

Stochastic Electrodynamics of the Photon: Maxwell Equations, Malus's Law, and the Origin of Photon Randomness from the Electromagnetic Vacuum

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Abstract

Papers 1–7 of this series derived the Schrödinger equation, the Dirac equation, quantum entanglement, the Coulomb force, the Lamb shift, the anomalous magnetic moment, the hydrogen hyperfine splitting, and the resolution of the double-slit paradox—all for the *electron*, whose randomness originates in the Zero-Point Field (ZPF) of its own Coulomb interaction.

This paper extends the stochastic framework to the *photon*. The photon has no charge and no rest mass, so the electron's Abraham–Lorentz equation does not apply directly. We identify the correct photon analogue: the photon's relativistic (effective) mass $m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ replaces the electron rest mass m , and the photon's own optical cycle $\tau_0^\gamma = 1/\omega_0$ replaces the electron radiation reaction time $\tau_0 = e^2/6\pi\epsilon_0 mc^3$.

From this photon Abraham–Lorentz equation, equating two independent expressions for the transverse momentum variance gives the photon diffusion coefficient:

$$D_\gamma = \frac{c^2}{2\omega_0} = \frac{\hbar}{2m_\gamma^{\text{eff}}},$$

which has exactly the same form as $D = \hbar/2m$ for the electron, with m_γ^{eff} replacing m .

Three major results follow:

1. **Maxwell equations** are derived from the photon Fokker–Planck equation via the Madelung transformation using the Riemann–Silberstein vector $\mathbf{F} = \sqrt{\epsilon_0}\mathbf{E} + (i/\sqrt{\mu_0})\mathbf{B}$ as the photon wave function. This is the exact photon analogue of deriving the Schrödinger equation from the electron Fokker–Planck in Paper 1.
2. **Malus's law** $P(\text{pass}) = \cos^2 \alpha$ is derived from Brownian motion of the polarization angle on the Poincaré sphere, without assuming quantum mechanics.

3. **Photon randomness** originates in the electromagnetic vacuum ZPF—the same field of which the photon is itself a quantum. The coherence volume of a photon equals $2D_\gamma/\omega_0 = c^2/\omega_0^2 = \bar{\lambda}^2$, consistent with the van Cittert–Zernike theorem.

The complete hierarchy of effective descriptions is: Schrödinger (spin- $\frac{1}{2}$, massive) $\rightarrow$ Dirac (spin- $\frac{1}{2}$, relativistic) $\rightarrow$ Maxwell (spin-1, massless)—all emerging from stochastic processes driven by the electromagnetic ZPF, differing only in the particle’s mass and spin.

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1 Introduction

Paper 7 of this series [7] identified a fundamental open question:

The electron’s randomness originates in its Coulomb ZPF, giving $D = \hbar/2m$ via the Abraham–Lorentz equation. The photon has no Coulomb field and no rest mass. Where does photon randomness come from?

This question is not merely philosophical. It has concrete experimental manifestations:

1. A 45° polarized photon hitting a 0° polarizer passes or is absorbed with probability exactly 50%—with no hidden variable determining the outcome.
2. The coherence length of sunlight ($\sim 1\ \mu\text{m}$) differs from that of a laser ($\sim 300\ \text{km}$) by twelve orders of magnitude— despite both being photons.
3. A single photon produces an interference pattern in the double-slit experiment, building up dot by dot on the screen.

All three phenomena require an explanation of photon randomness from first principles. The present paper provides it.

1.1 The Strategy

For the electron, the chain was:

$$\underbrace{\text{Coulomb field}}_{\text{source}} \xrightarrow{\text{Einstein 1905}} \text{ZPF} \xrightarrow{\text{Boyer 1969}} u(\omega) \propto \hbar\omega^3 \xrightarrow{\text{Casimir}} \hbar \xrightarrow{\text{AL equation}} D = \frac{\hbar}{2m} \xrightarrow{\text{Nelson}} \text{Schrödinger.} \quad (1)$$

For the photon, we establish the analogous chain:

$$\underbrace{\text{EM vacuum ZPF}}_{\text{source}} \xrightarrow{\text{photon AL equation}} D_\gamma = \frac{\hbar}{2m_\gamma^{\text{eff}}} \xrightarrow{\text{Nelson (spin-1)}} \text{Maxwell equations.} \quad (2)$$

The key difference is that the photon *is* a quantum of the electromagnetic field—its “source field” is the same electromagnetic vacuum ZPF that provides the electron’s ZPF. This self-referential structure gives the photon its unique properties.

1.2 Context in the Series

This is Paper 8 of the series *Quantum Mechanics from the Coulomb Field*. Full publication details (ai.viXra and/or Zenodo identifiers, as applicable) for each paper are given in the bibliography [1–7]. The papers in the series are:

Paper	Title (abbreviated)	Status
1	QM from Coulomb Field	Published
2	Entanglement + Coulomb Force	Published
3	Ergodicity	Published
4	Dirac Equation	Published
5	Schwinger term + Lamb shift	Published
6	Hydrogen Hyperfine Splitting	Published
7	Which Slit? Double-slit paradox	Published
8	Stochastic Photon (this paper)	New

2 The Photon as a ZPF Excitation

2.1 The Electron’s ZPF vs the Photon’s ZPF

The electron has a Coulomb field $E(r) = e/4\pi\epsilon_0 r^2$. Its ZPF is the electromagnetic quantum field arising from this Coulomb interaction, with spectral density (Paper 1 [1]):

$$S_E(\omega) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3}. \quad (3)$$

The photon has no charge and therefore no Coulomb field. However, the photon *is* a quantum of the electromagnetic field—it is a single excitation of a mode $(\mathbf{k}_0, \lambda_0)$ above the vacuum. The vacuum itself has spectral density (3)—the same ZPF that the electron interacts with.

The photon therefore propagates *immersed in* the electromagnetic vacuum ZPF. This is its “source field”:

$$\mathbf{E}_{\text{total}} = \underbrace{\mathbf{E}_\gamma}_{\text{photon}} + \underbrace{\mathbf{E}_{\text{ZPF}}}_{\text{vacuum}}. \quad (4)$$

The photon shares all modes of the electromagnetic field with the vacuum ZPF. This is the physical origin of photon randomness.

2.2 The Photon Wave Packet

A single photon of frequency ω_0 propagating in the x -direction is a wave packet:

$$\mathbf{E}_\gamma(\mathbf{x}, t) = \sqrt{\frac{\hbar\omega_0}{2\epsilon_0 V}} \hat{\mathbf{e}} f(\mathbf{x} - ct \hat{\mathbf{x}}) e^{i(k_0 x - \omega_0 t)} + \text{c.c.}, \quad (5)$$

where $k_0 = \omega_0/c$, $\hat{\mathbf{e}} \perp \hat{\mathbf{x}}$ is the polarization unit vector, f is the envelope function, and V is the normalization volume.

The monochromatic limit is unphysical and is excluded. Equation (5) is written with an explicit envelope f precisely because the opposite limit—a perfectly monochromatic plane wave, $f \equiv 1$ over all space and time—does not describe a physical single photon. Such a wave is not normalizable (its total field energy, integrated over infinite volume, diverges), and it does not correspond to a one-photon state in the usual sense: a field mode of exactly one frequency, persisting for all time, corresponds to the classical limit of a coherent superposition over photon number, not to a localized single quantum. A real, detectable single photon always has a finite envelope f , hence a finite coherence time τ_{coh} and coherence radius r_{coh} (Section 7 below); the textbook “photon of definite frequency ω , delocalized over all space” is a convenient idealization for computing energy and momentum eigenvalues, never a description of an actual emitted photon. This also pre-empts an apparent tension with relativity: a literal infinite, delocalized monochromatic state would assign equal detection probability to every point in the universe, which might seem to conflict with causality. In fact it does not—an extended, stationary probability amplitude carries no information and permits no superluminal signalling, exactly as for an ordinary non-relativistic plane-wave eigenstate $\psi = e^{i(kx - \omega t)}$ in elementary quantum mechanics, where the same delocalization is well known to be unproblematic. The point is moot in any case for the present framework, since the physical photon states considered throughout this paper are always the finite, normalizable wave packets of (5), never the monochromatic limit.

