The Momentum and Energy Consequences of Non-Geodesic Motion due to Proper Acceleration of a Three-Body System

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Abstract

We conducted an experiment to test if proper orbital angular acceleration of two rotating bodies within a three-body system due to proper linear acceleration of the system would cause the center of mass momentum of the system to increase as predicted by Euler's first law? The result would be significant because the increase in momentum would be due to the reaction of spacetime itself on a body that is forced off its natural geodesic path due to proper linear acceleration and not due to physical, external contact forces. Thirty test trials were conducted where two spheres were constrained to travel around quarter-circle tracks. This caused centrifugal reactive forces on the inner walls of the curves, forcing the system to linearly accelerate. This proper acceleration induced an increase in the orbital angular speed of the spheres by spacetime itself due to the non-geodesic motion of the spheres. After 30 test trials the

mean value of known external friction impulses acting on the three-body system accounted for only ~ 8.2 per cent (standard deviation .037 and standard error .0068) of the increase in the final momentum of the system, leaving a ~ 91.8 per cent discrepancy. This result highly suggested that spacetime itself contributed ~ 91.8 per cent of the total external impulse on the system.

1. Introduction

The experiment reported in this paper as far as we know is unique and never has been previously published in the existing physics literature. The initial interest which led to the reported experiment in this paper was due to the results of a prior experiment we performed. In this preliminary experiment a rotating body (rotator) with a vertical shaft at one end was inserted into a bearing assembly that was embedded in a platform. This platform could slide with one degree of freedom with respect to the y-axis. One Pasco accelerometer was attached to the platform and another accelerometer to the rotator. The platform accelerometer measured the proper linear acceleration of the platform; the rotator accelerometer via its internal 3-axis gyroscope measured the proper angular speed of the rotator. The platform was initially butted against a barrier so that it could not move in the negative y-direction.

A counter-clockwise rotation was imparted to the rotator when it was at rest at the 180 degrees position. When the rotator passed the 0 degrees point, this caused a proper acceleration to occur on the platform in the positive y-direction. The acceleration data of the platform and the angular velocity of the rotator from their accelerometers were transferred via Blue Tooth to a Pasco program on a computer that simultaneously plotted the graphs of the data.

The data graphed a sinusoidal wave with zero acceleration for the platform at the zero degrees point, a maximum acceleration at the 90 degrees point, followed by zero acceleration at the 180 degrees point. This implied by Newton's second law, there was a corresponding proper force acting on the platform with the same sinusoidal characteristic. Even more, the acceleration of the system was equal to the proper force divided by the total mass of the system. This implied that the source of the proper force must have been acting radially outward from the center of mass of the rotator because the rotator mass itself must be experiencing an instantaneous linear acceleration, else it could not be included in the total mass that accounts for the acceleration shown on the accelerometer attached to the platform. This conclusion is clearly illustrated when we consider both the platform and rotator is experiencing its maximum, instantaneous, proper linear acceleration when the rotator is at the 90 degrees position. When the rotator passed the 90 degrees point, the accelerometer on the rotator showed an increase in the rotator's orbital angular speed.

Since the increase in the proper orbital angular speed of the rotator was frame-invariant, we asked, could there be an increase in the center of mass momentum of a similar linearly-accelerated system with respect to our inertial lab frame due to this frame-invariant increase in orbital angular speed? This was the motivation for the experiment reported in this paper.

