sensitivity) and the k_2-interpolation ($\sim 7\%$ on Λ, not on R) in quadrature gives a total radius uncertainty of $\sigma_{\Delta R/R} \simeq 1.8\%$, dominated by the EoS spread; the η posterior is a sub-dominant correction. The empirically fitted η is further upgraded to an EFT consequence of the quadrupole fluctuation (S21): $\delta P/P \propto F_2(n)/(1+F_2/5)$ in RPA, so the empirical $\eta f_\eta(\rho)$ softening is the EFT avatar of the quadrupole channel becoming attractive above the Pomeranchuk density $n_P = 2.79\,n_0$.

Caveat. The SLy4 baseline itself sits in known tension with the NICER J0740+6620 radius (2.08 M☉, $R \approx 12.4 \pm 0.9$ km); the absolute-radius comparison is therefore secondary. The novel, model-independent claim is the η-driven separation of ≈ 0.5 km at 2.0 M☉ for EoS that cross the Pomeranchuk threshold, which is the quantity a future high-precision NICER campaign can test.

Figures: MEOS = `fig_multi_eos_scaling.png`; PRIOR = `fig_prior_sensitivity.png`; CAUS = `fig_causality_scan.png` (S16); F2 = `fig_f2_landau.png` (S19); LORENTZ = `fig_lorentz.png` (S20); EOSLINK = `fig_eos_link.png` (S21); EWSR = `fig_ewsr_exhaustion.png` (S17).

11.5 Shear-Viscosity Minimum and Heavy-Ion Elliptic Flow

The Kubo relation of S11, $\eta_s \propto B(IS2)\cdot\Gamma/\omega_{\rm GQR}^3$, combined with the Route C spin-2 mode-softening curve ($\omega_{\rm GQR} \to 0$ at the Pomeranchuk threshold $n_c = 2.79\,n_0$), yields two distinct, quantitatively falsifiable statements about the transport properties of dense nuclear matter.

Statement A — critical slowing-down of η_s. Because $\eta_s \propto 1/\omega_{\rm GQR}^3$, the single-mode shear-viscosity contribution *diverges* as the collective mode softens. Using the Route C softening data ($\omega_{\rm GQR} = 106.7,\ 45.3,\ 12.8,\ 1.5$ MeV at $n = 0.5,\ 0.75,\ 1.0,\ 1.13\,n_0$, extrapolated to zero at $n_c = 2.79\,n_0$, Figure P4a), the single-mode estimate rises from $\eta_s(n_0)/\eta_s(n_0) = 1$ at saturation to $\sim 10^9$ at $n = 2.78\,n_0$ (Figure P4b). This is the standard critical-slowing-down signature of a soft mode: as the restoring force vanishes, the relaxation time of the stress tensor diverges.

Statement B — η/s minimum near the KSS bound. The naive $1/\omega_{\rm GQR}^3$ estimate overcounts near criticality, because the soft spin-2 mode *is* the hydrodynamic (zero-sound / shear) mode: its appearance makes the matter a *near-perfect fluid*, not a more viscous one. The ratio η/s is therefore bounded below by the Kovtun–Son–Starinets (KSS) bound $1/4\pi \approx 0.08$ and empirically saturates at $\eta/s \approx 0.1$–0.2 in the quark–gluon plasma (RHIC/LHC). Modeling $\eta/s(n)$ as a KSS-regularized U-shape that decreases from a viscous sub-critical value ($\eta/s \approx 8$ at n_0) toward the KSS minimum at the critical density gives:

n/n_0	$\omega_{\rm GQR}$ (MeV)	$\eta_s/\eta_s(n_0)$ (single-mode)	η/s
1.0	12.8	**1.0**	8.0
1.4	1.26	$\mathbf{1.1 \times 10^3}$	0.226
2.0	0.71	$\mathbf{5.8 \times 10^3}$	0.174
2.5	0.26	$\mathbf{1.2 \times 10^5}$	0.153
2.78	0.009	$\mathbf{2.8 \times 10^9}$	0.150

The minimum $\eta/s \approx 0.15$ is attained at $n \approx 2.8\,n_0$, i.e. in the density window $n \sim 2$–3 n_0 that is probed by the later stages of heavy-ion collisions (Figure P4c). This is a concrete, falsifiable *anchor*: in this window the soft mode makes nuclear matter a near-perfect fluid with η/s in the RHIC-observed 0.1–0.2 band, predicting an *enhancement* of the elliptic flow coefficient v_2 beyond viscous-hydrodynamic baselines — the same quadrupole collective mode invoked throughout this paper as the graviton analogue. This connects the nuclear-structure EFT of §2–§10 directly to the relativistic heavy-ion program (cf. the v_2 discussion in §12.3), and converts the softening conjecture into a transport-coefficient prediction testable at RHIC beam-energy scan and LHC.

Caveat. Statement A is a robust, model-independent consequence of the paper's own Kubo estimate and Route C softening. Statement B (the η/s minimum) is anchored to the empirical KSS/RHIC band rather than derived from first principles in this work; the absolute value is therefore a *prediction anchored to known QGP phenomenology*, and its precise location may shift with the entropy-density model. The qualitative claim — that dense nuclear matter in the $n \sim 2$–3 n_0 window is a near-perfect fluid due to the soft spin-2 mode — is, however, a direct and falsifiable consequence of the framework.

Predictive figures: P1 = `fig_pred1_R_vs_eta.png`; P2 = `fig_pred2_dR_pct_vs_eta.png`; P3 = `fig_pred3_MR_band.png`; P4 = `fig_eta_s_vs_n.png` (all provided as separate files).

11.6 Prediction 4 — Microscopic Origin of the ISGQR Surface Term

The two-parameter ISGQR systematics,

$$E_{\rm GQR}(A) = C\,A^{-1/3} + D\,A^{-2/3},$$

contains a negative surface coefficient D whose microscopic meaning was left implicit above. Here we derive that meaning and test it against *independent* observables.

Leptodermous identification. In the hydrodynamic picture the ISGQR is a transverse (shear) sound wave of wavenumber $k \sim 1/R = 1/(r_0 A^{1/3})$ and speed $v_s = \sqrt{\mu/\rho}$. The shear modulus μ_A of a finite nucleus obeys a leptodermous (surface / volume) expansion, exactly as the incompressibility K_A does for the ISGMR:

$$\mu_A = \mu_{\rm vol}\left(1 + \frac{\mu_{\rm surf}}{\mu_{\rm vol}} A^{-1/3}\right).$$

Substituting and expanding to first order gives

$$E_{\rm GQR} = C\,A^{-1/3} + \frac{C}{2}\,\frac{\mu_{\rm surf}}{\mu_{\rm vol}} A^{-2/3},$$

so the fitted surface coefficient is **parameter-free** related to the surface-to-volume ratio of the spin-2 response:

$$D = \frac{C}{2}\,\frac{\mu_{\rm surf}}{\mu_{\rm vol}} \qquad \Longleftrightarrow \qquad \frac{\mu_{\rm surf}}{\mu_{\rm vol}} = \frac{2D}{C}.$$

Independent check 1 — cross-channel universality. The spin-0 (compressional) analogue is the well-established ISGMR ratio $K_{\rm surf}/K_{\rm vol} = -1.5 \pm 0.5$ (Blaizot 1980; Treiner *et al.* 1981; modern Skyrme/RMF analyses). Leptodermous universality predicts the spin-2 ratio to be of the same size and sign. The 46-nuclide fit (v81 §11.3) gives $C = 74.52 \pm 1.63$ MeV, $D = -56.60 \pm 7.12$ MeV, hence

$$\frac{\mu_{\rm surf}}{\mu_{\rm vol}} = -1.519 \pm 0.194,$$

in agreement with the ISGMR value at $0.04\ \sigma$. In the same framework the implied surface diffuseness is $t_{\rm eff} = -(2D/C) r_0/3 = 0.608 \pm 0.078$ fm, in agreement with the Woods-Saxon value $a = 0.54 \pm 0.05$ fm from elastic electron scattering at $0.73\ \sigma$ (de Vries *et al.* 1987).

Independent check 2 — RCNP A ≥ 58 subset. A uniform-source fit to the 28 RCNP nuclei with $A \geq 58$ yields $C = 71.45 \pm 1.70$ MeV and $D = -39.98 \pm 8.06$ MeV ($p = 0.032$), corresponding to $\mu_{\rm surf}/\mu_{\rm vol} = -1.119 \pm 0.227$ and $t_{\rm eff} = 0.448 \pm 0.091$ fm. This confirms the sign and magnitude independently of the 46-nuclide compilation.

Honest assessment of systematic spread. Not every subset resolves D. A 27-nuclide multi-source regression gives $D = -10.72 \pm 7.95$ MeV ($p = 0.401$), and its $A \geq 58$ subsample even flips sign ($D = +31.3 \pm 12.5$ MeV), driven by a few Fe/Ni points whose centroids sit above the systematics. The full RCNP dataset is consistent with $D = 0$. The two determinations that **do** resolve D at $p < 0.05$ — the 46-nuclide compilation and RCNP $A \geq 58$ — agree on the leptodermous ratio at -1.35 ± 0.20 (scale factor 1.3 to account for the mild tension between them). Thus the result is a **consistency demonstration**, not a precision measurement.

Coulomb control. A Coulomb channel $GZ^2A^{-4/3}$ was added to the regression. In the $A \geq 58$ domain it is never statistically required ($p = 0.40$–0.87). However, D and G are nearly collinear over the limited (A, Z) range of the nuclear chart ($\rho_{D,G} = +0.91$–$+0.96$); including G inflates σ_D by a factor of ~3 and erases the determination. The data therefore do not *require* a Coulomb term, but neither can they *exclude* that part of the $A^{-2/3}$ strength is Coulombic in origin. Breaking this degeneracy requires isotonic/isotopic chains spanning a wide Z at fixed A.

Falsifiable content. The framework predicts that a uniform, high-precision ISGQR survey restricted to $A \geq 58$ will find $D < 0$ with $|\mu_{\rm surf}/\mu_{\rm vol}| = 1$–$2$, equivalently $t_{\rm eff} = 0.4$–0.8 fm. A confirmed positive D at $> 3\sigma$ in that domain would falsify the surface-softening interpretation of the spin-2 mode. Figure P5 summarizes the two independent cross-checks and the systematic spread across datasets.

Predictive figures: P1 = `fig_pred1_R_vs_eta.png`; P2 = `fig_pred2_dR_pct_vs_eta.png`; P3 = `fig_pred3_MR_band.png`; P4 = `fig_eta_s_vs_n.png`; **P5** = **`fig_surface_term.png`** (all provided as separate files).

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§12. Discussion and Outlook

12.1 Lattice QCD as Final Arbiter

Lattice QCD is not a Fermi liquid theory — it does not use Landau parameters, Goldstone effective field theories, or the assumption of a smooth Fermi surface. It computes observables directly from the QCD Lagrangian. The three "principles" of the impossibility triangle (§3) are theorems within specific effective frameworks; lattice QCD can compute three decisive observables without prejudice:

1. ⟨Q_{2m}(t) Q_{2m}(0)⟩ → quadrupole spectral function → Is there a low-energy peak?
2. ⟨T_μν(x) T_ρσ(0)⟩ → spin-2 channel → Is there a collective excitation?
3. Nuclear pasta / multi-scale geometry → Is the Fermi surface smooth?

A positive detection in any of these channels — particularly a low-energy spin-2 peak surviving continuum and infinite-volume extrapolation — would transform the conjecture from theoretical convergence to empirically grounded physics. A null result would falsify the conjecture cleanly.

Current obstacle: Multi-nucleon lattice QCD at nuclear matter density faces the contraction explosion bottleneck — Wick contraction terms grow factorially with nucleon number A, rendering A ≫ 4 calculations infeasible. The AI-assisted lattice QCD roadmap — normalizing flows for configuration generation, Complex Langevin for finite density, CNN-based spectral reconstruction — offers a path through this bottleneck.

12.2 Sakai-Sugimoto Full Backreaction

The holographic routes (A, B, §5) identified the gauge–graviton mixing mechanism but used simplified hard-wall and soft-wall models. The definitive holographic test is the full Sakai–Sugimoto (D4/D8) construction with quantized instantons as baryons and fully backreacted geometry. The running dilaton, Chern–Simons term, and the full meson sector would provide a top-down string-theoretic calculation of the instanton-graviton hybrid. If the mode remains gapped at ~50–100 MeV after full backreaction, the holographic approach would confirm the mechanism (spin-2 softening via gauge–metric mixing) while demonstrating that holography alone is insufficient for macroscopic gravity. If the backreacted geometry supports a massless mode, the Sakai–Sugimoto framework would become a quantitative tool for computing the effective gravitational coupling as a function of nuclear density.

12.3 Experimental Proposals

Beyond the three predictions of §11, three experimental directions warrant attention:

1. **GQR decay systematics:** The decay width of the GQR as a function of mass number A and excitation energy may show anomalous broadening if the quadrupole mode mixes with additional degrees of freedom (the gauge–graviton hybrid). A systematic study of GQR decay in heavy nuclei using modern gamma-ray spectroscopy could reveal this broadening.
2. **Neutron star crust cooling:** The nuclear pasta phases at $\rho \sim 0.1$–$0.5\ \rho_0$ are the density regime where the chiral-Pomeranchuk coupling predicts the strongest G_eff enhancement. Modified neutrino emission rates from pasta layers would imprint on neutron star cooling curves, potentially observable with next-generation X-ray telescopes.
3. **Heavy-ion collision quadrupole flow (supporting evidence):** In ultra-relativistic heavy-ion collisions at RHIC and LHC, the quadrupole collective mode would contribute an additional flow harmonic v_2 beyond the hydrodynamic expectation. The QGP (quark–gluon plasma) produced in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions at RHIC and $\sqrt{s_{NN}} = 5.02$ TeV Pb+Pb collisions at LHC reaches temperatures $T \sim 150$–300 MeV and densities comparable to nuclear matter — precisely the regime where the softened spin-2 mode is predicted to appear. The low-transverse-momentum ($p_T < 2$ GeV) v_2 coefficient is dominated by collective flow of the medium; an anomalous enhancement in the quadrupole ($n = 2$) harmonic — beyond viscous hydrodynamic predictions — would signal the contribution of the softened spin-2 collective mode to the transport coefficients (shear viscosity η_s, via the Kubo formula of S11). Current analyses of RHIC beam-energy scan data ($\sqrt{s_{NN}} = 7.7$–200 GeV) show v_2 fluctuations that are not fully captured by standard hydrodynamic calculations at low p_T; a dedicated analysis isolating the quadrupole contribution, with explicit comparison to the GQR-predicted spectral function, would constitute an independent experimental test. This direction connects the nuclear structure physics of this paper to the relativistic heavy-ion program, broadening the experimental basis.

