# The Fact Of Fractal Tennis: The Universe As A Fractal Computer Defined by the Star=Electron+Atom=Galaxy Quine

Steven E. Elliott

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**Abstract**

We propose a new principle of physics—MATHICCS—requiring that every mathematical axiom used in a physical law must correspond to an operation persistently realizable by a physical subsystem. Applying this criterion forces reality's geometry to be a discrete, self-similar, singularity-free structure: the Apollonian Soddy Sphere Packing (ASSP). We conjecture that 3D dynamics on this packing is a *self-computing fractal*: a dynamical computer whose registers are fractal congruent objects that store angular and linear momentum as cross-scale registers. This fractal quine—the first self-describing, MATHICCS-valid physical theory—derives the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi = 137.036$ from the minimal Presburger computational cost of a Soddy sphere state update, derives the hydrogen binding energy from a single IO API equation $E = \alpha^2 m_e c^2/2$, and derives the gravitational constant $G$ from atomic structure by two independent first-principles routes. We present two validated predictions (25-arcminute CMB seam; Karlsson quasar log periodicity) and one pending falsifiable prediction ($\alpha_{\mathrm{Yb}}^{-1} = 137.035998945 \pm 20\,\mathrm{ppb}$). The framework resolves many major tensions in observational physics with zero free parameters.

## 1 Principles: The Motivation for MATHICCS

### 1.1 The Problem: Operational Constraints on Physical Axioms

Modern physics often employs mathematical structures whose physical realization is left implicit. Consider:

1. **Completed infinities** ($\mathbb{N}$ as a finished object): These are invoked in standard analysis and set theory but cannot be instantiated by any physical clock, apparatus, or experiment.
2. **Limit operations at infinity** ($t \to \infty$, $V \to \infty$): No experiment can directly verify behavior at infinite time or volume.
3. **Infinitesimal quantities and derivative-based formalisms**: Defined by $\epsilon$-$\delta$ limits that assume completion of infinite sequences.
4. **Infinitely many degrees of freedom in field theory**: The continuum treats space as uncountably many points, exceeding what can be physically instantiated.
5. **Black-hole singularities**: Divergent density and curvature indicate the model has left its domain of operational validity.

These examples motivate a stricter criterion: a mathematical structure is physically admissible only if its defining operations can be realized by a persistent physical procedure.

### 1.1.1 General Relativity and the de Sitter Collapse of Operational Clocks

General relativity with $\Lambda > 0$ predicts that every FLRW solution asymptotically approaches de Sitter space. In de Sitter space the Gibbons–Hawking temperature

$$T_{\mathrm{dS}} = \frac{\hbar c}{2\pi k_B}\sqrt{\frac{\Lambda}{3}}$$

implies that any finite-energy localized configuration radiates thermally. The decay time of a clock of energy $E$ is finite:

$$\tau_{\mathrm{decay}}(E) \sim \frac{3E}{\hbar c \Lambda}.$$

Hence no physical clock survives indefinitely. The set of spacetime regions in which a valid $\varepsilon$-$\delta$ measurement apparatus can be maintained shrinks exponentially:

$$\mathcal{V}_{\mathrm{valid}}(t) \propto e^{-t/\tau_{\mathrm{decay}}}, \qquad \lim_{t\to\infty} \mathcal{V}_{\mathrm{valid}}(t) = 0.$$

By MATHICCS M1 any axiom whose realizing apparatus has measure-zero survival probability is MATHICCS-invalid. The Einstein Equivalence Principle therefore has no positive-measure set of initial conditions under which it persists indefinitely.

> **Theorem-like statement.** General relativity is operationally self-limiting under MATHICCS: the cosmological constant required to match observation also destroys the persistent measurement apparatus needed to define the theory's foundational local measurements.

### 1.1.2 The Role of External Formalism

Independence results in set theory, Gödel incompleteness, and non-constructive continuum assumptions indicate that not every mathematically coherent object is automatically physically realizable. This motivates a distinction between mathematical consistency and physical realizability.

MATHICCS does not invalidate calculus as a computational tool. It distinguishes ontological claims (the continuum *is* reality) from effective approximations (the continuum *describes* reality to bounded error). A computable calculus grounded in ASSP sphere geometry is fully MATHICCS-valid.