The photon carries energy $\hbar\omega_0$ and momentum $\hbar k_0 = \hbar\omega_0/c$. Its effective (relativistic) mass is:

$$m_{\gamma}^{\text{eff}} \equiv \frac{\hbar\omega_0}{c^2}. \quad (6)$$

This is not a rest mass—the photon has $m_{\text{rest}} = 0$. It is the *inertial mass* associated with the photon’s energy, resisting transverse acceleration. It plays the role of m in all subsequent calculations.

2.3 The Photon Probability Density

For the electron, $\rho = |\psi|^2$. For the photon, the natural probability density is proportional to the electromagnetic energy density:

$$\rho_{\gamma}(\mathbf{x}, t) = u_{\text{EM}}(\mathbf{x}, t) = \epsilon_0 |\mathbf{E}|^2 + \frac{1}{\mu_0} |\mathbf{B}|^2. \quad (7)$$

This is the *photon Born rule*: the probability of detecting a photon at $\mathbf{x}$ is proportional to the electromagnetic energy density there. In the present framework this is not a postulate—it is the stationary distribution of the photon SDE, following from Nelson’s ergodic theorem (Paper 3 [3]), exactly as $|\psi|^2$ follows for the electron.

2.4 Postulates, Definitions, and Derived Results

Before proceeding to the photon Abraham–Lorentz equation, it is important to be explicit about the epistemic status of each ingredient used in this section and the next. Not everything stated above is on the same logical footing: some items are genuine postulates (assumed, not derived), some are definitions (notational conventions with no independent physical content), and some are theorems (derived from more basic principles already established elsewhere in this series). Conflating these categories would overstate what has actually been proven. Table 1 summarizes the classification used throughout the rest of this paper.

Table 1: Epistemic status of the propositions used to extend the stochastic electrodynamic framework from the electron (Papers 1–7) to the photon (this paper). Only the first row is a postulate inherited from the rest of the series; the two remaining postulates (photon–ZPF coupling and the photon AL equation) are specific to the photon extension and are flagged here so that the reader can judge which results below rest on solid derivational ground and which rest on physically motivated but unproven assumptions.

Proposition	Status	Remark
The electromagnetic field carries energy in discrete quanta $\hbar\omega$	Postulate	The single foundational postulate of the entire series (Paper 1 [1]). Not derived from anything more basic here.
The photon has a well-defined transverse position $\mathbf{r}_\perp(t)$	Definition	$\mathbf{r}_\perp(t)$ is <i>defined</i> as the centroid of the transverse envelope $f(\mathbf{x} - ct\hat{\mathbf{x}})$ in (5). Treating the photon as a localized wave packet with a sharp position is an idealization; a massless spin-1 field does not admit an exact position operator in the usual (Newton–Wigner) sense. The idealization is adopted for calculational convenience and is expected to be valid when the packet is narrow compared to the scales of interest.
Constant longitudinal speed c along the propagation axis	Derived	Follows from the dispersion relation $\omega = ck_0$ for a massless quantum, itself a consequence of the Maxwell equations derived in Section 5 below. Not an independent assumption.

Table 1 continued from previous page

Proposition	Status	Remark
Finite transverse speed, driven by the vacuum ZPF	Consequence	Not independent: follows automatically once the ZPF driving force (below) acts on a degree of freedom with finite effective mass m_γ^{eff} (6) through an equation of motion. Listed separately here only because it is often discussed as if it were a distinct assumption.
The photon's transverse motion is driven by the vacuum ZPF (photon–photon interaction via the vacuum)	Postulate	This is the central new assumption of this paper. Unlike the electron's ZPF driving (justified microscopically via the Coulomb field in Paper 1), the photon has no charge to couple to the ZPF through. The coupling mechanism asserted in Section 3 is motivated by analogy with the electron case and is <i>not</i> derived from a more microscopic photon–vacuum interaction. This should be regarded as the least secure assumption in the present framework and the primary target for future microscopic justification.
Effective mass $m_\gamma^{\text{eff}} = \hbar\omega/c^2$	Definition	Pure relativistic bookkeeping ($E = mc^2$ applied to $E = \hbar\omega$), with no independent physical content beyond notation.
The photon's transverse motion obeys an Abraham–Lorentz-type equation	Postulate / ansatz	Motivated by analogy and dimensional analysis with the electron AL equation (Section 3), <i>not</i> derived from the Maxwell equations and vacuum ZPF from first principles. A rigorous derivation, analogous to the electron AL derivation from the Coulomb field, remains an open problem (see Section 11.3).

Table 1 continued from previous page

Proposition	Status	Remark
The photon is a spin-1 (vector) field, with generators $\mathbf{S}$ satisfying $(\mathbf{S} \cdot \nabla)\mathbf{F} = i\nabla \times \mathbf{F}$ (introduced in Section 5)	Imported fact	Not derived anywhere in this framework. This is an external input, justified either by Wigner’s classification of massless representations of the Poincaré group (helicity ± 1 for the photon), or more elementarily by the classical observation that $\mathbf{E}$ and $\mathbf{B}$ are 3-vectors, so their polarization transforms under the spin-1 (vector) representation of the rotation group. The role played by $\mathbf{S}$ here is purely algebraic: it repackages the curl operator acting on a transverse field in a form parallel to the electron’s Pauli matrices, exactly as in the Pauli identity $(\boldsymbol{\sigma} \cdot \mathbf{p})^2 = p^2$ used for the Dirac–Pauli reduction. The postulate of Paper 1 (energy quanta $\hbar\omega$) does not by itself imply spin-1; that the photon carries spin-1 is presupposed, not derived.

Provenance of the spin-1 assignment. Since this fact is imported rather than derived, it is worth recording explicitly where it comes from. The theoretical determination is due to Wigner’s 1939 classification of the unitary representations of the Poincaré group [15]: massless particles are labelled by helicity, and requiring the electromagnetic field to transform as a Lorentz four-vector (or equivalently, a rank-2 antisymmetric tensor $F^{\mu\nu}$) forces the photon’s helicity to be ± 1 —this is the rigorous reason the photon is spin-1 specifically, rather than some other allowed massless representation. The physical existence of this angular momentum was anticipated earlier and purely classically by Poynting (1909), who showed that circularly polarised light carries angular momentum according to Maxwell’s equations alone, with no quantum input. It was confirmed quantitatively—one unit of $\hbar$ per photon—first indirectly via atomic spectroscopic selection rules (absorption/emission of a photon changes an atomic electron’s orbital angular momentum by exactly $\hbar$), and then directly and mechanically by Beth’s 1936 torsion-balance experiment [16], which measured the torque exerted by a beam of circularly polarised light on a birefringent wave plate and extracted exactly $\hbar$ of angular momentum per photon. None of these results are derivable from the single postulate of Paper 1 (discrete energy quanta $\hbar\omega$); they require either the full representation theory of the Poincaré group or an independent classical/experimental input external to the present stochastic framework.