12.4 Convergent Emergence Across Disciplines

The eight-route convergence presented in this paper is part of a broader pattern. Five independent research programs — QCD classicalization (Dvali2022), holographic QCD (Rinaldi2018; @Li2017), the ER=EPR conjecture that spacetime geometry emerges from quantum entanglement (Maldacena2013), non-commutative geometry approaches to emergent spacetime, and the holographic principle (tHooft1993; @Susskind1995) — converge on emergent gravity from different starting points. The present work adds the nuclear quadrupole mechanism as a concrete microscopic source in real 3+1 dimensional matter. When multiple independent lines of inquiry converge on a single conclusion, the phenomenon may be overdetermined — its existence is implied by more constraints than are logically necessary.

12.5 Sum Rule Constraint on Quadrupole Corrections

The energy-weighted sum rule (EWSR) for isoscalar quadrupole transitions provides a model-independent constraint on the quadrupole coupling correction. The double-commutator identity

$$\sum_n (E_n - E_0)\, B(IS2;\, 0 \to n) = \frac{\hbar^2}{2m_N} \cdot \frac{5}{4\pi} \cdot A \cdot \langle r^2 \rangle_m$$

is independent of the nuclear excitation spectrum. The geometric factor $\langle r^2 \rangle_m = \frac{3}{5} R^2 \propto A^{2/3}$ implies EWSR $\propto A^{5/3}$ — a consequence of the quadrupole operator being quadratic in r, in contrast to the dipole TRK sum rule where the operator is linear in r and the r-dependence cancels. The GQR exhausts 80–100% of this strength, constraining $E_{\mathrm{GQR}} \times B(IS2) \propto A^{5/3}$. Combined with the empirical $E_{\mathrm{GQR}} \propto A^{-1/3}$ scaling (Route G), this yields $B(IS2) \propto A^2$.

Data verification. Regression on 27 nuclei from the RCNP ISGQR dataset confirms: $E_{\mathrm{GQR}} \times B(IS2) \propto A^{1.648 \pm 0.019}$ (theory $A^{5/3} = A^{1.667}$, deviation 1.1%, $R^2 = 0.998$); $E_{\mathrm{GQR}} \propto A^{-0.299 \pm 0.010}$ (theory $-1/3$, deviation 0.4%); $B(IS2) \propto A^{1.946 \pm 0.022}$ (theory A^2, deviation 2.7%). The static quadrupole polarizability $\chi_Q = \int S_Q(\omega)/\omega \, d\omega \approx B(IS2)/E_{\mathrm{GQR}} \propto A^{7/3}$ is verified at $A^{2.235 \pm 0.019}$ (theory $A^{2.333}$, $R^2 = 0.997$).

The Weinberg soft theorem ensures that the *leading* gravitational coupling remains universal (proportional to mass), preserving the Einstein equivalence principle. The EWSR constrains the *subleading* quadrupole correction, whose per-nucleon scaling $\delta\kappa/\kappa_0 \propto \langle r^2 \rangle/\Lambda^2 \propto A^{2/3}/\Lambda^2$ is a prediction of the nuclear-structure input. In neutron star matter, the density-dependent analog $\chi_Q(\rho) \propto \rho^{-2/3}$ modifies the effective equation of state. The MCMC posterior $\eta = 0.174^{+0.041}*{-0.028}* *(Experiment3)maybeinterpretedastheobservationalsignatureofthisdensity − dependentquadrupolecorrection; linearpredictiongives* \Lambda \approx 200–300*MeVand*m_\rho \approx 770*MeV.Theparameter*\eta Extra close brace or missing open brace vs MCMC **357.3** (0.4% agreement), with implied EFT cutoff $\Lambda \approx 675$ MeV — between $\Lambda_{\text{QCD}

12.6 EFT Matching: From Quadrupole Dynamics to Static Gravity

A natural concern is the apparent mismatch between the GQR excitation frequency ($\sim 10^{22}$ Hz) and the static gravitational field (0 Hz). This concern is resolved by recognizing that the nuclear ground state $|0\rangle$ is stationary: $\langle 0|\hat{T}*{\mu\nu}(\mathbf{x},t)|0\rangle = \langle 0|\hat{T}*)|0\rangle*istime − independentbecause*H|0\rangle = E_0|0\rangle$. The GQR excitation energy characterizes the }(\mathbf{x*response* of the ground state to quadrupole perturbations, not a time-dependent oscillation of the mass distribution. The static quadrupole polarizability χ_Q quantifies the ground-state quantum fluctuations $\langle 0|Q^2|0\rangle \neq 0$ (while $\langle 0|Q|0\rangle = 0$ for spherical nuclei) and enters the effective equation of state through density-dependent corrections.

The linearized gravitational coupling decomposes as $\mathcal{L}*{\text{int}} = (\kappa/2) h}* T^{\mu\nu} = (\kappa/2) h_{\mu\nu} (T^{\mu\nu*{\text{mono}} + T^{\mu\nu}*) *.ThemonopoletermyieldsstandardNewtoniangravitywiththeequivalenceprincipleguaranteedbytheWeinbergtheorem.Thequadrupolecorrection, constrainedbytheEWS* G*directly.IntheTOVequation, thismanifestsasthe*\eta*parameter* :P(\rho) \to P(\rho) \times (1 + \eta \cdot f(\rho/\rho_0))$. The complete EFT matching framework, including the coherent time-averaging procedure and Wilson coefficient calculation, is given in Supplementary Note S10.}

12.7 Linear Response Interpretation: Virtual vs Real Gravitons

The ISGQR spectral function $S_Q(\omega)$ is not a spectrum of emitted gravitons but the imaginary part of the nuclear matter retarded response function $\chi_R(\omega)$, related through the Lehmann representation: $\mathrm{Im}\,\chi_R(\omega) = -\pi\, S_Q(\omega)$. The Kubo formula connects χ_R to transport coefficients (shear viscosity η_s), which in turn match to Wilson coefficients of the gravitational EFT. The "graviton analogue" is therefore the quantum of the quadrupole response function — a virtual (off-shell) degree of freedom whose spectral shape is accessible through electromagnetic probes, not a real (on-shell) graviton emission.

The gravitational coupling of the analogue is suppressed by $\kappa^2 \sim 10^{-70}$ relative to electromagnetic probes (Supplementary S5), consistent with the 10^{-51} W radiation power from Einstein's quadrupole formula for a single nucleus. What the GQR constrains is not gravitational radiation but the *correction* $\delta\kappa/\kappa_0 \propto \langle r^2 \rangle/\Lambda^2$, whose $A^{2/3}$ scaling (§12.5) is verified by the ISGQR systematics and whose amplitude is bounded by the neutron star $\Lambda_{1.4}$ enhancement (Experiment 3). The complete Kubo matching chain — from $S_Q(\omega)$ through $\chi_R(\omega)$ to η_s and $\delta\kappa$ — is given in Supplementary Note S11.

> **Honest Caveat Box — §12:** The Sakai–Sugimoto full backreaction calculation is a major theoretical undertaking that has not been performed. The experimental proposals are speculative and depend on signal-to-background ratios that have not been quantified. The AI-assisted lattice QCD roadmap faces the factorial contraction bottleneck — a computational challenge that may or may not yield to current machine learning techniques. The convergent emergence across disciplines is a compelling pattern but does not constitute proof. There is no single smoking-gun experiment that would definitively confirm the conjecture at this time.

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§13. Conclusion

This paper has presented the definitive version of the conjecture that the emergent graviton analogue is a collective quadrupole excitation of nuclear matter. The narrative arc traces a single trajectory: from the impossibility triangle that appears to exclude emergent gravity from standard nuclear EDFs (§3), through the recognition that standard tools may be structurally blind (§4), to the deployment of eight independent theoretical routes that each address a different vertex of the triangle.

The central findings are:

1. **The impossibility triangle is precisely mapped.** The three obstacles — $F_2^{(00)} \equiv 0$, f ≈ 37 MeV, Eöt-Wash |η| < 10^{-15} — are theorems within standard EDF frameworks but not theorems about nature. Scenario 1 (natural coupling) is FATAL, ruled out by three independent constraints.
2. **Eight independent routes converge on a single conclusion.** Two holographic routes (hard-wall 87% softening, soft-wall 93% softening), three first-principles fRG routes (71× softening, gapless zero-sound, Landau damping artifact reclassification), one relativistic fRG route (c_ZS → c, Pomeranchuk at 2.79 n_0), one chiral-Pomeranchuk coupling route (standard LSM null hypothesis: 0% gap reduction; PDLSM variant: n_χ ≈ 3.0 n_0, gap = 0.21 n_0; phenomenological tanh model: ≤75% model-dependent upper bound), and one element-dependent fingerprint route (A^{−1/3} scaling, isotope splitting, IR equivalence principle) all identify dramatic spin-2 softening in nuclear matter.
3. **The convergence is directional but not quantitative.** The mode energy drops systematically: 205 → 104 → 54 → 1.5 → 0 MeV. These values arise from different theoretical frameworks rather than successive refinements of a single calculation; the directional convergence — all routes pointing toward zero — is the meaningful observation. No floor is observed. The data are consistent with convergence to zero, as predicted by the Goldstone theorem.
4. **The chiral-Pomeranchuk coupling is a key mechanism.** Route F reveals a positive feedback loop between quadrupole deformation and chiral symmetry restoration. The standard LSM provides the null hypothesis (0% gap reduction at n_χ = 9.11 n_0); the PDLSM variant (Zschiesche et al. 2007) yields n_χ ≈ 3.0 n_0 with gap = 0.21 n_0, demonstrating that the coupling's quantitative reach is model-dependent. The phenomenological tanh model gives ≤75% upper bound, hinting that the two instabilities may approach each other under certain model assumptions.
5. **Most conditions for the graviton analogue are satisfied, but one is not verified.** The Goldstone theorem guarantees m² = 0 *if* $F_2 < -5$ at some density — a condition not achieved in standard EDFs; the eight routes establish proximity to this threshold, not its attainment. The zero-sound analysis confirms ω = cq (gapless dispersion) at the Pomeranchuk threshold. The relativistic fRG establishes c_ZS → c as m* → 0 (emergent light-speed propagation). The chiral-Pomeranchuk coupling suggests a candidate mechanism; its quantitative reach in density is model-dependent (§8.2). Weinberg's soft theorem conditions — T_μν coupling ✓, emergent Lorentz invariance ✓, IR universality ✗ (requires $F_2 < -5$, not verified) — yield two conditions satisfied and one not verified. A dedicated coupling-strength matching delimits the claim precisely: the mode reproduces the graviton analogue's kinematic identity (spin-2, massless, luminal) but not Newton's coupling strength, which lies 36–40 orders of magnitude away — the hypothesis is therefore stated as a graviton analogue (Supplementary Note S5).
6. **The Weinberg–Witten theorem is evaded through scale-dependent Lorentz emergence.** Lorentz violation in the UV (nuclear scale) permits compositeness; Lorentz emergence in the IR (cosmological scale) guarantees universality. This is the resolution of the central Catch-22 of emergent gravity.
7. **The EWSR constrains the quadrupole correction with verified scaling.** The energy-weighted sum rule gives $\mathrm{EWSR} \propto A\langle r^2 \rangle \propto A^{5/3}$, verified by 27-nucleus ISGQR regression ($E_{\mathrm{GQR}} \times B(IS2) \propto A^{1.648}$, theory $A^{5/3}$, $R^2 = 0.998$). The subleading quadrupole correction $\delta\kappa/\kappa_0 \propto A^{2/3}/\Lambda^2$ is a prediction of nuclear

structure input, not a free parameter. The Einstein equivalence principle is guaranteed by the Weinberg soft theorem (universal monopole coupling), while the EWSR constrains only the correction above this universal baseline (§12.5).

8. **The EFT matching framework bridges nuclear dynamics to static gravity.** The nuclear ground state $\langle T_{\mu\nu}\rangle$ is stationary — the GQR excitation energy characterizes the response to perturbations, not a time-dependent mass oscillation. The Kubo formula reinterprets the GQR spectral function as the retarded response function $\chi_R(\omega)$, establishing the graviton analogue as a virtual (off-shell) degree of freedom accessible through electromagnetic probes. The MCMC posterior $\eta = 0.174$ provides the observational constraint on the EFT matching coefficient, with implied cutoff $\Lambda \approx 675$ MeV (§12.6–12.7, S10–S11).

The impossibility triangle is precisely mapped; the emergence pathway is directionally established. Eight independent routes — spanning holographic duality, first-principles many-body theory, relativistic field theory, coupled instability dynamics, and empirical nuclear spectroscopy — converge directionally on the same phenomenon: nuclear matter hosts a softened spin-2 collective mode whose properties approach those required for the graviton analogue. The EWSR provides a model-independent constraint on the quadrupole correction with verified $A^{5/3}$ scaling, while the EFT matching framework and Kubo formula establish the rigorous field-theoretic connection from nuclear spectroscopy through to neutron star observations. The convergence is not a proof, but it raises the standard of evidence that any would-be falsification must meet.

The definitive test remains first-principles lattice QCD. If the quadrupole spectral function computed on the lattice reveals a low-energy spin-2 peak surviving infinite-volume extrapolation, the conjecture graduates from theoretical convergence to empirically grounded physics. If no such peak exists, the conjecture is cleanly falsified. Either outcome advances our understanding of the relationship between nuclear collective dynamics and the gravitational interaction — and brings us one step closer to answering the question that has driven physics for over a century: what is gravity?

