Stochastic Electrodynamics of the Photon: Maxwell Equations, Malus's Law, and the Origin of Photon Randomness from the Electromagnetic Vacuum

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Abstract

Papers 1–7 derived the Schrödinger equation, the Dirac equation, quantum entanglement, the Coulomb force, the Lamb shift, the anomalous magnetic moment, hydrogen hyperfine splitting, and the double-slit paradox—all for the electron, whose randomness originates in the Zero-Point Field (ZPF) of its own Coulomb interaction.

This paper extends the framework to the photon. Since the photon has no charge or rest mass, the electron's Abraham–Lorentz equation cannot apply directly; we identify the photon analogue via its effective mass $m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ and optical-cycle reaction time $\tau_0^\gamma = 1/\omega_0$, giving diffusion coefficient $D_\gamma = c^2/2\omega_0 = \hbar/2m_\gamma^{\text{eff}}$, the same form as $D = \hbar/2m$ for the electron.

Four results follow. (1) Maxwell's equations emerge from the photon Fokker–Planck equation via the Madelung transform, using the Riemann–Silberstein vector as photon wavefunction—the exact analogue of deriving Schrödinger's equation in Paper 1. (2) Malus's law $P(\text{pass}) = \cos^2\alpha$ is derived from Brownian motion on the Poincaré sphere, with no quantum postulate. (3) Photon randomness originates in the vacuum ZPF of which the photon is itself a quantum; the intrinsic coherence radius at formation, $r_{\text{coh}} = \bar{\lambda}$, is distinct from the smaller observed van Cittert–Zernike radius of sunlight, reconciled via a diffusive-to-diffractive crossover. (4) The electron–photon coupling, evaluated with the exact stochastic-mechanical dipole moment guaranteed by Nelson's theorem, reproduces the measured natural linewidths of hydrogen Lyman-α and Balmer-α to within 1%, zero fitted parameters, matching both QED and direct spectroscopic measurement.

The hierarchy—Schrödinger (spin-$\frac{1}{2}$, massive) → Dirac (relativistic) → Maxwell (spin-1, massless)—emerges from stochastic processes driven by the electromagnetic ZPF, differing only in mass and spin.

Contents

1 Introduction

Paper 7 of this series [7] identified a fundamental open question:

> *The electron's randomness originates in its Coulomb ZPF, giving $D = \hbar/2m$ via the Abraham–Lorentz equation. The photon has no Coulomb field and no rest mass. Where does photon randomness come from?*

This question is not merely philosophical. It has concrete experimental manifestations:

1. A 45° polarized photon hitting a 0° polarizer passes or is absorbed with probability exactly 50%—with no hidden variable determining the outcome.
2. The coherence length of sunlight ($\sim 1\,\mu$m) differs from that of a laser (~ 300 km) by twelve orders of magnitude— despite both being photons.
3. A single photon produces an interference pattern in the double-slit experiment, building up dot by dot on the screen.

All three phenomena require an explanation of photon randomness from first principles. The present paper provides it.

1.1 The Strategy

For the electron, the chain was:

$$\underbrace{\text{Coulomb field}}_{\text{source}} \xrightarrow{\text{Einstein 1905}} \text{ZPF} \xrightarrow{\text{Boyer 1969}} u(\omega) \propto \hbar\omega^3 \xrightarrow{\text{Casimir}} \hbar \xrightarrow{\text{AL equation}} D = \frac{\hbar}{2m} \xrightarrow{\text{Nelson}} \text{Schrödinger}. \tag{1}$$

For the photon, we establish the analogous chain:

$$\underbrace{\text{EM vacuum ZPF}}_{\text{source}} \xrightarrow{\text{photon AL equation}} D_\gamma = \frac{\hbar}{2m_\gamma^{\text{eff}}} \xrightarrow{\text{Nelson (spin-1)}} \text{Maxwell equations}. \tag{2}$$

The key difference is that the photon *is* a quantum of the electromagnetic field—its "source field" is the same electromagnetic vacuum ZPF that provides the electron's ZPF. This self-referential structure gives the photon its unique properties.

1.2 Context in the Series

This is Paper 8 of the series *Quantum Mechanics from the Coulomb Field*. Full publication details (ai.viXra and/or Zenodo identifiers, as applicable) for each paper are given in the bibliography [1–7]. The papers in the series are:

Paper	Title (abbreviated)	Status
1	QM from Coulomb Field	Published
2	Entanglement + Coulomb Force	Published
3	Ergodicity	Published
4	Dirac Equation	Published
5	Schwinger term + Lamb shift	Published
6	Hydrogen Hyperfine Splitting	Published
7	Which Slit? Double-slit paradox	Published
8	**Stochastic Photon (this paper)**	**New**

2 The Photon as a ZPF Excitation

2.1 The Electron's ZPF vs the Photon's ZPF

The electron has a Coulomb field $E(r) = e/4\pi\epsilon_0 r^2$. Its ZPF is the electromagnetic quantum field arising from this Coulomb interaction, with spectral density (Paper 1 [1]):

$$S_E(\omega) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3}. \tag{3}$$

The photon has no charge and therefore no Coulomb field. However, the photon *is* a quantum of the electromagnetic field—it is a single excitation of a mode $(\mathbf{k}_0, \lambda_0)$ above the vacuum. The vacuum itself has spectral density (3)—the same ZPF that the electron interacts with.

The photon therefore propagates *immersed in* the electromagnetic vacuum ZPF. This is its "source field":

$$\mathbf{E}_{\text{total}} = \underbrace{\mathbf{E}_\gamma}_{\text{photon}} + \underbrace{\mathbf{E}_{\text{ZPF}}}_{\text{vacuum}}. \tag{4}$$

The photon shares all modes of the electromagnetic field with the vacuum ZPF. This is the physical origin of photon randomness.

2.2 The Photon Wave Packet

A single photon of frequency ω_0 propagating in the x-direction is a wave packet:

$$\mathbf{E}_\gamma(\mathbf{x}, t) = \sqrt{\frac{\hbar\omega_0}{2\epsilon_0 V}}\, \hat{\boldsymbol{\epsilon}}\, f(\mathbf{x} - ct\,\hat{\mathbf{x}})\, e^{i(k_0 x - \omega_0 t)} + \text{c.c.}, \tag{5}$$

where $k_0 = \omega_0/c$, $\hat{\boldsymbol{\epsilon}} \perp \hat{\mathbf{x}}$ is the polarization unit vector, f is the envelope function, and V is the normalization volume.

The monochromatic limit is unphysical and is excluded. Equation (5) is written with an explicit envelope f precisely because the opposite limit—a perfectly monochromatic plane wave, $f \equiv 1$ over all space and time—does not describe a physical single photon. Such a wave is not normalizable (its total field energy, integrated over infinite volume, diverges), and it does not correspond to a one-photon state in the usual sense: a field mode of exactly one frequency, persisting for all time, corresponds to the classical limit of a coherent superposition over photon number, not to a localized single quantum. A real, detectable single photon always has a finite envelope f, hence a finite coherence time τ_{coh} and coherence radius r_{coh} (Section 7 below); the textbook "photon of definite frequency ω, delocalized over all space" is a convenient idealization for computing energy and momentum eigenvalues, never a description of an actual emitted photon. This also pre-empts an apparent tension with relativity: a literal infinite, delocalized monochromatic state would assign equal detection probability to every point in the universe, which might seem to conflict with causality. In fact it does not—an extended, stationary probability amplitude carries no information and permits no superluminal signalling, exactly as for an ordinary non-relativistic plane-wave eigenstate $\psi = e^{i(kx-\omega t)}$ in elementary quantum mechanics, where the same delocalization is well known to be unproblematic. The point is moot in any case for the present framework, since the physical photon states considered throughout this paper are always the finite, normalizable wave packets of (5), never the monochromatic limit.

The photon carries energy $\hbar\omega_0$ and momentum $\hbar k_0 = \hbar\omega_0/c$. Its effective (relativistic) mass is:

$$m_\gamma^{\text{eff}} \equiv \frac{\hbar\omega_0}{c^2}. \tag{6}$$

This is not a rest mass—the photon has $m_{\text{rest}} = 0$. It is the *inertial mass* associated with the photon's energy, resisting transverse acceleration. It plays the role of m in all subsequent calculations.

2.3 The Photon Probability Density

For the electron, $\rho = |\psi|^2$. For the photon, the natural probability density is proportional to the electromagnetic energy density:

$$\rho_\gamma(\mathbf{x}, t) = u_{\text{EM}}(\mathbf{x}, t) = \epsilon_0|\mathbf{E}|^2 + \frac{1}{\mu_0}|\mathbf{B}|^2. \tag{7}$$

This is the *photon Born rule*: the probability of detecting a photon at $\mathbf{x}$ is proportional to the electromagnetic energy density there. In the present framework this is not a postulate—it is the stationary distribution of the photon SDE, following from Nelson's ergodic theorem (Paper 3 [3]), exactly as $|\psi|^2$ follows for the electron.

2.4 Postulates, Definitions, and Derived Results

Before proceeding to the photon Abraham–Lorentz equation, it is important to be explicit about the epistemic status of each ingredient used in this section and the next. Not everything stated above is on the same logical footing: some items are genuine postulates (assumed, not derived), some are definitions (notational conventions with no independent physical content), and some are theorems (derived from more basic principles already established elsewhere in this series). Conflating these categories would overstate what has actually been proven. Table 1 summarizes the classification used throughout the rest of this paper.

Table 1: Epistemic status of the propositions used to extend the stochastic electrodynamic framework from the electron (Papers 1–7) to the photon (this paper). Only the first row is a postulate inherited from the rest of the series; the two remaining postulates (photon–ZPF coupling and the photon AL equation) are specific to the photon extension and are flagged here so that the reader can judge which results below rest on solid derivational ground and which rest on physically motivated but unproven assumptions.

Proposition	**Status**	**Remark**
The electromagnetic field carries energy in discrete quanta $\hbar\omega$	**Postulate**	The single foundational postulate of the entire series (Paper 1 [1]). Not derived from anything more basic here.
The photon has a well-defined transverse position $\mathbf{r}_\perp(t)$	**Definition**	$\mathbf{r}_\perp(t)$ is *defined* as the centroid of the transverse envelope $f(\mathbf{x} - ct\hat{\mathbf{x}})$ in (5). Treating the photon as a localized wave packet with a sharp position is an idealization; a massless spin-1 field does not admit an exact position operator in the usual (Newton–Wigner) sense. The idealization is adopted for calculational convenience and is expected to be valid when the packet is narrow compared to the scales of interest.
Constant longitudinal speed c along the propagation axis	**Derived**	Follows from the dispersion relation $\omega = ck_0$ for a massless quantum, itself a consequence of the Maxwell equations derived in Section 5 below. Not an independent assumption.

Table 1 continued from previous page

Proposition	**Status**	**Remark**
Finite transverse speed, driven by the vacuum ZPF	**Consequence**	Not independent: follows automatically once the ZPF driving force (below) acts on a degree of freedom with finite effective mass m_γ^{eff} (6) through an equation of motion. Listed separately here only because it is often discussed as if it were a distinct assumption.
The photon's transverse motion is driven by the vacuum ZPF (photon–photon interaction via the vacuum)	**Postulate**	This is the central new assumption of this paper. Unlike the electron's ZPF driving (justified microscopically via the Coulomb field in Paper 1), the photon has no charge to couple to the ZPF through. The coupling mechanism asserted in Section 3 is motivated by analogy with the electron case and is *not* derived from a more microscopic photon–vacuum interaction. This should be regarded as the least secure assumption in the present framework and the primary target for future microscopic justification.
Effective mass $m_\gamma^{\text{eff}} = \hbar\omega/c^2$	**Definition**	Pure relativistic bookkeeping ($E = mc^2$ applied to $E = \hbar\omega$), with no independent physical content beyond notation.
The photon's transverse motion obeys an Abraham–Lorentz-type equation	**Postulate / ansatz**	Motivated by analogy and dimensional analysis with the electron AL equation (Section 3), *not* derived from the Maxwell equations and vacuum ZPF from first principles. A rigorous derivation, analogous to the electron AL derivation from the Coulomb field, remains an open problem (see Section 11.3).

Table 1 continued from previous page

Proposition	Status	Remark
The photon is a spin-1 (vector) field, with generators $\mathbf{S}$ satisfying $(\mathbf{S}\cdot\nabla)\mathbf{F} = i\nabla\times\mathbf{F}$ (introduced in Section 5)	**Imported fact**	Not derived anywhere in this framework. This is an external input, justified either by Wigner's classification of massless representations of the Poincaré group (helicity ±1 for the photon), or more elementarily by the classical observation that $\mathbf{E}$ and $\mathbf{B}$ are 3-vectors, so their polarization transforms under the spin-1 (vector) representation of the rotation group. The role played by $\mathbf{S}$ here is purely algebraic: it repackages the curl operator acting on a transverse field in a form parallel to the electron's Pauli matrices, exactly as in the Pauli identity $(\boldsymbol{\sigma}\cdot\mathbf{p})^2 = p^2$ used for the Dirac–Pauli reduction. The postulate of Paper 1 (energy quanta $\hbar\omega$) does not by itself imply spin-1; that the photon carries spin-1 is presupposed, not derived.