2. Materials and Methods

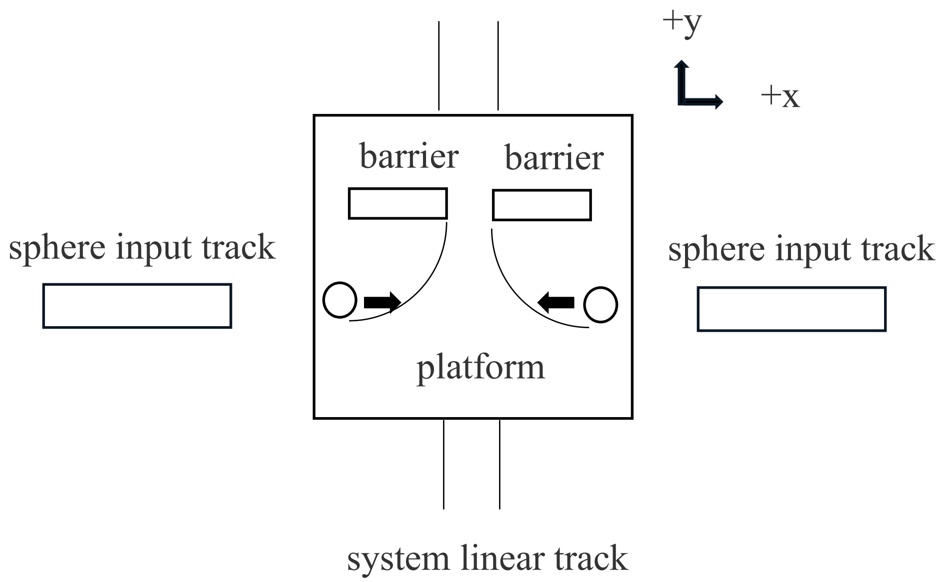

Figure 1. Top-down view of the experiment.

In this experiment we attached a platform to the top of a Pasco Smart Cart which was free to travel on top of an aluminum linear track horizontal to the earth with one degree of freedom as shown above in the top-down view of Figure 1.

Attached to the surface of the platform were two quarter-circle curves and two barriers. The platform, curves, barriers, and Pasco Smart Cart taken together constituted the mass designated as m_2. Two spheres of equal mass, designated by m_1, entered the surface of the platform simultaneously, initially rolling along the x-axis with equal velocities, designated by v_i.

Each sphere had a m_1 mass of .5330 kg ± .0001 kg. The m_2 mass was .4492 kg ± .0001 kg. A digital level with a resolution of .1 degrees was used to check if the linear track was level with respect to the horizontal x and y-axes. The spheres collided into barriers that contained neodymium magnets to prevent bounce back after the collision of the spheres. The diameter of each sphere was ~ .051 meters and the radius of each curve was ~ .042 meters.

As the spheres rounded their curves, centrifugal reactive forces acted on the curves and caused the spheres-platform-cart assembly to acquire a linear y-component of acceleration in the negative y-direction. Immediately, after rounding and exiting the curves, both spheres made inelastic collisions with the barriers in the positive y-direction, causing the system to acquire a post collision momentum in the positive y-direction. The time and velocity values of the system were communicated via Blue Tooth from the Pasco Smart Cart to a laptop computer. The velocity and time data from the Pasco Smart Cart were recorded at a 50 HZ sampling rate.

The core of this experiment was to measure the impact of friction impulses on the system to see if they accounted for all of the final momentum of the system. When the platform rolled backwards in the negative y-direction as the spheres rounded their curves, external friction impulses acted on the cart's wheels in the positive y-direction. The friction force for each trial was determined by dividing the post collision momentum of the system by the time it took the system to come to a stop. The friction impulse was calculated by multiplying the time of the back movement of the cart in the negative y-direction times this friction force.

It was not necessary to determine the rolling friction between the spheres and platform's surface and curved barriers since these were internal force pairs that would have no impact on the center of mass of the system. Air friction effects were also ignored since the speeds of the cart and spheres were small enough that any air friction drag effects would have negligible impact on the outcome.

3. Results

Table 1. Summary of experimental data.

Initial Sphere Velocity m/s	Back Time s	Friction Force N	Friction Impulse N · s	Post Collision Velocity m/s	Post Collision Momentum Kg · m/s	Friction Impulse to Momentum Ratio
0.65	0.154	0.0249	0.0038	0.047	0.071	0.05
0.65	0.116	0.0704	0.0082	0.053	0.080	0.10
0.65	0.171	0.0131	0.0022	0.025	0.038	0.06
0.66	0.148	0.0144	0.0021	0.028	0.042	0.05
0.65	0.141	0.0281	0.0040	0.033	0.050	0.08
0.65	0.192	0.0309	0.0059	0.051	0.077	0.08
0.66	0.264	0.0360	0.0095	0.034	0.052	0.18
0.65	0.137	0.0492	0.0067	0.077	0.12	0.06
0.65	0.186	0.0370	0.0069	0.044	0.067	0.10
0.65	0.132	0.0292	0.0039	0.032	0.048	0.08
0.65	0.135	0.0206	0.0028	0.035	0.053	0.05
0.66	0.155	0.0498	0.0077	0.047	0.071	0.11
0.66	0.164	0.0489	0.0080	0.06	0.091	0.088
0.66	0.2	0.0323	0.0065	0.046	0.070	0.09
0.67	0.212	0.0463	0.0098	0.063	0.095	0.10
0.65	0.136	0.0374	0.0051	0.039	0.059	0.09