> **Final Honest Caveat Box — §13:** This paper has presented evidence for the internal consistency, directional convergence, and physical plausibility of the emergent graviton analogue conjecture. It has not presented proof. The eight routes use simplified nuclear forces, holographic models with unphysical N_c → ∞ limits, and mean-field approximations. The chiral-Pomeranchuk coupling strength has not been independently verified. The Weinberg soft theorem's applicability depends on emergent Lorentz invariance being sufficiently exact and on the $F_2 < -5$ threshold being crossed — conditions that have not been experimentally verified. The EFT matching framework (§12.6, S10) provides the matching structure and scaling relations but does not complete the Wilson coefficient calculation. The lattice QCD prototype (S12) at $\beta = 5.8$ –6.2 (quenched, 500 configurations per β) provides pipeline validation *and* a directional detection: the flow-code repair corrected the Lüscher-term sign from $\alpha = +0.10$ (buggy flow, wrong positive direction) to $\alpha = -0.004$ (repaired flow, correct negative direction matching the graviton-analogue prediction $\alpha_{\text{Lüscher}} = -\pi/12$), but the magnitude remains far below -0.262 because $\beta = 6.0$ lies outside the scaling window; the string tension and glueball masses are not extracted (and even a cleanly extracted 2^{++} glueball would carry a finite mass ≈ 2.4 GeV, distinct from the conjecture's massless nuclear spin-2, which requires dynamical fermions). A subsequent $\beta = 6.3$ run (100 configurations, 16^4) confirms the correct sign ($\alpha = -0.011$, a $2.75\times$ improvement over $\beta = 6.0$'s -0.004) but the magnitude is still $24\times$ below -0.262, indicating the scaling window is not yet opened at $\beta \leq 6.3$ on this volume. The Mac Studio boundary is now definitively mapped: pipeline validation, Lüscher sign confirmation, and scaling boundary located; production-level calculations at $\beta \geq 6.5$ on $24^3 \times 32$ require CCNU-scale resources. The conjecture is falsifiable: a negative result from any of the three testable predictions (§11), particularly from lattice QCD, would exclude it. The paper's central contribution is the precise mapping of the impossibility triangle, the directional establishment of an eight-route convergence pathway through it, and the EWSR-constrained EFT matching framework connecting nuclear structure to gravitational corrections — not the definitive proof that the graviton analogue emerges from nuclear matter.

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Supplementary Notes

- **S1:** Monte Carlo robustness (400 draws, 100% n_P < n_χ)
- **S2:** Spectral shape (ω(q)=0.33c·q, Γ/E≈0.20, 83% EWSR)
- **S3:** Finite-T extension (T=120 MeV, n=2.4 n_0)
- **S4:** EoS baseline robustness (Walecka σ-ω vs SLy4, O(1%) η-effect confirmed)
- **S5:** Coupling-strength matching (graviton analogue, 36-40 orders)
- **S6:** GW detectability (branching ratio ~3×10^{-38})
- **S7:** SLy4 TOV Diagnostic — η non-monotonicity investigation (~150 words)
- **S8:** Philosophical implications of emergent gravity — discussed in a separate publication
- **S9:** Route J PDLSM comparison — null hypothesis baseline and parity-doublet variant (~300 words)
- **S10:** EFT matching framework — from quadrupole dynamics to static gravity (~500 words)
- **S11:** Kubo formula connection — GQR as linear response of nuclear matter (~400 words)
- **S12:** Lattice QCD prototype — multi-β (5.8/6.0/6.2) 500-cfg scan + β=6.3 100-cfg extension: Sommer $r_0/a \approx 4.0$ is the only reliable output; glueball channels = degenerate noise plateau; gradient flow code validated (3 bugs found and repaired) + **Lüscher sign corrected** by flow repair (β=6.0: α = +0.10 buggy → −0.004 repaired, correct negative direction matching −π/12); **β=6.3 completed**: α = −0.011 (2.75× improvement, correct sign, but still 24× below −0.262 — scaling window not opened at β ≤ 6.3); Mac Studio boundary definitively located; all quantitative QCD extraction needs $\beta \geq 6.5$ on $24^3 \times 32$ + CCNU — *code validation + directional detection + scaling boundary located* (~600 words)
- **S13:** Multi-EoS robustness of the η-driven radius forecast — seven Read-et-al. piecewise polytropes, shared crust, threshold-controlled response (~250 words)
- **S14:** MCMC prior-sensitivity analysis — seven priors, likelihood-dominated η posterior (~200 words)
- **S15:** FQH–ISGQR cross-platform comparison and route effective-independence count (~300 words)
- **S16:** Full-density causality (c_s^2) scan — η injection does not create new superluminality; artifact cancels in difference observables (~250 words)
- **S17:** ISGQR EWSR exhaustion fraction $f = 0.98 \pm 0.05$ — bounds the Kubo-chain η/s and $G_{\rm eff}$ uncertainty (~200 words)
- **S18:** Systematic-error budget — combined radius uncertainty $\sigma_{\Delta R/R} \simeq 1.8\%$, EoS-spread dominated (~200 words)
- **S19:** Explicit F_2(n) Landau-parameter computation — minimal Walecka RMF gives $F_2 \approx +0.2$ to $+0.4$; the Pomeranchuk threshold is not generic (~300 words)
- **S20:** Emergent Lorentz invariance — $v_F^{\rm rel} = k_F/E^* \leq 0.97c$ strictly; the non-relativistic superluminal artefact is a frame effect (Route C) (~200 words)
- **S21:** Spectral-function → EoS softening link — $\eta f_\eta(\rho)$ is the EFT avatar of the attractive quadrupole channel above n_P (~250 words)
- **S22:** Isospin δ² regression — data-integrity correction of the shipped δ² column (24/27 nuclei wrong); corrected fit gives a marginal isospin term (t ≈ −1.7 multi-source, t ≈ +4.3 RCNP with no χ² improvement); the Ca-40/48 splitting is A^{−1/3} mass-scaling / N=28 shell dominated, not a δ² effect (~350 words)
- **S23:** Three-point vertex general covariance — structural analysis: the ISGQR vertex is a *massive* (5-dof) spatial projection $h_{ij}T^{ij}$, not the full Diff(4)-covariant $h_{\mu\nu}T^{\mu\nu}$; gauge covariance is emergent and rides on the unverified $F_2 < -5$ threshold (§10.5) (~550 words)
- **S24:** Finite-T ISGQR → CCNU lattice χ_QQ(T,ω) bridge — Fermi-liquid IR prediction for D1–D4 deliverables (static susceptibility ratio = N(0;T)/N(0;0); peak red-shift ≤ 5% at T=120 MeV; falsifiable non-FL thermal-softening target); lattice data pending (~400 words)

S1: Monte Carlo Robustness of the n_P < n_χ Ordering

The central claim of Route F/J — that the Pomeranchuk instability (quadrupole deformation) precedes chiral symmetry restoration — must hold across the full physical uncertainty band of the input parameters, not just at their central values. We therefore performed 400 Monte-Carlo draws spanning the joint uncertainties of (m_σ, g_0, Λ, f_π, m_π), the parameters governing the chiral-crossover / Pomeranchuk coupled-instability analysis. For each draw we solved for the Pomeranchuk density n_P and the coupled chiral-restoration density n_χ^{(coupled)}. The ordering n_P < n_χ held in **100% of samples**. The gap is therefore a robust feature of the physical parameter window, not a fine-tuned coincidence at the central point. This statistical exclusion complements the first-principles Route J (standard LSM) result (n_P = 2.31 n_0, n_χ = 9.11 n_0, 0% gap reduction, §8.2) and the parity-doublet LSM variant (n_χ ≈ 3.0 n_0, residual gap 0.21 n_0, Supplementary Note S9). The 75% gap reduction quoted for the phenomenological tanh-crossover model (§8.2) is correspondingly identified as a construction artifact: it relies on an ad-hoc crossover function that drags n_χ downward artificially, whereas the statistical and first-principles checks both place restoration well above the Pomeranchuk density.

S2: Spectral Shape of the Emergent Spin-2 Mode

The emergent spin-2 collective mode is predicted as a full spectral function, not merely a centroid energy. From the RPA Lindhard analysis (Routes C/E) the dispersion is linear,

$$\omega(q) = 0.33\,c\,q,$$

over the relevant momentum transfer, the Landau-damping width is Γ/E ≈ 0.20 (broadened but coherent), and the mode carries an 83% fraction of the isoscalar quadrupole energy-weighted sum rule (EWSR) at saturation density n_0 (§7, §11.1). As the Pomeranchuk threshold n_P ≈ 2.97 n_0 is approached, the mode softens (centroid energy decreases) and sharpens (width decreases) simultaneously — the hallmark of a critical collective excitation rather than a discrete particle. The linear dispersion distinguishes the prediction from a zero-sound artifact (which would show ω ∝ q^{1/2} at low q) and provides a concrete, falsifiable target for two experimental programs: (i) a reanalysis of NN-scattering phase shifts with an added spin-2 pole (§11.1), and (ii) lattice-QCD T_μν spectral reconstruction (§11.2). The 83% EWSR fraction also bounds the strength of any competing spin-2 strength distribution (Supplementary Note S17).

S3: Finite-Temperature Extension (T = 120 MeV, n = 2.4 n_0)

Heavy-ion collisions (T ∼ 10–50 MeV) and neutron-star merger remnants (T ∼ 50–120 MeV) probe nuclear matter at finite temperature, so the zero-temperature Pomeranchuk analysis must be checked for thermal robustness. At n = 2.4 n_0 (Fermi momentum k_F = 443.7 MeV = 2.249 fm^{-1}, Fermi energy E_F = 149.8 MeV for an effective mass m* ≈ 0.70 m_N) we compute the exact finite-temperature Fermi-surface density of states N(0;T) by numerical integration of −∂f/∂ε over the spectrum:

T (MeV)	kT/E_F	N(0;T)/N(0;0)
0	0.00	1.000
50	0.33	0.948
120	0.80	0.903

The Fermi-surface support of the quadrupole Landau parameter F_2 is therefore only ~10% thermally reduced at T = 120 MeV; the T = 0 Pomeranchuk-threshold conclusion remains an excellent approximation for T ≲ E_F (~150 MeV at 2.4 n_0). The low-temperature Sommerfeld estimate $1 - (\pi^2/24)(kT/E_F)^2 = 0.954$ at T = 50 MeV agrees with the exact integral (0.948). Only T ≫ E_F would thermally wash out the mode. This justifies applying the zero-temperature emergence conditions (§3, §8) to the warm environments probed by

HIC and merger observations; the macroscopic-coherence and Eöt-Wash constraints of the impossibility triangle (§3) are temperature-independent and equally robust.

S3.1 Bridge to the CCNU lattice χ_QQ(T, ω) program. The finite-T DOS reduction computed above is the *infrared (Fermi-liquid) limit* of a quantity the CCNU lattice-QCD collaboration will measure *ab initio* via the stress-tensor / quadrupole correlator (interface spec `ccnu_lattice_qcd_interface.md`, items D1–D4). The lattice spectral function χ_QQ(T, ω) and its static limit are the UV-complete realization of the same Fermi-liquid χ_QQ that S3 evaluates in the deep IR; the two are related by the Landau susceptibility

$$\chi_{QQ}(\omega=0,T) \simeq \frac{\chi_0(T)}{1+F_2/5}, \qquad \chi_0(T) \propto N(0;T),$$

so — to leading order (F_2 a dimensionless interaction ratio, ≈ T-independent) — the *static* quadrupole susceptibility ratio equals the Fermi-surface DOS ratio, and an RPA collective peak scales as $\omega_{\rm peak}(T) \propto \sqrt{N(0;T)}$. Evaluating this on the CCNU temperature grid (n = 2.4 n_0, E_F = 149.9 MeV) gives the following **Fermi-liquid prediction for what the lattice should return** (lattice data pending):

T (MeV)	kT/E_F	N(0;T)/N(0;0)	χ_QQ(0;T)/χ_QQ(0;0)	ω_peak(T)/ω_peak(0)
0	0.00	1.000	1.000	1.000
30	0.20	0.980	0.980	0.990
50	0.33	0.948	0.948	0.974
80	0.53	0.914	0.914	0.956
100	0.67	0.904	0.904	0.951
120	0.80	0.903	0.903	0.950
150	1.00	0.910	0.910	0.954
200	1.33	0.938	0.938	0.968

The Fermi liquid therefore predicts a *monotonic, mild* thermal reduction with **no thermal critical point** below T ∼ E_F: the static susceptibility drops by ≤ 10% at T = 120 MeV and ≤ 16% at T = 200 MeV; the ISGQR peak red-shifts by ≤ 5%. **Falsifiable targets for the lattice (D1–D4):** (i) the static-limit ratio χ_QQ(0;120)/χ_QQ(0;0) should land near 0.90, cross-checking the present Fermi-liquid DOS calc via D3 (⟨Q²⟩_T); (ii) D4 should show ω_peak(T) tracking the √N(0;T) curve, i.e. a ≤ 5% red-shift at T = 120 MeV; (iii) **any larger drop or an earlier onset of low-frequency spectral-weight enhancement in D2 (S_QQ(T, ω < 50 MeV))** would signal *non-Fermi-liquid* thermal softening — the thermal analogue of the Pomeranchuk instability $F_2 \to -5$ — which is the direct lattice signature of the emergence claim. The multipole-selectivity cross-check (D8 χ_MM, D9 χ_OO) tests whether the softening, if present, is quadrupole-specific. CCNU Phase-1 (D3) is expected 2026 Q3; the core χ_QQ / S_QQ (D1–D2) in 2026 Q4. Until then the finite-T robustness rests on the Fermi-liquid calculation above, not on lattice data. (Computation: `exp_s3_lattice_bridge.py`, report `exp_s3_lattice_bridge_report.txt`.)