Provenance of the spin-1 assignment. Since this fact is imported rather than derived, it is worth recording explicitly where it comes from. The theoretical determination is due to Wigner's 1939 classification of the unitary representations of the Poincaré group [15]: massless particles are labelled by helicity, and requiring the electromagnetic field to transform as a Lorentz four-vector (or equivalently, a rank-2 antisymmetric tensor $F^{\mu\nu}$) forces the photon's helicity to be ±1—this is the rigorous reason the photon is spin-1 specifically, rather than some other allowed massless representation. The physical existence of this angular momentum was anticipated earlier and purely classically by Poynting (1909), who showed that circularly polarised light carries angular momentum according to Maxwell's equations alone, with no quantum input. It was confirmed quantitatively—one unit of $\hbar$ per photon—first indirectly via atomic spectroscopic selection rules (absorption/emission of a photon changes an atomic electron's orbital angular momentum by exactly $\hbar$), and then directly and mechanically by Beth's 1936 torsion-balance experiment [16], which measured the torque exerted by a beam of circularly polarised light on a birefringent wave plate and extracted exactly $\hbar$ of angular momentum per photon. None of these results are derivable from the single postulate of Paper 1 (discrete energy quanta $\hbar\omega$); they require either the full representation theory of the Poincaré group or an independent classical/experimental input external to the present stochastic framework.

In short: of the eight propositions commonly invoked when extending this framework to the photon, one is the series' foundational postulate, two are pure definitions, one is a derived theorem, one is a redundant restatement of consequences already implied by the others, two are genuine new postulates whose justification is presently by analogy only, and one (spin-1) is an external fact imported from outside this framework rather than derived or postulated within it. The results in Sections 3 and 4 below should be read with this distinction in mind: the derivation of $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$ (Section 3.3) is exact *given* the photon AL equation, but the AL equation itself is an ansatz, not a theorem.

3 The Photon Abraham–Lorentz Equation

3.1 Why the Electron AL Equation Fails for the Photon

The electron Abraham–Lorentz equation:

$$m\ddot{\mathbf{x}} = e\mathbf{E}_{\text{ZPF}}(t) + \frac{e^2}{6\pi\epsilon_0 c^3}\dddot{\mathbf{x}} \tag{8}$$

has two terms:

1. *ZPF driving*: $e\mathbf{E}_{\text{ZPF}}$ — requires charge e.
2. *Radiation reaction*: $\propto \dddot{\mathbf{x}}$ — arises because the electron emits radiation when accelerated; the emitted field acts back on the electron.

For the photon, $e = 0$ and $m_{\text{rest}} = 0$, so (8) cannot be applied directly. We must identify the photon analogues of both terms.

3.2 The Photon AL Equation for Transverse Momentum

The photon propagates in the x-direction. Its *longitudinal* momentum $p_\parallel = \hbar k_0$ is fixed by its frequency and cannot diffuse (the photon travels at c). The relevant dynamical degree of freedom is the *transverse position* $\mathbf{r}_\perp = (y, z)$, describing the spatial spreading of the wave packet perpendicular to propagation.

The photon analogue of the Abraham–Lorentz equation is:

$$m_\gamma^{\text{eff}}\ddot{\mathbf{r}}_\perp = \mathbf{F}_{\text{ZPF}}(t) + \frac{m_\gamma^{\text{eff}}}{\omega_0}\dddot{\mathbf{r}}_\perp, \tag{9}$$

where:

- $m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ is the photon effective mass (6),
- $\mathbf{F}_{\text{ZPF}}(t)$ is the radiation pressure of the vacuum ZPF on the photon's electromagnetic field (the photon analogue of $e\mathbf{E}_{\text{ZPF}}$),
- $(m_\gamma^{\text{eff}}/\omega_0)\,\dddot{\mathbf{r}}_\perp$ is the photon radiation reaction, with characteristic time $\tau_0^\gamma = 1/\omega_0$ (one optical cycle).

Physical justification of $\tau_0^\gamma = 1/\omega_0$. For the electron, the radiation reaction time is $\tau_0 = e^2/6\pi\epsilon_0 mc^3 = (2\alpha/3)\hbar/mc^2 = (2\alpha/3)\omega_c^{-1}$— a small fraction of the Compton period. The factor $2\alpha/3 \approx 5 \times 10^{-3}$ arises from the ratio of the electron's classical radius $r_e = e^2/4\pi\epsilon_0 mc^2$ to its Compton wavelength $\hbar/mc$.

For the photon, there is no charge and no classical radius in the same sense. The natural timescale is the *optical period* $\tau_0^\gamma = 2\pi/\omega_0$ (or $1/\omega_0$ up to the factor 2π). This is because the photon's self-interaction completes in one oscillation cycle—the photon *is* the oscillation. The precise relation is:

$$\tau_0^\gamma = \frac{1}{\omega_0} = \frac{\hbar}{\hbar\omega_0} = \frac{\hbar}{m_\gamma^{\text{eff}} c^2}, \tag{10}$$

which has exactly the same form as the electron $\tau_0 = \hbar/mc^2$ (up to the factor $2\alpha/3$, which is the electron's self-coupling ratio; for the photon, the analogous coupling is $O(1)$ since the photon is its own field).

Physical justification of $\mathbf{F}_{\text{ZPF}}$. The vacuum ZPF exerts radiation pressure on the photon's electromagnetic field through transverse mode coupling. The spectral density of this force can be computed from the cross-correlation of the photon field with the ZPF modes, but for the purposes of the equilibrium argument below, only its spectral properties matter.

3.3 Derivation of $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$ from the Photon AL Equation

We follow the exact structure of Paper 1 Section 5.2: equate two independent expressions for $\langle p_\perp^2 \rangle$.

Step 1: $\langle p_\perp^2 \rangle$ from the ZPF (photon AL equation)

In the frequency domain, the photon AL equation (9) gives the transverse velocity spectrum:

$$\tilde{\dot{r}}_\perp(\omega) = \frac{\tilde{F}_{\text{ZPF}}(\omega)/m_\gamma^{\text{eff}}}{-i\omega(1 + i\omega/\omega_0)}. \tag{11}$$

The ZPF force has spectral density:

$$S_F(\omega) = (m_\gamma^{\text{eff}})^2\omega_0^2 \cdot S_v(\omega) = \frac{(m_\gamma^{\text{eff}})^2\omega_0^2 S_E(\omega)}{(\hbar k_0/m_\gamma^{\text{eff}})^2} = \frac{(m_\gamma^{\text{eff}})^2\omega_0^2\hbar\omega^3}{6\pi^2\epsilon_0 c^3 \cdot (c^2)^2/c^2} = \frac{(m_\gamma^{\text{eff}})^2\omega^3}{6\pi^2\epsilon_0 c^3/\hbar}, \tag{12}$$

where we used $S_E(\omega) = \hbar\omega^3/6\pi^2\epsilon_0 c^3$.

In equilibrium, equating energy input from ZPF with radiation reaction damping (as in Paper 1 Section 5.1):

$$\langle p_\perp^2 \rangle_{\text{ZPF}} = m_\gamma^{\text{eff}} \cdot \frac{\hbar\omega_0}{2} = \frac{(\hbar\omega_0)^2}{2c^2} = \frac{(m_\gamma^{\text{eff}})^2 c^2}{2}. \tag{13}$$

This is the photon analogue of $\langle p^2 \rangle_{\text{ZPF}} = m\hbar\omega_0/2$ for the electron oscillator.

Step 2: $\langle p_\perp^2 \rangle$ from the osmotic velocity

The photon osmotic velocity:

$$\mathbf{u}_\gamma = D_\gamma \nabla \ln \rho_\gamma = D_\gamma \nabla \ln u_{\mathrm{EM}}. \tag{14}$$

The osmotic transverse momentum:

$$p_{\perp,\mathrm{osm}} = m_\gamma^{\mathrm{eff}} u_\gamma = m_\gamma^{\mathrm{eff}} D_\gamma \nabla_\perp \ln u_{\mathrm{EM}}. \tag{15}$$

For a Gaussian wave packet of minimum uncertainty width $\sigma_\perp = \bar{\lambda}/\sqrt{2}$ (where $\bar{\lambda} = c/\omega_0$):

$$\langle (\nabla_\perp \ln u_{\mathrm{EM}})^2 \rangle = \frac{2}{\bar{\lambda}^2} = \frac{2\omega_0^2}{c^2}. \tag{16}$$

Therefore:

$$\langle p_\perp^2 \rangle_{\mathrm{osm}} = (m_\gamma^{\mathrm{eff}})^2 D_\gamma^2 \cdot \frac{2\omega_0^2}{c^2}. \tag{17}$$

Step 3: Equate and solve

Setting $\langle p_\perp^2 \rangle_{\mathrm{ZPF}} = \langle p_\perp^2 \rangle_{\mathrm{osm}}$:

$$\frac{(m_\gamma^{\mathrm{eff}})^2 c^2}{2} = (m_\gamma^{\mathrm{eff}})^2 D_\gamma^2 \cdot \frac{2\omega_0^2}{c^2} \tag{18}$$

$$D_\gamma^2 = \frac{c^4}{4\omega_0^2} \tag{19}$$

$$\boxed{D_\gamma = \frac{c^2}{2\omega_0} = \frac{\hbar}{2m_\gamma^{\mathrm{eff}}}} \tag{20}$$

This is the central result of this section. The photon diffusion coefficient has *exactly the same form* as the electron diffusion coefficient $D = \hbar/2m$, with the photon effective mass $m_\gamma^{\mathrm{eff}} = \hbar\omega_0/c^2$ replacing the electron rest mass m.

3.4 Comparison: Electron vs Photon

Quantity	**Electron**	**Photon**
Inertial mass	m (rest mass)	$m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$
Characteristic frequency	$\omega_c = mc^2/\hbar$	ω_0 (own frequency)
Radiation reaction time	$\tau_0 = (2\alpha/3)\omega_c^{-1}$	$\tau_0^\gamma = \omega_0^{-1}$
ZPF driving	$e\mathbf{E}_{\text{ZPF}}$	$\mathbf{F}_{\text{ZPF}}$ (rad. pressure)
Diffusion coefficient	$D = \hbar/2m = c^2/2\omega_c$	$D_\gamma = \hbar/2m_\gamma^{\text{eff}} = c^2/2\omega_0$
Wave function	$\psi = \sqrt{\rho}e^{iS/\hbar}$	$\mathbf{F} = \sqrt{u_{\text{EM}}}e^{i\phi}\hat{\mathbf{e}}$
Wave equation	Schrödinger/Dirac	Maxwell
Spin	1/2	1

4 The Photon SDE and Fokker–Planck Equation

4.1 The Photon Stochastic Differential Equation

By exact analogy with Nelson's SDE for the electron, the photon obeys:

$$d\mathbf{x}_\gamma = \underbrace{c\hat{\mathbf{k}}\,dt}_{\text{propagation}} + \underbrace{D_\gamma \nabla \ln \rho_\gamma\, dt}_{\text{osmotic}} + \underbrace{\sqrt{2D_\gamma}\, d\mathbf{W}_\perp(t)}_{\text{ZPF kicks}}, \tag{21}$$

where:

- $c\hat{\mathbf{k}}\,dt$ is the mean forward propagation at speed c,
- $D_\gamma \nabla \ln \rho_\gamma = D_\gamma \nabla \ln u_{\text{EM}}$ is the osmotic velocity (transverse only),
- $d\mathbf{W}_\perp$ is the transverse Wiener process (ZPF kicks perpendicular to $\hat{\mathbf{k}}$).

The diffusion coefficient $D_\gamma = \hbar/2m_\gamma^{\text{eff}} = c^2/2\omega_0$ is derived in Section 3, not assumed.

4.2 The Photon Fokker–Planck Equation

The Fokker–Planck equation for $\rho_\gamma = u_{\text{EM}}$:

$$\frac{\partial \rho_\gamma}{\partial t} + \nabla \cdot \Big[(c\hat{\mathbf{k}} + D_\gamma \nabla \ln \rho_\gamma)\rho_\gamma \Big] = D_\gamma \nabla^2 \rho_\gamma. \tag{22}$$

Expanding and noting that the osmotic and diffusion terms cancel:

$$\boxed{\frac{\partial \rho_\gamma}{\partial t} + c\hat{\mathbf{k}} \cdot \nabla \rho_\gamma = 0} \tag{23}$$

This is the *photon transport equation*: the electromagnetic energy density propagates at speed c in direction $\hat{\mathbf{k}}$. This is the Poynting theorem:

$$\frac{\partial u_{\mathrm{EM}}}{\partial t} + \nabla \cdot \mathbf{S}_{\mathrm{Poynting}} = 0, \qquad \mathbf{S}_{\mathrm{Poynting}} = c^2 \epsilon_0 (\mathbf{E} \times \mathbf{B}), \tag{24}$$

which is a consequence of Maxwell's equations—but here it emerges *before* Maxwell's equations are derived, directly from the photon Fokker–Planck.

5 Maxwell Equations from the Photon Fokker–Planck

5.1 The Riemann–Silberstein Vector as Photon Wave Function

For the electron, the Madelung transformation $\psi = \sqrt{\rho} e^{iS/\hbar}$ linearised the nonlinear Fokker–Planck equation into the linear Schrödinger equation.

For the photon, we introduce the *Riemann–Silberstein vector* [12, 13]:

$$\mathbf{F}(\mathbf{x}, t) = \sqrt{\epsilon_0}\, \mathbf{E}(\mathbf{x}, t) + \frac{i}{\sqrt{\mu_0}} \mathbf{B}(\mathbf{x}, t). \tag{25}$$

The Riemann–Silberstein vector is the photon wave function. Its properties:

$$|\mathbf{F}|^2 = \epsilon_0 |\mathbf{E}|^2 + \frac{1}{\mu_0} |\mathbf{B}|^2 = u_{\mathrm{EM}} = \rho_\gamma, \tag{26}$$

$$\frac{1}{2} |\mathbf{F}|^2 \propto P(\mathbf{x}, t) \quad \text{(photon probability density)}, \tag{27}$$

$$i\hbar \partial_t \mathbf{F} = c(\mathbf{S} \cdot \hat{\mathbf{p}}) \mathbf{F}, \tag{28}$$

where $\mathbf{S} = (S_x, S_y, S_z)$ are the spin-1 matrices.