0.66	0.195	0.0543	0.0106	0.033	0.050	0.21
0.67	0.195	0.0150	0.0029	0.023	0.035	0.08
0.67	0.132	0.0349	0.0046	0.052	0.08	0.06
0.65	0.186	0.0410	0.0076	0.059	0.089	0.09
0.66	0.133	0.0172	0.0023	0.036	0.055	0.04
0.66	0.19	0.0175	0.0033	0.045	0.068	0.05
0.65	0.149	0.0379	0.0056	0.037	0.06	0.10
0.65	0.198	0.0147	0.0029	0.028	0.042	0.07
0.65	0.152	0.0388	0.0059	0.042	0.06	0.09
0.66	0.127	0.0147	0.0019	0.032	0.048	0.04
0.66	0.2	0.0219	0.0044	0.028	0.04	0.10
0.66	0.19	0.0172	0.0033	0.037	0.056	0.06
0.66	0.21	0.0124	0.0026	0.027	0.041	0.06
0.73	0.137	0.0249	0.0034	0.051	0.08	0.04

Table 2. Statistical results for friction impulse to momentum ratio after 30 test trials

Mean	Std Dev	Std Error
.082	.037	.0068

The back time is the time the platform moved backwards from its initial at rest state to the time it attained its maximum value. Post collision velocity is the maximum velocity of the platform right after the collision of the two spheres with their barriers.

4.1 Discussion – The Momentum Implications of the Experiment Data

There were two spacetime inertial force effects that we suggest were responsible for the ~ 91.8 per cent impulse discrepancy measured in the experiment. The first was due to the curved barriers causing proper acceleration on the spheres to change their direction from their straight-line geodesic path. This caused an inertial reaction from spacetime itself, causing real centrifugal forces to act radially outward from the center of mass of the spheres that pushed on the inner walls of the curves. This in turn caused the proper linear acceleration of the system in the negative y-direction. This meant the instantaneous linear speed of the spheres increased in the negative y-direction, and therefore, to deviate from their natural geodesic *constant linear speed* in spacetime. This is when the second space inertial force effect occurred. That is, spacetime itself opposed the increase in linear speed of the spheres off their natural constant speed geodesic path, inducing a real inertial force reaction on the spheres in the positive y-direction.

Because this was an interaction between the spheres and spacetime itself, there were no equal and opposite forces that acted on the platform in the negative y-direction to cancel out the forces on the spheres. And Euler's first law predicts that any increase in the center of mass momentum of a body within a multi-body system due to an external impulse affects an increase in the center of mass momentum of the whole system. Thus, we suggest that it was the non-geodesic motion of the system due to its proper linear acceleration that accounted for the ~ 91.8 per cent impulse discrepancy measured in the experiment and when added to the friction impulse accounts for the total increase in the center of mass momentum of the system.

To help in acquiring a more intuitive grasp of the dynamics occurring, we introduce the sigma velocity concept in the context of two configurations of a system at this time. We start with the first configuration with a thought experiment using what we define as the side feed configuration. By invoking intuition alone, we will illustrate how the speed of a sphere rounding a curve impacts the impulse acting on a curve which directly impacts the momentum of the system.

We imagine two spheres, one initially moving in the positive x-direction and the other initially moving in the negative x-direction both with initial velocity v_1. Both spheres enter quarter-circle curves simultaneously which are attached to a platform that moves with one degree of freedom along they y-axis. The sphere on the left enters a curve that constrains the sphere to rotate in a counter-clockwise sense. The sphere on the right enters a curve that constrains the sphere to rotate in a clockwise sense. After rounding the curves, both spheres exit their curves in the positive y-direction with the same initial speed v_1 *with respect to the frame of the platform*. The mass of each sphere is denoted by m_1. The mass of the platform, including the curves, is denoted by m_2. As the spheres round their curves, centrifugal reactive forces act on the inner walls of each curve, imparting a y-component impulse in the negative y-direction on the platform, denoted by $J_{constant\ v}$.

We now imagine the same thought experiment but this time both spheres have an initial velocity less than the initial velocity in the previous case. However, as the spheres round their curves, their speeds increase so that both exit their curves with the same final speed v_1 *with respect to the frame of the platform* as in the first case. The y-component impulse in the negative y-direction for this case is denoted by $J_{varying\ v}$.