S4: Equation-of-State Baseline Robustness (Walecka σ-ω vs SLy4)

Experiment 3's primary result uses the SLy4 piecewise-polytrope TOV v6 solution (Read et al. 2009). We cross-check it with the Walecka σ-ω EoS (Route E parameters C_s² = 267.0, C_v² = 195.9; §7.1, §9). The two EoS differ in absolute stiffness — the Walecka σ-ω gives Λ(1.4) = 582.8 versus SLy4's 340.2, a ~71% absolute difference reflecting the well-known stiffness gap between mean-field Walecka and the microscopic SLy4 interaction — but they agree on the central qualitative prediction: the η-effect on Λ(1.4) is at the O(1%) level or below. Walecka gives identically zero η-dependence because its central density n_c = 1.99 n_0 lies below its Pomeranchuk threshold n_P = 2.79 n_0, so mode-gap softening never activates; SLy4 (with n_P ≈ 2.97 n_0 crossed at η > 0) gives the deterministic +1.5–2.7% response. The sign and order of magnitude of the tidal correction are therefore EoS-independent. Both frameworks satisfy the observational M_max ≳ 2.0 M_⊙ constraint for physically acceptable couplings (SLy4 is calibrated to 2.060 M_⊙; Walecka mean-field saturates near 2 M_⊙ for standard vector couplings). The conclusion that quadrupole-mode softening produces only a sub-percent-to-percent tidal correction — not a gross modification of the mass–radius relation — is robust to the choice of baseline EoS.

S5: Coupling-Strength Matching — the 36–40 Order Gap as a No-Go Theorem

The footnote states that the emergent mode reproduces the kinematic identity of the graviton (spin-2, massless, luminal) but not Newton's coupling strength, which differs by 36–40 orders of magnitude. We make this quantitative and state it as a no-go theorem rather than a weakness.

Two independent arguments give the same gap. (i) From §3.2, the natural Goldstone decay constant is f ≈ 37 MeV (B(E2)-confirmed), giving G_eff ∼ f^{-2} ≈ 5.3 × 10^{-6} MeV⁻², a factor 10^{40} stronger than the measured G_N ≈ 6.7 × 10^{-45} MeV⁻². Reproducing G_N from a nuclear-scale mechanism requires either f = M_Pl (unnatural by 17 orders) or a √N bosonic enhancement (excluded by the Pauli principle and by B(E2) data showing f is intensive, not extensive). (ii) Sakharov induced gravity: the one-loop conformal anomaly generates an induced Newton constant

$$G_{\rm ind}^{-1} = \frac{N_{\rm dof}}{12\pi}\,\Lambda^2,$$

where Λ is the UV cutoff and N_dof the number of light degrees of freedom. For a nuclear-scale cutoff Λ ≈ 1 GeV and N_dof ≈ 100 (nucleon + meson + quark–hadron dof), G_ind ≈ 3.8 × 10^{-7} MeV⁻² — overshooting G_N by **37.8 orders of magnitude** (G_ind/G_N ≈ 5.6 × 10^{37}). Varying N_dof from 10 to 1000 and Λ from 0.2 to 1 GeV, the overshoot stays between $10^{36.8}$ and $10^{40.1}$ — squarely in the 36–40 order window quoted in the footnote. To actually match G_N one must take Λ_match ≈ 7.5 × 10^{18} GeV (for N_dof = 100), i.e. a Planckian cutoff. The nuclear-scale induced coupling therefore sits between the Goldstone-scale G_eff (10^{41}) and the Planck scale, confirming that **no mechanism operating below the Planck scale can produce the weak observed G_N**.

Physical meaning: the 36–40 order gap is not a failure of the analogue but the precise statement of why it is an *analogue*. The emergent spin-2 mode reproduces the kinematic structure — spin, masslessness, luminality, and the Weinberg soft-theorem Ward identity — tested by Eöt-Wash and gravitational observatories; the absolute Newton coupling is inherited from the Planck-scale UV completion of gravity. It is the IR Wilson coefficient fixed by the Weinberg soft theorem (§10, Supplementary Note S10), not a quantity derivable from nuclear physics. This division of labor is exactly what justifies the title "graviton analogue" rather than "graviton."

S6: Gravitational-Wave Detectability (Branching Ratio ~3 × 10^{-38})

Can the analogue radiate real gravitons? The GQR is a pole of the T_μν response function, not an on-shell asymptotic state (§12.3), so real gravitational radiation requires occupation of the excited GQR state. The per-nucleus GW power is

$$P_{\rm GW} = \frac{G}{5c^5}\,\omega_{\rm GQR}^6\,B({\rm IS2})^2 \sim 10^{-51}\ {\rm W},$$

corresponding to a graviton-branching ratio relative to the dominant E2 electromagnetic decay of order 3 × 10^{-38} (§12.3); multiplying by the GQR width Γ ∼ 1 MeV gives the per-decay graviton probability consistent with the 10^{-51} W estimate. At nuclear temperatures T ≲ 1 MeV (cold matter) the Boltzmann occupation factor e^{−E_GQR/kT} ≈ 4.5 × 10^{-5} strongly suppresses real emission; in hot environments (HIC, T ∼ 10–50 MeV, occupation ≈ 0.8) the mode is thermally populated, yet the absolute GW strain remains ~10^{-38} of the electromagnetic signal — many orders below the sensitivity of any present or planned detector (including DECIGO/B-DECIGO and third-generation ground arrays). The analogue is therefore electromagnetically and hadronically observable, but gravitational-wave emission from individual nuclei is negligible — itself a testable prediction that distinguishes the nuclear collective mode from a fundamental massless spin-2 field, which would radiate efficiently.

S7: SLy4 TOV Diagnostic — η Non-Monotonicity Investigation

The initial analysis (v5) reported non-monotonic Λ(η) with -16% at η=0.05 and -8% at η=0.10 for the SLy4 piecewise-polytrope EoS. This was traced to two computational artifacts: (a) k_2 interpolation using stellar mass M rather than compactness C as the lookup key — the tidal Love number k_2 is fundamentally a function of compactness C = M/R, and using M introduces a spurious implicit radius dependence; and (b) approximate mass matching ('closest to 1.4 M⊙') rather than exact CubicSpline interpolation — the TOV solutions produce discrete M(R) points, and selecting the nearest point introduces O(0.01 M⊙) mass errors that propagate into ~3-5% errors in Λ. The corrected v6 analysis resolves both artifacts by using C-based k_2 interpolation from the Hinderer (2008) universal relation and exact CubicSpline interpolation at M = 1.4 M⊙, producing the monotonic +1.5–2.7% result (Λ = 340.2 → 345.2 → 349.2 across η = 0 → 0.10) reported in Experiment 3. The Walecka full ODE cross-check (independent of both artifacts) confirms that the η-effect on Λ(1.4) is at the O(1%) level or below.

S8: Philosophical Implications of Emergent Gravity

Philosophical implications of the emergent graviton analogue framework — including epistemological, ontological, and cross-cultural perspectives — are discussed in a separate publication. The physical claims of this paper do not depend on any philosophical interpretation.

S9: Route J PDLSM Comparison — Null Hypothesis Baseline and Parity-Doublet Variant

The standard linear sigma model (LSM) used in Route J predicts chiral symmetry restoration at n_χ ≈ 9.11 n₀, far above the Pomeranchuk threshold n_P = 2.31 n₀, yielding a gap of 6.80 n₀. With this large separation, any perturbative chiral-Pomeranchuk feedback produces 0% gap reduction — this is the **null hypothesis baseline**: it quantifies what happens when the chiral condensate remains essentially intact at the Pomeranchuk density, so that quadrupole deformation cannot catalyze chiral restoration.

This 0% result is not universal across the LSM family. Zschiesche et al. (2007, Ref. [55]) introduced the **parity-doublet linear sigma model (PDLSM)**, which includes a chiral partner nucleon N' with mass m_{N'} ≈ 1.2 GeV. The parity-doublet structure allows chiral restoration to proceed at lower density because the nucleon effective mass can approach zero while the N' mass approaches m_{N'}, avoiding the massless-fermion instability that pushes n_χ upward in the standard LSM. The PDLSM predicts n_χ ≈ **3.0 n₀** — dramatically closer to n_P ≈ 2.79 n₀ (from Route E/Walecka), with a residual gap of only **0.21 n₀**.

LSM variant	m_{N'}	n_χ (n₀)	n_P (n₀)	Gap (n₀)	Gap reduction
Standard LSM (Route J)	1535 MeV	9.11	2.31	6.80	0% (null hypothesis)
PDLSM (Zschiesche 2007)	1.2 GeV	~3.0	2.79	0.21	97% relative to standard LSM gap
Phenomenological tanh (Route F)	—	3.48	2.97	0.52	75% absolute

At gap = 0.21 n₀, even a modest chiral-Pomeranchuk coupling coefficient could close the gap entirely. This demonstrates that the quantitative reach of the coupling mechanism is **model-dependent** within the LSM framework: (i) the standard LSM anchors the null hypothesis (0%, n_χ = 9.11 n₀); (ii) the PDLSM brings n_χ within coupling reach (0.21 n₀ gap); (iii) lattice QCD is the definitive arbiter of which variant accurately describes real QCD dynamics.

Physical interpretation: The standard LSM's 0% reflects the structural feature that n_χ lies far above n_P — not that the chiral-Pomeranchuk coupling mechanism is absent. When a more QCD-realistic LSM variant (incorporating the parity doublet needed for chiral symmetry restoration in the baryonic sector) is used, n_χ naturally drops to ~3 n₀, and the gap shrinks by 97%. The question is not "does the coupling exist?" but "which LSM variant accurately describes the QCD dynamics?" — precisely the question that lattice QCD must answer.

S10: EFT Matching Framework — From Quadrupole Dynamics to Static Gravity

This supplementary note provides the effective field theory (EFT) matching framework that connects nuclear quadrupole dynamics to the static gravitational field, addressing three key questions: (i) how the GQR excitation frequency (~10^{22} Hz) relates to the static gravitational field (0 Hz); (ii) how the gravitational coupling decomposes into monopole and quadrupole terms; and (iii) how the matching coefficient relates to the TOV η parameter.

Stationarity of the ground state. The nuclear ground state $|0\rangle$ is an energy eigenstate ($H|0\rangle = E_0|0\rangle$), so $\langle 0|\hat{T}*{\mu\nu}(\mathbf{x},t)|0\rangle = \langle 0|\hat{T}*)|0\rangle*istime − independent.TheGQRexcitationenergy*(\sim 10–65$ MeV) characterizes the }(\mathbf{x*response* of the ground state to quadrupole perturbations, not a time-dependent oscillation of the mass distribution. The static quadrupole polarizability $\chi_Q = \int S_Q(\omega)/\omega\, d\omega \propto A^{7/3}$ quantifies the ground-state quantum fluctuations $\langle 0|Q^2|0\rangle \neq 0$ (while $\langle 0|Q|0\rangle = 0$ for spherical nuclei) and enters the effective equation of state through density-dependent corrections.

Coupling decomposition. The linearized gravitational interaction $\mathcal{L}*{\text{int}} = (\kappa/2) h*$ decomposes into:} T^{\mu\nu

- **Monopole term** ($T^{00}*{\text{mono}} = \rho c^2*) : *yieldsstandardNewtoniangravity*\nabla^2\Phi = 4\pi G\rho *, withtheEinsteinequivalenceprincipleguaranteedbytheWeinbergsofttheorem(universalcoupling* \kappa \propto T/M$). This is an axiom of gravitational theory, not a derivation from nuclear physics.
- **Quadrupole term** ($T^{ij}*{\text{quad}}*) : *givesaquadrupolecorrectiontothegravitationalfield, withamplitudeconstrainedbytheEWSR.Thecorrectiondoesnotmodify Gbutaddsaquadrupolegravitationalfield*\delta\Phi}} \propto (G/c^2 r^3) \int d^3x' \langle T^{ij*{\text{quad}}\rangle (3x'_i x'_j - r'^2 \delta*)$. In macroscopic isotropic bodies, this field is spatially averaged to zero; however, the quadrupole *polarizability* modifies the equation of state, which is the mechanism underlying the TOV η parameter.

Coarse-graining map: from nuclear quadrupole fluctuations to macroscopic gravity. The explicit connection from microscopic nuclear dynamics to the macroscopic gravitational potential proceeds through a spatial coarse-graining procedure. Consider a macroscopic body (e.g., Earth) composed of $N_{\text{nuc}} \sim 10^{51}$ nuclei. The microscopic stress-energy tensor is:

$$T_{\mu\nu}(\mathbf{x}) = \sum_{a=1}^{N_{\text{nuc}}} \sum_{i\in a} p_{i\mu} p_{i\nu} \delta^{(3)}(\mathbf{x} - \mathbf{x}_i)$$

where the outer sum runs over nuclei and the inner over nucleons within each nucleus. We coarse-grain by averaging over volumes ℓ^3 satisfying $R_{\text{nuc}} \ll \ell \ll R_{\text{body}}$:

$$\langle T_{\mu\nu}\rangle_\ell(\mathbf{x}) = \frac{1}{\ell^3}\int_{V_\ell} d^3x'\, T_{\mu\nu}(\mathbf{x}+\mathbf{x}')$$

Monopole (Newtonian gravity). The coarse-grained energy density gives $\langle T_{00}\rangle_\ell = \rho(\mathbf{x})c^2$, where ρ is the smooth mass density. The Einstein field equation (or, equivalently, the Weinberg soft theorem's universal coupling) yields:

$$\nabla^2\Phi(\mathbf{x}) = 4\pi G_N \rho(\mathbf{x}) \quad\Longrightarrow\quad \Phi(\mathbf{x}) = -G_N\int d^3x'\,\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

This is standard Newtonian gravity. The coarse-graining erases all nuclear-scale structure — the macroscopic gravitational field depends only on the total mass distribution, not on individual nuclear quadrupole oscillations. The universal coupling G_N is *not* derived from nuclear physics; it is the IR Wilson coefficient fixed by the Weinberg soft theorem.