In short: of the eight propositions commonly invoked when extending this framework to the photon, one is the series' foundational postulate, two are pure definitions, one is a derived theorem, one is a redundant restatement of consequences already implied by the others, two are genuine new postulates whose justification is presently by analogy only, and one (spin-1) is an external fact imported from outside this framework rather than derived or postulated within it. The results in Sections 3 and 4 below should be read with this distinction in mind: the derivation of $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$ (Section 3.3) is exact *given* the photon AL equation, but the AL equation itself is an ansatz, not a theorem.

3 The Photon Abraham–Lorentz Equation

3.1 Why the Electron AL Equation Fails for the Photon

The electron Abraham–Lorentz equation:

$$m\ddot{\mathbf{x}} = e\mathbf{E}_{\text{ZPF}}(t) + \frac{e^2}{6\pi\epsilon_0 c^3} \ddot{\mathbf{x}} \quad (8)$$

has two terms:

1. *ZPF driving*: $e\mathbf{E}_{\text{ZPF}}$ — requires charge e .
2. *Radiation reaction*: $\propto \ddot{\mathbf{x}}$ — arises because the electron emits radiation when accelerated; the emitted field acts back on the electron.

For the photon, $e = 0$ and $m_{\text{rest}} = 0$, so (8) cannot be applied directly. We must identify the photon analogues of both terms.

3.2 The Photon AL Equation for Transverse Momentum

The photon propagates in the x -direction. Its *longitudinal* momentum $p_{\parallel} = \hbar k_0$ is fixed by its frequency and cannot diffuse (the photon travels at c). The relevant dynamical degree of freedom is the *transverse position* $\mathbf{r}_{\perp} = (y, z)$, describing the spatial spreading of the wave packet perpendicular to propagation.

The photon analogue of the Abraham–Lorentz equation is:

$$m_\gamma^{\text{eff}} \ddot{\mathbf{r}}_{\perp} = \mathbf{F}_{\text{ZPF}}(t) + \frac{m_\gamma^{\text{eff}}}{\omega_0} \ddot{\mathbf{r}}_{\perp}, \quad (9)$$

where:

- $m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ is the photon effective mass (6),
- $\mathbf{F}_{\text{ZPF}}(t)$ is the radiation pressure of the vacuum ZPF on the photon's electromagnetic field (the photon analogue of $e\mathbf{E}_{\text{ZPF}}$),
- $(m_\gamma^{\text{eff}}/\omega_0) \ddot{\mathbf{r}}_{\perp}$ is the photon radiation reaction, with characteristic time $\tau_0^\gamma = 1/\omega_0$ (one optical cycle).

Physical justification of $\tau_0^\gamma = 1/\omega_0$. For the electron, the radiation reaction time is $\tau_0 = e^2/6\pi\epsilon_0 mc^3 = (2\alpha/3)\hbar/mc^2 = (2\alpha/3)\omega_c^{-1}$ —a small fraction of the Compton period. The factor $2\alpha/3 \approx 5 \times 10^{-3}$ arises from the ratio of the electron’s classical radius $r_e = e^2/4\pi\epsilon_0 mc^2$ to its Compton wavelength $\hbar/mc$.

For the photon, there is no charge and no classical radius in the same sense. The natural timescale is the *optical period* $\tau_0^\gamma = 2\pi/\omega_0$ (or $1/\omega_0$ up to the factor 2π). This is because the photon’s self-interaction completes in one oscillation cycle—the photon *is* the oscillation. The precise relation is:

$$\tau_0^\gamma = \frac{1}{\omega_0} = \frac{\hbar}{\hbar\omega_0} = \frac{\hbar}{m_\gamma^{\text{eff}} c^2}, \quad (10)$$

which has exactly the same form as the electron $\tau_0 = \hbar/mc^2$ (up to the factor $2\alpha/3$, which is the electron’s self-coupling ratio; for the photon, the analogous coupling is $O(1)$ since the photon is its own field).

Physical justification of $\mathbf{F}_{\text{ZPF}}$. The vacuum ZPF exerts radiation pressure on the photon’s electromagnetic field through transverse mode coupling. The spectral density of this force can be computed from the cross-correlation of the photon field with the ZPF modes, but for the purposes of the equilibrium argument below, only its spectral properties matter.

3.3 Derivation of $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$ from the Photon AL Equation

We follow the exact structure of Paper 1 Section 5.2: equate two independent expressions for $\langle p_\perp^2 \rangle$.

Step 1: $\langle p_\perp^2 \rangle$ from the ZPF (photon AL equation)

In the frequency domain, the photon AL equation (9) gives the transverse velocity spectrum:

$$\tilde{r}_\perp(\omega) = \frac{\tilde{F}_{\text{ZPF}}(\omega)/m_\gamma^{\text{eff}}}{-i\omega(1 + i\omega/\omega_0)}. \quad (11)$$

The ZPF force has spectral density:

$$S_F(\omega) = (m_\gamma^{\text{eff}})^2 \omega_0^2 \cdot S_v(\omega) = \frac{(m_\gamma^{\text{eff}})^2 \omega_0^2 S_E(\omega)}{(\hbar k_0/m_\gamma^{\text{eff}})^2} = \frac{(m_\gamma^{\text{eff}})^2 \omega_0^2 \hbar \omega^3}{6\pi^2 \epsilon_0 c^3 \cdot (c^2)^2/c^2} = \frac{(m_\gamma^{\text{eff}})^2 \omega^3}{6\pi^2 \epsilon_0 c^3/\hbar}, \quad (12)$$

where we used $S_E(\omega) = \hbar \omega^3/6\pi^2 \epsilon_0 c^3$.

In equilibrium, equating energy input from ZPF with radiation reaction damping (as in Paper 1 Section 5.1):

$$\langle p_\perp^2 \rangle_{\text{ZPF}} = m_\gamma^{\text{eff}} \cdot \frac{\hbar \omega_0}{2} = \frac{(\hbar \omega_0)^2}{2c^2} = \frac{(m_\gamma^{\text{eff}})^2 c^2}{2}. \quad (13)$$

This is the photon analogue of $\langle p^2 \rangle_{\text{ZPF}} = m\hbar\omega_0/2$ for the electron oscillator.

Step 2: $\langle p_{\perp}^2 \rangle$ from the osmotic velocity

The photon osmotic velocity:

$$\mathbf{u}_{\gamma} = D_{\gamma} \nabla \ln \rho_{\gamma} = D_{\gamma} \nabla \ln u_{\text{EM}}. \quad (14)$$

The osmotic transverse momentum:

$$p_{\perp, \text{osm}} = m_{\gamma}^{\text{eff}} u_{\gamma} = m_{\gamma}^{\text{eff}} D_{\gamma} \nabla_{\perp} \ln u_{\text{EM}}. \quad (15)$$

For a Gaussian wave packet of minimum uncertainty width $\sigma_{\perp} = \bar{\lambda}/\sqrt{2}$ (where $\bar{\lambda} = c/\omega_0$):

$$\langle (\nabla_{\perp} \ln u_{\text{EM}})^2 \rangle = \frac{2}{\bar{\lambda}^2} = \frac{2\omega_0^2}{c^2}. \quad (16)$$

Therefore:

$$\langle p_{\perp}^2 \rangle_{\text{osm}} = (m_{\gamma}^{\text{eff}})^2 D_{\gamma}^2 \cdot \frac{2\omega_0^2}{c^2}. \quad (17)$$

Step 3: Equate and solve

Setting $\langle p_{\perp}^2 \rangle_{\text{ZPF}} = \langle p_{\perp}^2 \rangle_{\text{osm}}$:

$$\frac{(m_{\gamma}^{\text{eff}})^2 c^2}{2} = (m_{\gamma}^{\text{eff}})^2 D_{\gamma}^2 \cdot \frac{2\omega_0^2}{c^2} \quad (18)$$

$$D_{\gamma}^2 = \frac{c^4}{4\omega_0^2} \quad (19)$$

$$\boxed{D_{\gamma} = \frac{c^2}{2\omega_0} = \frac{\hbar}{2m_{\gamma}^{\text{eff}}}} \quad (20)$$

This is the central result of this section. The photon diffusion coefficient has *exactly the same form* as the electron diffusion coefficient $D = \hbar/2m$, with the photon effective mass $m_{\gamma}^{\text{eff}} = \hbar\omega_0/c^2$ replacing the electron rest mass m .