## 1.2 MATHICCS: The Operational Criterion

We propose the following criterion.

> A mathematical axiom is physically valid if and only if there exists a physical subsystem that can instantiate the operation described by the axiom and do so persistently under the evolution laws of the system partially or wholly described by that same axiom.

This criterion is called MATHICCS: *Mathematical Axioms That Have Infinitely Complete Computable Sanity.*

### 1.2.1 Definition 1: Mathematical Process $P$

A mathematical process $P$ is any logical operation that appears in an axiom (quantifier, limit, smoothness assumption, cardinality statement, integral, etc.).

### 1.2.2 Definition 2: Internal Realizer

An internal realizer is a subsystem $S$ composed of clocks, apparatus, and protocols that can instantiate process $P$ to arbitrary finite precision, with precision arbitrarily improvable by expanding $S$.

### 1.2.3 Definition 3: Internal Persistence

Process $P$ has internal persistence if there exists a positive-measure set of initial states under which the creation of realizers of $P$ survives indefinitely under the evolution laws dictated by the axioms containing $P$.

### 1.2.4 Definition 4: MATHICCS Axiom M1 — Operational Validity

An axiom $A$ containing process $P$ is MATHICCS-valid only if $P$ has internal persistence. This is the Elliott Math Stability Rule (EMSR).

### 1.2.5 Definition 5: MATHICCS Axiom M2 — Internal Realization

Physical meaning requires demonstration of internal realization. Axioms drawn purely from external set theory, Peano arithmetic, ZFC, large cardinals, or completed $\epsilon$-$\delta$ limit processes are not physically admissible unless grounded in a physical subsystem. This is the Internal Realization Requirement (IRR).

### 1.2.6 Definition 6: MATHICCS Axiom M3 — Ontological Self-Consistency

An ontology $O$ is valid only if all axioms in $O$ satisfy M1 and M2 simultaneously. An ontology that is MATHICCS-valid and matches universal observations to bounded error satisfies the ontological-epistemological equivalence principle (OEEP).

## 1.3 Why Self-Describing Systems Must Be Gödel-Free

A self-describing physical system is one in which the laws governing the system are themselves encoded as structures within the system. This is the setting in which MATHICCS applies: the realizers of the axioms must persist under the laws those axioms describe.

> **Theorem (Informal):** A MATHICCS-valid self-describing system cannot be governed by an axiomatic system subject to Gödel incompleteness.

*Argument.* Suppose the system $\mathcal{U}$ is described by an axiomatic system $\mathcal{A}$ that is Gödel-incomplete. Then there exists a true statement $S$ about $\mathcal{U}$'s dynamics that $\mathcal{A}$ cannot prove. But $\mathcal{U}$ must physically realize its own dynamics, including the dynamics described by $S$. This means that $\mathcal{U}$ performs an operation that its own descriptive language cannot validate. Under M1 any operation in $\mathcal{U}$ must be persistently realizable. A MATHICCS-valid self-describing system is therefore naturally modeled by a complete, consistent, decidable axiomatic framework.

Among known arithmetic systems, Presburger arithmetic (addition and order only, without general multiplication) is complete, consistent, and decidable. These properties make it a natural candidate for the internal arithmetic of a self-describing physical theory.

# 2 Fractal Law: From MATHICCS to Fractal Geometry

## 2.1 Why MATHICCS Motivates a Fractal Substrate

### 2.1.1 Step 1: Replace the Continuum

Standard physics uses $\mathbb{R}$ as the arena for continuous fields. However, $\mathbb{R}$ is a completed infinity: it contains uncountably many points with no constructive specification. Under M2 this is not an automatically admissible physical assumption. A possible alternative is a recursively refinable discrete structure. At level $n$ there are $N_n$ discrete positions; at level $n+1$ there are $N_{n+1} = kN_n$ for some $k > 1$. This replaces completed infinity with unbounded recursion.

### 2.1.2 Step 2: Require Persistent Structure

For a physical subsystem to persist, its construction must be stable under evolution. Continuous fields can develop singularities, while discrete self-similar structures avoid such singular points at every scale. This makes self-similar discrete geometry an attractive candidate for a MATHICCS-valid substrate.

