A Speculative Mathematical Ontology for Pre-Temporal Structure - Logical Construction from Set Theory, Category Theory, and Algebra

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Abstract

We construct a *timeless mathematical template* that builds structures in a purely logical sequence: from the empty set, a distinguished pair {something, anti-something}, an infinite alternating 1D lattice, higher-dimensional parity lattices, a conjectured collapse to dimensions 1–3, complex-valued fields with a $U(1)$ symmetry, the division algebra ladder $\mathbb{C} \to \mathbb{H} \to \mathbb{O}$, and finally a representation of the Standard Model gauge group via Furey's embedding [3, 4]. We then show that if one adds a monoid action $\mathbb{R}_{\geq 0}$ compatible with the $U(1)$ symmetry, the resulting formalism matches the kinematics of quantum field theory. The entire pre-temporal phase is a static logical dependency; time does *not* emerge logically but can be *consistently attached*. We explicitly separate formal mathematics from heuristic interpretation, highlight open conjectures (notably the dimensional collapse), and frame the work as a speculative mathematical ontology rather than a derived physical theory.

1 Introduction

The question "what happened before the Big Bang?" is often dismissed on the grounds that time itself may have begun with the Bang [5]. Here we explore a different kind of answer: a *timeless mathematical construction* that exists logically prior to any temporal parameter. "Development" in this context means *logical dependency*, not temporal evolution. We use only standard mathematics: ZF set theory, category theory, division algebras, and monoid actions.

The work belongs to the tradition of *pregeometric* cosmology [10], *structural realism* [11, 2], and *mathematical Platonism* [8, 9]. However, unlike mature physical theories, it currently lacks empirical content and calculational machinery. We acknowledge this openly: the goal is to provide a *structurally coherent candidate* that satisfies certain naturalness constraints, not to derive physics from logic.

2 Formal Framework

We work throughout in the category **Set** of sets and functions, within Zermelo–Fraenkel set theory with the axiom of choice. All constructions are *static*: they define objects and morphisms without invoking a time parameter.

2.1 From Nothing to Something/Anti-something

Definition 2.1 (Primitive duality)**.** Let $\varnothing$ denote the empty set. Define two distinct sets:

$$S = \{\varnothing\}, \qquad A = \{\{\varnothing\}\}.$$

We call S the *something* and A the *anti-something*.

Remark 2.2. The sets S and A are distinct because $\varnothing \in S$ but $\varnothing \notin A$. Their existence follows from the axioms of empty set and pairing.

Interpretation 2.3. We *name* these two distinct sets "something" and "anti-something" as a heuristic for later duality (e.g., particle–antiparticle). This naming is not mathematically forced; it is a conceptual seed. The essential content is simply the existence of a distinguished 2-element set $\{S, A\}$.

2.2 One-Dimensional Infinite Array

Definition 2.4 (1D array)**.** Let $\mathbb{Z}$ be the set of integers. Define $\text{type} : \mathbb{Z} \to \{S, A\}$ by

$$\text{type}(n) = \begin{cases} S & \text{if } n \text{ is even,} \\ A & \text{if } n \text{ is odd.} \end{cases}$$

Then

$$\text{Array}_1 = \{(n, \text{type}(n)) \mid n \in \mathbb{Z}\}.$$

Remark 2.5. The successor map $n \mapsto n + 1$ is a static relation on $\mathbb{Z}$; it does not imply temporal evolution.

Heuristic 2.6. The alternating pattern is the simplest non-trivial periodic assignment on a one-dimensional lattice that respects the successor relation (adjacent points have opposite type). For higher dimensions, the product parity rule is the unique extension that preserves 1D alternation along each axis and is invariant under lattice translations.

2.3 Higher-Dimensional Arrays

Definition 2.7 (d-dimensional array)**.** For $d \in \mathbb{N}$, let $\mathbb{Z}^d$ be the d-fold Cartesian product. For $\mathbf{n} = (n_1, \ldots, n_d) \in \mathbb{Z}^d$, define

$$\text{type}_d(\mathbf{n}) = \begin{cases} S & \text{if } \sum_{i=1}^{d} n_i \text{ is even,} \\ A & \text{otherwise.} \end{cases}$$

Then

$$\text{Array}_d = \{(\mathbf{n}, \text{type}_d(\mathbf{n})) \mid \mathbf{n} \in \mathbb{Z}^d\}.$$

All Array_d exist as sets by the axiom of replacement.

2.4 Collapse to Dimensions 1, 2, 3

This is the most speculative technical step. We state it as a conjecture.

