Euclidean Quantum Field Theory and Euclidean Quantum Mechanics

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Abstract

A formulation of quantum mechanics and quantum field theory is presented in which Euclidean spacetime, real interference kernels, and action–difference dynamics replace the conventional complex Hilbert space and Lorentzian structure.

Groupthink quantum mechanics is a shortcut using complex space, which actually dates back to mathematician Weyl in his 1918 gauge theory of quantum gravity, which falsely claimed to unify electromagnetism with gravity (or rather, the metric of Einstein's physically false "curved spacetime" quantum gravity approximation, i.e.. classical general relativity). Weyl "quantized" the metric by setting it proportional to Euler's complex function of X, exp(iX). Although physically plain wrong for gravity (Weyl's unification was debunked by Einstein) it was later in 1926 used by Schroedinger to "explain" Bohr's discrete energy levels in the atom. What all these guys missed is:

(1) the correct relativistic theory is Feynman's path integrals, where exp(iX) is a path amplitude and you have to sum (or integrate) all possible path amplitudes to allow for multipath interference, to get resultant amplitude and

(2) Euler's expression for exp(iX) = (i sin X) + (cos X), and you don't actually need the complex (i) terms at all when you use multipath phase interference! IT IS EXACTLY LIKE OLD 1960s HF RADIO MULTIPATH INTERFERENCE FOR TRANSATLANTIC COMMUNICATIONS: the radio waves take ALL possible paths, bouncing off the various high electron density layers of the ionosphere (D, E, F layers at altitudes 60-100 km or so). So you can REPLACE exp(iX) with cos X, and simply use Euclidean spacetime for quantum field theory and quantum mechanics; a "real" electron is an uncancelled resultant amplitude! It's all simple. But heresy!

The fundamental object is a real interference functional over pairs of Euclidean histories,

$$\mathcal{I}_E[x, y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right),$$

from which all transition probabilities are constructed. The usual complex phase $e^{iS/\hbar}$ is shown to be a compressed encoding of this real structure. A Euclidean Schrödinger equation emerges as an approximation to the double–path interference dynamics. The hydrogen atom spectrum is derived within this framework, demonstrating equivalence with standard quantum mechanics while eliminating the need for complex numbers in the axioms.

1 Foundational Principles

1.1 Euclidean spacetime

The fundamental geometry is taken to be Euclidean with coordinates $(\tau, \mathbf{x})$ and metric

$$ds^2 = d\tau^2 + d\mathbf{x}^2.$$

Lorentzian structure is not assumed; it will emerge as an effective description.

1.2 Euclidean action

A history $x(\tau)$ carries a Euclidean action

$$S_E[x] = \int d\tau \left[\frac{m}{2}\dot{x}^2 + V(x)\right].$$

1.3 Interference principle

Quantum behaviour arises from interference between *pairs* of histories. Define the real interference functional

$$\mathcal{I}_E[x,y] = \cos\!\left(\frac{S_E[x] - S_E[y]}{\hbar}\right).$$

1.4 Probability postulate

The transition probability between boundary points a and b is

$$P(b,a) = \mathcal{N}\int_{x(a)=y(a)=a}^{x(b)=y(b)=b} \mathcal{D}x\,\mathcal{D}y\ \cos\!\left(\frac{S_E[x] - S_E[y]}{\hbar}\right), \tag{1}$$

with normalization $\mathcal{N}$ fixed by total probability.

2 Emergent Wave Dynamics

2.1 Coarse–grained amplitude

Define a coarse–grained real amplitude

$$\psi(x,\tau) = \int \mathcal{D}y\ \cos\!\left(\frac{S_E[x] - S_E[y]}{\hbar}\right).$$

2.2 Euclidean Schrödinger equation

Under locality and Markovian coarse–graining, ψ satisfies

$$\hbar\,\frac{\partial\psi}{\partial\tau} = \left(\frac{\hbar^2}{2m}\nabla^2 - V\right)\psi. \tag{2}$$

Analytic continuation $\tau \to it$ yields the usual Schrödinger equation. Thus the complex structure is emergent rather than fundamental.

3 Euclidean Quantum Field Theory

For a field $\phi(x)$ with Euclidean action $S_E[\phi]$, define the double–field generating functional

$$\mathcal{Z}_E = \int \mathcal{D}\phi_+\,\mathcal{D}\phi_-\ \cos\!\left(\frac{S_E[\phi_+] - S_E[\phi_-]}{\hbar}\right).$$

Correlation functions follow by functional differentiation with respect to sources coupled symmetrically to ϕ_+ and ϕ_-.

4 Hydrogen Atom in Euclidean Quantum Mechanics

4.1 Euclidean action for the Coulomb problem

For a particle in a Coulomb potential $V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$, the Euclidean action is

$$S_E = \int d\tau \left[\frac{m}{2}(\dot{r}^2 + r^2\dot{\Omega}^2) - \frac{e^2}{4\pi\epsilon_0 r} \right].$$

4.2 Separation of variables

Assume a stationary solution of the Euclidean Schrödinger equation (2) of the form

$$\psi(r, \theta, \phi, \tau) = e^{-E\tau/\hbar} R(r) Y_{\ell m}(\theta, \phi).$$

Substituting into (2) yields the radial equation

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} - \frac{\ell(\ell+1)}{r^2}R + \frac{2m}{\hbar^2}\left(E + \frac{e^2}{4\pi\epsilon_0 r}\right)R = 0. \tag{3}$$

4.3 Dimensionless variables

Define

$$\rho = \frac{2r}{na_0}, \qquad a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2},$$

and write

$$R(r) = \rho^\ell e^{-\rho/2} L(\rho).$$

Substituting into (3) yields the associated Laguerre equation

$$\rho\frac{d^2L}{d\rho^2} + (2\ell + 2 - \rho)\frac{dL}{d\rho} + (n - \ell - 1)L = 0.$$

4.4 Quantization

Polynomial solutions require

$$n = 1, 2, 3, \ldots$$

and the energy spectrum is

$$E_n = -\frac{1}{2}\frac{me^4}{(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2}.$$

4.5 Interpretation

The hydrogen spectrum arises entirely from the Euclidean interference principle and the Euclidean Schrödinger equation. No complex numbers are required in the axioms; the usual complex wavefunction is an emergent representation of the real double–path interference dynamics.

5 Conclusion

Quantum mechanics and quantum field theory can be formulated entirely within Euclidean spacetime using real interference between pairs of histories. The cosine of action differences is the fundamental dynamical object. The hydrogen atom spectrum follows directly from the Euclidean Schrödinger equation derived from this interference principle. Lorentzian structure and complex Hilbert space arise as effective representations rather than foundational ingredients.