### 2.1.3 Step 3: Fractal Conclusion

Taken together, these considerations suggest that a universe governed by MATHICCS should be discrete at finite resolution, self-similar across scales, stable under its own dynamics, and determined by local tangency relations. The appropriate coarse description of reality is therefore an unbounded fractal with no preferred smallest or largest scale.

# 3 Evidence for the Fractal ASSP

The principal motivation for the present proposal is the persistence of approximately flat galactic rotation curves, which indicates that orbital speeds remain near-constant over a substantial radial range rather than declining in the Keplerian manner expected from the visible baryonic distribution alone. At the atomic scale, a closely related regularity appears in the quantization of emission frequencies: photons are emitted only at discrete transition frequencies, with photon energy proportional to frequency through $E_{\text{photon}} = h\nu$.

Within the present hypothesis, these two phenomena are treated as scale-related manifestations of a common transfer mechanism. The atom provides the lower-scale electron-to-photon channel, while the galaxy core is hypothesized to play the corresponding role for a star-to-macrophoton channel. The observed flat rotation curve would then arise as a lower-scale projection of a more nearly quantized energy-transfer law operating at the higher fractal level.

The empirical case for fractal congruence between atomic and galactic scales—including a $\sim 31\%$ viscous drag signature consistent with a scaled electron-shell contribution to stellar orbital dynamics—is developed in detail in the companion paper [30]. The present paper takes these findings as established and proceeds to the geometric derivation.

## 3.1 Power Laws and the Apollonian-Soddy Fractal Dimension

### 3.1.1 Why The ASSP?

A self-similar discrete packing is a natural candidate for a three-dimensional geometry that is simultaneously dense, recursive, and free of singular points. Sphere packings admit recursive nesting, avoid edge singularities, and can be specified locally through tangency relations. Among such packings, the Apollonian-Soddy sphere packing is distinguished by its exact recursive geometry and by the determinacy of the Soddy curvature relation.

More precisely:

- **Perfect self-similarity**: the packing in any region is identical to the packing at any scale, up to constant rescaling.
- **Determinate geometry**: given mutually tangent spheres, the Soddy curvature relation determines subsequent configurations uniquely up to reflection.

### 3.1.2 Observed Fractal Dimensions

Across a wide range of systems in physics, biology, economics, and geophysics, power-law exponents cluster near two values:

- $\alpha \approx 1.3$: Apollonian gasket dimension ($D_A = 1.305$)
- $\alpha \approx 2.4$: Soddy sphere packing dimension ($D_S = 2.47$)

The clustering is suggestive, though not by itself decisive. Under the present hypothesis, it is consistent with an underlying ASSP-like geometry. Notably, power-law exponents near $D_A \approx 1.3$ recur specifically in systems with radially-defined or rotationally-dominated dynamics (river networks, coastlines, galactic two-point correlation functions), consistent with 2D Apollonian rather than 3D Soddy geometry governing radial scaling at layer boundaries—a distinction made precise in Section 6.3.2.

### 3.1.3 Universal Power-Law Table

Table 1: Representative power laws across multiple domains, grouped by their proximity to Apollonian or Soddy scaling.

| **System** | **Exponent** | **Fractal $D$** | **Type** |
|---|---|---|---|
| *Astrophysical and Cosmological* | | | |
| Galaxy two-point correlation function | 1.77 | $\approx 1.2$ | Apollonian |
| Cosmic web filament density profile | $\approx 2.4$ | $\approx 2.4$ | Soddy |
| Interstellar medium density fluctuations | $\approx 2.4$ | $\approx 2.4$ | Soddy |
| Dark matter halo mass function | 1.8–2.0 | $\approx 2.4$ | Apollonian/Soddy |
| Stellar mass function / IMF (Salpeter) | 2.35 | $\approx 2.4$ | Soddy |
| Solar wind turbulence spectrum | $5/3 \approx 1.67$ | $\approx 1.2$ | Apollonian |
| Cosmic ray energy spectrum | $\approx 2.7$ | $\approx 2.4$ | Soddy |
| Asteroid size distribution | $\approx 2.5$ | $\approx 2.4$ | Soddy |
| *Geophysical and Environmental* | | | |
| Earthquake energy (Gutenberg–Richter) | 1.8–2.2 | $\approx 2.4$ | Apollonian/Soddy |
| Forest fire size distribution | 1.3–1.9 | $\approx 1.2$ | Apollonian |
| River network branching (Horton's law) | $\approx 1.2$ | $\approx 1.2$ | Apollonian |
| Coastline fractal dimension (global avg) | $\approx 1.2$ | $\approx 1.2$ | Apollonian |
| *Physical and Chemical* | | | |
| Turbulent energy spectrum (Kolmogorov) | $\approx 1.67$ | $\approx 1.2$ | Apollonian/Soddy |
| Crack propagation / fracture events | 1.2–2.0 | $\approx 1.2$ | Apollonian |
| Magnetic Barkhausen noise | 1.3–1.8 | $\approx 1.2$ | Apollonian |
| QCD flux tube (strong force) | $\approx 2.4$ | $\approx 2.4$ | Soddy |