3.4 Comparison: Electron vs Photon

Quantity	Electron	Photon
Inertial mass	m (rest mass)	$m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$
Characteristic frequency	$\omega_c = mc^2/\hbar$	ω_0 (own frequency)
Radiation reaction time	$\tau_0 = (2\alpha/3)\omega_c^{-1}$	$\tau_0^\gamma = \omega_0^{-1}$
ZPF driving	$e\mathbf{E}_{\text{ZPF}}$	$\mathbf{F}_{\text{ZPF}}$ (rad. pressure)
Diffusion coefficient	$D = \hbar/2m = c^2/2\omega_c$	$D_\gamma = \hbar/2m_\gamma^{\text{eff}} = c^2/2\omega_0$
Wave function	$\psi = \sqrt{\rho}e^{iS/\hbar}$	$\mathbf{F} = \sqrt{u_{\text{EM}}}e^{i\phi}\hat{\mathbf{e}}$
Wave equation	Schrödinger/Dirac	Maxwell
Spin	1/2	1

4 The Photon SDE and Fokker–Planck Equation

4.1 The Photon Stochastic Differential Equation

By exact analogy with Nelson’s SDE for the electron, the photon obeys:

$$d\mathbf{x}_\gamma = \underbrace{c\hat{\mathbf{k}}dt}_{\text{propagation}} + \underbrace{D_\gamma \nabla \ln \rho_\gamma dt}_{\text{osmotic}} + \underbrace{\sqrt{2D_\gamma} d\mathbf{W}_\perp(t)}_{\text{ZPF kicks}}, \quad (21)$$

where:

- $c\hat{\mathbf{k}}dt$ is the mean forward propagation at speed c ,
- $D_\gamma \nabla \ln \rho_\gamma = D_\gamma \nabla \ln u_{\text{EM}}$ is the osmotic velocity (transverse only),
- $d\mathbf{W}_\perp$ is the transverse Wiener process (ZPF kicks perpendicular to $\hat{\mathbf{k}}$).

The diffusion coefficient $D_\gamma = \hbar/2m_\gamma^{\text{eff}} = c^2/2\omega_0$ is derived in Section 3, not assumed.

4.2 The Photon Fokker–Planck Equation

The Fokker–Planck equation for $\rho_\gamma = u_{\text{EM}}$:

$$\frac{\partial \rho_\gamma}{\partial t} + \nabla \cdot \left[(c\hat{\mathbf{k}} + D_\gamma \nabla \ln \rho_\gamma) \rho_\gamma \right] = D_\gamma \nabla^2 \rho_\gamma. \quad (22)$$

Expanding and noting that the osmotic and diffusion terms cancel:

$$\boxed{\frac{\partial \rho_\gamma}{\partial t} + c\hat{\mathbf{k}} \cdot \nabla \rho_\gamma = 0} \quad (23)$$

This is the *photon transport equation*: the electromagnetic energy density propagates at speed c in direction $\hat{\mathbf{k}}$. This is the Poynting theorem:

$$\frac{\partial u_{\text{EM}}}{\partial t} + \nabla \cdot \mathbf{S}_{\text{Poynting}} = 0, \quad \mathbf{S}_{\text{Poynting}} = c^2 \epsilon_0 (\mathbf{E} \times \mathbf{B}), \quad (24)$$

which is a consequence of Maxwell's equations—but here it emerges *before* Maxwell's equations are derived, directly from the photon Fokker–Planck.

5 Maxwell Equations from the Photon Fokker–Planck

5.1 The Riemann–Silberstein Vector as Photon Wave Function

For the electron, the Madelung transformation $\psi = \sqrt{\rho} e^{iS/\hbar}$ linearised the nonlinear Fokker–Planck equation into the linear Schrödinger equation.

For the photon, we introduce the *Riemann–Silberstein vector* [12, 13]:

$$\mathbf{F}(\mathbf{x}, t) = \sqrt{\epsilon_0} \mathbf{E}(\mathbf{x}, t) + \frac{i}{\sqrt{\mu_0}} \mathbf{B}(\mathbf{x}, t). \quad (25)$$

The Riemann–Silberstein vector is the photon wave function. Its properties:

$$|\mathbf{F}|^2 = \epsilon_0 |\mathbf{E}|^2 + \frac{1}{\mu_0} |\mathbf{B}|^2 = u_{\text{EM}} = \rho_\gamma, \quad (26)$$

$$\frac{1}{2} |\mathbf{F}|^2 \propto P(\mathbf{x}, t) \quad (\text{photon probability density}), \quad (27)$$

$$i\hbar \partial_t \mathbf{F} = c(\mathbf{S} \cdot \hat{\mathbf{p}}) \mathbf{F}, \quad (28)$$

where $\mathbf{S} = (S_x, S_y, S_z)$ are the spin-1 matrices.

The analogy with the electron is exact:

	Electron	Photon
Wave function	$\psi = \sqrt{\rho} e^{iS/\hbar}$	$\mathbf{F} = \sqrt{u_{\text{EM}}} e^{i\phi} \hat{\mathbf{e}}$
$ \text{wf} ^2$	$ \psi ^2 = \rho$	$ \mathbf{F} ^2 = u_{\text{EM}} = \rho_\gamma$
Spin matrices	Pauli $\boldsymbol{\sigma}$ (spin- $\frac{1}{2}$)	$\mathbf{S}$ (spin-1)
Wave equation	$i\hbar \partial_t \psi = \hat{H} \psi$	$i\hbar \partial_t \mathbf{F} = c(\mathbf{S} \cdot \hat{\mathbf{p}}) \mathbf{F}$

5.2 The Spin-1 Matrix Identity

The key algebraic identity connecting the spin-1 matrices to the curl is:

$$(\mathbf{S} \cdot \nabla) \mathbf{F} = i \nabla \times \mathbf{F} \quad (29)$$

for any transverse vector field $\mathbf{F}$ (satisfying $\nabla \cdot \mathbf{F} = 0$). This identity is the photon analogue of the Pauli identity $(\boldsymbol{\sigma} \cdot \mathbf{p})^2 = p^2$ used in deriving the Pauli equation from the Dirac equation.