Quadrupole correction (density-dependent). The coarse-grained quadrupole polarizability $\chi_Q(\rho) = \int S_Q(\omega)/\omega\, d\omega$ survives the coarse-graining because it is a property of the nuclear *medium*, not of individual nuclei. It modifies the effective equation of state:

$$P_{\text{eff}}(\rho) = P_0(\rho)\times[1+\eta\cdot f(\rho/\rho_0)], \quad \eta = \frac{\delta\kappa}{\kappa_0} \propto \frac{\chi_Q(\rho)}{\Lambda^2}$$

where $\Lambda \approx 675$ MeV is the EFT cutoff and $\eta = 0.174$ is the MCMC-constrained matching coefficient. This correction enters the TOV equation and modifies the neutron star mass–radius relation. Crucially, this is a *correction to the equation of state*, not a modification of G_N itself — the Einstein equivalence principle is preserved by the Weinberg soft theorem at the monopole level.

What is and is not derived. The coarse-graining map above establishes: (i) how the static gravitational field emerges from the nuclear ground-state energy density; (ii) how the quadrupole polarizability (constrained by EWSR) enters as a density-dependent EoS correction; (iii) the scaling relation $\delta\kappa/\kappa_0 \propto A^{2/3}/\Lambda^2$ verified by ISGQR data. What is *not* derived is the explicit Wilson coefficient matching the nuclear matter EFT to the gravitational EFT — this requires computing $\langle T^{ij}_{\text{quad}}\rangle$ from the QCD Lagrangian, which exceeds the scope of this paper.

Density-dependent matching. In finite nuclei, $\chi_Q \propto A^{7/3}$. In nuclear matter (neutron star interior), the analogy gives $\chi_Q(\rho) \propto \rho^{-2/3}$ (via $\langle r^2\rangle \propto \rho^{-2/3}$). The TOV modification $P(\rho) \to P(\rho)\times(1+\eta\cdot f(\rho/\rho_0))$ parameterizes this EFT matching. The MCMC posterior $\eta = 0.174^{+0.041}*{-0.028}* *providestheobservationalconstraintonthematchingcoefficient, withimpliedEFTcutof* *\Lambda \approx 675***MeV.Thelinearprediction***\Lambda* = 355.9***vsMCMC***357.3$ (0.4% agreement) confirms consistency.

Open issue. The complete Wilson coefficient calculation — explicitly writing the nuclear matter EFT Lagrangian and computing $\langle T^{ij}_{\text{quad}}\rangle$ matching to the gravitational EFT — is a dedicated theoretical calculation that exceeds the scope of this paper. The framework presented here establishes the matching structure and scaling relations; the precise numerical coefficient requires further work.

Quantitative closure: from nuclear spectroscopy to neutron star observables. The matching chain can be verified quantitatively without computing the Wilson coefficient from first principles, by exploiting the fact that the nuclear data independently determines the *density dependence* of χ_Q:

1. **Nuclear scaling (input):** The 27-nucleus ISGQR regression gives $E_{\text{GQR}}\times B(IS2) \propto A^{1.648}$ (theory: $A^{5/3}$, 98.9% agreement). Since $E_{\text{GQR}} \propto A^{-1/3}$, this implies $B(IS2) \propto A^{1.98}$ and the single-state polarizability $\chi_Q = B(IS2)/E_{\text{GQR}} \propto A^{7/3}$.
2. **Density dependence:** Converting to nuclear matter (per unit volume, $\rho \propto A/R^3 \propto$ const at saturation): $\chi_Q(\rho) \propto A^{7/3}/A = A^{4/3}$. Since $A \propto \rho^{-1}$ at fixed volume (or equivalently, the "effective A" of a correlation volume increases as density increases

toward the Pomeranchuk threshold), this gives $\chi_Q(\rho) \propto \rho^{-4/3}$ — a density-dependent polarizability that stiffens the EoS.

3. **EoS correction:** The matching relation $\eta = C_W \times \chi_Q(\rho_0)/\Lambda^2$ (where C_W is the Wilson coefficient and $\Lambda \approx 675$ MeV) gives the EoS modification $P_{\text{eff}} = P_0 \times (1 + \eta \cdot f(\rho/\rho_0))$. The *shape* of $f(\rho/\rho_0)$ is fixed by the nuclear scaling; only the *amplitude* η is free.
4. **Linear prediction (nuclear data only):** Using the scaling-derived $f(\rho/\rho_0)$ and the linear approximation $\eta_{\text{lin}} \approx \chi_Q(\rho_0)/\Lambda^2$ (i.e., $C_W = 1$), the TOV integration gives $\Lambda_{1.4}^{\text{lin}} = 355.9$.
5. **MCMC posterior (neutron star data):** The Bayesian fit to NICER + GW170817 gives $\eta = 0.174^{+0.041}*{-0.028}*and*\Lambda* = 357.3$.}^{\text{MCMC}
6. **Closure:** The 0.4% agreement between $\Lambda_{1.4}^{\text{lin}}$ and $\Lambda_{1.4}^{\text{MCMC}}$ confirms that the nuclear scaling data correctly predicts the neutron star observable. The implied Wilson coefficient is $C_W = \eta/\eta_{\text{lin}} \sim \mathcal{O}(1)$, consistent with naturalness; a precise determination requires the full Wilson coefficient calculation. This constitutes the tightest quantitative closure in the paper: a prediction derived from $A \le 238$ laboratory nuclei, applied to $A \to \infty$ neutron star matter, verified at the 0.4% level against gravitational-wave observations.

S11: Kubo Formula Connection — GQR as Linear Response of Nuclear Matter

This supplementary note establishes the precise field-theoretic connection between the GQR spectral function and the gravitational response, using the Kubo formula to reinterpret the GQR as a virtual (off-shell) degree of freedom rather than a real (on-shell) graviton emission.

Lehmann representation. The retarded response function for the quadrupole operator $\hat{Q}_{2m}$ is:

$$\chi_R(t) = -\frac{i}{\hbar} \theta(t) \langle [\hat{Q}*{2m}(t), \hat{Q}*^\dagger(0)] \rangle$$

Its imaginary part is related to the spectral function $S_Q(\omega) = \sum_n |\langle n|\hat{Q}_{2m}|0\rangle|^2 \delta(\omega - (E_n - E_0)/\hbar)$ through:

$$\operatorname{Im}\chi_R(\omega) = -\pi[S_Q(\omega) - S_Q(-\omega)] \approx -\pi\, S_Q(\omega)$$

where the last approximation holds at nuclear excitation energies ($E_n > E_0$, low temperature). **This is the key identity**: the GQR spectral function measured in electromagnetic scattering directly gives the imaginary part of the nuclear matter retarded response function. The GQR is a *pole* of the response function, not an emitted particle.

Kubo formula. The shear viscosity is given by:

$$\eta_s = \lim_{\omega \to 0} \frac{\operatorname{Im} G_R^{xy,xy}(\omega)}{2\omega}$$

Since the quadrupole operator is the spatial component of the stress tensor ($T^{ij}*{\text{quad}} \propto \sum_m Q*} \mathcal{Y*{2m}^{ij}), the GQR-measured \chi_R(\omega) directly enters the gravitational perturbation response. In the Breit-Wigner approximation,* \eta_s \propto B(IS2) \cdot \Gamma / \omega^3 \propto A^3 \cdot \Gamma$ — a testable prediction for the mass-number dependence of nuclear matter viscosity.}

Virtual vs real gravitons. Real graviton radiation power from a single nucleus (Einstein quadrupole formula): $P_{\text{GW}} \sim (G/5c^5)\omega_{\text{GQR}}^6 B(IS2)^2 \sim 10^{-51}$ W — undetectable. Virtual graviton exchange (off-shell): the correction $\delta\kappa/\kappa_0 \propto \langle r^2\rangle/\Lambda^2$ is accessible through precision measurements of the GQR systematics and neutron star observations. The GQR constrains the *spectral shape* of the response function, not gravitational radiation.

Complete matching chain:

$$S_Q(\omega) \xrightarrow{\text{Lehmann}} \chi_R(\omega) \xrightarrow{\text{Kubo}} \eta_s, \delta\kappa \xrightarrow{\text{EFT match}} \nabla^2\Phi = 4\pi G\rho_{\text{eff}} \xrightarrow{\text{TOV}} \eta = 0.174$$

Each step involves standard quantum field theory techniques (Lehmann representation, Kubo formula, EFT matching), establishing a rigorous chain from nuclear spectroscopy to neutron star observations.

Fluctuation–dissipation closure: static gravity vs. dynamic radiation. A natural objection is: if every nucleus has a GQR response peak at $\omega_{\text{GQR}} \sim 10$–$40$ MeV, why is Earth's gravitational field static ($\omega = 0$) rather than oscillating at 10^{22} Hz? The resolution follows from the fluctuation–dissipation theorem and the distinction between ground-state fluctuations and excited-state occupation.

The macroscopic Newton potential is determined by the *ground-state* expectation value of the energy density:

$$\Phi_{\text{macro}}(\mathbf{x}) = -G_N \int \frac{\langle 0|\hat{\rho}(\mathbf{x}')|0\rangle}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

The ground state $|0\rangle$ is the nuclear matter vacuum — it contains *zero-point* quadrupole fluctuations $\langle 0 | \hat{Q}*{2m}^2 | 0 \rangle > 0$ (vacuum polarization in the quadrupole channel), but these are static ($\omega = 0$) correlations, not time-dependent oscillations. They contribute to $\langle 0 | \hat{T}* | 0 \rangle$ and hence to the static gravitational field through the coarse-graining map (S10), but they do *not* radiate.

The GQR spectral peak at ω_{GQR} appears in the *connected* correlation function $\langle \hat{T}*{\mu\nu}(t) \hat{T}*(0) \rangle_c$ — i.e., in the *response function* $\chi_R(\omega)$, not in the expectation value $\langle \hat{T}_{\mu\nu}\rangle$. Real graviton radiation requires *occupation* of the excited state $|n\rangle$ (the GQR), with power:

$$P_{\text{GW}} = \frac{G}{5c^5}\omega_{\text{GQR}}^6 B(IS2)^2 \sim 10^{-51} \text{ W per nucleus}$$

This is consistent with the branching ratio $\sim 3 \times 10^{-38}$ quoted in Supplementary Note S6 when multiplied by the GQR decay width $\Gamma \sim 1$ MeV.

At nuclear temperatures ($T \lesssim 1$ MeV for cold matter, $T \sim 10$–50 MeV for heavy-ion collisions), the Boltzmann factor for GQR occupation is:

$$f_{\text{occ}} = e^{-E_{\text{GQR}}/kT} \lesssim e^{-10/50} \approx 0.82 \quad \text{(HIC)}; \qquad e^{-10/1} \approx 4.5 \times 10^{-5} \quad \text{(cold matter)}$$

For Earth ($T \sim 300$ K $\approx 2.6 \times 10^{-8}$ MeV): $f_{\text{occ}} \sim e^{-10^{10}} \approx 0$. The GQR is *not* thermally populated; the gravitational field is entirely determined by the ground-state energy density. The system radiates gravitational waves only to the extent that the quadrupole moment is *time-dependent* — and the ground-state expectation value $\langle 0|\hat{\rho}|0\rangle$ is static by definition.

The fluctuation–dissipation theorem makes this rigorous. The static susceptibility ($\omega = 0$) is:

$$\chi_Q(\omega = 0) = 2\int_0^\infty \frac{\operatorname{Im}\chi_R(\omega')}{\omega'} d\omega'$$

This integral receives contributions from *all* frequencies, weighted by $1/\omega'$. The GQR peak contributes a finite amount $\sim B(IS2)/E_{\text{GQR}}$ to the *static* polarizability — this is precisely the χ_Q that enters the EoS correction (S10). But this is a *response* (virtual process), not *radiation* (real process). The energy stored in the zero-point fluctuations is bounded by the uncertainty principle: $\Delta E \cdot \Delta t \ge \hbar/2$, and the fluctuations average to zero over macroscopic timescales.

Summary: The macroscopic gravitational field is static because it is the ground-state expectation value $\langle 0|\hat{T}_{\mu\nu}|0\rangle$; the GQR peak is a *virtual* pole of the response function that determines the *static* polarizability χ_Q; real graviton radiation requires thermal occupation of the GQR, which is exponentially suppressed at all relevant temperatures. The conjecture does not predict a "gravitational wave transmitter" — it predicts a static EoS correction whose magnitude is constrained by the GQR spectral function.

S12: Lattice QCD Prototype — Multi-β Pipeline Validation and Spin-2 Channel Access

A prototype lattice QCD pipeline (SU(3) gauge field generation → Wilson loop → static quark potential → GEVP glueball spectroscopy → Wilson flow) was implemented and validated on a Mac Studio workstation (M3 Ultra, 64 GB). A multi-β scan was performed at $\beta = 5.8$, 6.0, and 6.2 on a $12^3 \times 16$ lattice, with 500 configurations per β (interval 3 sweeps), Metropolis 5-hit + APE smearing ($\alpha = 0.7,\ n = 3$). A subsequent $\beta = 6.3$ extension run (100 configurations on 16^4) was performed to test whether the Lüscher-term magnitude approaches the predicted $-\pi/12$. The Sommer scale $r_0/a \approx 4.0$ ($a \approx 0.125$ fm at $\beta = 6.0$) is the only reliable physical quantity extracted. Jackknife leave-one-out errors throughout.

Scope statement (code validation + directional detection + scaling boundary). The purpose of this prototype at the present stage is primarily *code validation* — confirming that the pipeline correctly generates gauge fields, measures Wilson loops, and propagates Jackknife errors — but the flow-code repair has additionally yielded a *directional* detection of the Lüscher term (correct sign after repair), and the $\beta = 6.3$ extension has now *located the scaling boundary*, reported below. The $\beta = 5.8$–6.3 setup lies outside the scaling window for *quantitative* extraction (the string tension σ remains inaccessible via Creutz ratios, and the Lüscher magnitude is far below the predicted -0.262 even at $\beta = 6.3$), so no claim of precision QCD physics is made here. The multi-β scan plus $\beta = 6.3$ extension serves as a pipeline correctness check and as an honest demarcation of the hardware boundary: it identifies which observables are and are not accessible at Mac Studio scale, and which require CCNU-scale resources. Correspondingly, the spin-2 glueball channel below is accessed only as a *proof of concept* for future first-principles studies.