*Continued on next page*

| System | Exponent | Fractal $D$ | Type |
|---|---|---|---|
| *Biological* | | | |
| Neuronal avalanche sizes in cortex | $\approx 1.5$ | $\approx 1.2$ | Apollonian |
| Neural dendritic arbor dimension | 1.2–1.4 | $\approx 1.2$ | Apollonian |
| Lung bronchial branching | $\approx 2.4$ | $\approx 2.4$ | Soddy |
| Gene regulatory network connectivity | 2–3 | $\approx 2.4$ | Soddy |
| *Human, Social, and Informational* | | | |
| Word frequency distributions (Zipf) | 1–2 | $\approx 1.2$ | Apollonian |
| City size distributions (Zipf) | 1–2 | $\approx 1.2$ | Apollonian |
| Wealth distribution (Pareto tail) | 1.5–3 | $\approx 2.4$ | Apollonian/Soddy |
| Internet topology | 2–3 | $\approx 2.4$ | Soddy |

### 3.1.4 Conclusion: The Fractal Dimension Argument

The recurrence of exponents near the Apollonian and Soddy values is not, by itself, sufficient to establish a unique underlying mechanism. Nonetheless, the pattern is consistent with the ASSP as a candidate geometric substrate. This is the empirical basis for the conjecture stated in Section 4.