The spin-1 matrices are:

$$S_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (30)$$

5.3 Derivation of Maxwell Equations

Step 1: The photon Fokker–Planck as a vector equation

For a superposition of plane waves (general field), the transport equation (23) summed over all propagation directions $\hat{\mathbf{k}}$ with the transverse (spin-1) structure gives:

$$\frac{\partial \mathbf{F}}{\partial t} = -c(\mathbf{S} \cdot \nabla) \mathbf{F}. \quad (31)$$

Using the identity (29):

$$\frac{\partial \mathbf{F}}{\partial t} = -ic \cdot i \nabla \times \mathbf{F} = c \nabla \times \mathbf{F}. \quad (32)$$

However, the sign depends on helicity. For the positive-helicity component $\mathbf{F}^+ = \sqrt{\epsilon_0} \mathbf{E} + i\mathbf{B}/\sqrt{\mu_0}$, the correct equation is:

$$\frac{\partial \mathbf{F}^+}{\partial t} = -c \nabla \times \mathbf{F}^+. \quad (33)$$

Step 2: Separate real and imaginary parts

Substituting $\mathbf{F}^+ = \sqrt{\epsilon_0} \mathbf{E} + (i/\sqrt{\mu_0}) \mathbf{B}$:

Left side:

$$\frac{\partial \mathbf{F}^+}{\partial t} = \sqrt{\epsilon_0} \frac{\partial \mathbf{E}}{\partial t} + \frac{i}{\sqrt{\mu_0}} \frac{\partial \mathbf{B}}{\partial t}. \quad (34)$$

Right side:

$$-c \nabla \times \mathbf{F}^+ = -c \sqrt{\epsilon_0} \nabla \times \mathbf{E} - \frac{ic}{\sqrt{\mu_0}} \nabla \times \mathbf{B}. \quad (35)$$

Real part (coefficients of $\sqrt{\epsilon_0}$):

$$\sqrt{\epsilon_0} \frac{\partial \mathbf{E}}{\partial t} = -\frac{ic}{\sqrt{\mu_0}} \nabla \times \mathbf{B} \cdot \frac{1}{i} \quad (36)$$

More carefully, equating real parts:

$$\sqrt{\epsilon_0} \frac{\partial \mathbf{E}}{\partial t} = \frac{c}{\sqrt{\mu_0}} \nabla \times \mathbf{B}. \quad (37)$$

Dividing by $\sqrt{\epsilon_0}$ and using $c = 1/\sqrt{\epsilon_0 \mu_0}$:

$$\boxed{\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0 \mu_0} \nabla \times \mathbf{B} = c^2 \nabla \times \mathbf{B}} \quad (38)$$

This is Ampère's law (in vacuum, no current). ✓

Imaginary part (coefficients of $i/\sqrt{\mu_0}$):

$$\frac{1}{\sqrt{\mu_0}} \frac{\partial \mathbf{B}}{\partial t} = -c\sqrt{\epsilon_0} \nabla \times \mathbf{E}. \quad (39)$$

Multiplying by $\sqrt{\mu_0}$ and using $c\sqrt{\epsilon_0\mu_0} = 1$:

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}} \quad (40)$$

This is Faraday's law. ✓

Step 3: Gauss's laws from transversality

The ZPF consists of transverse modes (Lorentz invariance, Boyer 1969, Paper 1 Section 3.4). Each mode satisfies $\hat{\mathbf{k}} \cdot \tilde{\mathbf{E}} = 0$ and $\hat{\mathbf{k}} \cdot \tilde{\mathbf{B}} = 0$. For a superposition of transverse modes:

$$\boxed{\nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{B} = 0.} \quad (41)$$

Gauss's laws. ✓

5.4 Summary: Maxwell from Fokker–Planck

$$\underbrace{\frac{\partial \mathbf{F}}{\partial t} = -c \nabla \times \mathbf{F}}_{\text{photon Fokker–Planck}} \xrightarrow{\mathbf{F} = \sqrt{\epsilon_0} \mathbf{E} + \frac{i}{\sqrt{\mu_0}} \mathbf{B}} \left\{ \begin{array}{l} \partial_t \mathbf{E} = c^2 \nabla \times \mathbf{B} \\ \partial_t \mathbf{B} = -\nabla \times \mathbf{E} \\ \nabla \cdot \mathbf{E} = 0 \\ \nabla \cdot \mathbf{B} = 0 \end{array} \right. \quad (42)$$

Maxwell equations in vacuum

This is the exact photon analogue of Paper 1:

$$\underbrace{\frac{\partial \rho}{\partial t} + \nabla \cdot (\mathbf{v} \rho) = D \nabla^2 \rho}_{\text{electron Fokker–Planck}} \xrightarrow{\psi = \sqrt{\rho} e^{iS/\hbar}} \underbrace{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi}_{\text{Schrödinger equation}} \quad (43)$$

6 Malus's Law from Brownian Motion on the Poincaré Sphere

6.1 Setup

A photon in polarization state α (angle from 0°) hits a linear polarizer at 0° . Standard quantum mechanics gives $P(\text{pass}) = \cos^2 \alpha$. We derive this from the photon SDE.

6.2 Polarization Space as the Poincaré Sphere

The polarization state of a photon is described by a point on the *Poincaré sphere*. For linear polarizations, the relevant coordinate is the polarization angle $\theta \in [0^\circ, 90^\circ)$, where $\theta = 0^\circ$ is horizontal (0° polarization) and $\theta = 90^\circ$ is vertical.

The Poincaré sphere has a non-Euclidean metric in polarization space. For linear polarizations, the natural arc length is:

$$ds = 2 d\theta \quad (44)$$

because linear polarizations have period 180° (not 360°).

6.3 The Polarizer as a ZPF Boundary Condition

The polarizer at 0° physically selects the x -component of the ZPF and absorbs the y -component. Inside the polarizer, the effective energy density is:

$$u_{\text{EM}}^{(\text{pol})}(\theta) = u_{\text{EM}} \cos^2 \theta \quad (45)$$

(only the 0° component survives).

The osmotic velocity in polarization space:

$$u_\theta = D_\gamma \frac{\partial}{\partial \theta} \ln \cos^2 \theta = -2D_\gamma \tan \theta. \quad (46)$$

6.4 The Critical Timescale Argument

There are two timescales:

- $\tau_{\text{Brownian}} \sim \theta_0^2 / D_\gamma$ — time for random walk to reach a boundary, starting from θ_0 .
- $\tau_{\text{osmotic}} \sim 1 / D_\gamma |\partial_\theta \ln \cos^2 \theta|$ — time for osmotic velocity to develop a preference.

At $\theta_0 = 45^\circ$, both timescales are comparable. However, the key observation is:

Before the osmotic drift has time to develop, the photon performs pure Brownian motion in polarization space, starting at $\theta_0 = 45^\circ$. The outcome (pass or absorb) is determined by which boundary the Brownian motion hits first.

6.5 First-Passage Probability: 50/50 Result

For pure Brownian motion $d\theta = \sqrt{2D_\gamma} dW$ on $[0^\circ, 90^\circ]$ with absorbing boundaries at $\theta = 0^\circ$ (pass) and $\theta = 90^\circ$ (absorb), starting at $\theta_0 = 45^\circ$:

$$P(\text{pass}) = \frac{90^\circ - 45^\circ}{90^\circ - 0^\circ} = \frac{1}{2}. \quad (47)$$

The 50% probability follows from the *symmetry of the starting position*: $\theta_0 = 45^\circ$ is exactly midway between the two boundaries.

6.6 General Angle: Deriving Malus's Law

For a photon at angle α hitting a 0° polarizer, the starting position in polarization space is $\theta_0 = \alpha$.

On the Poincaré sphere with metric (44), the natural coordinate is $u = \sin^2 \theta$. In this coordinate, the Brownian motion becomes uniform, and the first-passage probability from $u_0 = \sin^2 \alpha$ to $\{u = 0, u = 1\}$ is:

$$P(\text{pass}) = \frac{1 - \sin^2 \alpha}{1 - 0} = \cos^2 \alpha. \quad (48)$$

$$\boxed{P(\text{pass}) = \cos^2 \alpha} \quad (49)$$

This is Malus's law—derived from Brownian motion on the Poincaré sphere, without any quantum postulate.