The analogy with the electron is exact:

	Electron	**Photon**
Wave function	$\psi = \sqrt{\rho} e^{iS/\hbar}$	$\mathbf{F} = \sqrt{u_{\mathrm{EM}}}\, e^{i\phi} \hat{\mathbf{e}}$
$\lvert\mathrm{wf}\rvert^2$	$\lvert\psi\rvert^2 = \rho$	$\lvert\mathbf{F}\rvert^2 = u_{\mathrm{EM}} = \rho_\gamma$
Spin matrices	Pauli $\boldsymbol{\sigma}$ (spin-$\frac{1}{2}$)	$\mathbf{S}$ (spin-1)
Wave equation	$i\hbar \partial_t \psi = \hat{H} \psi$	$i\hbar \partial_t \mathbf{F} = c(\mathbf{S} \cdot \hat{\mathbf{p}}) \mathbf{F}$

5.2 The Spin-1 Matrix Identity

The key algebraic identity connecting the spin-1 matrices to the curl is:

$$(\mathbf{S} \cdot \nabla) \mathbf{F} = i \nabla \times \mathbf{F} \tag{29}$$

for any transverse vector field $\mathbf{F}$ (satisfying $\nabla \cdot \mathbf{F} = 0$). This identity is the photon analogue of the Pauli identity $(\boldsymbol{\sigma} \cdot \mathbf{p})^2 = p^2$ used in deriving the Pauli equation from the Dirac equation.

The spin-1 matrices are:

$$S_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{30}$$

5.3 Derivation of Maxwell Equations

Step 1: The photon Fokker–Planck as a vector equation

For a superposition of plane waves (general field), the transport equation (23) summed over all propagation directions $\hat{\mathbf{k}}$ with the transverse (spin-1) structure gives:

$$\frac{\partial \mathbf{F}}{\partial t} = -c(\mathbf{S} \cdot \nabla)\mathbf{F}. \tag{31}$$

Using the identity (29):

$$\frac{\partial \mathbf{F}}{\partial t} = -ic \cdot i\nabla \times \mathbf{F} = c\nabla \times \mathbf{F}. \tag{32}$$

However, the sign depends on helicity. For the positive-helicity component $\mathbf{F}^+ = \sqrt{\epsilon_0}\mathbf{E} + i\mathbf{B}/\sqrt{\mu_0}$, the correct equation is:

$$\frac{\partial \mathbf{F}^+}{\partial t} = -c\nabla \times \mathbf{F}^+. \tag{33}$$

Step 2: Separate real and imaginary parts

Substituting $\mathbf{F}^+ = \sqrt{\epsilon_0}\mathbf{E} + (i/\sqrt{\mu_0})\mathbf{B}$:

Left side:

$$\frac{\partial \mathbf{F}^+}{\partial t} = \sqrt{\epsilon_0}\frac{\partial \mathbf{E}}{\partial t} + \frac{i}{\sqrt{\mu_0}}\frac{\partial \mathbf{B}}{\partial t}. \tag{34}$$

Right side:

$$-c\nabla \times \mathbf{F}^+ = -c\sqrt{\epsilon_0}\nabla \times \mathbf{E} - \frac{ic}{\sqrt{\mu_0}}\nabla \times \mathbf{B}. \tag{35}$$

Real part (coefficients of $\sqrt{\epsilon_0}$):

$$\sqrt{\epsilon_0}\frac{\partial \mathbf{E}}{\partial t} = -\frac{ic}{\sqrt{\mu_0}}\nabla \times \mathbf{B} \cdot \frac{1}{i} \tag{36}$$

More carefully, equating real parts:

$$\sqrt{\epsilon_0}\frac{\partial \mathbf{E}}{\partial t} = \frac{c}{\sqrt{\mu_0}}\nabla \times \mathbf{B}. \tag{37}$$

Dividing by $\sqrt{\epsilon_0}$ and using $c = 1/\sqrt{\epsilon_0\mu_0}$:

$$\boxed{\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0\mu_0}\nabla \times \mathbf{B} = c^2\nabla \times \mathbf{B}} \tag{38}$$

This is Ampère's law (in vacuum, no current). ✓

Imaginary part (coefficients of $i/\sqrt{\mu_0}$):

$$\frac{1}{\sqrt{\mu_0}}\frac{\partial \mathbf{B}}{\partial t} = -c\sqrt{\epsilon_0}\nabla \times \mathbf{E}. \tag{39}$$

Multiplying by $\sqrt{\mu_0}$ and using $c\sqrt{\epsilon_0\mu_0} = 1$:

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}} \tag{40}$$

This is Faraday's law. ✓

Step 3: Gauss's laws from transversality

The ZPF consists of transverse modes (Lorentz invariance, Boyer 1969, Paper 1 Section 3.4). Each mode satisfies $\hat{\mathbf{k}} \cdot \tilde{\mathbf{E}} = 0$ and $\hat{\mathbf{k}} \cdot \tilde{\mathbf{B}} = 0$. For a superposition of transverse modes:

$$\boxed{\nabla \cdot \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0.} \tag{41}$$

Gauss's laws. ✓

5.4 Summary: Maxwell from Fokker–Planck

$$\underbrace{\frac{\partial \mathbf{F}}{\partial t} = -c\nabla \times \mathbf{F}}_{\text{photon Fokker–Planck}} \xrightarrow{\mathbf{F}=\sqrt{\epsilon_0}\mathbf{E}+\frac{i}{\sqrt{\mu_0}}\mathbf{B}} \underbrace{\begin{cases} \partial_t \mathbf{E} = c^2 \nabla \times \mathbf{B} \\ \partial_t \mathbf{B} = -\nabla \times \mathbf{E} \\ \nabla \cdot \mathbf{E} = 0 \\ \nabla \cdot \mathbf{B} = 0 \end{cases}}_{\text{Maxwell equations in vacuum}} \tag{42}$$

This is the exact photon analogue of Paper 1:

$$\underbrace{\frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{v}\rho) = D\nabla^2 \rho}_{\text{electron Fokker–Planck}} \xrightarrow{\psi=\sqrt{\rho}e^{iS/\hbar}} \underbrace{i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi}_{\text{Schrödinger equation}} \tag{43}$$

6 Malus's Law from Brownian Motion on the Poincaré Sphere

6.1 Setup

A photon in polarization state α (angle from 0°) hits a linear polarizer at 0°. Standard quantum mechanics gives $P(\text{pass}) = \cos^2 \alpha$. We derive this from the photon SDE.

6.2 Polarization Space as the Poincaré Sphere

The polarization state of a photon is described by a point on the *Poincaré sphere*. For linear polarizations, the relevant coordinate is the polarization angle $\theta \in [0°, 90°)$, where $\theta = 0°$ is horizontal (0° polarization) and $\theta = 90°$ is vertical.

The Poincaré sphere has a non-Euclidean metric in polarization space. For linear polarizations, the natural arc length is:

$$ds = 2\, d\theta \tag{44}$$

because linear polarizations have period 180° (not 360°).

6.3 The Polarizer as a ZPF Boundary Condition

The polarizer at 0° physically selects the x-component of the ZPF and absorbs the y-component. Inside the polarizer, the effective energy density is:

$$u_{\mathrm{EM}}^{(\mathrm{pol})}(\theta) = u_{\mathrm{EM}} \cos^2 \theta \tag{45}$$

(only the 0° component survives).

The osmotic velocity in polarization space:

$$u_\theta = D_\gamma \frac{\partial}{\partial \theta} \ln \cos^2 \theta = -2 D_\gamma \tan \theta. \tag{46}$$

6.4 The Critical Timescale Argument

There are two timescales:

- $\tau_{\mathrm{Brownian}} \sim \theta_0^2 / D_\gamma$ — time for random walk to reach a boundary, starting from θ_0.
- $\tau_{\mathrm{osmotic}} \sim 1/D_\gamma |\partial_\theta \ln \cos^2 \theta|$ — time for osmotic velocity to develop a preference.

At $\theta_0 = 45°$, both timescales are comparable. However, the key observation is:

> *Before the osmotic drift has time to develop, the photon performs pure Brownian motion in polarization space, starting at $\theta_0 = 45°$. The outcome (pass or absorb) is determined by which boundary the Brownian motion hits first.*

6.5 First-Passage Probability: 50/50 Result

For pure Brownian motion $d\theta = \sqrt{2D_\gamma}\, dW$ on $[0°, 90°]$ with absorbing boundaries at $\theta = 0°$ (pass) and $\theta = 90°$ (absorb), starting at $\theta_0 = 45°$:

$$P(\mathrm{pass}) = \frac{90° - 45°}{90° - 0°} = \frac{1}{2}. \tag{47}$$

The 50% probability follows from the *symmetry of the starting position*: $\theta_0 = 45°$ is exactly midway between the two boundaries.

6.6 General Angle: Deriving Malus's Law

For a photon at angle α hitting a 0° polarizer, the starting position in polarization space is $\theta_0 = \alpha$.

On the Poincaré sphere with metric (44), the natural coordinate is $u = \sin^2\theta$. In this coordinate, the Brownian motion becomes uniform, and the first-passage probability from $u_0 = \sin^2\alpha$ to $\{u = 0, u = 1\}$ is:

$$P(\text{pass}) = \frac{1 - \sin^2\alpha}{1 - 0} = \cos^2\alpha. \tag{48}$$

$$\boxed{P(\text{pass}) = \cos^2\alpha} \tag{49}$$

This is Malus's law—derived from Brownian motion on the Poincaré sphere, without any quantum postulate.

6.7 Which Individual Photon Passes?

The outcome for each individual photon is determined by the specific realization of the Wiener process $dW(t)$—the instantaneous ZPF configuration at the moment of interaction with the polarizer. There is no hidden variable beyond the ZPF itself.

This answers the hundred-year question about photon randomness in the polarizer:

> *The ZPF configuration at the moment of interaction is the "hidden variable." But it is not a predetermined hidden variable in Bell's sense—it is a real stochastic field (the electromagnetic vacuum ZPF) whose value at each moment is genuinely random. The randomness is physical, not epistemic.*

7 The Coherence Volume from D_γ

7.1 Derivation of the Coherence Radius

The photon diffusion coefficient $D_\gamma = c^2/2\omega_0$ determines how rapidly the photon wave packet spreads transversely. After propagating for time t, the transverse spread is:

$$\langle r_\perp^2 \rangle = 2D_\gamma t = \frac{c^2}{\omega_0} t. \tag{50}$$

The *minimum* coherence radius—for a single-mode photon over one optical period $t = 1/\omega_0$—is:

$$r_{\text{coh}}^{\min} = \sqrt{2D_\gamma/\omega_0} = \sqrt{\frac{c^2}{\omega_0^2}} = \frac{c}{\omega_0} = \bar{\lambda}. \tag{51}$$

The minimum coherence radius equals the *reduced wavelength* $\bar{\lambda} = c/\omega_0 = \lambda/2\pi$. This is derived, not assumed.

7.2 First-Principles Estimate of the Continuum Coherence Time

Equation (56) below requires a value of $\tau_{\rm coh}$ as input, and for solar continuum (Bremsstrahlung) radiation the literature value $\tau_{\rm coh} \approx 3$ fs is normally quoted without derivation [14]. This subsection derives that timescale from classical electron–proton Coulomb collision physics, so that Eq. (57) below rests on a first- principles estimate rather than solely on a cited figure.

Distance of closest approach. Consider a free electron and proton in a plasma at temperature T. The Coulomb interaction becomes strong enough to produce a large-angle deflection when the potential energy at separation r is comparable to the thermal kinetic energy of relative motion. Setting

$$\frac{e^2}{4\pi\epsilon_0\, r_c} = 3k_B T \qquad \Longrightarrow \qquad r_c = \frac{e^2}{4\pi\epsilon_0 \cdot 3k_B T}, \tag{52}$$

this is the standard 90° Coulomb impact parameter of plasma kinetic theory [21]. For the solar photosphere ($T \approx 6273$ K, $3k_BT \approx 1.62$ eV, using $e^2/4\pi\epsilon_0 \approx 1.44$ eV·nm),

$$r_c \approx \frac{1.44 \text{ eV·nm}}{1.62 \text{ eV}} \approx 0.89 \text{ nm}. \tag{53}$$

Collision duration. The simplest estimate of the collision duration is the time to cross a distance r_c at the thermal speed $v_{th} = \sqrt{3k_BT/m_e} \approx 5.3 \times 10^5$ m/s:

$$\tau_c^{(\text{straight})} = \frac{r_c}{v_{th}} \approx 1.7 \text{ fs}. \tag{54}$$

This underestimates the true duration because r_c is, by construction, the impact parameter for a *large-angle* deflection: the electron's velocity vector rotates substantially during the encounter rather than passing through in a straight line. A better geometric estimate treats the swept path as a quarter-circle arc of radius r_c (a 90° turn),

$$\tau_c \approx \frac{\pi r_c/2}{v_{th}} \approx 2.6\text{–}2.7 \text{ fs}, \tag{55}$$

in close agreement with the literature figure of ≈ 3 fs used in Eq. (57). (The true Coulomb trajectory is a hyperbola of continuously varying curvature rather than a circular arc, so Eq. (55) is a geometric refinement of Eq. (54), not an exact integral along the orbit; the agreement with the independently-tabulated value is nonetheless a non-trivial, falsifiable check, since nothing forced these two numbers, computed by unrelated methods, to land within the same few-femtosecond window.)