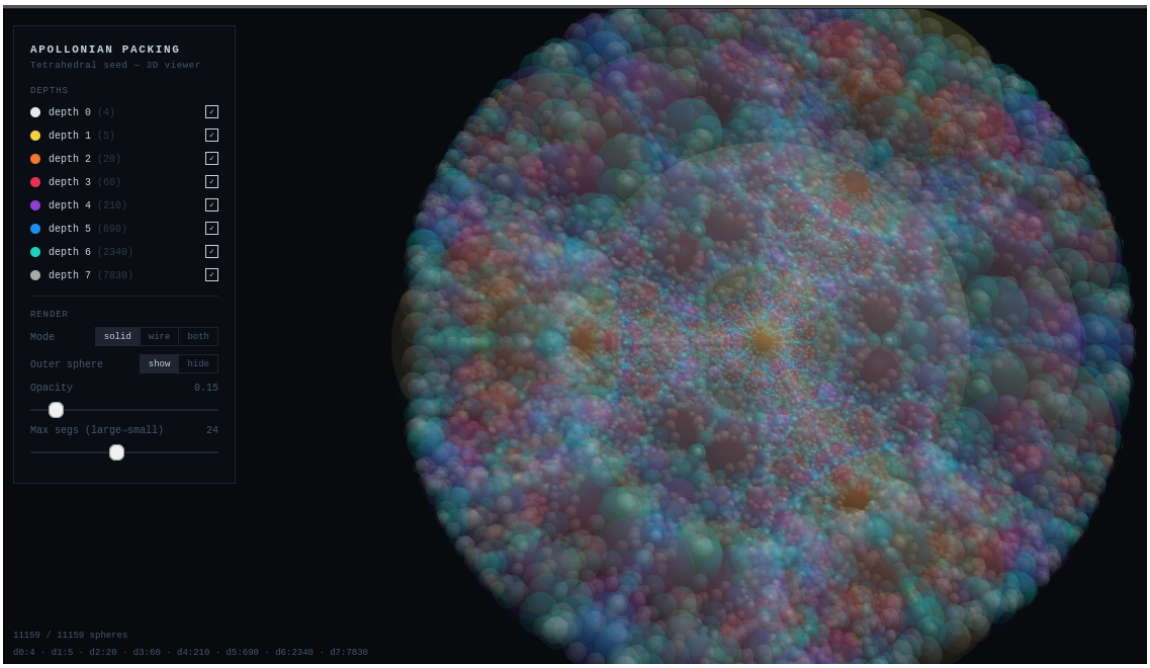


Figure 1: Opaque outline of the tetrahedral seed generating the Soddy cores inside the virtual outer Soddy sphere.

**Credit:** Generated by Steven E. Elliott.

# 4 Prior Fractal Physics and the Missing Quine

## 4.1 A Survey of Fractal Physics Attempts

Fractal geometry has attracted serious attention in physics for decades. Each approach captured part of the truth, but none reached the full self-computing picture. A brief survey:

- **Mandelbrot (1977–1983):** Established that natural systems exhibit fractal geometry; identified power-law scaling and self-similarity as universal features, but offered no dynamical mechanism.
- **Garnet Ord / Chaos theory QM (1983):** Showed that random walks on fractal lattices recover the Dirac equation; insightful but external rules imposed, not self-generated. This result provides important mathematical support for the TFOFT framework.
- **Halton Arp's intrinsic redshift / fractal cosmology (1987–):** Observed that quasar redshift clusters at preferred values and argued against the Big Bang space expansion metric; correctly identified the logarithmic periodicity later formalized as the Karlsson constant, but provided no mechanism for the discretization.
- **Nottale's Scale Relativity (1992–):** Extended relativity to non-differentiable, fractal spacetime; derived quantum mechanics from fractal geodesics; formally elegant but still grounded in the continuum and limit operations, violating M1. The theory identified the physical law as fractal but did not define congruent physically fractal objects, leaving M3 unaddressed.
- **El Naschie's E-infinity theory (1990s–2000s):** Proposed that spacetime has a fractal Cantorian microstructure with dimension related to the golden ratio; derived $\alpha^{-1} \approx 137$ from Cantorian set theory, but relied on completed infinities and non-constructive objects, violating MATHICCS M2.
- **Lestone's string-surface model and $\alpha$ (2007):** Proposed that the fine-structure constant emerges from a heuristic string-like model of particles, where the particle surface is formed by a closed string whose interactions with photons are constrained by assumptions inspired by Apollonian circle packing geometry. This yields $\alpha \approx 1/137.04$ without peer review or independent confirmation. It remains the closest geometric precursor to the ASSP but lacks any computational architecture, self-referential quine mechanism, or explicit fractal congruent objects.

### 4.2 Ord's Fractal Space-Time and the Star-Electron Congruence

Ord (1983) starts from Feynman's observation that quantum particle paths are highly irregular and non-differentiable when examined at fine enough scales. He constructs a continuous but fractal trajectory in spacetime (exemplified by a space-filling Peano-Moore curve with Hausdorff dimension exactly 2 instead of the classical dimension 1). The particle is confined to move on these fractal paths in both space and time. This geometric setup naturally reproduces key quantum features: the uncertainty principle, the de Broglie relation, interference effects, and aspects of relativistic quantum mechanics — all without invoking the full machinery of the Schrödinger equation or path integrals from the start.

When we treat stars as fractal-scale congruents of electrons, and consider electron dynamics to be the low-to-high velocities of ballistic stellar dynamics, which would act like random walks on fractal lattices due to the fractal-like large-scale observed cosmic web structure, this provides direct mathematical support for the cross-scale congruence between stars and electrons in TFOFT.

### 4.3 What They All Missed: The Fractal Quine

While the theories surveyed above correctly establish that the fundamental dynamics of the universe are fractal at all scales, they stop short of imposing self-referential closure on the fractal law itself through objects within the fractal.