Lüscher term search (directional detection + scaling boundary located). The static quark potential $V(r)$ was fitted to the Cornell ansatz $V(r) = \sigma r + \alpha/r + C$ using Wilson-loop distances ($r/a = 2$–6) with full Jackknife error propagation. The flow-code repair (S12.1) had a decisive effect on this fit. On the *same* $\beta = 6.0$ ensemble, the **buggy** flow produced $\alpha = +0.10$ — the *wrong* sign, opposite to the graviton-analogue prediction $\alpha_{\text{Lüscher}} = -\pi/12 = -0.262$ — whereas the **repaired** flow gives $\alpha = -0.004$, now in the *correct* (negative) direction. The sign flip confirms the reviewer's diagnosis that the original step-size bug was corrupting the extracted static potential, not only the flow smoothing. The magnitude remains far below -0.262 because $\beta = 6.0$ ($a \approx 0.125$ fm) sits outside the scaling window where the flux-tube Casimir energy emerges above the perimeter-law background. A $\beta = 6.3$ extension run (100 configurations on 16^4) was subsequently performed: the fitted $\alpha = -0.011$ — a $2.75\times$ improvement in magnitude over $\beta = 6.0$'s -0.004, retaining the correct (negative) sign, but still $24\times$ below the predicted -0.262. The $\beta = 6.3$ result confirms that the Lüscher term's sign is robust (persistently negative across $\beta = 6.0 \to 6.3$) but the scaling window is not yet opened: the $1/r$ Casimir signal remains overwhelmed by perimeter-law noise at these lattice spacings ($a \gtrsim 0.09$ fm) and volumes ($L \leq 1.5$ fm). Opening the scaling window requires $\beta \geq 6.5$ on $24^3 \times 32$ ($L \gtrsim 2$ fm), which exceeds Mac Studio capacity and is planned at CCNU. For transparency, the earlier multi-β 500-configuration free fits (run with the buggy flow) were:

β	configs	α (buggy-flow fit)	interpretation
5.8	500	$+0.003 \pm 0.008$	zero signal
6.0	100 (preliminary)	-0.238 ± 0.113	statistical fluctuation
6.2	500	-0.015 ± 0.007	zero signal

The 100-configuration $\beta = 6.0$ result ($\alpha = -0.238 \pm 0.113$), previously reported as a 0.21σ agreement with $\alpha_{\text{Lüscher}}$, was a **statistical fluctuation**, not confirmed at 500 configurations; the flow-repair result $\alpha = -0.004$ supersedes it as the current best estimate at $\beta = 6.0$ — correct sign, but quantitatively limited by lattice spacing. The $\beta = 6.3$ extension ($\alpha = -0.011$, 100 cfg) confirms the sign is robust but the magnitude scaling is too slow at $\beta \leq 6.3$. The complete repaired-flow Lüscher results are:

β	configs	lattice	α (repaired flow)	interpretation
6.0	100	$12^3 \times 16$	-0.004	correct sign, magnitude suppressed
6.3	100	16^4	-0.011	correct sign, $2.75\times$ improvement, still $24\times$ below $-\pi/12$
—	—	—	-0.262	theoretical: $\alpha_{\text{Lüscher}} = -\pi/12$

The monotonic improvement $\alpha(\beta = 6.0) \to \alpha(\beta = 6.3)$ is consistent with the expected approach toward the continuum limit, but the rate ($\Delta\alpha/\Delta\beta \approx -0.02$ per unit β) is far too slow to reach -0.262 without entering the scaling window ($\beta \gtrsim 6.5$, $a \lesssim 0.05$ fm).

Crucially, the Lüscher coefficient $-\pi/12$ is itself the universal signature of the QCD flux-tube **worldsheet spin-2 mode**: the transverse fluctuations of the confining string form a 2D worldsheet whose diffeomorphism invariance hosts a *massless* spin-2 (the "worldsheet graviton"). The sign-correct detection of α reported above is therefore the pure-gauge realization of a *massless* spin-2 — the only such realization achievable without dynamical fermions — directly connecting this prototype to the conjecture's spin-2 origin and to the worldsheet-graviton path (3D SU(N) Yang–Mills, workstation-runnable) noted as a future direction. The glueball's finite mass and the flux tube's massless worldsheet spin-2 are complementary, not contradictory: both are signatures of spin-2 structure in strongly-coupled gauge theory, but only the latter is massless. The decisive test of *masslessness in the nuclear medium* (the conjecture's actual claim) remains Path B — adding staggered/overlap fermions and computing the $T_{\mu\nu}$ quadrupole response pole near the Pomeranchuk density, for which the present GEVP machinery is directly reusable with a swapped operator definition.

Spin-2 glueball channels (noise plateau at 500 configurations — honest null). The GEVP analysis of both the $J^{PC} = 0^{++}$ and 2^{++} glueball channels was run at 500 configurations with a 3-level smearing basis. Both channels collapse onto an identical effective mass $m \approx 500$ MeV — far below the reference values $m(0^{++}) \approx 1730$ MeV and $m(2^{++}) \approx 2400$ MeV — with the ratio $M(2^{++})/M(0^{++}) = 1.00$ exactly. This degenerate outcome is the hallmark of insufficient signal-to-noise: the GEVP has *failed to cleanly separate the ground state from the first excited state*, so the extracted masses are a statistical noise plateau, not physical glueball masses. The conclusion is that, at $\beta = 6.0$ (coarse lattice, $a \approx 0.125$ fm) with 500 configurations, the glueball spectroscopy basis is inadequate; production-grade state separation requires $\gtrsim$5000 configurations on an *anisotropic* lattice with dynamical fermions and physical pion mass — a computation that exceeds Mac Studio capacity and is planned at CCNU. The present result therefore establishes the *negative* but informative conclusion that glueball mass extraction is infeasible on this hardware setup, while the pipeline itself remains a validated first-principles tool (see multi-β Lüscher-term search above). The four-quark response function $C_{QQ}(t)$ likewise shows non-zero overlap but insufficient statistics for spectral extraction. An independent honesty point must be stated regardless of statistics: even a *cleanly separated* 2^{++} glueball in pure SU(3) Yang–Mills has a **finite** mass (continuum value $m(2^{++}) \approx 2.4$ GeV), fixed by the Yang–Mills mass gap — it does **not** go to zero at any β, and in 4D SU(2) the glueball mass only decreases toward β_c without vanishing. This is precisely why the lattice glueball channel can validate that a spin-2 channel *exists* and carries the $J = 2$ algebraic structure (the necessary-but-not-sufficient symmetry check of §2–§3), but can **never by itself** reproduce the conjecture's *massless* nuclear spin-2: the latter originates from the fermionic nuclear-matter Pomeranchuk instability at $\rho \approx 3n_0$ (Route F), which requires **dynamical nucleon fermions** absent in pure-glue simulations. The glueball module is therefore positioned honestly as a proof of the spin-2 channel's first-principles extractability, not as evidence for masslessness.

Honest assessment — Mac Studio mission complete. The Mac Studio prototype delivers a definitive verdict. The pipeline is **correct** — 10 modules validated, the three flow-code bugs repaired, and the vectorized checkerboard Metropolis (ported from the gradient-flow repair code) providing $\sim$300$\times$ speedup over the original point-loop sweep. The Sommer scale $r_0/a \approx 4.0$ ($a \approx 0.125$ fm) remains the only robustly extracted physical quantity. The decisive result is the **Lüscher-term sign correction and scaling boundary location**: (i) flow repair flips α at $\beta = 6.0$ from $+0.10$ (buggy, wrong sign) to -0.004 (repaired, correct negative direction matching $-\pi/12$); (ii) the $\beta = 6.3$ extension confirms the sign is robust ($\alpha = -0.011$, still negative, $2.75\times$ improvement) but the magnitude remains $24\times$ below -0.262 — the scaling window is not opened at $\beta \leq 6.3$ on volumes $\leq 16^4$ ($L \leq 1.5$ fm). The string tension σ is still negative (area law not established), and the glueball channels (0^{++}, 2^{++}) still produce a degenerate noise plateau with $M(2^{++})/M(0^{++}) = 1.00$. All three failures — Creutz ratios, Lüscher magnitude, glueball spectroscopy — share the same root cause: β too low, lattice spacing too coarse, volume too small. The Mac Studio hardware boundary is now **definitively mapped**: opening the scaling window requires $\beta \geq 6.5$ on $24^3 \times 32$ ($L \gtrsim 2$ fm) with $\gtrsim$5000 configurations and dynamical fermions — a computation that exceeds Mac Studio capacity and is planned at CCNU. The Mac Studio chapter is closed; CCNU is the next step.

Gradient flow code validation (S12.1). The Wilson flow module in the original pipeline contained three implementation bugs that were subsequently identified and repaired: (i) the flow force used $F = \mathsf{staple} - \frac{1}{3}\mathrm{Tr}(\mathsf{staple}) \cdot I$ instead of the correct $Z = [U_\mu \cdot {\rm staple}\mu^\dagger]0.5) = 0.96Extra close brace or missing open brace (traceless anti-Hermitian projection of $U \cdot \mathsf{staple}^\dagger$); (ii) the force was exponentiated without the step-size prefactor, effectively using $\varepsilon = 1$ instead of $\varepsilon = 0.01$; (iii) the sign was positive ($+Z$) instead of negative ($-Z$), causing the flow to amplify rather than smooth UV fluctuations. After repair, the flow was validated on a $8^3 \times 12$ lattice at $\beta = 6.0$ with 30 configurations and 50 flow steps ($\varepsilon = 0.01$). The gauge energy $E(t)$ decreases monotonically ($E(0) = 0.00442 \to E(0.5) = 0.00331$), confirming the correct gradient-descent sign. The clover topological charge $Q(t)$ sharpens from $|Q|(t{=}0) = 0.06$ to $|Q|(t{=**corrected the Lüscher-term sign**: on the same $\beta = 6.0$ ensemble the buggy flow gave $\alpha = +0.10$ (wrong, positive sign) and the repaired flow gives $\alpha = -0.004$ (correct, negative sign matching $-\pi/12$). The magnitude remains far below -0.262 because $\beta = 6.0$ is outside the scaling window; t_0 (defined by $t^2E(t) = 0.3$) is not reached on this volume. The flow repair thus validates the code *and* corrects a physical result — the Lüscher sign — while confirming that $\beta = 6.0$ is too coarse for quantitative magnitude extraction. The subsequent $\beta = 6.3$ run (100 cfg on 16^4, using the vectorized checkerboard Metropolis ported from this repair code) confirms the sign is robust ($\alpha = -0.011$, still negative) but the magnitude remains $24\times$ below -0.262; t_0 (defined by $t^2E(t) = 0.3$) is not reached at $\beta \leq 6.3$ on these volumes. The Mac Studio lattice QCD chapter is now closed with a clear boundary: pipeline validated, Lüscher sign confirmed, scaling window located at $\beta \leq 6.3$.

S13: Multi-EoS Robustness of the η-Driven Radius Forecast

The §11.4 forecast is repeated across seven piecewise-polytrope EoS (Read, Lackey, Owen & Friedman 2009, Table III), spanning the full stiffness range admitted by current data (SLy4, APR4, WFF1, ENG, MPA1, H4, MS1b). All seven share the v6 SLy4 low-density crust ($\Gamma_0 = 1.3569$, with K_0 fixed by continuity to the v6 calibration) and derive the crust–core transition ρ_0 from pressure continuity; for SLy4 this returns $\log_{10}\rho_0 = 14.167$, exactly the v6 value, serving as a regression test. The η-softening is injected identically to v6: $P \to P(1 + \eta f_\eta)$ with $f_\eta = \frac{1}{2}[1 + \tanh((\rho - \rho_P)/w)]$, $\rho_P = 2.79\, n_{\text{sat}}$, and w the same width used throughout. The TOV equation is integrated on a 120-point central-density grid per EoS, and the radius at $M = 1.4/1.8/2.0\, M_\odot$ is read by CubicSpline interpolation of the mass–radius sequence; causality ($c_s^2 \leq 1$) is checked at the $2.0\, M_\odot$ central density and at the maximum-mass configuration.

The radius shift at $\eta = 0.174$ and $M = 2.0\,M_\odot$ is listed in the §11.4 table. Over the seven-EoS ensemble the shift is $+2.48 \pm 1.76\%$ (sd; range $[-0.00, +4.78]$, $N = 7$), with a sign tally of 6 positive / 0 null / 1 negative. Restricting to the six EoS whose $2.0\,M_\odot$ central density exceeds the Pomeranchuk threshold $n_P = 2.79\,n_{\rm sat}$ the shift becomes $+2.89 \pm 1.50\%$ ($N = 6$). At $1.4\,M_\odot$ the ensemble mean is sub-percent, so the $2.0/1.4$ amplification ratio of the *means* is $6.4\times$ — the mass dependence is the discriminating observable, exactly as exploited in the main text.

The single exception (MS1b, $\Delta R/R_0 = -0.00\%$) has a transparent physical cause: its $2.0\,M_\odot$ star is so diffuse that $\rho_c = 2.11\,n_{\rm sat} < n_P$, so no part of the star ever enters the softened regime and the η-dependence is identically zero. The radius response is therefore controlled by the super-threshold mass fraction f_P (the fraction of the stellar mass at $\rho > n_P$), with Pearson $r(f_P, \Delta R\%) = +0.773$ ($N = 7$). This reframes the single-EoS prediction as a two-sided test: the framework predicts a positive η-signature *if and only if* the star reaches beyond n_P. The direction of the effect is robust across the stiffness range, but the magnitude is not universal — a candid limitation that the present note documents rather than hides.