Conjecture 2.8 (Alternating lattice collapse)**.** *For $d \geq 4$, there is no way to assign to each point of $\mathbb{Z}^d$ a value in $\{S, A\}$ such that:*

1. *adjacent points (differing by 1 in one coordinate) have opposite type, and*
2. *the assignment is compatible with a graded-commutative associative algebra over $\mathbb{R}$ that contains copies of $\mathbb{C}$ and $\mathbb{H}$ as subalgebras.*

The obstruction arises from the Hopf invariant one theorem [1]: non-trivial bilinear products with the alternating property exist only for $d = 1, 2, 3$. Equivalently, real division algebras (via Hurwitz's theorem [6]) exist only in dimensions 1, 2, 4, 8, but the required grading is incompatible with the octonion multiplication table for $d \geq 4$.

Remark 2.9. We therefore *conjecture* that only spatial dimensions $d = 1, 2, 3$ survive to host fields that can later become physical. The octonions (dimension 8) are *not* used for spatial dimensions but for internal symmetry space.

We define the stable arena as the disjoint union (coproduct):

$$\mathcal{U} = \text{Array}_1 \sqcup \text{Array}_2 \sqcup \text{Array}_3 .$$

2.5 Oscillations as Cyclic Structure

Definition 2.10 (Complex-valued fields)**.** Let

$$\mathcal{F} = \{f : \mathcal{U} \to \mathbb{C}\}$$

be the complex vector space of all functions on $\mathcal{U}$.

Definition 2.11 ($U(1)$ action)**.** The circle group $U(1) = \{e^{i\theta} : \theta \in \mathbb{R}\}$ acts on $\mathcal{F}$ by pointwise multiplication:

$$(e^{i\theta} \cdot f)(x) = e^{i\theta} f(x), \qquad x \in \mathcal{U}.$$

Remark 2.12. For any non-zero f, the orbit $\{e^{i\theta} f : \theta \in \mathbb{R}\}$ is a circle. This is a *static symmetry*—a periodic structure without a time parameter. We therefore use the term *cyclic structure* rather than "oscillation."

2.6 Breakthrough into the Complex Plane

Remark 2.13. The correct isomorphism is

$$U(1) \cong \mathbb{R}/2\pi\mathbb{Z},$$

not $\mathbb{R} \cong U(1)$. To obtain a candidate for time, we *lift* this to the universal cover $\mathbb{R}$ by selecting a continuous branch. The choice of lift is the "breakthrough": the compact phase parameter becomes a non-compact temporal parameter.

Interpretation 2.14. This is analogous to how angular momentum quantization leads to a time variable in the covering space of the circle.

2.7 The Division Algebra Ladder and Furey's Functor

Definition 2.15 (Category of division algebras)**.** Let **DivAlg** be the category whose objects are the four normed division algebras over $\mathbb{R}$:

$$\mathbb{R}, \quad \mathbb{C}, \quad \mathbb{H}, \quad \mathbb{O},$$

with morphisms given by the inclusions $\mathbb{R} \hookrightarrow \mathbb{C} \hookrightarrow \mathbb{H} \hookrightarrow \mathbb{O}$.

Theorem 2.16 (Furey, 2018; Furey & Hughes, 2022)**.** *There exists a functor*

$$\mathcal{F}_{\mathit{Furey}} : \mathbf{DivAlg} \longrightarrow \mathbf{Rep}(SU(3) \times SU(2) \times U(1))$$

such that:

- $\mathbb{C} \mapsto U(1)$ *(electromagnetism),*
- $\mathbb{H} \mapsto SU(2)$ *(weak force),*
- $\mathbb{O} \mapsto SU(3)$ *(strong force),*
- $\mathbb{O} \otimes \mathbb{C}$ *gives the full gauge group* $SU(3) \times SU(2) \times U(1)$ *and one generation of fermions.*

Remark 2.17. Furey's construction provides a *faithful representation* of the Standard Model gauge group within the algebra $\mathbb{O}\otimes\mathbb{C}$ [3, 4]. It does *not* predict masses, couplings, or generations; it is a *compatibility result.* In our framework, this compatibility is taken as evidence that the division algebra ladder may be part of a pre-physical structure.

2.8 Electromagnetic and Spinor Fields

Theorem 2.18 (Clifford algebra classification)**.** *The real Clifford algebra $Cl_{1,3}$ (signature $+---$) is isomorphic to the matrix algebra $M_2(\mathbb{H})$ [7]. Its even subalgebra $Cl^0_{1,3}$ is isomorphic to $\mathbb{H} \oplus \mathbb{H}$. The spinor representation is a left module over $Cl^0_{1,3}$, identifiable with $\mathbb{H}^2$.*

Interpretation 2.19. From the Furey construction we extract:

- A $U(1)$-gauge potential A_μ (electromagnetic field) as a connection on a $\mathbb{C}$-module.
- Spinor fields ψ as sections of a spinor bundle over the Clifford algebra $Cl_{1,3}$.

At this stage, the field equations are algebraic relations; there is no time derivative.