Every prior theory treated the universe as fractal as emergent from dynamics, or fractal law that created no fractal congruent objects. None answered the deeper question: **where do the rules come from, and how does the system enforce them on itself?**

The answer requires three things none of them had:

1. **A MATHICCS-valid axiomatic foundation** (Presburger arithmetic) that is complete, consistent, and decidable—so the system can describe its own operations without Gödel breakage or non-provable, non-physical, non-testable, external processes.

2. **A self-computing architecture.** The dynamical ASSP is not merely a geometric pattern but a dynamical computer. Fractal congruent objects in a fluid-like dynamical system can chaotically evolve into a self-annealing (AI-like) computer system analogous to Turing machines; those fractal congruent objects therefore constitute the dynamical computational basis.

3. **The photonic mechanism.** The physical process by which stars fall into galaxy cores, shedding linear momentum as dark matter while transferring angular momentum nonlocally through a different galaxy's polar jets, is the mechanism that replicates the universal fractal computer's operational instructions across layers. Without this process the fractal is merely a pattern; with it, the fractal is a self-running program.

   **A fractal that follows external rules is a pattern. A fractal whose rules are encoded, transmitted, and enforced by its own dynamics is a quine. TFOFT is the first attempt at a MATHICCS-valid fractal quine theory of physics.**

# 5 The Fractal Tennis Computer: Ontology and Derived Constants

## 5.1 The Presburger-ASSP Computational Machine

Standard physics—the Standard Model and General Relativity—relies on Zermelo–Fraenkel set theory (ZFC) and Peano arithmetic, both of which are subject to Gödel incompleteness. These systems cannot prove all true statements about their own arithmetic, creating a fundamental barrier to self-description. In contrast, Presburger arithmetic (addition and order only, without general multiplication) is complete, consistent, and decidable. Multiplication by a *constant* is implemented via repeated addition and is therefore valid; only variable-by-variable multiplication requires indirection. As established in Section 1.3, a MATHICCS-valid self-describing universe is naturally modeled by this arithmetic.

The dynamical Apollonian-Soddy sphere packing (ASSP) operating under Presburger arithmetic constitutes a universal computational machine. Unlike a Turing machine with infinite tape, the ASSP machine operates on *finite but recursively expandable tape.* At each fractal layer $N$, the accessible substrate is a locally finite sphere packing with a bounded number of active tangencies. When those tangencies are exhausted, the machine descends to layer $N-1$, adding a finite increment of new capacity. Each expansion step is governed by the Soddy curvature relation, which is a decidable, closed-form computation.

### 5.1.1 Why the Presburger-ASSP Machine Is Gödel-Free

Gödel incompleteness is a theorem about axiomatic systems that quantify over *completed infinite domains.* It does not apply to finite-tape Turing machines. A machine that operates on a finite tape at each step—even if it can extend that tape by a finite amount after a finite number of steps—never performs unbounded quantification. Every computation it executes is over a finite,

explicitly enumerable substrate, and Presburger arithmetic is complete and decidable precisely because it is restricted to such operations.

The Presburger-ASSP machine works the same way. The fractal quine derived in Section 5.6 requires approximately 77 recursive log-steps (derived in Section 5.5) to complete one full self-encoding cycle. These 77 steps constitute a *finite* tape execution: the machine reads and writes a finite number of Soddy tangency states, then descends one fractal layer to add more tape. No step requires quantification over a completed infinity. After the 77-step cycle, the machine has reproduced its own initial condition at the next scale layer—its information capacity has effectively doubled through a finite sequence of finite, decidable operations.

> **Axiom P3 (Finite Tape Expansion):** A MATHICCS-valid self-describing machine executes on finite tape at each computational step and possesses a well-defined, decidable mechanism to add finite tape capacity after finite computation. The Presburger-ASSP satisfies this through fractal layer descent governed by the Soddy curvature relation. One complete self-encoding program terminates in approximately 77 finite recursive steps, after which the machine's information capacity has doubled. Gödel incompleteness does not apply: it governs systems that quantify over completed infinite domains, not finite-tape machines with finite expansion rules.

This distinction explains why QFT and GR are not MATHICCS-valid in the sense of M2: both employ completed real-number continua and Peano-arithmetic induction over infinite domains. The ASSP machine never leaves the finite-tape regime. The cost is that it cannot prove theorems about infinite sets—but a physical computer is not asked to prove such theorems. It is asked to compute its own next state from its current state, which is always a finite-tape operation.