6.7 Which Individual Photon Passes?

The outcome for each individual photon is determined by the specific realization of the Wiener process $dW(t)$ —the instantaneous ZPF configuration at the moment of interaction with the polarizer. There is no hidden variable beyond the ZPF itself.

This answers the hundred-year question about photon randomness in the polarizer:

The ZPF configuration at the moment of interaction is the “hidden variable.” But it is not a predetermined hidden variable in Bell's sense—it is a real stochastic field (the electromagnetic vacuum ZPF) whose value at each moment is genuinely random. The randomness is physical, not epistemic.

7 The Coherence Volume from D_γ

7.1 Derivation of the Coherence Radius

The photon diffusion coefficient $D_\gamma = c^2/2\omega_0$ determines how rapidly the photon wave packet spreads transversely. After propagating for time t , the transverse spread is:

$$\langle r_\perp^2 \rangle = 2D_\gamma t = \frac{c^2}{\omega_0} t. \quad (50)$$

The *minimum* coherence radius—for a single-mode photon over one optical period $t = 1/\omega_0$ —is:

$$r_{\text{coh}}^{\text{min}} = \sqrt{2D_\gamma/\omega_0} = \sqrt{\frac{c^2}{\omega_0^2}} = \frac{c}{\omega_0} = \bar{\lambda}. \quad (51)$$

The minimum coherence radius equals the *reduced wavelength* $\bar{\lambda} = c/\omega_0 = \lambda/2\pi$. This is derived, not assumed.

7.2 Coherence Volume and the Emission Process

The actual coherence volume of a photon depends on the emission process through the coherence time τ_{coh} :

$$r_{\text{coh}}(\tau_{\text{coh}}) = \sqrt{2D_\gamma \tau_{\text{coh}}} = \bar{\lambda} \sqrt{\omega_0 \tau_{\text{coh}}}. \quad (52)$$

For sunlight ($\tau_{\text{coh}} \approx 3$ fs, $\omega_0 \approx 3 \times 10^{15}$ rad/s):

$$r_{\text{coh}}^{(\text{sun})} = \bar{\lambda} \sqrt{3 \times 10^{-15} \times 3 \times 10^{15}} = 3\bar{\lambda} \approx 3 \times \frac{600 \text{ nm}}{2\pi} \approx 286 \text{ nm}. \quad (53)$$

This gives the intrinsic single-photon coherence radius. The *observed* transverse coherence radius of sunlight at Earth is a distinct, much larger quantity, set not by τ_{coh} but by the van Cittert–Zernike geometric factor from the finite angular size of the Sun [14]. The full transverse coherence *diameter* is

$$d_{\text{coh}}^{(\text{sun})} = \frac{\lambda z_1}{D_\odot}, \quad (54)$$

where $z_1 = 1.496 \times 10^{11}$ m is the Sun–Earth distance and $D_\odot = 1.391 \times 10^9$ m is the solar diameter. For $\lambda = 0.52 \mu\text{m}$ this gives

$$d_{\text{coh}}^{(\text{sun})} = \frac{(0.52 \times 10^{-6})(1.496 \times 10^{11})}{1.391 \times 10^9} \approx 56 \mu\text{m}. \quad (55)$$

The corresponding coherence *radius* is half this value, $r_{\text{coh}}^{(\text{sun,obs})} \approx 28 \mu\text{m}$, in close agreement with the standard quoted figure of $\sim 27 \mu\text{m}$ for the spatial coherence radius of sunlight [14].

For a single-mode laser ($\tau_{\text{coh}} \sim 10^{-3}$ s):

$$r_{\text{coh}}^{(\text{laser})} = \bar{\lambda} \sqrt{3 \times 10^{12}} \approx 10^6 \bar{\lambda} \sim \text{mm}, \quad (56)$$

consistent with the observed millimetre-scale coherence radius of a well-collimated laser beam.

7.3 Physical Interpretation

The coherence volume $V_{\text{coh}} = \pi r_{\text{coh}}^2 \cdot c\tau_{\text{coh}}$ is the region of the electromagnetic vacuum ZPF that is correlated with the photon. It is the photon’s analogue of the electron’s Coulomb field extent:

	Electron	Photon
“Field extent”	Coulomb field ($\propto 1/r^2$, infinite)	Coherence volume (V_{coh} , finite)
Set by	Rest mass m	Emission process (τ_{coh})
Minimum scale	Compton wavelength $\hbar/mc$	Reduced wavelength $\bar{\lambda} = c/\omega_0$
ZPF extent	Infinite (Coulomb)	V_{coh} (wave packet)

8 The Quantum Eraser for Photons

8.1 Setup

The quantum eraser was originally proposed as a thought experiment by Scully and Drühl [10]. The first real experimental realisation with photons was achieved by Kim et al. [11], who demonstrated the delayed-choice version with entangled photon pairs. The experiment provides the most direct test of whether randomness is controlled by the physical ZPF state or by the observer's knowledge. The setup is:

1. A photon passes through a double slit. A which-path marker entangles the signal photon's path with the polarization of an idler photon: slit A $\rightarrow |V\rangle_{\text{idler}}$, slit B $\rightarrow |H\rangle_{\text{idler}}$.
2. The joint state is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_A\rangle_{\text{signal}}|V\rangle_{\text{idler}} + |\psi_B\rangle_{\text{signal}}|H\rangle_{\text{idler}}). \quad (57)$$

3. Measuring the idler in the V/H basis reveals which path $\rightarrow$ no interference. Measuring in the $\pm 45^\circ$ basis erases which-path information $\rightarrow$ interference restored in each sub-ensemble.

8.2 Explanation in the ZPF Framework

Why the marker destroys fringes

The marker interaction at the slit physically entangles the signal photon's ZPF with the idler photon. The signal photon's probability density is no longer $|\psi_A + \psi_B|^2$ but the mixed state:

$$\rho_{\text{signal}}(\mathbf{x}) = \frac{1}{2}|\psi_A(\mathbf{x})|^2 + \frac{1}{2}|\psi_B(\mathbf{x})|^2. \quad (58)$$

The osmotic velocity becomes:

$$u(\mathbf{x}) = D_\gamma \nabla \ln\left(\frac{1}{2}|\psi_A|^2 + \frac{1}{2}|\psi_B|^2\right), \quad (59)$$

which has no interference term. No fringes.

Why erasure restores fringes

Measuring the idler in the $\pm 45^\circ$ basis gives outcomes:

$$|\pm 45^\circ\rangle = \frac{1}{\sqrt{2}}(|V\rangle \pm |H\rangle). \quad (60)$$

For the $+45^\circ$ outcome, the signal photon's state is *post-selected* to:

$$\rho_{\text{signal}}(\mathbf{x} | +45^\circ) = |\psi_A(\mathbf{x}) + \psi_B(\mathbf{x})|^2. \quad (61)$$

The osmotic velocity restores to $u = D_\gamma \nabla \ln |\psi_A + \psi_B|^2$. Interference fringes reappear. For the -45° outcome the fringes are shifted by π , so the two sub-ensembles show complementary patterns that cancel when added—confirming no-signalling.

8.3 Does the Idler Measurement Instantly Affect the Signal Photon?

This is the key question. The answer in the present framework is:

No. The idler measurement does not physically disturb the signal photon's ZPF. It is a Bayes update of the joint probability distribution—a change in our knowledge of a pre-existing correlation, not a physical influence.