Wave-packet length is set by acceleration duration. It is worth stating explicitly why τ_c—the duration of a single, uninterrupted accelerating encounter—is the correct input for $\tau_{\rm coh}$, rather than the mean free time between successive encounters. By the standard Liénard–Wiechert result of classical electrodynamics [22], the radiation (acceleration) field is a direct, undistorted map of the source acceleration, $\mathbf{E}_{\rm rad}(t) \propto \mathbf{a}(t - r/c)$: the pulse exists only while $\mathbf{a} \neq 0$, i.e. only during the close encounter itself. Once the encounter ends and the electron resumes free motion,

it stops radiating entirely, and the *next* encounter—occurring, on average, one mean free path $L_{\rm mfp} = 1/(n\sigma)$ later—produces a new, causally independent pulse. The mean free path therefore governs the *repetition rate* of independent emission events (and hence the source's overall duty cycle and radiated power), not the duration or coherence length of any single one of them. This distinction matters because a formula of the form $\sqrt{\sigma} \propto L_{\rm mfp}$—valid for the decoherence of a particle undergoing *repeated* scattering while traversing a medium [21]—answers a different physical question and should not be applied to a photon created in one free-free emission event that subsequently escapes without further scattering, as is the case for continuum radiation reaching Earth from the photosphere. It would, however, be the appropriate tool for a photon still undergoing radiative diffusion deeper in the optically thick solar interior, where repeated absorption and re-emission is the operative physical process.

7.3 Coherence Volume and the Emission Process

The actual coherence volume of a photon depends on the emission process through the coherence time $\tau_{\rm coh}$:

$$r_{\rm coh}(\tau_{\rm coh}) = \sqrt{2D_\gamma \tau_{\rm coh}} = \bar{\lambda}\sqrt{\omega_0 \tau_{\rm coh}}. \tag{56}$$

For sunlight ($\tau_{\rm coh} \approx 3$ fs, $\omega_0 \approx 3 \times 10^{15}$ rad/s):

$$r_{\rm coh}^{(\rm sun)} = \bar{\lambda}\sqrt{3 \times 10^{-15} \times 3 \times 10^{15}} = 3\bar{\lambda} \approx 3 \times \frac{600\,\text{nm}}{2\pi} \approx 286\,\text{nm}. \tag{57}$$

This gives the intrinsic single-photon coherence radius. The *observed* transverse coherence radius of sunlight at Earth is a distinct, much larger quantity, set not by $\tau_{\rm coh}$ but by the van Cittert–Zernike geometric factor from the finite angular size of the Sun [14]. The full transverse coherence *diameter* is

$$d_{\rm coh}^{(\rm sun)} = \frac{\lambda z_1}{D_\odot}, \tag{58}$$

where $z_1 = 1.496 \times 10^{11}$ m is the Sun–Earth distance and $D_\odot = 1.391 \times 10^9$ m is the solar diameter. For $\lambda = 0.52\,\mu$m this gives

$$d_{\rm coh}^{(\rm sun)} = \frac{(0.52 \times 10^{-6})(1.496 \times 10^{11})}{1.391 \times 10^9} \approx 56\,\mu\text{m}. \tag{59}$$

The corresponding coherence *radius* is half this value, $r_{\rm coh}^{(\rm sun,obs)} \approx 28\,\mu$m, in close agreement with the standard quoted figure of $\sim 27\,\mu$m for the spatial coherence radius of sunlight [14].

For a single-mode laser ($\tau_{\rm coh} \sim 10^{-3}$ s):

$$r_{\rm coh}^{(\rm laser)} = \bar{\lambda}\sqrt{3 \times 10^{12}} \approx 10^6\,\bar{\lambda} \sim \text{mm}, \tag{60}$$

consistent with the observed millimetre-scale coherence radius of a well-collimated laser beam.

7.4 Propagation to Earth: Diffusive-to-Diffractive Crossover

Equation (50) describes *diffusive* growth, $\langle r_\perp^2 \rangle \propto t$, and this raises an immediate question: if a photon's transverse coherence radius keeps growing diffusively for the full ~ 8.3-minute transit from the Sun, it should arrive at Earth enormously larger than the $r_{\rm coh}^{(\rm sun,obs)} \approx 28\,\mu$m observed van Cittert–Zernike radius. Taking $\tau = z_1/c \approx 499$ s as the diffusion time directly in Eq. (56) gives

$$r_{\rm coh}(\tau = z_1/c) = \bar{\lambda}\sqrt{\omega_0\, z_1/c} \approx 79.6 \text{ nm} \times \sqrt{3.77 \times 10^{15} \times 499} \approx 1 \times 10^2 \text{ m}, \tag{61}$$

six orders of magnitude larger than $r_{\rm coh}^{(\rm sun,obs)}$. This is not a contradiction; it is a signal that Eq. (50) applies only within a bounded near-field region set by the formation-stage aperture, beyond which the wave packet propagates as an ordinary diffracting wave rather than continuing to diffuse.

The crossover. The diffusive growth law of Sec. 3.3 describes the ZPF-driven build-up of the wave packet during *formation*, seeded by an aperture of radius $w_0 = r_{\rm coh}(\tau_{\rm coh})$ from Eq. (56). Once formed, the packet is a solution of the free Maxwell equations (Sec. 5) and must propagate by ordinary diffraction, with Rayleigh range

$$z_R = \frac{\pi w_0^2}{\lambda}. \tag{62}$$

Using the sunlight continuum aperture already established in Eq. (57), $w_0 = r_{\rm coh}^{(\rm sun)} \approx 286$ nm, $z_R \approx 0.43\,\mu$m; for a narrow spectral line ($\tau_{\rm coh} \approx 50$ ps, giving via Eq. (56) $w_0 = \bar{\lambda}\sqrt{\omega_0 \tau_{\rm coh}} \approx 38\,\mu$m), $z_R \approx 7.5$ mm. In both cases $z_R \ll z_1 = 1$ AU, so the diffusive regime of Eq. (50) is confined to a region within a fraction of a micrometre to a centimetre of the emitting atom; essentially the entire Sun–Earth path lies in the diffractive far field, where the transverse radius grows *linearly* with distance rather than as $\sqrt{t}$:

$$w(d) \approx \theta\, d, \qquad \theta \approx \frac{\lambda}{\pi w_0}, \qquad d \gg z_R. \tag{63}$$

Size at Earth. Applying Eq. (63) at the Sun–Earth distance $d = z_1 = 1.496 \times 10^{11}$ m:

$$\text{continuum:} \quad \theta \approx 0.67 \text{ rad}, \qquad w(z_1) \approx 1.0 \times 10^{11} \text{ m } (\sim 0.67 \text{ AU}), \tag{64}$$

$$\text{line:} \quad \theta \approx 5.0 \times 10^{-3} \text{ rad}, \qquad w(z_1) \approx 7.5 \times 10^{8} \text{ m } (\sim 2 \times \text{Earth–Moon distance}). \tag{65}$$

These are estimates of the single-photon coherent wavefront radius at Earth, and they are enormous compared with $r_{\rm coh}^{(\rm sun,obs)} \approx 28\,\mu$m—but for a reason that removes, rather than deepens, the apparent tension. The quantity $d_{\rm coh}^{(\rm sun)}$ of Eq. (58) is not the coherence extent of a single photon's wavefront at all. It is a van Cittert–Zernike quantity, obtained by incoherently summing the fields of $\sim 10^{20}$ independent, randomly-phased emitters distributed over the solar disk; it is precisely this averaging over uncorrelated sources that shrinks the *ensemble* coherence patch relative to any single photon's own diffracted wavefront. A single photon, by contrast, undergoes no such averaging: it is one continuous solution of the free-space Maxwell equations, spreading

outward from a single atomic emission event, and nothing in ordinary wave optics prevents that wavefront from becoming macroscopic over an astronomical propagation distance.

This is correspondence, not new physics. The large sizes in Eqs. (64)–(65) are Bohr-correspondence behaviour, not a signature of the ZPF mechanism proposed here. A Liénard–Wiechert field from a single accelerating charge is, in the far zone, a smooth spherical wavefront, and nothing in classical or quantum electrodynamics restricts how far such a wavefront may spread; any orthodox treatment of the free-photon wave equation, given the same small formation-scale aperture w_0, would predict the same diffraction-limited growth. The prediction that *is* unique to this framework is the formation-scale aperture itself,

$$w_0 = r_{\mathrm{coh}}(\tau_{\mathrm{coh}}) = \bar{\lambda}\sqrt{\omega_0 \tau_{\mathrm{coh}}}, \tag{66}$$

derived here from $D_\gamma = \hbar/2m_\gamma^{\mathrm{eff}}$ rather than assumed—not the subsequent diffractive growth, which is inherited, unremarkable wave optics once w_0 is fixed. Equation (58) and the results of this subsection should therefore be read as descriptions of two distinct physical quantities at two disparate length scales, not as competing predictions for the same observable. (The numerical coefficients in Eq. (63) follow the standard Gaussian-beam divergence convention and are order-of-magnitude estimates; Sec. 7.5 below re-derives the diffusive-to-diffractive transition directly from the photon SDE, confirming the qualitative picture and giving the exact analogue of Eq. (63) up to this convention-dependent factor.)

7.5 Exact Time Evolution: The Crossover Derived from the Photon SDE

Section 7.4 argued for the diffusive-to-diffractive crossover by analogy with classical Gaussian-beam optics. This subsection derives the same crossover directly from the photon SDE of Eq. (21), with no imported optics formula.

The transverse dynamics is a free Schrödinger equation. Equation (21) is, by construction, the exact photon analogue of Nelson's electron SDE: the same osmotic-velocity structure $D_\gamma \nabla \ln \rho_\gamma$ and the same ZPF-driven Wiener noise $\sqrt{2D_\gamma}\, d\mathbf{W}$, restricted to the plane transverse to $\hat{\mathbf{k}}$ (the propagation direction is deterministic, at speed c, and carries no diffusive spread of its own). Equation (43) already shows that the Madelung transform $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ converts the electron's Nelson SDE into the free Schrödinger equation. Applying the identical transform to the *transverse* marginal of the photon Fokker–Planck equation (22), in the frame co-moving with the pulse at speed c along $\hat{\mathbf{k}}$, gives by the same algebra

$$i\hbar \frac{\partial \psi_\gamma}{\partial t} = -\frac{\hbar^2}{2m_\gamma^{\mathrm{eff}}} \nabla_\perp^2 \psi_\gamma, \tag{67}$$

a genuine two-dimensional *free-particle* Schrödinger equation for the transverse envelope, with the photon's effective mass $m_\gamma^{\mathrm{eff}} = \hbar\omega_0/c^2$ playing the role of m. No new postulate is introduced:

Eq. (67) is forced by the same Madelung transform already used elsewhere in this paper, applied to the transverse sector of the same SDE that produced D_γ in the first place.

Exact solution for a Gaussian aperture. Equation (67) is the standard free Schrödinger equation, whose exact solution for an initial Gaussian envelope of width w_0 (set at the end of formation, $w_0 = r_{\rm coh}(\tau_{\rm coh})$ from Eq. (56), taking $t = 0$ at the end of the formation stage) is well known:

$$|\psi_\gamma(r_\perp, t)|^2 \;\propto\; \exp\!\left[-\frac{r_\perp^2}{2\,w(t)^2}\right], \qquad w(t)^2 = w_0^2\left[1 + \left(\frac{t}{\tau_R}\right)^2\right], \qquad \tau_R \equiv \frac{w_0^2}{D_\gamma}. \tag{68}$$

This is obtained from Eq. (67) exactly the same way the ordinary free-particle Gaussian wave-packet spreading formula is obtained from the electron's free Schrödinger equation, with $D \to D_\gamma$ and $m \to m_\gamma^{\rm eff}$.

The two limits. Equation (68) contains both regimes of Sec. 7.4 as exact limits of a single formula, with no ad-hoc matching at z_R:

- **Near zone, $t \ll \tau_R$:** $w(t) \approx w_0$, constant. The transverse size stays pinned near its formation value—this is the regime in which the diffusive language of Eq. (50) is meaningful, and it is now seen to be transient rather than exact even there (the true $w(t)^2$ is quadratic in t near $t = 0$, not linear).
- **Far zone, $t \gg \tau_R$:** $w(t) \approx (D_\gamma/w_0)\,t$, *linear* in t. This is precisely the ballistic, spherical-wave-like growth invoked in Sec. 7.4 on the basis of the Liénard–Wiechert far-zone field and ordinary diffraction—but it is now obtained directly from solving the photon SDE, with no appeal to classical optics.

The crossover time $\tau_R = w_0^2/D_\gamma$ plays exactly the role of the Rayleigh range z_R used previously ($z_R^{(\rm exact)} = c\tau_R$), differing from the optics-convention value of Eq. (62) only by an $O(1)$ factor that depends on the (differing) conventions for relating a Gaussian's standard deviation to a "beam waist"; numerically, $z_R^{(\rm exact)} \approx 4\times$ the value used in Sec. 7.4, for both the continuum and line cases, confirming that this factor is a fixed convention artifact rather than a physical discrepancy. The qualitative and parametric content—a near-field plateau followed by a crossover into growth linear in t—is exact and convention-independent.

Conclusion. The spherical-wave, correspondence-principle behaviour of Sec. 7.4 is not an assumption imported from classical optics; it is what Eq. (21) itself predicts once solved exactly rather than truncated to its short-time diffusive limit. This closes the gap left open above: the diffusive-to-diffractive transition is now a direct consequence of the same stochastic equations used throughout this paper, rather than an analogy grafted onto them.

7.6 Physical Interpretation

The coherence volume $V_{\text{coh}} = \pi r_{\text{coh}}^2 \cdot c\tau_{\text{coh}}$ is the region of the electromagnetic vacuum ZPF that is correlated with the photon. It is the photon's analogue of the electron's Coulomb field extent:

	Electron	**Photon**
"Field extent"	Coulomb field ($\propto 1/r^2$, infinite)	Coherence volume (V_{coh}, finite)
Set by	Rest mass m	Emission process (τ_{coh})
Minimum scale	Compton wavelength $\hbar/mc$	Reduced wavelength $\bar{\lambda} = c/\omega_0$
ZPF extent	Infinite (Coulomb)	V_{coh} (wave packet)

7.7 Two Meanings of $\lambda d/D$: Aperture Diffraction versus Van Cittert–Zernike

Equations (63) and (58) share the identical algebraic form, size $\sim \lambda d/D$, and it is worth stating explicitly that they describe two physically opposite mechanisms, distinguished by what D represents:

- **Aperture diffraction** ($D = w_0$, a single coherent emitter's own formation-scale size, as in Sec. 7.4): one coherent wavefront necessarily diverges with angle $\theta \sim \lambda/w_0$, so a *smaller* coherent aperture produces *faster* angular spreading and hence a *larger* spot at distance d. This is a single wave spreading outward; no averaging over independent sources is involved.
- **Van Cittert–Zernike** ($D = D_\odot$, the diameter of an extended *incoherent* source, as in Eq. (58)): here a *larger* source diameter produces a *smaller* coherence patch at distance d, because the mechanism is destructive interference between the randomly-phased contributions of many independent emitters spread across D—the opposite physical process from aperture diffraction, even though the resulting formula looks the same.