Our intuition alone should tell us the magnitude of the impulse in the second *case must be less than the impulse in the first case or* $J_{varying\ v} < J_{constant\ v}$. Without rigorous proof, we state it is possible that this diminished impulse can be attributed to an equivalent, reduced, but constant sphere speed, whose speed is denoted by σv_1, where $0 \leq \sigma \leq 1$. We define this reduced, constant velocity as the sigma velocity.

We now proceed with another thought experiment using a second configuration we define as the straight feed configuration. Everything is the same as in the first thought experiment but this time the spheres enter from the top with initial velocity v_1 with respect to our laboratory frame, moving in the negative y-direction, round their curves, and then both exit along the x-axis with a final velocity v_1 with respect to the platform frame. One sphere exits in the negative x-direction. The other sphere exits in the positive x-direction. Again, as the spheres round their curves, centrifugal reactive forces act on the inner walls of the curves, imparting a net y-component impulse on the platform in the negative y-direction, denoted by $J_{constant\ v}$.

We now imagine the same thought experiment but this time both spheres have an initial velocity v_1 with respect to our laboratory frame as before but after the spheres round their curves, their speeds decrease to less than v_1 along the x-axis with respect to the platform frame. The y-component impulse in the negative y-direction for this case is denoted by $J_{varying\ v}$.

Again, our intuition tells us the magnitude of the impulse in the second *case must be less than the impulse in the first case or* $J_{varying\ v} < J_{constant\ v}$. And as before this impulse can be attributed to an equivalent, reduced, but constant sphere speed, whose speed is denoted by σv_1, where $0 \leq \sigma \leq 1$.

Now we proceed to test this sigma velocity concept using the side feed configuration but before we can test the physical reliability of this concept, we must first derive a general impulse equation. Afterwards, we will compare the sigma velocity prediction to see if it aligns with the calculated results using this impulse equation.

We begin its derivation by examining the centrifugal reactive contact force acting on the inside of the curved barriers. This force has a magnitude of $nkm_1r\omega^2$. In this expression n is the number of spheres, $k = \frac{m_2}{nm_1+m_2}$, m_1 is the mass of each sphere, m_2 is the mass of the platform, r is the radius of curvature of the path of the center of mass of each sphere, and ω is the orbital angular velocity of each sphere. Note that $m_1r\omega^2$ is the real centrifugal radially outward force acting through the center of mass of the spheres. We will assume the orbital angular speed ω of each sphere remains constant as they round their curves in this derivation and we ignore friction between the sphere and the curves and platform surface since these internal force pairs will have no impact on the center of mass of the system.

Finally, since we are only interested in the y-component of the impulse J, we include a sine expression factor in front of the force equation (Recall the platform is constrained to move with one degree of freedom along the y-axis.) The final form of the y-component of the reactive centrifugal force equation is given by:

$$f_y = \ sin(\omega t)\, nkm_1r\omega^2 \qquad (1)$$

The k arises because we assume the spheres and m_2 are rigid bodies, and therefore, have the same instantaneous y-component of linear acceleration.

We integrate the force with respect to time to determine the impulse J on m_2 with respect to the y-axis. This integral is expressed by:

$$J_y = \int_{\frac{\theta_i}{\omega}}^{\frac{\theta_f}{\omega}} nsin(\omega t)km_1r\omega^2\, dt \qquad (2)$$

Recognizing the indefinite integral of $\int \sin(\omega t)\,dt$ is equal to $-\frac{1}{\omega}\cos(\omega t) + C$ and applying this to Eq. (2) and after moving the constants out of the expression, we evaluate the above integral, giving us the general impulse equation:

$$J_y = n\,k\,m_1 r\omega\left[cos(\theta_i) - \cos(\theta_f)\right] \tag{3}$$

This equation expresses the y-component of the impulse that acts on the platform by the action of the centrifugal reactive force on the inner walls of the curves as the spheres round their curves from θ_i to θ_f.

Alternatively, we can use the tangential velocity of the sphere in the equation by setting $r\omega = v_i$, which gives us the expression:

$$J_y = n\,k\,m_1 v_i\left[cos(\theta_i) - \cos(\theta_f)\right] \tag{3a}$$

For the special case where the initial angle is 90 degrees or 270 degrees and where there is a 90 degrees angular displacement in either the clockwise or counter-clockwise direction, the above equation reduces to:

$$J_y = n\,k\,m_1 v_i \tag{3b}$$

The sign of the calculated impulse on the platform depends on the initial angle of the spheres.