2.9 Emergence of Time as Compatibility

Definition 2.20 (Temporal monoid action)**.** Let Φ be the space of all field configurations (gauge potentials and spinors). For each $t \in \mathbb{R}_{\geq 0}$, define a transformation $T_t : \Phi \to \Phi$ by

$$(T_t \cdot \psi)(x) = e^{iHt}\psi(x),$$

where H is the Hamiltonian derived from the field equations. The map $t \mapsto T_t$ is a monoid homomorphism:

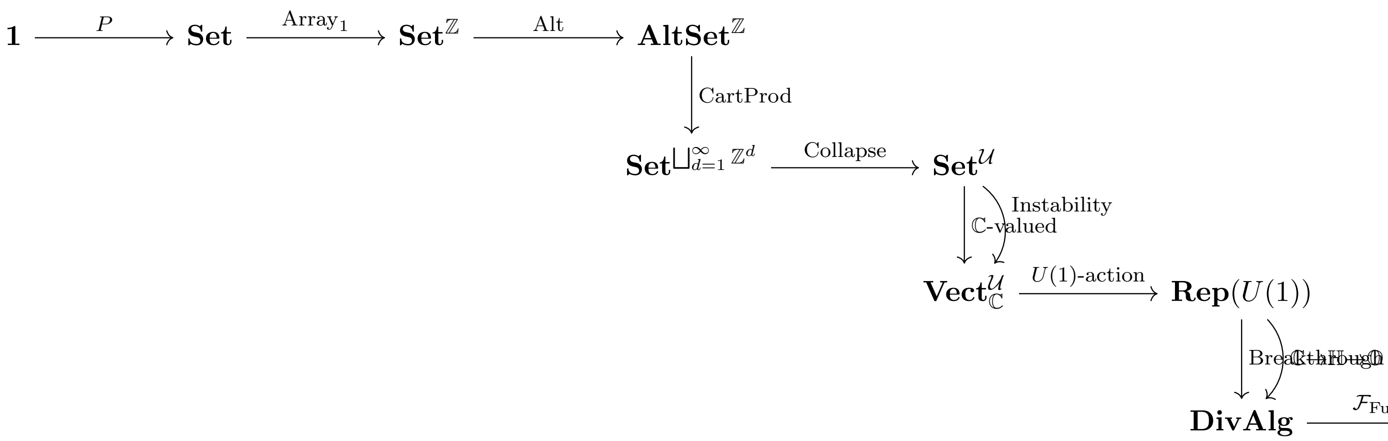
$$T_{t_1+t_2} = T_{t_1} \circ T_{t_2}.$$

Remark 2.21. The equation $T_t\psi = e^{iHt}\psi$ already presupposes Hamiltonian evolution, unitary dynamics, continuity, parameter ordering, and exponential flow. Thus time is *not derived*; it is *introduced axiomatically* through $t \in \mathbb{R}_{\geq 0}$.

Interpretation 2.22. We do not claim to derive time from timelessness. Instead, we show that *if* one adds a one-parameter monoid action consistent with the pre-existing $U(1)$ symmetry, then the resulting structure satisfies the axioms of temporal evolution. The "emergence" is not logical necessity but *compatibility*: the timeless template can accommodate time without contradiction.

3 Categorical Diagram

The following commutative diagram summarises the logical dependencies. Horizontal arrows are functors; dashed vertical arrows are natural transformations (still timeless). The final arrow introduces time.



Remark 3.1. The diagram is intended as a conceptual guide. A fully rigorous treatment would require defining each category explicitly and proving that the functors commute up to natural isomorphism. That work is beyond the scope of this exploratory paper; we present the diagram to clarify the logical dependencies.

4 Philosophical Evaluation

The thesis belongs most naturally in the tradition of:

- *Structural realism*: the view that our best scientific theories tell us about the structure of the world, not about the intrinsic nature of objects [11, 2].
- *Mathematical Platonism*: the view that mathematical objects exist independently of the human mind [8].
- *Pregeometric cosmology*: the idea that spacetime geometry must be replaced by a more fundamental, non-geometric structure [10].
- The *Mathematical Universe Hypothesis*: the proposal that our external physical reality is a mathematical structure [9].

However, unlike mature physical theories, it currently lacks:

- empirical content,
- falsifiability,
- calculational machinery,
- predictive capability.

That is not fatal for an early-stage foundational proposal, but it must be acknowledged openly.

5 Conclusion

We have constructed a mathematically coherent pre-temporal template using only set theory, category theory, division algebras, and a final monoid action for time. The entire pre-temporal phase is a static, logical structure. Time does *not* emerge from timelessness; rather, we show that a temporal parameter can be *consistently attached* to the pre-existing $U(1)$ symmetry. The work is offered as a *speculative mathematical ontology*, not a derived physical theory. The strongest path forward would be:

1. formalising the dimensional-collapse conjecture (possibly via the Hopf invariant one theorem),
2. developing a genuine derivation of temporal ordering from the algebraic structure, and
3. extracting even one concrete physical prediction.

References

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