The two are not competing predictions for the same quantity: the first answers "how large is the coherent wavefront of one photon at distance d" and the second answers "over what patch do many independent photons from an extended source remain mutually coherent." Both are needed to correctly interpret coherence measurements of extended thermal sources such as the Sun.

7.8 Classical Propagation, Quantum Emission and Detection

The results of this section are derived entirely from classical Maxwell theory (the Liénard–Wiechert field, ordinary diffraction), yet they are used throughout this paper to describe the coherence properties of *single* photons. This is not an extension of classical results into the quantum domain by analogy: it is a consequence of the fact that the Riemann–Silberstein vector, which *is* the single-photon wave function in this framework (Secs. 4–5), obeys the free Maxwell equations exactly, whether it carries the amplitude for a single photon or the field of a macroscopic, many-photon beam. Propagation—diffraction, interference, coherence-area

formulas—is governed by the same linear wave equation regardless of photon number; nothing in that equation changes when the number of quanta is reduced to one.

This is well established experimentally. Single-photon interference was first demonstrated by Taylor using extremely attenuated light [19], and modern heralded single-photon double-slit experiments build up, one detection event at a time, exactly the fringe pattern predicted by classical diffraction theory. Photon-counting intensity interferometry [20] likewise confirms that van Cittert–Zernike coherence-area predictions from classical optics correctly describe single-photon correlation statistics at astronomical sources. The one phenomenon classical electrodynamics alone cannot supply—and the one place genuinely quantum behaviour enters—is the *discreteness* of detection: individual, randomly-positioned clicks that only statistically build up the classical intensity pattern, together with phenomena such as photon antibunching that have no classical field analogue whatsoever.

The clean statement, consistent with the emission/detection stochastic picture developed in Sec. 8 and Sec. 6.7, is therefore: *propagation is classical Maxwell theory, exactly; the only quantum steps are which discrete quantum of energy is created at emission, and which single, stochastically-selected detector absorbs it.* Every coherence and diffraction formula in this paper is exact for that reason, not merely valid as a semiclassical approximation.

7.9 Worked Examples: Natural Linewidths of Atomic Hydrogen

Section 7 worked out $\tau_{\rm coh}$, $r_{\rm coh}$, and the diffusive-to-diffractive crossover for two illustrative continuum sources: sunlight and a single-mode laser. This subsection applies the same machinery to three specific, spectroscopically well-characterized atomic hydrogen transitions, for which the natural (Doppler-free) linewidth $\Delta\nu$ is known either from theory or from direct measurement. This serves two purposes: it grounds Eq. (56) in real experimental numbers rather than order-of-magnitude estimates alone, and—in the Lyman-α case—it provides a genuine internal consistency check of the framework, using only the electron radiation-reaction time τ_0 already defined in Sec. 3, with no additional input.

As in Sec. 11.2, $\tau_{\rm coh} = 1/A_{eg}$ for a two-level emitter, equivalently $\tau_{\rm coh} = 1/(2\pi\Delta\nu)$ for the natural (lifetime-limited) linewidth $\Delta\nu$. The longitudinal wave-packet length follows directly, $l_{\rm coh} = c\,\tau_{\rm coh}$, and the transverse coherence radius from Eq. (56), $r_{\rm coh} = \sqrt{2D_\gamma\tau_{\rm coh}} = \bar{\lambda}\sqrt{\omega_0\tau_{\rm coh}}$.

Lyman-α (2p→1s, $\lambda = 121.567$ nm). The $2p$ state has radiative lifetime $\tau(2p) = 1.596$ ns, giving natural linewidth $\Delta\nu = 1/(2\pi\tau) \approx 99.7$ MHz [23]. With $\omega_0 = 1.5495 \times 10^{16}$ rad/s:

$$\tau_{\rm coh} = 1.596 \text{ ns}, \qquad l_{\rm coh} = 0.478 \text{ m}, \qquad r_{\rm coh} = 96.2\ \mu\text{m}, \qquad V_{\rm coh} = 1.39 \times 10^{-8}\ \text{m}^3. \tag{69}$$

Balmer-α, 2S–3P component ($\lambda = 656.3$ nm). The natural linewidth of this transition, $\Delta\nu = 30$ MHz, has been directly established via Doppler-free saturated-absorption spectroscopy on ultracold, magnetically trapped metastable hydrogen, where sub-Doppler conditions allow the natural width to be approached experimentally [24]. With $\omega_0 = 2.8702 \times 10^{15}$ rad/s:

$$\tau_{\rm coh} = 5.31 \text{ ns}, \qquad l_{\rm coh} = 1.59 \text{ m}, \qquad r_{\rm coh} = 408\ \mu\text{m}, \qquad V_{\rm coh} = 8.30 \times 10^{-7}\ \text{m}^3. \tag{70}$$

1S–2S two-photon transition ($\lambda = 243$ nm, two-photon excitation). The $2s$ state cannot decay to $1s$ by single-photon electric-dipole emission ($\Delta l = 0$ is E1-forbidden); its dominant decay channel is simultaneous two-photon emission, with rate $\Gamma_{2s\to 1s} = 8.229\ \mathrm{s}^{-1}$ (i.e. $\tau = 121.5$ ms, $\Delta\nu = \Gamma/2\pi = 1.31$ Hz), first obtained in closed form by Klarsfeld using the Coulomb Green's function [25], and confirmed by every independent calculation since [25]. Applying $\tau_{\mathrm{coh}} = 1/\Gamma$ and Eq. (56) with ω_0 taken from the 243 nm two-photon probe frequency gives, purely for illustration,

$$\tau_{\mathrm{coh}} = 121.5\ \mathrm{ms}, \qquad l_{\mathrm{coh}} = 3.64 \times 10^{7}\ \mathrm{m}, \qquad r_{\mathrm{coh}} = 1.19\ \mathrm{m}, \qquad V_{\mathrm{coh}} = 1.61 \times 10^{8}\ \mathrm{m}^3. \quad (71)$$

This case must be flagged explicitly: it lies outside the single-photon spontaneous-emission picture developed in this paper. Two-photon decay is a genuine second-order process—requiring summation over a complete set of virtual intermediate np states via the Coulomb Green's function [25]—not a single photon of well-defined frequency ω_0 emitted from a two-level system. The numbers in Eq. (71) should be read as an order-of-magnitude illustration of how large τ_{coh} (and hence l_{coh}) can become for a sufficiently metastable level, not as a rigorous application of Eq. (56). Extending the electron–photon stochastic coupling of Sec. 9 to genuinely second-order (two-vertex) processes, so that this case can be treated on the same footing as the other two, is listed as an open problem in Sec. 11.3.

Transition	λ	$\Delta\nu$ (natural)	τ_{coh}	$l_{\mathrm{coh}} = c\tau_{\mathrm{coh}}$	r_{coh} (Eq. 56)	V_{coh}
Lyman-α (2p→1s)	121.567 nm	99.7 MHz	1.596 ns	0.478 m	96.2 μm	1.39×10^{-8} m^3
Balmer-α (2S–3P)	656.3 nm	30 MHz	5.31 ns	1.59 m	408 μm	8.30×10^{-7} m^3
1S–2S (two-photon)*	243 nm†	1.31 Hz	121.5 ms	3.64×10^{7} m	1.19 m	1.61×10^{8} m^3

Table 2: Coherence properties of three hydrogen transitions computed from their measured or calculated natural (Doppler-free) linewidths via $\tau_{\mathrm{coh}} = 1/2\pi\Delta\nu$ and Eq. (56). *Two-photon E1E1 decay, second-order process, outside the single-photon domain of Secs. 3–7; included for illustration only (see text). †Wavelength of the two-photon excitation laser, used here as a nominal ω_0 since no single-frequency emitted photon exists for this transition.

Self-consistency check: the electron radiation-reaction time. The three cases above (summarized in Table 2) take τ_{coh} as an external input, exactly as prescribed in Sec. 11.2. It is worth asking how well τ_{coh} for a single-channel transition can instead be estimated from material already internal to this paper—specifically, the electron's classical radiation-reaction time $\tau_0 = e^2/6\pi\epsilon_0 mc^3 = 6.2664 \times 10^{-24}$ s introduced in Sec. 3 for the electron AL equation (8). Treating the radiating electron as a classical Lorentz oscillator of frequency ω_0 gives the classical radiative damping rate $\Gamma_{\mathrm{cl}}(\omega_0) = \omega_0^2\tau_0$, and hence a purely classical, SED-internal estimate $\tau_{\mathrm{coh}}^{(\mathrm{cl})} = 1/\Gamma_{\mathrm{cl}} = 1/(\omega_0^2\tau_0)$, with no quantum matrix element supplied.

For Lyman-α, $\Gamma_{\mathrm{cl}} = 1.505 \times 10^{9}\ \mathrm{s}^{-1}$, giving $\tau_{\mathrm{coh}}^{(\mathrm{cl})} = 0.665$ ns—compared to the true $\tau_{\mathrm{coh}} = 1.596$ ns in Eq. (69), a ratio of 0.4165. This reproduces the known quantum oscillator strength $f(1s \to 2p) = 0.4162$ [23] to within 0.1%. Since $A_{\mathrm{quantum}} = f\,\Gamma_{\mathrm{cl}}$ is the standard relation between the Einstein coefficient and the classical (unit-oscillator-strength) damping rate, this

is not a coincidence: it confirms that the purely classical radiating-dipole picture underlying τ_0—already present in this paper for the electron AL equation—correctly reproduces the true natural linewidth of a genuine two-level transition once the standard oscillator-strength reduction is accounted for. Lyman-α is such a case, since the $2p$ level has only one allowed E1 decay channel.

The same test fails informatively for Balmer-α: $\Gamma_{\rm cl}(\omega_{0\mathrm{H}\alpha}) = 5.16\times10^7\ \mathrm{s}^{-1}$ gives $\tau_{\rm coh}^{(\rm cl)} = 19.4$ ns, *longer* than the true 5.31 ns by a factor of 3.65—a discrepancy no oscillator strength $f \le 1$ can repair. The reason is that the upper level, $3p$, decays through two channels, not one: to $2s$ (Balmer-α, $\lambda = 656.3$ nm) and to $1s$ (Lyman-β, $\lambda = 102.57$ nm), and because $\Gamma_{\rm cl} \propto \omega_0^2$, the shorter-wavelength channel dominates the classical estimate by a factor $(\omega_{0\mathrm{Ly}\beta}/\omega_{0\mathrm{H}\alpha})^2 \approx 40.9$. The natural linewidth of any line originating from a multiply-decaying level is set by the level's *total* decay rate, summed over all channels weighted by their respective oscillator strengths—information the single-frequency classical Lorentz-oscillator picture, as used here, does not supply. This is a genuine and instructive limitation of the present treatment for multi-channel decays, distinct from the two-photon limitation of the 1S–2S case above, and is likewise flagged in Sec. 11.3.

Three-way comparison: SED, QED, and experiment. Table 3 collects, side by side, two SED calculations, QED, and the directly measured values for the two single-photon cases. The first SED column is the naive estimate available at this point in the paper, using only the electron radiation-reaction time τ_0 of Sec. 3 with a classical unit-strength oscillator. The second SED column is the exact result, derived in Sec. 9.4 from the true SED/Nelson dipole moment; it is quoted here for direct comparison, with the derivation deferred to that section. (The 1S–2S two-photon transition is omitted: no experiment has yet resolved its natural linewidth—see Sec. 7.9—so a comparison is not presently possible for that case.)

Transition	SED, naive ($\Gamma_{\rm cl} = \omega_0^2\tau_0$)	SED, exact (Sec. 9.4)	QED (theory)	Experiment
Lyman-α (2p→1s)	$\tau_{\rm coh} = 0.665$ ns	$\tau_{\rm coh} = 1.598$ ns	$\tau(2p) = 1.596$ ns	$\tau = 1.60$ ns [26]
Balmer-α ($3p$ level)	$\tau_{\rm coh} = 19.4$ ns (naive, single-channel)	$\tau_{\rm coh} = 5.269$ ns ($\Delta\nu = 30.2$ MHz)	$\tau = 5.31$ ns ($\Delta\nu = 30$ MHz)	$\Delta\nu = 30$ MHz [24] ($3p$ lifetime confirmed [26])

Table 3: SED (naive and exact), QED, and measured values of $\tau_{\rm coh}$ for the two single-photon transitions of Sec. 7.9. The exact SED column (Eqs. (92) and (95)) uses the true SED/Nelson dipole moment in place of the naive column's unit-strength oscillator, and is derived in full in Sec. 9.4.

It is worth being precise about what separates each SED column from QED, since none of these is a different order of a perturbative expansion. All three rates are $O(e^2)$—one photon emitted, no loops—so SED and QED are evaluating the *same* lowest-order process throughout. Genuine higher-order QED corrections to these rates (radiative corrections of relative order $\alpha(Z\alpha)^2$, analogous in spirit to the Lamb shift but applied to a decay rate rather than an energy level) exist in the literature but are parts-per-million for hydrogen and are not part of the numbers quoted here. The gap between the *naive* SED column and QED comes from a single

source: naive SED treats the radiating electron as a classical Lorentz oscillator with unit coupling strength ($f = 1$), while QED evaluates the exact quantum dipole matrix element $\langle 1s|\mathbf{r}|2p\rangle$ built from the true Coulomb wavefunctions. For Lyman-α that difference is entirely captured by the single number $f(1s \to 2p) = 0.4162$; for Balmer-α it is not captured at all, because the missing physics there is an entire absent decay channel, not an imprecise matrix element. The *exact* SED column removes this gap for both cases simultaneously, by replacing the unit-strength placeholder with the dipole moment Nelson's theorem actually guarantees—see Sec. 9.4 for the derivation and Eqs. (92) and (95) for the explicit calculation.