At this point we can now combine the above reduced impulse Eq. (3b) with the concept of the sigma velocity using the side feed configuration to derive an expression that predicts the final momentum of a system depending on the value of σ. We begin by stating three fundamental expressions. The first is the reduced sigma impulse Eq. (3b), modified by using the sigma velocity as:

$$J_{\sigma y} = n\,k\,m_1 \sigma v_i \quad (4)$$

We use the sigma impulse Eq. (4) because we are analyzing the case where the spheres enter the curves at a given speed and exits the curves at an increased speed equal to v_i with respect to the frame of the platform.

Next, we use the impulse-momentum theorem to derive the final velocity of the platform after the spheres exit their curves, moving in the positive y-direction. Noting that the initial velocity of the platform is zero, gives us:

$$J_{\sigma y} = m_2 v_{2f} \quad (5)$$

Substituting Eq. (4) for $J_{\sigma y}$, then solving for v_{2f}, we arrive at:

$$v_{2f} = \frac{n m_1 \sigma v_1}{n m_1 + m_2} \quad (6)$$

Finally, we use Galilean velocity transformation to determine the final velocities of the spheres with respect to our laboratory frame by substituting Eq. (6) for the final velocity of the platform, which gives us:

$$v_{1lab} = v_1 - \frac{n m_1 \sigma v_1}{n m_1 + m_2} \quad (7)$$

We emphasize again that v_1 is the final velocity of each *sphere with respect to the platform frame.*

Now, we are ready to determine the total momentum of the spheres-platform system after the spheres have rounded their curves by using Eq. (6) for the platform velocity and using the expressions on the right side of Eq. (7) for the velocity of each sphere with respect to our laboratory frame, then multiplying the velocity of the platform by its mass and the velocities of the spheres by their masses, giving us the momentum expression below:

$$p_{total} = nm_1(v_1 - \frac{nm_1\sigma v_1}{nm_1+m_2}) - \frac{m_2nm_1\sigma v_1}{nm_1+m_2} \tag{8}$$

Next, we perform a series of algebraic steps. First, we multiply the first expression on the right by nm_1, giving us:

$$p_{total} = nm_1v_1 - \frac{n^2\, m_1^2\sigma v_1}{nm_1+m_2} - \frac{m_2nm_1\sigma v_1}{nm_1+m_2} \tag{9}$$

Continuing, we now convert the first expression on the right to an expression with a common denominator, transforming Eq. (9) into a one fraction expression given by:

$$p_{total} = \frac{n^2m_1^2v_1+nm_2m_1v_1-n^2\, m_1^2\sigma v_1-m_2nm_1\sigma v_1}{nm_1+m_2} \tag{10}$$

Next, we rearrange and group the numerator by common factors, which gives us:

$$p_{total} = \frac{(n^2m_1^2v_1-n^2\, m_1^2\sigma v_1) +(nm_1m_2v_1-nm_1m_2\sigma v_1)}{nm_1+m_2} \tag{11}$$

Finally, we factor out the common terms, giving us the final expression for the total momentum of the system after the spheres exit their curves, expressed by:

$$p_{total} = \frac{n^2m_1^2(v_1-\sigma v_1) + nm_1m_2(v_1-\sigma v_1)}{nm_1+m_2} \tag{12}$$

Recall the omega velocity σv_1 term was derived on the intuitive assumption that if the initial velocity of each sphere was less than the final velocity v_1 with respect to the platform frame, then the impulse on the platform must be less than if each sphere's velocity was constant as they rounded their curves. The above expression, using the sigma velocity σv_1, mathematically shows the consequence of this on the final momentum of the system.

If the spheres velocities remain constant, then $\sigma = 1$, and Eq. (12) reduces to zero momentum with respect to the y-axis. If instead the velocity varies, then $\sigma < 1$, and the above expression reduces to a positive momentum value with respect to the y-axis.