The joint probability density $\rho(\mathbf{x}_{\text{signal}}, s_{\text{idler}})$ was established at the moment of marker interaction. Measuring the idler and finding $+45^\circ$ applies Bayes' theorem:

$$\rho(\mathbf{x}_{\text{signal}} | +45^\circ) = \frac{\rho(\mathbf{x}_{\text{signal}}, +45^\circ)}{\int \rho(\mathbf{x}, +45^\circ) d\mathbf{x}} = |\psi_A + \psi_B|^2. \quad (62)$$

No-signalling. The marginal distribution of the signal photon is unchanged by any idler measurement:

$$\rho(\mathbf{x}_{\text{signal}}) = \int \rho(\mathbf{x}_{\text{signal}}, s_{\text{idler}}) ds_{\text{idler}} = \frac{1}{2}|\psi_A|^2 + \frac{1}{2}|\psi_B|^2. \quad (63)$$

No fringes, no dependence on the idler measurement. Only after the idler result is *classically communicated* and the signal data *post-selected* do sub-ensemble fringes appear.

Situation	Signal photon ZPF	Result
No marker	$\rho = \psi_A + \psi_B ^2$, undisturbed	Interference fringes
Marker, idler unmeasured	$\rho = \frac{1}{2} \psi_A ^2 + \frac{1}{2} \psi_B ^2$, mixed	No fringes
Marker, idler = V/H	Post-selected to $ \psi_A ^2$ or $ \psi_B ^2$	No fringes (single-slit)
Marker, idler = $\pm 45^\circ$	Post-selected to $ \psi_A \pm \psi_B ^2$	Fringes in sub-ensemble

The quantum eraser thus confirms the central claim of the present paper: fringes are controlled by the physical ZPF state, not by the observer's knowledge. The “instant non-local collapse” is an artefact of the Copenhagen language—in the ZPF framework it is simply Bayes' theorem applied to a pre-existing joint distribution.

8.4 The Delayed-Choice Quantum Eraser

The *delayed-choice* quantum eraser [11] is the most striking version of the experiment. The decision of whether to erase the which-path information is made *after* the signal photon has already arrived at the screen. This appears to imply retrocausality: a future choice determines a past event.

Why there is no retrocausality

The present framework resolves the apparent paradox completely through a single observation:

The full signal photon data—all events recorded at the screen—never shows interference fringes, regardless of what the idler measurement will be. Fringes appear only in a sub-ensemble post-selected on the idler outcome. Post-selection at a later time does not change the past; it sorts data that was always structured that way.

Timeline and ZPF state:

t	Event	ZPF framework
t_0	Photon pair created	Joint ZPF correlation $\rho(\mathbf{x}_s, s_i)$ established
t_1	Signal hits screen	Full data recorded. Marginal $\rho(\mathbf{x}_s) = \frac{1}{2} \psi_A ^2 + \frac{1}{2} \psi_B ^2$ — <i>no fringes</i>
$t_2 > t_1$	Choice: erase or not?	Classical decision; no physical effect on $\rho(\mathbf{x}_s)$
$t_3 > t_2$	Idler measured; data sorted	Bayes post-selection reveals sub-ensemble structure that was always present in $\rho(\mathbf{x}_s, s_i)$

Mathematical proof of no retrocausality.

For all times $t \geq t_0$, the signal photon marginal satisfies:

$$\rho(\mathbf{x}_s) = \int \rho(\mathbf{x}_s, s_i) ds_i = \frac{1}{2}|\psi_A(\mathbf{x}_s)|^2 + \frac{1}{2}|\psi_B(\mathbf{x}_s)|^2. \quad (64)$$

This is independent of:

- *When* the idler is measured (t_2 can be years after t_1),
- *Whether* the idler is measured at all,
- *Which basis* is chosen for the idler.

The post-selected sub-ensembles:

$$\rho(\mathbf{x}_s | +45^\circ) = |\psi_A(\mathbf{x}_s) + \psi_B(\mathbf{x}_s)|^2 \quad (\text{fringes}), \quad (65)$$

$$\rho(\mathbf{x}_s | -45^\circ) = |\psi_A(\mathbf{x}_s) - \psi_B(\mathbf{x}_s)|^2 \quad (\text{complementary fringes}). \quad (66)$$

Adding (65) and (66) with equal weight recovers (64):

$$\frac{1}{2}|\psi_A + \psi_B|^2 + \frac{1}{2}|\psi_A - \psi_B|^2 = |\psi_A|^2 + |\psi_B|^2. \quad (67)$$

The two sub-ensemble fringe patterns are exactly complementary and cancel—which is why the full (unsorted) data never shows fringes.

The paradox dissolved.

The delayed-choice quantum eraser does not show retrocausality. It shows that:

1. The joint ZPF correlation $\rho(\mathbf{x}_s, s_i)$ was fully determined at t_0 .
2. The signal data at t_1 already *contained* the sub-ensemble structure—it was not yet *sorted*.
3. Sorting at t_3 reveals structure that was always present. The future does not change the past.

9 QED from Electron-Photon Stochastic Coupling

9.1 The Two Stochastic Processes

The electron SDE (Paper 4 [4]):

$$du^\mu = \frac{e}{m} F_{\text{ZPF}}^{\mu\nu} u_\nu d\tau + \sqrt{\frac{\hbar}{m}} dW_{\text{electron}}^\mu(\tau). \quad (68)$$

The photon SDE (this paper):

$$d\mathbf{x}_\gamma = c\hat{\mathbf{k}} dt + D_\gamma \nabla \ln u_{\text{EM}} dt + \sqrt{2D_\gamma} d\mathbf{W}_\gamma(t). \quad (69)$$

9.2 The Coupling

The electromagnetic field $F_{\text{ZPF}}^{\mu\nu}$ in the electron SDE includes both the vacuum ZPF and the photon field:

$$F_{\text{total}}^{\mu\nu} = F_{\text{vacuum ZPF}}^{\mu\nu} + F_{\text{photon}}^{\mu\nu}. \quad (70)$$

The coupling between the two SDEs is:

$$\mathcal{H}_{\text{int}} = e j^\mu A_\mu = e \underbrace{\bar{\psi} \gamma^\mu \psi}_{\text{electron current}} \underbrace{A_\mu}_{\text{photon field}}. \quad (71)$$

This is the QED interaction vertex—emerging naturally from the coupling of the electron and photon stochastic processes. The Feynman rules of QED correspond to the perturbative expansion of this stochastic coupling in powers of $e^2 = 4\pi\alpha\hbar c$.

9.3 Propagators from the SDEs

Electron propagator. The Green’s function of the electron SDE gives the Feynman electron propagator:

$$S_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}. \quad (72)$$

Photon propagator. The cross-correlation $S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega)$ derived in Paper 2 [2] is the photon propagator:

$$S_E^{(12)} = S_E(\omega) \cdot \text{sinc}\left(\frac{\omega r_{12}}{c}\right) \leftrightarrow D^{\mu\nu}(k) = \frac{-ig^{\mu\nu}}{k^2 + i\epsilon}. \quad (73)$$

The sinc factor is the position-space photon propagator. The coherence volume of the photon equals the spatial extent of this propagator.

9.4 Loop Corrections

The one-loop QED corrections—Lamb shift (Paper 5 [5]), anomalous magnetic moment $g - 2 = \alpha/\pi$ —arise from the leading non-Markovian correction to the Markovian (Fokker–Planck) approximation. Papers 5 and 6 have already derived these corrections within the stochastic framework, confirming the consistency of the electron-photon coupling described here.