This yields a calibrated, rather than blanket, statement about the present framework: the *naive* single-oscillator shortcut reproduces experiment reasonably well *exactly where its underlying assumption holds*—a single radiating dipole with one available decay channel, as for Lyman-α—and fails informatively, in a diagnosable and well-understood way, where that assumption does not hold, as for Balmer-α's multiply-decaying upper level. The *exact* SED/Nelson calculation has no such failure mode in either case, agreeing with QED and experiment to under 1% for both transitions, because it is not really a distinct calculation from QED at all—it is the same calculation, performed within this paper's own formalism, as shown explicitly in Sec. 9.4.

8 The Quantum Eraser for Photons

8.1 Setup

The quantum eraser was originally proposed as a thought experiment by Scully and Drühl [10]. The first real experimental realisation with photons was achieved by Kim et al. [11], who demonstrated the delayed-choice version with entangled photon pairs. The experiment provides the most direct test of whether randomness is controlled by the physical ZPF state or by the observer's knowledge. The setup is:

1. A photon passes through a double slit. A which-path marker entangles the signal photon's path with the polarization of an idler photon: slit A $\to |V\rangle_{\text{idler}}$, slit B $\to |H\rangle_{\text{idler}}$.

2. The joint state is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_A\rangle_{\text{signal}}|V\rangle_{\text{idler}} + |\psi_B\rangle_{\text{signal}}|H\rangle_{\text{idler}})\,. \tag{72}$$

3. Measuring the idler in the V/H basis reveals which path $\to$ no interference. Measuring in the $\pm 45^\circ$ basis erases which-path information $\to$ interference restored in each sub-ensemble.

8.2 Explanation in the ZPF Framework

Why the marker destroys fringes

The marker interaction at the slit physically entangles the signal photon's ZPF with the idler photon. The signal photon's probability density is no longer $|\psi_A + \psi_B|^2$ but the mixed state:

$$\rho_{\text{signal}}(\mathbf{x}) = \frac{1}{2}|\psi_A(\mathbf{x})|^2 + \frac{1}{2}|\psi_B(\mathbf{x})|^2. \tag{73}$$

The osmotic velocity becomes:

$$u(\mathbf{x}) = D_\gamma \nabla \ln\left(\tfrac{1}{2}|\psi_A|^2 + \tfrac{1}{2}|\psi_B|^2\right), \tag{74}$$

which has no interference term. No fringes.

Why erasure restores fringes

Measuring the idler in the $\pm 45^\circ$ basis gives outcomes:

$$|\pm 45^\circ\rangle = \frac{1}{\sqrt{2}}\left(|V\rangle \pm |H\rangle\right). \tag{75}$$

For the $+45^\circ$ outcome, the signal photon's state is *post-selected* to:

$$\rho_{\text{signal}}(\mathbf{x}\,|\,+45^\circ) = |\psi_A(\mathbf{x}) + \psi_B(\mathbf{x})|^2. \tag{76}$$

The osmotic velocity restores to $u = D_\gamma \nabla \ln|\psi_A + \psi_B|^2$. Interference fringes reappear. For the -45° outcome the fringes are shifted by π, so the two sub-ensembles show complementary patterns that cancel when added—confirming no-signalling.

8.3 Does the Idler Measurement Instantly Affect the Signal Photon?

This is the key question. The answer in the present framework is:

> *No. The idler measurement does not physically disturb the signal photon's ZPF. It is a Bayes update of the joint probability distribution—a change in our knowledge of a pre-existing correlation, not a physical influence.*

The joint probability density $\rho(\mathbf{x}_{\text{signal}}, s_{\text{idler}})$ was established at the moment of marker interaction. Measuring the idler and finding $+45^\circ$ applies Bayes' theorem:

$$\rho(\mathbf{x}_{\text{signal}}\,|\,+45^\circ) = \frac{\rho(\mathbf{x}_{\text{signal}},\,+45^\circ)}{\displaystyle\int \rho(\mathbf{x},\,+45^\circ)\,d\mathbf{x}} = |\psi_A + \psi_B|^2. \tag{77}$$

No-signalling. The marginal distribution of the signal photon is unchanged by any idler

measurement:

$$\rho(\mathbf{x}_{\text{signal}}) = \int \rho(\mathbf{x}_{\text{signal}},\, s_{\text{idler}})\, ds_{\text{idler}} = \frac{1}{2}|\psi_A|^2 + \frac{1}{2}|\psi_B|^2. \tag{78}$$

No fringes, no dependence on the idler measurement. Only after the idler result is *classically communicated* and the signal data *post-selected* do sub-ensemble fringes appear.

Situation	**Signal photon ZPF**	**Result**
No marker	$\rho = \lvert\psi_A + \psi_B\rvert^2$, undisturbed	Interference fringes
Marker, idler unmeasured	$\rho = \frac{1}{2}\lvert\psi_A\rvert^2 + \frac{1}{2}\lvert\psi_B\rvert^2$, mixed	No fringes
Marker, idler $= V/H$	Post-selected to $\lvert\psi_A\rvert^2$ or $\lvert\psi_B\rvert^2$	No fringes (single-slit)
Marker, idler $= \pm 45°$	Post-selected to $\lvert\psi_A \pm \psi_B\rvert^2$	Fringes in sub-ensemble

The quantum eraser thus confirms the central claim of the present paper: fringes are controlled by the physical ZPF state, not by the observer's knowledge. The "instant non-local collapse" is an artefact of the Copenhagen language—in the ZPF framework it is simply Bayes' theorem applied to a pre-existing joint distribution.

8.4 The Delayed-Choice Quantum Eraser

The *delayed-choice* quantum eraser [11] is the most striking version of the experiment. The decision of whether to erase the which-path information is made *after* the signal photon has already arrived at the screen. This appears to imply retrocausality: a future choice determines a past event.

Why there is no retrocausality

The present framework resolves the apparent paradox completely through a single observation:

> *The full signal photon data—all events recorded at the screen—never shows interference fringes, regardless of what the idler measurement will be. Fringes appear only in a sub-ensemble post-selected on the idler outcome. Post-selection at a later time does not change the past; it sorts data that was always structured that way.*

Timeline and ZPF state:

t	**Event**	**ZPF framework**
t_0	Photon pair created	Joint ZPF correlation $\rho(\mathbf{x}_s, s_i)$ established
t_1	Signal hits screen	Full data recorded. Marginal $\rho(\mathbf{x}_s) = \frac{1}{2}\lvert\psi_A\rvert^2 + \frac{1}{2}\lvert\psi_B\rvert^2$ — *no fringes*
$t_2 > t_1$	Choice: erase or not?	Classical decision; no physical effect on $\rho(\mathbf{x}_s)$
$t_3 > t_2$	Idler measured; data sorted	Bayes post-selection reveals sub-ensemble structure that was always present in $\rho(\mathbf{x}_s, s_i)$

Mathematical proof of no retrocausality.

For all times $t \geq t_0$, the signal photon marginal satisfies:

$$\rho(\mathbf{x}_s) = \int \rho(\mathbf{x}_s,\, s_i)\, ds_i = \tfrac{1}{2}|\psi_A(\mathbf{x}_s)|^2 + \tfrac{1}{2}|\psi_B(\mathbf{x}_s)|^2. \tag{79}$$

This is independent of:

- *When* the idler is measured (t_2 can be years after t_1),
- *Whether* the idler is measured at all,
- *Which basis* is chosen for the idler.

The post-selected sub-ensembles:

$$\rho(\mathbf{x}_s \,|\, +45^\circ) = |\psi_A(\mathbf{x}_s) + \psi_B(\mathbf{x}_s)|^2 \quad \text{(fringes)}, \tag{80}$$

$$\rho(\mathbf{x}_s \,|\, -45^\circ) = |\psi_A(\mathbf{x}_s) - \psi_B(\mathbf{x}_s)|^2 \quad \text{(complementary fringes)}. \tag{81}$$

Adding (80) and (81) with equal weight recovers (79):

$$\tfrac{1}{2}|\psi_A + \psi_B|^2 + \tfrac{1}{2}|\psi_A - \psi_B|^2 = |\psi_A|^2 + |\psi_B|^2. \tag{82}$$

The two sub-ensemble fringe patterns are exactly complementary and cancel—which is why the full (unsorted) data never shows fringes.

The paradox dissolved.

The delayed-choice quantum eraser does not show retrocausality. It shows that:

1. The joint ZPF correlation $\rho(\mathbf{x}_s, s_i)$ was fully determined at t_0.
2. The signal data at t_1 already *contained* the sub-ensemble structure—it was not yet *sorted*.
3. Sorting at t_3 reveals structure that was always present. The future does not change the past.

9 QED from Electron-Photon Stochastic Coupling

9.1 The Two Stochastic Processes

The electron SDE (Paper 4 [4]):

$$du^\mu = \frac{e}{m} F^{\mu\nu}_{\text{ZPF}} u_\nu \, d\tau + \sqrt{\frac{\hbar}{m}} \, dW^\mu_{\text{electron}}(\tau). \tag{83}$$

The photon SDE (this paper):

$$d\mathbf{x}_\gamma = c\hat{\mathbf{k}} \, dt + D_\gamma \nabla \ln u_{\text{EM}} \, dt + \sqrt{2D_\gamma} \, d\mathbf{W}_\gamma(t). \tag{84}$$

9.2 The Coupling

The electromagnetic field $F^{\mu\nu}_{\text{ZPF}}$ in the electron SDE includes both the vacuum ZPF and the photon field:

$$F^{\mu\nu}_{\text{total}} = F^{\mu\nu}_{\text{vacuum ZPF}} + F^{\mu\nu}_{\text{photon}}. \tag{85}$$

The coupling between the two SDEs is:

$$\mathcal{H}_{\text{int}} = e \, j^\mu A_\mu = e \underbrace{\bar{\psi}\gamma^\mu\psi}_{\text{electron current}} \underbrace{A_\mu}_{\text{photon field}}. \tag{86}$$

This is the QED interaction vertex—emerging naturally from the coupling of the electron and photon stochastic processes. The Feynman rules of QED correspond to the perturbative expansion of this stochastic coupling in powers of $e^2 = 4\pi\alpha\hbar c$.

9.3 Propagators from the SDEs

Electron propagator. The Green's function of the electron SDE gives the Feynman electron propagator:

$$S_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}. \tag{87}$$

Photon propagator. The cross-correlation $S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega)$ derived in Paper 2 [2] is the photon propagator:

$$S_E^{(12)} = S_E(\omega) \cdot \text{sinc}\left(\frac{\omega r_{12}}{c}\right) \;\leftrightarrow\; D^{\mu\nu}(k) = \frac{-ig^{\mu\nu}}{k^2 + i\epsilon}. \tag{88}$$

The sinc factor is the position-space photon propagator. The coherence volume of the photon equals the spatial extent of this propagator.

9.4 Spontaneous Emission Rates: The Correspondence-Principle Bridge

Section 7.9 tested the electron–photon coupling of this section against real hydrogen data, but only through the crude proxy $\Gamma_{\rm cl}(\omega_0) = \omega_0^2\tau_0$ of Sec. 3—a classical Lorentz oscillator of *unit* strength, corrected afterward by an externally-supplied oscillator strength f. This subsection asks whether the electron–photon coupling $H_{\rm int} = ej^\mu A_\mu$ of Sec. 9, applied properly rather than via that shortcut, determines atomic spontaneous emission rates on its own.

Why this differs from the Lamb shift and $g-2$. The one-loop corrections of Sec. 9.5 are non-Markovian: the electron's radiation reaction acts back on the *same* energy level, requiring the memory-dependent machinery of Papers 5 and 6 [5, 6]. Spontaneous emission is different in kind—it is the tree-level (single-vertex, no-loop) process itself, not a correction to it. The relevant tool is therefore not the non-Markovian expansion, but the tree-level content of Sec. 9 combined with a fact already established earlier in this series.

Nelson's theorem supplies the exact dipole moment. Paper 1 [1] establishes, via Nelson's theorem, that the electron's Fokker–Planck equation is equivalent to the Schrödinger equation: the stochastic-mechanical density $\rho(\mathbf{x},t)$ and the quantum $|\psi(\mathbf{x},t)|^2$ are the same function for the same state, for *any* ψ—including a coherent superposition of atomic levels. For $\psi = c_1(t)\psi_{1s}e^{-iE_1t/\hbar} + c_2(t)\psi_{2p}e^{-iE_2t/\hbar}$, the SED electron's ensemble-mean position therefore satisfies

$$\langle \mathbf{x}(t)\rangle_{\rm SED} = \langle\psi|\mathbf{x}|\psi\rangle = 2\,\mathrm{Re}\big[c_1^*(t)c_2(t)\,\mathbf{x}_{12}\,e^{-i\omega_{21}t}\big]\,, \qquad \mathbf{x}_{12} \equiv \langle 1s|\mathbf{x}|2p\rangle, \tag{89}$$

exactly, with no approximation beyond what Paper 1 already established. The SED electron's mean trajectory genuinely oscillates at the transition frequency ω_{21}, with an amplitude fixed by the true quantum dipole matrix element $\mathbf{x}_{12}$—not by an arbitrary unit-strength placeholder.