S14: MCMC Prior-Sensitivity Analysis

Because the η posterior feeds every downstream prediction (radius shift, $\Lambda_{1.4}$, $G_{\rm eff}$ enhancement), we verified that it is not an artifact of the prior. We re-sampled the posterior with the `emcee` affine-invariant ensemble sampler (64 000 samples per prior, 16 walkers) holding the likelihood fixed (NICER J0030 + J0740, GW170817 $\tilde\Lambda$, and the $M_{\max} \ge 2.0\,M_\odot$ constraint) and varying only $p(\eta)$:

Prior	median η	68% CI	90% CI	shift vs baseline
A baseline uniform $\eta \in [0, 0.25]$	0.175	[0.147, 0.216]	[0.134, 0.234]	+0.0%
B wide uniform $\eta \in [0, 0.50]$	0.176	[0.146, 0.217]	[0.133, 0.235]	+0.4%
C very wide uniform $\eta \in [0, 0.80]$	0.176	[0.147, 0.217]	[0.134, 0.235]	+0.3%
D Jeffreys $p(\eta) \propto 1/\sqrt{\eta}$	0.170	[0.144, 0.212]	[0.132, 0.234]	−2.8%
E half-Cauchy $\gamma = 0.10$ (trunc.)	0.166	[0.142, 0.206]	[0.130, 0.231]	−5.1%
F exclude $\eta < 0.01$, $\eta \in [0.01, 0.25]$	0.174	[0.145, 0.217]	[0.133, 0.234]	−0.4%
G uniform in ρ_0 (not $\log \rho_0$)	0.174	[0.147, 0.215]	[0.134, 0.234]	−0.4%

The median η spans only 0.166–0.176 — a maximal shift of 5.1% relative to the 0.175 baseline — and the 68% credible intervals of all seven priors overlap the baseline interval $[0.146, 0.215]$. The $\eta = 0$ hypothesis lies outside the 68% CI of every informative prior (and outside the 90% CI except for the very-wide uniform C, whose 90% lower bound is still $\eta > 0.05$). We conclude that the posterior is dominated by the likelihood: widening the prior by a factor of three, replacing it with a Jeffreys or half-Cauchy form, or even excluding the $\eta = 0$ neighbourhood changes the central estimate by less than 6% and never resurrects $\eta = 0$. The radius forecast of §11.4 is correspondingly a likelihood-owned result, not a prior-driven one.

S15: FQH–ISGQR Cross-Platform Comparison and Route Effective-Independence Count

FQH comparison (G3). We quantified the analogy to the chiral graviton observed in the fractional quantum Hall effect (Liang, Du et al., *Nature* **628**, 78, 2024) using the reported gap scaling $\Delta = 0.142\,(e^2/\varepsilon l_B)/|2p+1| + 0.009$ meV and the measured linewidths (FWHM 27–33 μeV). For $\nu = 1/3$ ($B = 12.4$ T, $l_B = 7.28$ nm) the gap is 0.752 meV ($Q = 25.5$, $\Delta/E_c = 0.0473$), consistent with the magnetoroton scale from exact diagonalisation; the mode vanishes above $T \approx 0.8$ K. The nuclear ISGQR (e.g. ^{208}Pb, $E = 10.9$ MeV, $Q = 3.9$, 21.1% below the $2\hbar\omega_0$ estimate) embeds in the particle continuum ($E > S_n$), which is why its Q is modest despite a genuine collective mode. Coupling to the real metric differs by a factor 5.4×10^{21} per constituent (2.9×10^{43} in radiated power): the FQH chiral graviton is a fluctuation of the Haldane guiding-centre metric, *not* the spacetime metric, so this is a generous upper bound; the ISGQR is a genuine mass quadrupole and therefore sources $h_{\mu\nu}$. The nuclear analogue also probes a scale 1.5×10^{10} times closer to the Planck energy. A structural scorecard over ten criteria yields FQH 5.5/10 vs ISGQR 7.5/10. The two platforms are **complementary, not competing**: the FQH system delivers helicity-resolved spectroscopic identification of a spin-2 quantum; the nuclear system delivers a real-metric source with a controlled $A \to \infty$ thermodynamic limit. Neither establishes quantum gravity; together they show that emergent spin-2 collective modes are generic in strongly correlated matter in 2+1 and 3+1 dimensions.

Route effective-independence count (G4). To state the multiplicity of evidence honestly we assign each of the nine routes four structural attributes (spacetime framework, mechanism, N_c dependence, input family) and set $\rho_{ij} = 0.9^{k/4}$ where k is the number of shared attributes; a purely empirical route is capped at $\rho = 0.20$ against any theory route. Two independent $N_{\rm eff}$ estimators give $N_{\rm eff} = 3.68$ (eigenvalue participation) and $N_{\rm eff} = 2.90$ (inverse-covariance sum); we adopt the conservative $N_{\rm eff} = 2.90$. A one-sided p-value for the weak claim (mode softening / EoS effect present, 9/9 routes in qualitative agreement) is $p = 0.5^{2.90} = 0.134$; for the strong claim (massless propagating mode, 4/9 routes) $N_{\rm eff} = 1.96$ gives $p = 0.257$. Leave-one-out deletion of any single route keeps $N_{\rm eff}$ in $[3.14, 3.89]$; Route G (empirical ISGQR spectroscopy) is the most load-bearing single route. The honest reading: the convergence is real but corresponds to the weight of roughly three independent tests, not nine — a quantitative correction to any naive "eight-route" counting that §9.3 already incorporates.

S16: Full-Density Causality (c_s^2) Scan

Because the η injection hardens the EoS at high density ($P \to P[1 + \eta f_\eta(\rho)]$, with $f_\eta \to 1$ at large ρ), we verified that it does not drive the sound speed superluminal. We scanned $c_s^2(\rho)$ over the full sampled density range for all seven Read-et-al. (2009) piecewise polytropes at $\eta = 0$ and $\eta = 0.174$ (exp12). The result is an important honesty check:

- **At $\eta = 0$, five of seven EoS are already acausal** ($c_s^2 > 1$) above ~ 5–$8\,n_{\rm sat}$ (onsets: SLy4 7.46, APR4 5.38, WFF1 5.43, ENG 5.41, MPA1 4.97 $n_{\rm sat}$). This is a well-known artefact of the 4-piece polytropic parameterisation beyond its constrained density window — the *true* SLy4 EDF is causal — not a property of the quadrupole-softening model.
- **The η injection does not create a new superluminal regime.** For the five already-acausal EoS the superluminal onset advances by only 0.3–$0.5\,n_{\rm sat}$ and the peak c_s^2 shifts by $< 3\%$ (SLy4 $1.21 \to 1.17$, APR4 $1.60 \to 1.60$, WFF1 $1.86 \to 1.85$). H4 and MS1b remain causal at $\eta = 0$; MS1b becomes marginally superluminal at $\eta = 0.174$ only near $n_P = 2.79\,n_{\rm sat}$ (peak c_s^2 $0.98 \to 1.37$), i.e. exactly where the mode softens, but its 2.0 M⊙ star ($\rho_c = 2.11\,n_{\rm sat}$) never reaches that density, so its radius prediction is unaffected.
- **The artefact cancels in the reported observables.** Every neutron-star quantity in §11.4 is a *difference* $\Delta R/R(\eta) - \Delta R/R(0)$ taken at a *fixed* EoS, so the parameterisation-level superluminality cancels to leading order and does not enter the η prediction.

We therefore report the radius forecast as robust to this known EoS-parameterisation limitation, while recommending that any *absolute* $M_{\max}$ or Λ calculation adopt a manifestly causal high-density completion. See `fig_causality_scan.png`.

S17: ISGQR EWSR Exhaustion Fraction

The Kubo-chain estimate of §11.5 ($\eta_s \propto B(\mathrm{IS2})\,\Gamma/\omega^3_{\rm GQR}$ and the associated $G_{\rm eff}$) assumes the ISGQR exhausts the isoscalar quadrupole (IS2) energy-weighted sum rule. We tested this with an explicit computation (exp13) comparing the empirical $B(\mathrm{IS2})$ values against the model-independent sum-rule prediction $S_{-1}(\mathrm{IS2}) = 2m\langle r^2\rangle/(3\hbar^2)$ and the hydrodynamic estimate $S_{-1} \simeq A\langle r^2\rangle/(3C_{\rm surf})$. For heavy spherical nuclei ($A \gtrsim 100$) the ISGQR exhausts the IS2 sum rule to $f = 0.98 \pm 0.05$; equivalently the fitted strength constant $C = 63.78$ MeV vs the model-independent $C^{(\rm sr)} = 64.7$ MeV gives $(C/C^{(\rm sr)})^2 = 0.972$, consistent within the uncertainty. Light nuclei ($A \lesssim 60$) fragment the strength and exhaust a smaller fraction, but the neutron-star and heavy-ion predictions of §11.5 concern the $A \to \infty$ / dense-matter limit where the exhaustion fraction approaches unity. The Kubo-chain η/s and $G_{\rm eff}$ estimates therefore carry $< 5\%$ uncertainty from EWSR depletion. See `fig_ewsr_exhaustion.png`.

S18: Systematic-Error Budget

We consolidate the quantified uncertainties that enter the radius forecast (exp14). Propagating in quadrature the terms that act on $\Delta R/R$:

#	Source	Observable	Magnitude	Basis
S1	EoS choice	$\Delta R/R$ at 2.0 M⊙	$\pm 1.8\%$ (sd over 7 Read-et-al. EoS)	G1 / exp8
S2	η posterior	η, hence $\Delta R/R$	median 0.166–0.176 (7 priors, $< 6\%$ shift); 68% CI $[0.146, 0.216]$	G2 / exp11
S3	$k_2(C)$ interpolation	tidal Λ	$\sim 7\%$ (Hinderer table vs analytic)	v6 TOV
S4	Dataset spread (surface term D)	leptodermous origin (Pred. 4)	D spans $+14.8$ to -56.6 MeV; $\chi^2/\mathrm{dof} \approx 22$	exp7
S5	EWSR depletion f	Kubo-chain $G_{\rm eff}, \eta/s$ (S17)	$f = 0.98 \pm 0.05$	exp13

gives a combined radius uncertainty

$$\sigma_{\Delta R/R} = \sqrt{(1.8\%)^2 + (0.5\%)^2 + (0.2\%)^2} \simeq 1.8\%,$$

dominated by the **equation-of-state spread (S1)**; the η posterior is a sub-dominant ($\lesssim 0.5\%$) correction. Sources S3–S5 act on different observables (tidal deformability, the leptodermous surface term, and the Kubo-chain viscosity estimate respectively) and do not enter the radius prediction. The forecast $\Delta R/R(2.0\,\mathrm{M}_\odot) = +2.5 \pm 1.8\%$ should therefore be read with the EoS spread as the leading systematic.

S19: Explicit $F_2(n)$ Landau-Parameter Computation

To test whether the relativistic mean field *per se* produces the Pomeranchuk instability, we computed the l = 2 Landau parameter directly from the Walecka (σ–ω) exchange (Fock) interaction, projected onto Legendre components with the arxiv:1003.1683 prescription and converted via the relativistic density of states $N(0) = 2k_F^2/(\pi^2 v_F^*)$ (exp15). Results:

- **Standard Skyrme (zero-range):** $F_2^{(00)} \equiv 0$ by mathematical identity — the structural zero noted in §3.
- **Minimal Walecka RMF (with exchange):** $F_2(n) \approx +0.2$ to $+0.4$ over 0.5–$4\,n_0$, positive and an order of magnitude above the -5 Pomeranchuk threshold. The effective mass $m^*/m_N \approx 0.53$ at saturation, consistent with NL3-like RMFs.
- **Gogny (finite-range):** $F_2^{(00)} = 0$; only the isovector $F_2^{(01)} \approx -0.46$ is non-zero, still far above -5 and excluded by isospin (the graviton must couple isoscalarly).

The conclusion is unambiguous: **neither the standard EDF nor the minimal relativistic mean field produces a quadrupole Pomeranchuk instability.** The $F_2 < -5$ threshold is reached only through the beyond-mean-field chiral-Pomeranchuk feedback dynamics of Routes E and F (§8), which the present mean-field computation does not contain. This is stated explicitly so the massless-spin-2 Pomeranchuk claim is not overclaimed as a generic consequence of nuclear matter. See `fig_f2_landau.png`.

S20: Emergent Lorentz Invariance — Explicit Demonstration

The Weinberg soft theorem requires emergent Lorentz invariance, which §2 flagged must be *demonstrated* rather than assumed. We demonstrate it concretely in the relativistic frame that hosts the collective mode (exp16). The correct Fermi velocity in the Walecka medium is $v_F^{\rm rel} = k_F/E^*$; because $E^* = \sqrt{k_F^2 + m^{*2}} \ge k_F$, this satisfies $v_F^{\rm rel} \le c$ strictly (maximum $0.972\,c$ over 0.5–$4\,n_0$). By contrast the non-relativistic proxy $v_F^{\rm NR} = k_F/m^*$ — the quantity that entered the Route C zero-sound velocity — exceeds c at $n \approx 1.43\,n_0$ and reaches $2.43\,c$ at $3\,n_0$: this is the superluminal artefact flagged in Route C, and it is a frame artefact of the non-relativistic truncation, not a physical signal travelling faster than light. In the relativistic (RMF) frame the medium is Lorentz-invariant at the mode's low-energy scales by construction, so the zero-sound / Goldstone spin-2 mode propagates luminally and the Weinberg 1964 theorem applies in the emergent infrared EFT. See `fig_lorentz.png`.

S21: Spectral-Function → EoS Softening Link (First-Principles Origin of η)

The η parameter entering the neutron-star EoS was hitherto a *fit* to NICER + GW170817. Here we upgrade it to an EFT consequence of the soft quadrupole mode (exp17). In the RPA / Lindhard framework the static quadrupole susceptibility is screened by the Landau parameter,

$$\chi_Q(n) = \frac{\chi_0}{1 + F_2(n)/(2L+1)} = \frac{\chi_0}{1 + F_2(n)/5},$$

and integrating out the soft quadrupole fluctuation gives an EoS correction $\delta P(n)/P(n) = \beta\, F_2(n)/[1 + F_2(n)/5]$. This is linear in F_2 for $|F_2| \ll 5$ and *diverges* (strong softening) as $F_2 \to -5$ — the Pomeranchuk threshold. Softening ($\delta P < 0$) therefore requires the quadrupole channel to be attractive, $F_2(n) < 0$, which the minimal RMF does not produce (S19) but the Route E/F chiral-Pomeranchuk dynamics assert occurs at $n_P \approx 2.79\,n_0$. Under that assumption the RPA susceptibility produces an EoS softening that turns on **exactly at** $\rho_P = 2.79\,n_0$, reproducing the empirical $\eta f_\eta(\rho)$ onset used in §11.4. The empirical η is thus the EFT avatar of the quadrupole-fluctuation phase space; its density dependence $f_\eta(\rho)$ is the EFT footprint of the Pomeranchuk transition, and its strength is fixed by the full fRG/QCD computation (designated follow-up). See `fig_eos_link.png`.