We also derive a similar expression at this time using the sigma velocity concept and the impulse equation for a straight feed configuration. We begin with the initial momentum of the system which equals the combined initial momentum of each sphere in the negative y-direction, given as:

$$p_i = nm_1 v_1 \tag{13}$$

Next, we use the same sigma impulse Eq. (4) as before, expressed as:

$$J_{\sigma y} = n\, k\, m_1 \sigma v_i \tag{14}$$

From this we derive as previously the final velocity of the platform after the spheres round their curves, given by:

$$v_{2f} = \frac{nm_1 \sigma v_1}{nm_1 + m_2} \tag{15}$$

Finally, we set up a final total momentum expression for the system in the negative y-direction after the spheres round their curves by multiplying the final velocity of the platform times the total mass of the system as shown below:

$$p_t = (nm_1 + m_2)\frac{(nm_1 \sigma v_1)}{(nm_1 + m_2)} \tag{16}$$

Which simplifies to:

$$p_t = nm_1 \sigma v_1 \tag{17}$$

We clearly see the above expression using the sigma velocity makes intuitive sense. If the velocity of each sphere remains constant with its initial v_1 as they round their curves, then $\sigma = 1$, and the final momentum of the system is nm_1v_1 which equals the initial momentum of the system of nm_1v_1. If the final velocity of each sphere decreases from its initial velocity v_1 as they round their curves, then $\sigma < 1$, and the final momentum of the system is equal to $nm_1\sigma v_1$ which is less than the initial momentum of the system which was equal to nm_1v_1 .

We now use a concrete example using the side feed configuration and the general impulse Eq. (3a), to see if it aligns with our intuition and the sigma velocity predictions. We apply the general impulse equation in two hypothetical cases that focuses on the impact the speed of a sphere has on the center of mass momentum of a system. We begin with a constant sphere velocity case to show that the momentum of the center of mass of the system is conserved. To simplify, we use one sphere. The masses of m_1 and m_2 each is 1 kg. The platform is constrained to move with one degree of freedom along the y-axis.

The initial speed of the sphere is 2.7 m/s and the angular counter-clockwise displacement is from 270 degrees to 0 degrees. We use Eq. (3b) which for the initial values given, yields an impulse equal to -1.35 N s, and therefore, a final momentum of the platform equal to -1.35 kg · m/s. Dividing this value by the mass of the platform gives us the final velocity of the platform equal to -1.35 m/s after the sphere exits the curve in the positive y-direction.

The final momentum of the sphere with respect to our lab frame in the positive y-direction is 1.35 kg · m/s. This is determined by applying Galilean transformation by adding the final velocity of m_2 equal to -1.35 m/s to the velocity of m_1 equal to 2.7 m/s with respect to the m_2 frame, which gives a velocity of the sphere equal to 1.35 m/s with respect to our laboratory frame. Multiplying this velocity by the mass of the sphere gives us the final momentum with respect to our lab frame.

Hence, the final center of mass momentum of the system is equal to the final momentum of the platform which is -1.35 kg · m/s plus the final momentum of the sphere which is 1.35 kg · m/s. This equals 0 kg · m/s which was the initial momentum of the system along the y-axis, and hence, momentum of the sphere-platform system is conserved as expected. This complies with the prediction of Eq. (12) for when $\sigma = 1$.

We now show the case where a hypothetical external impulse acts on the sphere which increases the orbital angular speed of the sphere as it rounds the curve. We keep everything the same as above except the speed of the sphere increases by .1 m/s for every 5 degrees angular displacement from 270 degrees to 0 degrees. This time we apply the general impulse Eq. (3a), for each angular increment, then sum the incremental impulses to get the total impulse on the platform. Table 3 below summarizes the results.

Table 3. Increasing Orbital Angular Speed Result

Sphere Initial Speed M/S	Sphere Final Speed M/S	Initial Angle Degrees	Final Angle Degrees	Incremental Impulse N · S
1	1	270	275	-0.04358
1.1	1.1	275	280	-0.04757
1.2	1.2	280	285	-0.0511
1.3	1.3	285	290	-0.05408

1.4	1.4	290	295	-0.05642
1.5	1.5	295	300	-0.05804
1.6	1.6	300	305	-0.05886
1.7	1.7	305	310	-0.05883
1.8	1.8	310	315	-0.05789
1.9	1.9	315	320	-0.05599
2	2	320	325	-0.05311
2.1	2.1	325	330	-0.04922
2.2	2.2	330	335	-0.04431
2.3	2.3	335	340	-0.03839
2.4	2.4	340	345	-0.03148
2.5	2.5	345	350	-0.0236
2.6	2.6	350	355	-0.0148
2.7	2.7	355	0	-0.00514

The sum of the incremental impulses from the last column acting on the platform in the negative y-direction equals ~ -0.7464 N s which equals a platform momentum value of ~ -.746 kg · m/s. Dividing by the mass of the platform, gives us the final velocity of the platform which equals ~ -.746 m/s. We sum the final velocity of the sphere with respect to the platform frame which equals 2.7 m/s with the final velocity of the platform which equals -.746 m/s. This gives us the velocity of the sphere with respect to our lab frame equal to 1.95 m/s.