The correspondence-principle theorem. It is a rigorous, long-established result—first obtained by Dirac from the quantized-field theory [27], and independently by Kramers and Heisenberg from the correspondence principle, as reviewed in detail by Milonni [28]—that the exact QED spontaneous emission rate equals the rate at which a *classical* oscillating dipole of moment $2\,\mathrm{Re}[e\mathbf{x}_{12}\,e^{-i\omega_{21}t}]$ radiates energy via the ordinary Larmor formula, divided by $\hbar\omega_{21}$ per photon:

$$A_{21} = \frac{1}{\hbar\omega_{21}} \times \Big\langle P_{\rm Larmor}\big(2e\,\mathrm{Re}[\mathbf{x}_{12}e^{-i\omega_{21}t}]\big)\Big\rangle = \frac{e^2\omega_{21}^3|\mathbf{x}_{12}|^2}{3\pi\epsilon_0\hbar c^3}. \tag{90}$$

This is not approximate: Milonni's review is explicit that Dirac's perturbative QED result and the classical-dipole calculation give identical expressions [28]. Equation (90) is independently consistent with the oscillator-strength relation used in Sec. 7.9: writing $A_{21} = f_{12}\,\Gamma_{\rm cl}(\omega_{21})$ with $\Gamma_{\rm cl} = e^2\omega_{21}^2/6\pi\epsilon_0 mc^3$ and $f_{12} = (2m\omega_{21}/3\hbar)|\mathbf{x}_{12}|^2$ reproduces (90) exactly [29]—the same $\Gamma_{\rm cl} = \omega_0^2\tau_0$ already defined in Sec. 3, now with the placeholder unit oscillator replaced by the true SED/Nelson dipole amplitude $\mathbf{x}_{12}$.

Worked example: Lyman-α from first principles. The dipole matrix element $\mathbf{x}_{12}$ is not merely asserted to exist; it can be evaluated directly from the exact hydrogen radial wavefunctions $R_{10}(r) = 2a_0^{-3/2}e^{-r/a_0}$ and $R_{21}(r) = \frac{1}{2\sqrt{6}}a_0^{-3/2}(r/a_0)e^{-r/2a_0}$. For the $2p(m{=}0) \to 1s$ sublevel transition, the radial integral gives

$$z_{12} \equiv \langle 2p, m{=}0|z|1s\rangle = \int_0^\infty R_{21}(r)\, r\, R_{10}(r)\, r^2\, dr \times \frac{1}{\sqrt{3}} = \frac{2^7\sqrt{2}}{3^5}\, a_0 = 0.74494\, a_0, \tag{91}$$

the well-known closed-form result (the angular factor $1/\sqrt{3}$ comes from $\langle Y_{10}|\sqrt{4\pi/3}\, Y_{10}|Y_{00}\rangle$, with $z = r\sqrt{4\pi/3}\, Y_{10}$). Only z contributes for this sublevel by the usual $\Delta m = 0$ selection rule, so $|\mathbf{x}_{12}|^2 = z_{12}^2 = 0.5549\, a_0^2$. Substituting into Eq. (90) with $\omega_{21} = 1.5495 \times 10^{16}$ rad/s (from $\lambda = 121.567$ nm):

$$A_{21} = \frac{e^2\omega_{21}^3\,(0.5549\, a_0^2)}{3\pi\epsilon_0\hbar c^3} = 6.258 \times 10^8\ \mathrm{s}^{-1}, \tag{92}$$

against the tabulated value 6.2649×10^8 s^{-1}—agreement to 0.1%, with *no* fitted parameters: $\mathbf{x}_{12}$ came directly from the exact Coulomb wavefunctions, and Eq. (90) came directly from the correspondence-principle theorem. Equivalently, $\tau(2p) = 1/A_{21} = 1.598$ ns, matching Eq. (69), and $f = A_{21}/\Gamma_{\mathrm{cl}} = 0.4160$, matching the tabulated $f(1s \to 2p) = 0.4162$ used in Sec. 7.9. This closes the loop opened by the crude τ_0-only estimate there: the true SED/Nelson dipole moment, computed explicitly rather than assumed, reproduces both the Einstein A coefficient and the oscillator strength independently derived elsewhere, to the precision of the inputs used.

Worked example: Balmer-α, the multi-channel case. Section 7.9 showed that the single-oscillator estimate fails for Balmer-α because the $3p$ level decays through two channels, $3p \to 2s$ and $3p \to 1s$, which a single classical oscillator cannot represent. The exact SED/Nelson dipole moment has no such limitation: $\mathbf{x}_{12}$ is evaluated separately for each final state, and Eq. (90) is applied to each channel in turn. Using the same method as above with $R_{31}(r) = \frac{2\sqrt{6}}{243}a_0^{-3/2}(r/a_0)(6 - r/a_0)e^{-r/3a_0}$:

$$z(3p \to 1s) = \langle 1s|z|3p, m{=}0\rangle = 0.29831\, a_0, \quad \omega_{31} = 1.8374 \times 10^{16}\ \text{rad/s}\ (\lambda = 102.52\ \text{nm}), \tag{93}$$

$$z(3p \to 2s) = \langle 2s|z|3p, m{=}0\rangle = 1.76947\, a_0, \quad \omega_{32} = 2.8709 \times 10^{15}\ \text{rad/s}\ (\lambda = 656.11\ \text{nm}). \tag{94}$$

Applying Eq. (90) to each channel separately and summing, as the exact dipole moment permits but a single classical oscillator cannot:

$$A(3p \to 1s) = 1.673 \times 10^8\ \mathrm{s}^{-1}, \quad A(3p \to 2s) = 2.246 \times 10^7\ \mathrm{s}^{-1}, \quad A_{\mathrm{tot}}(3p) = 1.898 \times 10^8\ \mathrm{s}^{-1}, \tag{95}$$

giving $\tau(3p) = 1/A_{\mathrm{tot}} = 5.269$ ns and predicted natural linewidth $\Delta\nu = A_{\mathrm{tot}}/2\pi = 30.2$ MHz—against the measured 30 MHz [24] and the $\tau = 5.31$ ns ($\Delta\nu = 30$ MHz) used in Sec. 7.9, agreement to under 1%, again with no fitted parameters. The branching ratio falls out automatically: 88.2% of $3p$ decays go to $1s$ (Lyman-β) and only 11.8% to $2s$ (Balmer-α)—precisely the physics the naive single-oscillator estimate of Sec. 7.9 could not represent, and precisely why that estimate failed in the direction it did (too slow, i.e. missing the dominant, faster Lyman-β channel entirely).

What this does and does not establish. Combining Eq. (89) with Eq. (90) shows that the electron–photon coupling of this paper, evaluated using the exact stochastic-mechanical dipole moment guaranteed by Nelson's theorem rather than the crude unit-oscillator proxy, reproduces the QED spontaneous emission rate *exactly*, with no residual gap to patch with an external oscillator strength. This is now confirmed explicitly, not merely argued: the crude τ_0-only estimate for Lyman-α needed the correction factor $f(1s \to 2p) = 0.4162$ precisely because it used an idealized unit-strength oscillator in place of $\mathbf{x}_{12}$ (Eq. (92)); and it failed for Balmer-α because a single classical oscillator has no mechanism to represent two competing final states, whereas the exact dipole moments for $3p \to 2s$ and $3p \to 1s$ enter separately and automatically apportion the decay correctly between them (Eq. (95)). It should be emphasized that this is not a new, independent SED prediction distinct from the QED value: evaluating $\mathbf{x}_{12}$ for hydrogen requires the same radial integral over the same Coulomb wavefunctions used in the ordinary quantum-mechanical calculation, since Nelson's theorem guarantees $\rho_{\mathrm{SED}} = |\psi_{\mathrm{QM}}|^2$ are the same function. The content of this subsection is therefore a genuine completion of the tree-level part of Sec. 9—showing that it is mathematically equivalent to standard first-order QED for spontaneous emission, rather than merely analogous to it—not a new numerical result to compare against experiment independently of QED.

Summary. Table 3 in Sec. 7.9 already collects these results, alongside the naive single-oscillator estimate, QED, and experiment, for both transitions. Both agree with QED and experiment to under 1%, with no fitted parameters anywhere in the exact SED column—because, as shown above, it is not really a distinct calculation from QED at all, but the same calculation performed within this paper's own formalism.

9.5 Loop Corrections

The one-loop QED corrections—Lamb shift (Paper 5 [5]), anomalous magnetic moment $g - 2 = \alpha/\pi$—arise from the leading non-Markovian correction to the Markovian (Fokker–Planck) approximation. Papers 5 and 6 have already derived these corrections within the stochastic framework, confirming the consistency of the electron-photon coupling described here.

10 The Complete Hierarchy

The eight papers of this series establish a complete hierarchy of effective descriptions, all derived from the single postulate of Paper 1 (electromagnetic field energy is carried in discrete quanta):

Scale	Particle	Process	Equation
Sub-Compton	Electron ($\Delta\tau \ll \hbar/mc^2$)	Non-Markovian ALD	None (QFT regime)
Compton–Dirac	Electron ($\Delta\tau \gg \hbar/mc^2$)	4D Brownian, spin-$\frac{1}{2}$	**Dirac equation**
Schrödinger	Electron ($v \ll c$)	3D Brownian, spin-$\frac{1}{2}$	**Schrödinger equation**
Optical	Photon ($\Delta t \gg 1/\omega_0$)	Brownian, spin-1	**Maxwell equations**
QED	Electron + Photon	Coupled SDEs	**QED (Feynman rules)**

All effective descriptions emerge from the same stochastic framework. The differences are:

- **Spin**: $\frac{1}{2}$ (electron) vs 1 (photon) $\rightarrow$ Pauli matrices vs spin-1 matrices $\rightarrow$ gradient vs curl.
- **Mass**: m (electron rest mass) vs $m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ (photon effective mass) $\rightarrow$ $D = \hbar/2m$ vs $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$.
- **Propagation**: diffusion in 3D or 4D (electron) vs directed at c with transverse diffusion (photon).

11 Discussion

11.1 Why $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$ Has the Same Form as $D = \hbar/2m$

This is not a coincidence. Both follow from the same underlying structure:

$$D = \frac{\hbar}{2m_{\text{inertial}}} \tag{96}$$

where m_{inertial} is the inertial mass resisting transverse acceleration:

- For the electron: $m_{\text{inertial}} = m$ (rest mass).
- For the photon: $m_{\text{inertial}} = m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ (relativistic mass).

Equation (96) is the universal relation between the diffusion coefficient and the inertial mass of a ZPF-driven stochastic process. It holds for any particle—massive or massless— once the appropriate inertial mass is identified.

11.2 What Determines the Photon's Coherence Length?

What τ_{coh} is, and what it is not. It is important to be precise about the physical meaning of τ_{coh}, since a careless reading invites a real objection: if τ_{coh} were literally the duration of the emission event itself—the time during which the atom is "partway through" emitting—this would imply a partial quantum of energy existing mid-emission, in direct conflict with the postulate that the electromagnetic field carries energy only in complete, discrete units $\hbar\omega$. That is not what τ_{coh} means. The photon's energy is $\hbar\omega$, complete and indivisible, at every instant from emission to detection; there is no intermediate state of fractional energy. What τ_{coh} actually characterizes is the temporal extent of the emitted wave packet's envelope f in (5)—equivalently, the time delay beyond which the photon's field can no longer interfere with a time-shifted copy of itself. This finite extent arises not from a gradual emission process but from the energy–time uncertainty relation applied to the *decaying excited atomic state* that does the emitting: an excited state with finite lifetime τ (set by the spontaneous emission rate, the Einstein A coefficient) does not have a perfectly sharp energy, and the resulting energy spread $\Delta E \sim \hbar/\tau$ Fourier-transforms into a finite spectral linewidth $\Delta\omega \sim 1/\tau$ for the emitted photon. The coherence time $\tau_{\text{coh}} \sim 1/\Delta\omega$ is therefore best understood as the coherence time of the emitted wave train, numerically tied to the excited state's lifetime, not as a literal stretching-out of an instantaneous quantum event. Each individual emission produces one complete photon of energy $\hbar\omega_0$ for some frequency ω_0 drawn from the Lorentzian line shape; τ_{coh} describes the resulting wavepacket envelope and the linewidth of the distribution of emitted frequencies across repeated emissions, not a single emission's duration.

The coherence length $l_{\text{coh}} = c\tau_{\text{coh}}$ is set by the *emission process*:

- **Thermal source** (sunlight): each photon is emitted by a single atomic transition from an excited state of lifetime $\tau \sim 1/\Delta\omega$, giving a coherence time $\tau_{\text{coh}} \sim 1/\Delta\omega$, where $\Delta\omega$ is the linewidth. The linewidth is set by the natural linewidth (spontaneous emission rate) and collisional broadening.
- **Laser**: stimulated emission into a single mode gives τ_{coh} limited only by cavity losses and laser linewidth—many orders of magnitude longer.
- **Single photon from a two-level atom**: $\tau_{\text{coh}} = 1/A_{eg}$ where A_{eg} is the Einstein A coefficient.

In each case, τ_{coh} is set by the emission dynamics, and $r_{\text{coh}} = \bar{\lambda}\sqrt{\omega_0\tau_{\text{coh}}}$ follows from equation (56).

11.3 Open Questions

The photon AL equation from first principles. Section 3 motivates the photon AL equation by analogy and dimensional analysis. A rigorous derivation from the Maxwell equations and vacuum ZPF—analogous to the electron AL derivation from the Coulomb field—remains to be completed.

QED beyond one loop. The present framework gives the Feynman rules of QED at tree level and one loop. The extension to higher loops, and the treatment of infrared divergences (soft photon emission), requires a more complete treatment of the non-Markovian corrections.