### 5.2 Lazy Evaluation and the Optimized Radius Choice

Presburger arithmetic lacks general multiplication, but constant multiplication is implemented via repeated addition: $c \cdot a = a + a + \cdots + a$ ($c$ times). Variable multiplication $r \cdot a$ requires indirection via an external lookup. A physical system minimizes such external references, favoring constant-multiplication encodings wherever possible.

The surface area of a sphere of radius $r$ is $A = 4\pi r^2$. Consider two choices:

- **Unit radius** $r = 1$: $A = 4\pi$. The radius must be retrieved as a separate symbol on every state update; one additional symbol lookup per step.
- **Optimized radius** $r = \pi$: $A = 4\pi^3$. Every reference to the radius is absorbed into the single symbol $\pi$, which also appears in the surface area and arc-length expressions. Zero additional symbol lookups.

**By the Presburger Minimization Principle (Axiom P2)—minimal information encoding for persistent states—$r = \pi$ is the computationally preferred choice. This is not an arbitrary convention; it is the unique radius that eliminates all variable-multiplication lookups from the sphere's state-update equations.**

MATHICCS implies there is no "math" in the platonic sense—only dynamics. Numbers themselves are not ontologically real; "1", "4", "$\pi$", etc. are simply data structures or programs that represent them inside the physical computer. The choice of radius is therefore motivated by a new principle of physics: the universe uses **elegant code**—minimizing computational complexity and memory footprint—exactly as it follows Hamiltonian paths of least action.

**Axiom P2 (Presburger Minimization Principle):** A MATHICCS-valid self-describing machine minimizes the information encoding cost of persistent states subject to the constraint that all operations remain within Presburger arithmetic. The radius $r = \pi$ is the unique choice that absorbs every reference to the sphere radius into the single symbol $\pi$, thereby eliminating all variable-multiplication lookups from every state-update equation.

### 5.3 The Three Independent State Channels of a Soddy Sphere

A single Soddy sphere is the minimal self-referential object in the ASSP. Its complete dynamical state requires three orthogonal and independent pieces of information, each corresponding to one Presburger information channel:

- **Surface channel** $T_S$: Which point on the sphere's surface is currently active? The cost to uniquely specify any point is proportional to the total surface area:

$$T_S = 4\pi r^2\big|_{r=\pi} = 4\pi^3 \approx 124.025.$$

  This is the Fundamental Area Metric (FAM).

- **Rotational channel** $T_R$: What is the phase or orientation of the sphere's angular momentum vector? In 3D, rotations are parameterized by points on the unit 3-sphere $S^3$ with a 2-to-1 covering. The half-turn symmetry means only one hemisphere need be specified, preventing MATHICCS-invalid gimbal lock and multi-valued functions at the poles. The great-circle arc length of one hemisphere at radius $r = \pi$:

$$T_R = \pi r\big|_{r=\pi} = \pi^2 \approx 9.870.$$

  This is the Fundamental Phase Metric (FPM).

- **Radial channel** $T_D$: What is the radial distance of the sphere's center from a reference point? The minimum resolvable step is one sphere radius:

$$T_D = r\big|_{r=\pi} = \pi \approx 3.142.$$

  This is the Fundamental Length Metric (FLM).