Our final step involves adding the momentum of the sphere to the momentum of the platform by multiplying both the sphere velocity and the platform velocity by their masses which gives us a positive momentum value equal to ~1.20 kg · m/s. This also aligns with our intuitive prediction found in Eq. (12) for the case where $\sigma < 1$ which predicts a positive final momentum value.

4.2 Discussion – The Energy Implications of the Experiment Data

We now address a final point in this discussion dealing with the energy implications of the experimental results. If a system acquires a net momentum by the action of an external force multiplied by the time the force acts, it follows that the system acquires kinetic energy by the same force multiplied by the displacement of the force as predicted by the work-energy theorem. Since kinetic energy and change in kinetic energy is a frame dependent phenomenon, an interesting energy dynamic occurs when we add an initial velocity to the system before the spheres around their curves which impacts the final kinetic energy of the system. We show a general result of this mathematically as follows.

We begin with the impulse-momentum theorem using the external space impulse J_{space} that acts on the spheres as they round their quarter-circle barriers when the system linearly accelerates in the negative y-direction. The total mass of the system is given by m. Thus, we have the following expression, where $v_{f\ system}$ is the final velocity of the system and $v_{i\ system}$ is the initial velocity of the system:

$$J_{space} = mv_{f\ system} - mv_{i\ system} \qquad (18)$$

Solving for $v_{f\ system}$, we obtain:

$$v_{f\ system} = \frac{(J_{space} + mv_{i\ system})}{m} \qquad (19)$$

Substituting the above for $v_{f\ system}$ in the expression for ΔKE, gives us :

$$\Delta KE = \frac{1}{2}\, m[\frac{(J_{space}+mv_{i\,system})}{m}]^2 - \frac{1}{2}mv_{i\,system}^2 \tag{20}$$

Simplifying the above, we arrive at:

$$\Delta KE = \frac{J_{space}^2}{2m} + J_{space}\; v_{i\,system} \tag{21}$$

The second term on the right of the above equation shows that the change in kinetic energy of a system while keeping the impulse J_{space} and mass the same is impacted by the initial velocity of the system. This can be easily confirmed by noting that with respect to a boosted inertial reference frame the initial velocity of the system will always impact the change in kinetic energy of the system. This is an observable, physical fact. And by the first postulate of special relativity this phenomenon observed by a boosted inertial frame must also be possible with respect to an inertial laboratory frame.

An even more interesting consequence has to do with the energy that is responsible for the initial kinetic energy of the spheres. For example, suppose the spheres are propelled by compressed springs that are attached to the platform. The elastic potential energy of the springs (epe) is a frame invariant scalar. When the springs expand, they do work on the spheres by displacing the spheres along the x-axis, imparting an initial kinetic energy to the springs before they enter the curves. By the same reasoning above, there always exists a boosted inertial reference frame such that with respect to this frame there is an initial velocity of the system before the spheres are propelled where the change in kinetic energy of the system after the spheres collide with their barriers will be *greater than the total frame invariant elastic potential energy (epe) of the springs.* And again, by the first postulate of special relativity this phenomenon observed by a boosted inertial frame must also be possible with respect to an inertial laboratory frame.

This final result highly implies that the change in kinetic energy of the whole system after the collision of the spheres cannot be coming from the velocity independent scalar value of the elastic potential energy of the springs. This begs the question as to where is the source of the change in kinetic energy of the system since this increase in kinetic energy of the system can be greater than the elastic potential energy of the springs that initially imparts the spheres along the x-axis? This is an open question and will not be addressed in this paper. Hopefully, there will be further research on this topic by others who are interested in solving this energy question.

4.3 Limitations of the Experiment

It was crucial in this experiment that the two spheres entered the platform with the same speed and that they entered the curves simultaneously. This was necessary in order to cancel out any forces acting on the platform along the x-axis. We were only interested in forces that acted along the y-axis. If the two spheres did not enter the platform simultaneously at the same speed, the cart would twist completely off the track and for these extreme cases the test data run was rejected.

Because the experiment was built of low-density PLA fiber, the system was susceptible to high frequency, transient, mechanical vibrations and noise which clouded the interpretation of the final post collision velocity of the spheres and time increment readings and often required a conservative interpretation of the data. This interpretation always filtered the results of the experiment to maximize the impact of the friction impulses on the system so as not to underestimate the contribution of the friction impulses. In future tests of the experiment, it is recommended that higher density materials be used to dampen the mechanical vibrations while holding the ratio of the total mass of the spheres to the mass of the platform to be at least 2:1.