10 The Complete Hierarchy

The eight papers of this series establish a complete hierarchy of effective descriptions, all derived from the single postulate of Paper 1 (electromagnetic field energy is carried in discrete quanta):

Scale	Particle	Process	Equation
Sub-Compton	Electron $(\Delta\tau \ll \hbar/mc^2)$	Non-Markovian ALD	None (QFT regime)
Compton–Dirac	Electron $(\Delta\tau \gg \hbar/mc^2)$	4D Brownian, spin- $\frac{1}{2}$	Dirac equation
Schrödinger	Electron $(v \ll c)$	3D Brownian, spin- $\frac{1}{2}$	Schrödinger equation
Optical	Photon $(\Delta t \gg 1/\omega_0)$	Brownian, spin-1	Maxwell equations
QED	Electron + Photon	Coupled SDEs	QED (Feynman rules)

All effective descriptions emerge from the same stochastic framework. The differences are:

- **Spin:** $\frac{1}{2}$ (electron) vs 1 (photon) $\rightarrow$ Pauli matrices vs spin-1 matrices $\rightarrow$ gradient vs curl.
- **Mass:** m (electron rest mass) vs $m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ (photon effective mass) $\rightarrow D = \hbar/2m$ vs $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$.
- **Propagation:** diffusion in 3D or 4D (electron) vs directed at c with transverse diffusion (photon).

11 Discussion

11.1 Why $D_\gamma = \hbar/2m_\gamma^{\text{eff}}$ Has the Same Form as $D = \hbar/2m$

This is not a coincidence. Both follow from the same underlying structure:

$$D = \frac{\hbar}{2m_{\text{inertial}}} \quad (74)$$

where m_{inertial} is the inertial mass resisting transverse acceleration:

- For the electron: $m_{\text{inertial}} = m$ (rest mass).
- For the photon: $m_{\text{inertial}} = m_\gamma^{\text{eff}} = \hbar\omega_0/c^2$ (relativistic mass).

Equation (74) is the universal relation between the diffusion coefficient and the inertial mass of a ZPF-driven stochastic process. It holds for any particle—massive or massless—once the appropriate inertial mass is identified.

11.2 What Determines the Photon’s Coherence Length?

What τ_{coh} is, and what it is not. It is important to be precise about the physical meaning of τ_{coh} , since a careless reading invites a real objection: if τ_{coh} were literally the duration of the emission event itself—the time during which the atom is “partway through” emitting—this would imply a partial quantum of energy existing mid-emission, in direct conflict with the postulate that the electromagnetic field carries energy only in complete, discrete units $\hbar\omega$. That is not what τ_{coh} means. The photon’s energy is $\hbar\omega$, complete and indivisible, at every instant from emission to detection; there is no intermediate state of fractional energy. What τ_{coh} actually characterizes is the temporal extent of the emitted wave packet’s envelope f in (5)—equivalently, the time delay beyond which the photon’s field can no longer interfere with a time-shifted copy of itself. This finite extent arises not from a gradual emission process but from the energy–time uncertainty relation applied to the *decaying excited atomic state* that does the emitting: an excited state with finite lifetime τ (set by the spontaneous emission rate, the Einstein A coefficient) does not have a perfectly sharp energy, and the resulting energy spread $\Delta E \sim \hbar/\tau$ Fourier-transforms into a finite spectral linewidth $\Delta\omega \sim 1/\tau$ for the emitted photon. The coherence time $\tau_{\text{coh}} \sim 1/\Delta\omega$ is therefore best understood as the coherence time of the emitted wave train, numerically tied to the excited state’s lifetime, not as a literal stretching-out of an instantaneous quantum event. Each individual emission produces one complete photon of energy $\hbar\omega_0$ for some frequency ω_0

drawn from the Lorentzian line shape; τ_{coh} describes the resulting wavepacket envelope and the linewidth of the distribution of emitted frequencies across repeated emissions, not a single emission's duration.

The coherence length $l_{\text{coh}} = c\tau_{\text{coh}}$ is set by the *emission process*:

- **Thermal source** (sunlight): each photon is emitted by a single atomic transition from an excited state of lifetime $\tau \sim 1/\Delta\omega$, giving a coherence time $\tau_{\text{coh}} \sim 1/\Delta\omega$, where $\Delta\omega$ is the linewidth. The linewidth is set by the natural linewidth (spontaneous emission rate) and collisional broadening.
- **Laser**: stimulated emission into a single mode gives τ_{coh} limited only by cavity losses and laser linewidth—many orders of magnitude longer.
- **Single photon from a two-level atom**: $\tau_{\text{coh}} = 1/A_{eg}$ where A_{eg} is the Einstein A coefficient.

In each case, τ_{coh} is set by the emission dynamics, and $r_{\text{coh}} = \bar{\lambda}\sqrt{\omega_0\tau_{\text{coh}}}$ follows from equation (52).

11.3 Open Questions

The photon AL equation from first principles. Section 3 motivates the photon AL equation by analogy and dimensional analysis. A rigorous derivation from the Maxwell equations and vacuum ZPF—analogous to the electron AL derivation from the Coulomb field—remains to be completed.

QED beyond one loop. The present framework gives the Feynman rules of QED at tree level and one loop. The extension to higher loops, and the treatment of infrared divergences (soft photon emission), requires a more complete treatment of the non-Markovian corrections.

The photon in a medium. The present treatment is for the photon in vacuum. Extension to photons in a dielectric medium (refractive index n , group velocity $v_g < c$) would require replacing $c \rightarrow v_g$ in the photon SDE, which would give a modified diffusion coefficient $D_\gamma = v_g^2/2\omega_0$.

12 Conclusion

We have extended the stochastic electrodynamic framework of Papers 1–7 to the photon, deriving three major results without any quantum postulate:

1. The photon diffusion coefficient:

$$D_\gamma = \frac{c^2}{2\omega_0} = \frac{\hbar}{2m_\gamma^{\text{eff}}}, \quad m_\gamma^{\text{eff}} = \frac{\hbar\omega_0}{c^2}. \quad (20)$$

Derived by equating two independent expressions for $\langle p_\perp^2 \rangle$ from the photon Abraham–Lorentz equation. Same form as $D = \hbar/2m$ for the electron.

2. Maxwell equations from the photon Fokker–Planck:

$$\text{Photon Fokker–Planck} \xrightarrow{\mathbf{F}=\sqrt{\epsilon_0}\mathbf{E}+\frac{i}{\sqrt{\mu_0}}\mathbf{B}} \nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad \nabla \times \mathbf{B} = \frac{1}{c^2} \partial_t \mathbf{E}. \quad (42)$$

The Madelung transformation using the Riemann–Silberstein vector linearises the Fokker–Planck into Maxwell’s equations—the photon analogue of deriving Schrödinger from the electron Fokker–Planck.

3. Malus’s law from Brownian motion:

$$P(\text{pass}) = \cos^2 \alpha. \quad (49)$$

Derived from first-passage probability of Brownian motion on the Poincaré sphere. The 50/50 randomness at $\alpha = 45^\circ$ follows from the symmetry of the starting position.

4. **Photon randomness:** The photon’s randomness originates in the electromagnetic vacuum ZPF—the same field of which the photon is a quantum. There is no hidden variable beyond the ZPF itself. The coherence volume equals $V_{\text{coh}} = \pi \bar{\lambda}^2 \cdot c \tau_{\text{coh}}$, consistent with the van Cittert–Zernike theorem.
5. **QED:** The coupling of the electron and photon SDEs gives the QED interaction vertex $ej^\mu A_\mu$, with propagators given by the electron SDE Green’s function and the photon cross-correlation $S_E^{(12)}$ of Paper 2.

The universal relation $D = \hbar/2m_{\text{inertial}}$ holds for both the electron ($m_{\text{inertial}} = m$) and the photon ($m_{\text{inertial}} = \hbar\omega_0/c^2$). The difference between Schrödinger and Maxwell is spin: $\frac{1}{2}$ gives the gradient operator; 1 gives the curl.

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