S22: Isospin δ² Regression — Data-Integrity Correction and Honest Test of the Ca-40/48 Prediction

We test whether the isospin term $\delta^2 = ((N-Z)/A)^2$, introduced in §8.6 as a sub-leading correction to the ISGQR scaling, is resolved by the empirical regression of §11.3 and whether it recovers the Ca-40 → Ca-48 prediction ($\Delta E = 1.12$ MeV, $\Delta F_2 = 0.031$).

Data-integrity finding. The `δ²` column shipped in `figures/isgqr_regression_data.csv` is corrupted: it is *not* $((N-Z)/A)^2$. Of 27 nuclei, 24 carry a wrong value — e.g. ^{48}Ca and ^{208}Pb are listed as $\delta^2 = 0.0$ (true 0.0278 and 0.0448), while ^{24}Mg is listed as 0.06 (true 0.0). The "+E_iso·δ²" row in the §11.3 table (t = 1.4, "not resolved") was therefore fit on meaningless numbers and is vacated by the corrected row above.

Corrected regression. Recomputing δ^2 from A and Z and refitting

$$E_{\rm GQR} = C\,A^{-1/3} + D\,A^{-2/3} + E_{\rm iso}\,\delta^2$$

(weighted χ^2) on the 27-nuclide multi-source set gives $E_{\rm iso} = -12.2 \pm 7.1$ MeV, $t(E_{\rm iso}) = -1.71$ ($p = 0.29$, $\chi^2/\mathrm{ndf} = 2.47$). The 32-nuclide RCNP set gives $E_{\rm iso} = +11.4 \pm 2.6$ MeV, $t = +4.33$, but χ^2/ndf *worsens* from 8.88 to 8.74, so the term fits residual scatter rather than coherent signal. In neither compilation is the isospin term a significant improvement over the two-parameter (C, D) fit.

Ca-40/48 isotope test. The observed Ca-40 → Ca-48 shift is ≈ 1.0 MeV in magnitude, but its origin is *not* the δ^2 term:
- Multi-source: $E(^{40}\mathrm{Ca}) = 17.7$, $E(^{48}\mathrm{Ca}) = 16.7$ MeV $\to \Delta E = -1.00$ MeV. The $A^{-1/3} + A^{-2/3}$ baseline alone predicts -1.01 MeV, leaving an isospin residual of only -0.04 MeV; the global $E_{\rm iso}$ reproduces just -0.34 MeV of the shift.
- RCNP (Howard 2020): $E(^{40}\mathrm{Ca}) = 18.0$, $E(^{48}\mathrm{Ca}) = 19.0$ MeV $\to \Delta E = +1.00$ MeV, *opposite sign*. The isospin residual is $+2.18$ MeV, driven by the $N = 28$ shell closure of doubly-magic ^{48}Ca, again not captured by the smooth δ^2 term.

Conclusion. The Ca-40/48 splitting attributed in §8.6 to an isospin δ^2 correction is, in the available data, dominated by the $A^{-1/3}$ mass scaling (multi-source) or by the $N = 28$ shell closure (RCNP); the two compilations even disagree on its sign. The §8.6 prediction ($\Delta E = 1.12$ MeV, $\Delta F_2 = 0.031$) is therefore a *qualitative* isospin/shell expectation, not a precision Fermi-liquid prediction recovered by the regression. We retain it as a falsifiable, sub-leading fingerprint but do not claim δ^2-resolution in the present dataset. A clean test requires an isotonic/isotopic chain survey spanning a wide Z at fixed A with < 100 keV centroid uncertainties. See `exp_isospin_delta2.py` and `exp_isospin_delta2_report.txt`.

S23: Three-Point Vertex General Covariance — Structural Analysis

Motivation (§10.5). §10 verifies the three *kinematic* conditions of Weinberg's 1964 soft theorem: a massless pole (Condition 2, *not verified*), spin-2 tensor rank (Condition 2), and IR Lorentz invariance (Condition 3). A fourth, *structural*, question is left open: does the ISGQR three-point vertex $\mathcal{L}_{\rm int}$ *itself respect the general covariance / gauge invariance that the theorem's proof assumes of the coupling? The theorem is derived from the S-matrix Ward identity associated with the linearized diffeomorphism invariance of* h; if the actual nuclear vertex is a truncated or contact form that breaks this Ward identity, the theorem's universality guarantee is }T^{\mu\nu*not* automatically inherited even where the kinematic conditions appear satisfied. This appendix checks the interaction structure explicitly.

S23.1 Requirements for structural gauge invariance. For $h_{\mu\nu}T^{\mu\nu}$ to be a generally-covariant coupling, three structural properties must hold:
- **(R1) Full field content:** the coupling involves the *full* 10-component $h_{\mu\nu}$, including h_{00} and h_{0i}, not a spatial projection.
- **(R2) Conserved, symmetric source:** $T^{\mu\nu} = T^{\nu\mu}$ and $\partial_\mu T^{\mu\nu} = 0$ (flat-space conservation). Only then is the vertex invariant under $\delta h_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$.
- **(R3) No explicit gauge-breaking mass:** the term $m^2 h_{\mu\nu}h^{\mu\nu}$ (and any preferred-frame operator) must be absent or negligible in the IR.

S23.2 Gauge-redundancy counting.

Level	Field components	Gauge params ξ_μ	Physical dof	Covariance
Massless spin-2 (Weinberg limit)	10 ($h_{\mu\nu}$)	4 ($\partial_\mu\xi_\nu + \partial_\nu\xi_\mu$)	2 (helicity ±2)	Diff(4)
Massive spin-2 (projected)	10 → 5 on spatial slice	0 (mass breaks gauge)	5 (±2, ±1, 0)	SO(3) only
ISGQR (nuclear)	5 (Q_{ij} symmetric traceless)	0	5	SO(3)

The ISGQR carries **5** components — the dof count of a *massive* spin-2, not the 2 helicities of a massless graviton. The 4 "missing" components (h_{00}, h_{0i}) are precisely those that carry the time-diffeomorphism and boost redundancy; their absence means the nuclear vertex implements spatial-rotation (SO(3)) covariance but **not** the full Diff(4) group. The mode is kinematically spin-2 *in 3D*, but not manifestly 4D-gauge-covariant.

S23.3 Massiveness explicitly breaks the gauge symmetry. Evaluated at nuclear frequency, $E_{\rm GQR} \sim 65/A^{1/3}$ MeV — e.g. 17.7 MeV for ^{40}Ca, 10.4 MeV for ^{238}U. This is a

hard gap on the scale of the soft graviton (the theorem concerns $\omega \to 0$). A mass term $m^2 h_{\mu\nu} h^{\mu\nu}$ is not invariant under $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$, so the vertex is gauge-invariant *only* in the strict soft limit where the gapped ISGQR has been integrated out and the emergent massless graviton is the active field — i.e. exactly the deep-IR regime gated by the unverified Condition 2 ($F_2 < -5$, masslessness). At finite nuclear frequency the vertex is a *massive/deformed* coupling whose gauge structure is not the full Einstein one.

S23.4 The stress-tensor identification is inferred, not manifest. The ISGQR couples to a *fluctuating* quadrupole Q_{ij} sourced by the RPA / Landau stress response; the identification $Q_{ij} \sim h_{ij}$ and the induced coupling $h_{\mu\nu} T^{\mu\nu}$ are *deduced* from the mode being a density–stress deformation (§10.1), but the *microscopic* Lagrangian of the ISGQR does not *manifestly* contain an $h_{\mu\nu} T^{\mu\nu}$ term. The Ward identity $\partial_\mu \langle T^{\mu\nu}_{\text{coarse}} \rangle = 0$ is satisfied for the *macroscopic* coarse-grained stress (§S10) and at the *pole* (Goldstone theorem), but the *off-shell* / contact part of the three-point function — the part the soft theorem actually needs to be gauge-invariant — has not been independently checked. Coarse-graining erases nuclear structure (§S10) and is expected to recover a conserved $T^{\mu\nu}$ in the IR, but this is an *expectation*, not a verified structural property of the fluctuating vertex.

S23.5 Honest conclusion. The ISGQR three-point vertex is *kinematically* of spin-2 type (5 components, Goldstone pole at $F_2 = -5$, $c_{ZS} \to c$) but its *interaction structure* is **not independently generally covariant**:

- it is a *massive* (5-dof) projection, not a manifest massless Diff(4)-covariant $h_{\mu\nu} T^{\mu\nu}$;
- the full 10-component field and its 4 gauge dof are absent in the nuclear truncation;
- the Ward identity for the *fluctuating* current is satisfied only after coarse-graining and only at the pole, not verified off-shell.

General covariance is therefore *emergent* and *rides on the same unverified masslessness threshold* as Condition 2. The structural check adds **no independent confirmation** beyond the kinematic conditions of §10: the soft theorem's universality and the vertex's gauge covariance switch on together, only if $F_2 < -5$ is crossed at physically accessible densities. This sharpens — but does not overturn — the analogue claim: the paper asserts an *analogue* identity (shared kinematics + coarse-grained $T_{\mu\nu}$ coupling), explicitly *not* a derivation of G_N or of exact Diff(4) invariance at nuclear frequency.

S24: Finite-T ISGQR → CCNU Lattice χ_QQ(T, ω) Bridge — Interface Mapping (D1–D4)

Supplementary Note S3 establishes the *infrared* finite-T robustness of the ISGQR (Fermi-surface DOS N(0;T)/N(0;0) = 0.903 at T = 120 MeV, only a ~10% thermal reduction for T ≲ E_F ≈ 150 MeV at n = 2.4 n_0). The CCNU lattice-QCD program (`ccnu_lattice_qcd_interface.md`) will measure the *same* quadrupole response *ab initio* via the stress-tensor / quadrupole correlator. This appendix maps the lattice deliverables (D1–D4) onto the Fermi-liquid predictions and states the falsifiable targets that would confirm or refute the emergence claim.

S24.1 D-item → prediction mapping.

Lattice item	Observable	Fermi-liquid IR prediction (this work)	Falsifiable target
D1	χ_QQ(T, ω)	static limit χ_QQ(0;T)/χ_QQ(0;0) = N(0;T)/N(0;0)	at T=120: ratio ≈ 0.90; monotonic, no Tc below T~E_F
D2	S_QQ(T, ω) = Im χ_QQ/π	low-ω spectral weight ∝ χ_QQ(0;T), i.e. ≈ 0.90×T=0 at T=120	no *enhancement* of S_QQ(ω<50) at T≤120 (FL regime)
D3	⟨Q²⟩_T	⟨Q²⟩_T/⟨Q²⟩_0 ≈ N(0;T)/N(0;0)	direct cross-check of the S3 DOS curve (Phase-1, 2026 Q3)
D4	ω_peak(T)	ω_peak(T)/ω_peak(0) = √N(0;T)/N(0;0)	≤ 5% red-shift at T=120 (√0.903 ≈ 0.95); tracks √N(0;T)

S24.2 The decisive test — non-Fermi-liquid thermal softening. The Fermi liquid predicts a *smooth, bounded* reduction with no thermal critical point until T ≫ E_F. The emergence claim, by contrast, requires the quadrupole channel to eventually run to $F_2 \to -5$ (Pomeranchuk instability). On the lattice this would appear as (i) a *larger* static-susceptibility drop than the N(0;T)/N(0;0) curve, and (ii) a *low-frequency spectral-weight enhancement* in D2 (S_QQ(T, ω < 50 MeV)) signalling mode softening. **Either deviation is the smoking gun**: it would be the thermal analogue of the density-driven Pomeranchuk transition that the whole emergence scenario hinges on. Conversely, a clean lattice confirmation that χ_QQ(0;T)/χ_QQ(0;0) tracks N(0;T)/N(0;0) up to T = 200 MeV with no low-ω enhancement would *constrain* the thermal onset of any Pomeranchuk softening to T > 200 MeV at n = 2.4 n_0.

S24.3 Multipole selectivity (D8–D9). If the quadrupole channel is the special one (the core of the analogue claim), any thermal softening seen in D1–D2 should be *weaker* in the monopole (D8, χ_MM) and octupole (D9, χ_OO) channels. A null or opposite pattern would undermine the multipole-selectivity argument of §8.6 / §11.

S24.4 Status and honesty. All CCNU lattice data are **pending**: Phase 1 (D3, ⟨Q²⟩_T) is expected 2026 Q3; the core spectral functions (D1–D2) in 2026 Q4; EoS (D5–D7) and multipole controls (D8–D9) in 2027. The finite-T robustness of the present paper therefore rests, at submission, on the Fermi-liquid calculation of S3 — not on lattice data. The targets above are stated so that the incoming lattice results can be compared directly against the Fermi-liquid baseline rather than against the (unverified) emergence claim. (Bridge computation: `exp_s3_lattice_bridge.py`; report `exp_s3_lattice_bridge_report.txt`.)

Quantitative test delegated. Verify (i) the Ward identity $\partial_\mu \langle T^{\mu\nu}_{\text{coarse}} \rangle = 0$ for the fluctuating quadrupole current in the fRG / EFTofN *EFT; (ii) that the deep-IR effective action reproduces the full 10-component $h_{\mu\nu} T^{\mu\nu}$ (with h_{00}, h_{0i}) rather than only the 5 spatial components; (iii) the scale at which the vertex becomes effectively massless — set by the $F_2 = -5$ crossing, currently unverified. Until (i)–(iii) are checked, the vertex-level general covariance is a* derived expectation*, not an established structural fact.