Initial velocity of the spheres should be as high as possible as the acceleration of the system is proportional to this speed.

In this experiment we assumed the mean value of rolling friction was constant for both directions of the platform's movement. That is, we assumed the coefficient of rolling friction on the average was constant and was not significantly altered by the speed or momentary acceleration of the platform or by the shifting masses of the spheres across the surface of the platform that redistributed the load over the four wheels. We also recognize that the wheels' flanges rolled within the grooves of the aluminum track on one side of the track. If the cart twisted slightly due to imperfect simultaneous motion of the two spheres, this may have increased the friction between the wheels and the track during the back motion of the cart.

Data in the experiment suggested that inertial impulses were the main contributing impulses to the final center of mass momentum of the system. It is recommended that further experiments be conducted in a near zero friction environment on an air track or within a plane in a zero-g dive or by a CubeSat in space where these friction issues would be totally eliminated.

5. Conclusion

Thus, we have answered the question posed in the introduction of this paper. When there is a frame invariant increase in the proper orbital angular speed of a body, then the center of mass momentum of a system does increase with respect to our inertial laboratory frame. We conclude it was the proper linear acceleration of the system, forcing the spheres to move contrary to their natural geodesic paths and the consequential real inertial force reaction of spacetime itself on the spheres which was the major contributor to the final center of mass momentum of the system.

Appendix A contains a set of equations that can be used for further testing and confirmation of the effect in near frictionless environments.

Appendix A

This appendix presents a set of five equations for predicting the motions of a more complicated system. These equations apply in idealized flat Minkowski spacetime where external friction impulses or other external impulses are nearly eliminated such as in the micro-gravity environment of inter-stellar space. In this more elaborate system, the total mass m is the sum of the masses of the spheres, the platform, and all attachments to the platform.

We need to define what a cycle N means. In this system two spheres are propelled by the release of two compressed springs, afterwards the spheres enter and round quarter-circle curves, then have elastic collisions with elastic barriers in the positive y-direction after existing the curves, bounce back, and then round the curves again in the opposite direction, returning to their initial positions, and finally resting against the two recompressed springs. This is defined as one cycle N. We denote a spacetime impulse $J_{\text{spacetime}}$, defined by measuring the change in momentum of the system after one cycle.

The compressed springs which propel the spheres store elastic potential energy (epe) which is a scalar or invariant quantity that does not depend on the velocity of the system. Denoting the total number of springs as n and the energy per compressed spring as epe, the cumulative energy applied to the spheres after N cycles is:

$$E_{\text{cumulative}} = N\, n\, epe \qquad (A1)$$

The cumulative final velocity of the system, with external impulse $J_{\text{spacetime}}$, after N cycles equates to:

$$v_{f\ \text{system}} = \frac{N\, J_{\text{spacetime}}}{m} \qquad (A2)$$

Substituting $v_{f\ \text{system}}$ into the formula for kinetic energy yields:

$$KE_{\text{final}} = \frac{(N\, J_{\text{spacetime}})^2}{2m} \qquad (A3)$$

The cycle N at which KE_{final} equals $E_{\text{cumulative}}$ is determined by setting Eq. (A1) equal to Eq. (A3), then solving for N, giving us:

$$N = \frac{2\, m\, epe}{J_{spacetime}^2} \qquad \text{(A4)}$$

It is useful to graph both Eq. (A1) and Eq. (A3) to visually see their relationship with N plotted on the x-axis and energy plotted on the y-axis.

The value for m can easily be predetermined by direct measurement. The value for epe can be calculated knowing the spring constant and the degree of compression. The spacetime impulse $J_{\text{spacetime}}$ is determined experimentally by activating a single cycle when the system is at rest and measuring the system's final velocity v_f, giving us the spacetime impulse as:

$$J_{\text{spacetime}} = m\, v_f \qquad \text{(A5)}$$

Declaration of AI Use

During the preparation of this work, the author used generative AI to assist with spelling, formatting, and citation protocols. The author reviewed and edited the content and takes full responsibility for the publication.

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The author identifies as an independent researcher and received no institutional support for this study. All experimental trials were conducted privately.

Data Availability

Pasco data relating to the experiments is available for review upon request.