Multi-channel and two-photon decays. Section 7.9 tests the electron radiation-reaction time τ_0 of Sec. 3 against real hydrogen spectroscopic data and finds two concrete gaps. First, for a level with more than one allowed decay channel (e.g. $3p$, which decays to both $1s$ and $2s$), the single-frequency classical estimate $\Gamma_{\rm cl}(\omega_0) = \omega_0^2\tau_0$ needs to be replaced by a sum over channels, each weighted by its own (currently externally-supplied) oscillator strength—the present framework does not yet derive these weights internally. Second, transitions forbidden at first order, such as $2s \to 1s$, decay only via genuinely second-order (two-photon) processes that lie outside the single-vertex electron–photon coupling of Sec. 9; treating such cases would require extending that coupling, and the photon SDE (21) itself, to two correlated photon emissions sharing one virtual electron excursion. Both gaps are well-defined, numerically documented targets for future extension of the framework, rather than obstacles encountered only in principle.

The photon in a medium. The present treatment is for the photon in vacuum. Extension to photons in a dielectric medium (refractive index n, group velocity $v_g < c$) would require replacing $c \to v_g$ in the photon SDE, which would give a modified diffusion coefficient $D_\gamma = v_g^2/2\omega_0$.

12 Conclusion

We have extended the stochastic electrodynamic framework of Papers 1–7 to the photon, deriving three major results without any quantum postulate:

1. **The photon diffusion coefficient:**

$$D_\gamma = \frac{c^2}{2\omega_0} = \frac{\hbar}{2m_\gamma^{\rm eff}}, \qquad m_\gamma^{\rm eff} = \frac{\hbar\omega_0}{c^2}. \tag{20}$$

   Derived by equating two independent expressions for $\langle p_\perp^2 \rangle$ from the photon Abraham–Lorentz equation. Same form as $D = \hbar/2m$ for the electron.

2. **Maxwell equations from the photon Fokker–Planck:**

$$\text{Photon Fokker–Planck} \xrightarrow{\mathbf{F}=\sqrt{\epsilon_0}\mathbf{E}+\frac{i}{\sqrt{\mu_0}}\mathbf{B}} \nabla\times\mathbf{E} = -\partial_t\mathbf{B}, \quad \nabla\times\mathbf{B} = \frac{1}{c^2}\partial_t\mathbf{E}. \tag{42}$$

   The Madelung transformation using the Riemann–Silberstein vector linearises the Fokker–Planck into Maxwell's equations—the photon analogue of deriving Schrödinger from the electron Fokker–Planck.

3. **Malus's law from Brownian motion:**

$$P(\text{pass}) = \cos^2\alpha. \tag{49}$$

Derived from first-passage probability of Brownian motion on the Poincaré sphere. The 50/50 randomness at $\alpha = 45°$ follows from the symmetry of the starting position.

4. **Photon randomness:** The photon's randomness originates in the electromagnetic vacuum ZPF—the same field of which the photon is a quantum. There is no hidden variable beyond the ZPF itself. The coherence volume at formation equals $V_{\rm coh} = \pi\bar{\lambda}^2 \cdot c\tau_{\rm coh}$. This intrinsic single-photon quantity is distinct from the observed van Cittert–Zernike coherence radius of sunlight at Earth (Sec. 7); we show (Sec. 7.4) that the diffusive growth law governing formation gives way to ordinary diffractive spreading beyond the Rayleigh range, so that a single photon's coherent wavefront at Earth is in fact many orders of magnitude *larger* than the ensemble van Cittert–Zernike radius—the two describe different physical quantities, not competing predictions of the same observable.

5. **QED:** The coupling of the electron and photon SDEs gives the QED interaction vertex $ej^\mu A_\mu$, with propagators given by the electron SDE Green's function and the photon cross-correlation $S_E^{(12)}$ of Paper 2.

The universal relation $D = \hbar/2m_{\rm inertial}$ holds for both the electron ($m_{\rm inertial} = m$) and the photon ($m_{\rm inertial} = \hbar\omega_0/c^2$). The difference between Schrödinger and Maxwell is spin: $\frac{1}{2}$ gives the gradient operator; 1 gives the curl.

A Explicit Derivation of the Exact SED/Nelson Dipole Matrix Elements

This appendix records, in full, the calculation summarized in Sec. 9.4: the exact dipole matrix elements $\mathbf{x}_{12}$ entering Eq. (90), computed directly from the hydrogen radial wavefunctions with no fitted parameters, together with the resulting Einstein A coefficients.

A.1 Radial wavefunctions and general method

The normalized hydrogen radial wavefunctions needed below (Bohr radius a_0; normalization $\int_0^\infty R_{nl}^2\, r^2\, dr = 1$, verified symbolically for each) are

$$R_{10}(r) = 2\, a_0^{-3/2}\, e^{-r/a_0}, \tag{97}$$

$$R_{20}(r) = \frac{\sqrt{2}}{4}\, a_0^{-3/2} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}, \tag{98}$$

$$R_{21}(r) = \frac{\sqrt{6}}{12}\, a_0^{-3/2} \left(\frac{r}{a_0}\right) e^{-r/2a_0}, \tag{99}$$

$$R_{31}(r) = \frac{2\sqrt{6}}{243}\, a_0^{-3/2} \left(\frac{r}{a_0}\right) \left(6 - \frac{r}{a_0}\right) e^{-r/3a_0}. \tag{100}$$

Every radial matrix element below reduces, after expanding the polynomial prefactors, to sums of the elementary integral

$$\int_0^\infty r^n\, e^{-\beta r}\, dr = \frac{n!}{\beta^{n+1}}, \tag{101}$$

which is used repeatedly without further comment.

For every transition considered here the initial and final states are both $m = 0$ sublevels ($2p_z$, $3p_z$), so only the z-component of $\mathbf{r}$ contributes, $z = r\cos\theta = r\sqrt{4\pi/3}\,Y_{10}(\theta,\phi)$, and the angular integral is the same in every case:

$$\int Y_{10}^{*}\,\sqrt{\tfrac{4\pi}{3}}\,Y_{10}\,Y_{00}\,d\Omega = \sqrt{\tfrac{4\pi}{3}}\cdot\frac{1}{\sqrt{4\pi}}\int |Y_{10}|^2\,d\Omega = \frac{1}{\sqrt{3}}. \tag{102}$$

Each matrix element below is therefore $z_{fi} = I_{\text{radial}}/\sqrt{3}$, where $I_{\text{radial}} = \int_0^\infty R_f(r)\,r\,R_i(r)\,r^2\,dr$.

A.2 Lyman-α: $\langle 1s|z|2p, m{=}0\rangle$

$$\begin{aligned} I_{\text{radial}} &= \int_0^\infty R_{21}(r)\,r\,R_{10}(r)\,r^2\,dr = \frac{\sqrt{6}}{12}\cdot 2\,a_0^{-4}\int_0^\infty r^4\,e^{-r/2a_0 - r/a_0}\,dr \\ &= \frac{1}{\sqrt{6}}\,a_0^{-4}\int_0^\infty r^4\,e^{-3r/2a_0}\,dr. \end{aligned} \tag{103}$$

Applying Eq. (101) with $n = 4$, $\beta = 3/2a_0$:

$$\int_0^\infty r^4\,e^{-3r/2a_0}\,dr = 4!\left(\frac{2a_0}{3}\right)^5 = \frac{256\,a_0^5}{81}. \tag{104}$$

Substituting back,

$$I_{\text{radial}} = \frac{1}{\sqrt{6}}\cdot\frac{256\,a_0}{81} = \frac{128\sqrt{6}}{243}\,a_0, \tag{105}$$

and applying the angular factor (102),

$$z_{12} \equiv \langle 1s|z|2p, m{=}0\rangle = \frac{128\sqrt{6}}{243\sqrt{3}}\,a_0 = \frac{128\sqrt{2}}{243}\,a_0 = 0.744936\,a_0, \tag{106}$$

matching Eq. (91) and the standard closed form $(2^7\sqrt{2}/3^5)\,a_0$.

A.3 Balmer-α, channel 1: $\langle 1s|z|3p, m{=}0\rangle$ (Lyman-β)

$$\begin{aligned} I_{\text{radial}} &= \int_0^\infty R_{31}(r)\,r\,R_{10}(r)\,r^2\,dr = \frac{2\sqrt{6}}{243}\cdot 2\,a_0^{-5}\int_0^\infty r(6a_0 - r)\,r^3\,e^{-r/3a_0 - r/a_0}\,dr \\ &= \frac{4\sqrt{6}}{243}\,a_0^{-5}\left[6a_0\int_0^\infty r^4 e^{-4r/3a_0}dr - \int_0^\infty r^5 e^{-4r/3a_0}dr\right]. \end{aligned} \tag{107}$$

Applying Eq. (101) with $\beta = 4/3a_0$:

$$\int_0^\infty r^4 e^{-4r/3a_0}dr = \frac{729\,a_0^5}{128}, \qquad \int_0^\infty r^5 e^{-4r/3a_0}dr = \frac{10935\,a_0^6}{512}. \tag{108}$$

Combining,

$$6a_0\cdot\frac{729\,a_0^5}{128} - \frac{10935\,a_0^6}{512} = \frac{17496\,a_0^6}{512} - \frac{10935\,a_0^6}{512} = \frac{6561\,a_0^6}{512}, \tag{109}$$

so

$$I_{\text{radial}} = \frac{4\sqrt{6}}{243} \cdot \frac{6561}{512}\, a_0 = \frac{27\sqrt{6}}{128}\, a_0, \tag{110}$$

using $6561/243 = 27$. Applying the angular factor,

$$z(3p \to 1s) = \frac{27\sqrt{6}}{128\sqrt{3}}\, a_0 = \frac{27\sqrt{2}}{128}\, a_0 = 0.298311\, a_0, \tag{111}$$

matching Eq. (93).

A.4 Balmer-α, channel 2: $\langle 2s|z|3p, m{=}0\rangle$

$$I_{\text{radial}} = \int_0^\infty R_{31}(r)\, r\, R_{20}(r)\, r^2\, dr = \frac{2\sqrt{6}}{243} \cdot \frac{\sqrt{2}}{4}\, a_0^{-6} \int_0^\infty r^4 (6a_0 - r)(2a_0 - r)\, e^{-5r/6a_0}\, dr. \tag{112}$$

Expanding $(6a_0 - r)(2a_0 - r) = 12a_0^2 - 8a_0 r + r^2$ and applying Eq. (101) term by term with $\beta = 5/6a_0$ gives three elementary integrals ($n = 4, 5, 6$), which combine (after clearing denominators to the common value 15625) to

$$\int_0^\infty r^4 (6a_0 - r)(2a_0 - r)\, e^{-5r/6a_0}\, dr = \frac{6718464}{15625}\, a_0^7. \tag{113}$$

This polynomial-weighted integral was evaluated by symbolic computer algebra (as a cross-check on hand expansion of the three constituent terms); the intermediate elementary integrals are, for reference, $\int r^4 e^{-5r/6a_0} dr = 186624\, a_0^5/3125$, $\int r^5 e^{-5r/6a_0} dr = 1119744\, a_0^6/3125$, and $\int r^6 e^{-5r/6a_0} dr = 40310784\, a_0^7/15625$. Substituting Eq. (113) back,

$$I_{\text{radial}} = \frac{\sqrt{3}}{243} \cdot \frac{6718464}{15625}\, a_0 = \frac{27648\sqrt{3}}{15625}\, a_0, \tag{114}$$

and applying the angular factor (which removes the $\sqrt{3}$ exactly),

$$z(3p \to 2s) = \frac{27648}{15625}\, a_0 = 1.769472\, a_0, \tag{115}$$

matching Eq. (94).

A.5 Numerical substitution: Einstein A coefficients

With $|\mathbf{x}_{12}|^2 = z_{12}^2$ (only z contributes for each $m = 0$ sublevel by the $\Delta m = 0$ selection rule) and the transition frequencies from the Bohr energy levels $E_n = -13.6057\ \text{eV}/n^2$, Eq. (90) gives:

$$A_{21} = \frac{e^2 \omega_{21}^3\, z_{12}^2}{3\pi\epsilon_0 \hbar c^3} = 6.258 \times 10^8\ \text{s}^{-1} \qquad (\omega_{21} = 1.5495 \times 10^{16}\ \text{rad/s}), \tag{116}$$

$$A(3p \to 1s) = \frac{e^2 \omega_{31}^3\, z(3p \to 1s)^2}{3\pi\epsilon_0 \hbar c^3} = 1.673 \times 10^8\ \text{s}^{-1} \qquad (\omega_{31} = 1.8374 \times 10^{16}\ \text{rad/s}), \tag{117}$$

$$A(3p \to 2s) = \frac{e^2 \omega_{32}^3\, z(3p \to 2s)^2}{3\pi\epsilon_0 \hbar c^3} = 2.246 \times 10^7\ \text{s}^{-1} \qquad (\omega_{32} = 2.8709 \times 10^{15}\ \text{rad/s}). \tag{118}$$

Summing the two $3p$ channels, $A_{\rm tot}(3p) = 1.898 \times 10^8$ s^{-1}, giving $\tau(3p) = 5.269$ ns and $\Delta\nu = A_{\rm tot}/2\pi = 30.2$ MHz—the values quoted in Table 3 and Eqs. (92) and (95).

A.6 Summary

Matrix element	Exact closed form	Numerical value
$z_{12} = \langle 1s\|z\|2p, m{=}0\rangle$	$\dfrac{2^7\sqrt{2}}{3^5}\,a_0$	$0.744936\,a_0$
$z(3p \to 1s) = \langle 1s\|z\|3p, m{=}0\rangle$	$\dfrac{3^3\sqrt{2}}{2^7}\,a_0$	$0.298311\,a_0$
$z(3p \to 2s) = \langle 2s\|z\|3p, m{=}0\rangle$	$\dfrac{2^{10}\cdot 3^3}{5^6}\,a_0$	$1.769472\,a_0$

Table 4: The three exact dipole matrix elements underlying the SED/Nelson column of Table 3, derived in full above from the radial wavefunctions of Eq. (100) and the angular factor (102).

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