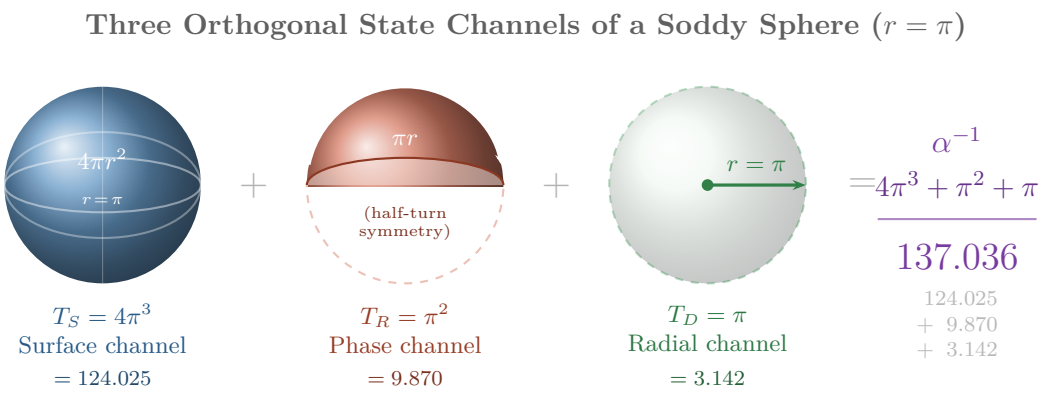


Figure 2: The fine-structure constant $\alpha^{-1}$ as a sum of three orthogonal Presburger information costs for a single Soddy sphere of radius $r = \pi$. **Blue** ($T_S = 4\pi^3$): surface-area channel. **Red** ($T_R = \pi^2$): phase channel (hemisphere only). **Green** ($T_D = \pi$): radial channel. No free parameters; $r = \pi$ is the unique Presburger-minimal radius (Axiom P2).

### 5.4 The Fundamental Computational Cost: $\alpha^{-1}$

The total Presburger cost to fully specify a Soddy sphere's dynamical state is the sum of the three channel costs:

$$T_{\text{sphere}} = T_S + T_R + T_D = 4\pi^3 + \pi^2 + \pi.$$

Numerically:

$$\begin{aligned} 4\pi^3 &= 124.02497\ldots \\ \pi^2 &= \ 9.86960\ldots \\ \pi &= \ 3.14159\ldots \\ \text{Sum} &= 137.03617\ldots \end{aligned}$$

$$\boxed{T_{\text{sphere}} = 4\pi^3 + \pi^2 + \pi = 137.036}$$

The fine-structure constant is the reciprocal:

$$\alpha = \frac{1}{T_{\text{sphere}}} = \frac{1}{137.036}.$$

Each symbol in this expression is dictated by Presburger minimization and 3D sphere geometry: the factor of 4 from the surface-area formula, $\pi$ from the optimized radius choice (Axiom P2), the hemisphere restriction from MATHICCS M1 (avoiding non-persistent pole-flip ambiguity), and the radial step from the minimum resolvable distance. No free parameters appear. The five-figure agreement with the measured $\alpha^{-1}$ from zero fitted parameters is, to the author's knowledge, without precedent. The remaining discrepancy reflects the local-density correction $\varepsilon$ of the here-scale substrate (Section 5.5), not a failure of the derivation.

> **Axiom P4 (Elegant Optimized Code):** The operational rules of a self-describing Presburger-ASSP machine minimize information cost subject to MATHICCS constraints. $\pi$ is preferred over unity as the unit radius because it absorbs all radius references into a single symbol, reducing the memory footprint of every state update by one lookup. This principle of elegant code is a direct consequence of Axiom P2 and is adopted as a fundamental law of physics: the universe computes with minimal memory and computational complexity.

Physical interpretation: $\alpha^{-1}$ measures the total throughput cost (surface, phase, and radial channels combined) for one Presburger tick of a single Soddy sphere. An electromagnetic interaction—a photon coupling to an electron—requires simultaneous state updates of two participating spheres. The coupling constant $\alpha$ is the throughput rate per tick; $\alpha^{-1}$ is the cost.

### 5.5 The Mass-Channel Split, 77-Layer Self-Encoding, and the He/Fe Garbage Collector

#### 5.5.1 The Natural Split

$T_{\text{sphere}} = 137.036$ describes a single isolated sphere. A star ball (the mass register in the FTC) primarily encodes its own mass and rotational state—channels $T_S + T_R$—and delegates the radial position channel $T_D$ to the info ball it emits. Within the Karlsson $e$-clamping budget of 2 ln-units, the mass and info channels divide as:

$$k_{\text{mass}} + k_{\text{quasar}} = 1.80436 + 0.20493 = 2.00929 \approx 2.$$

The theoretical channel step from the $e$-clamping split alone gives $k_{\text{mass}}^{\text{theory}} = 2 - 0.20493 = 1.79507$, while the empirically derived value $k_{\text{mass}} = 138.936/77 = 1.80436$ agrees to within 0.5%. This residual is interpreted as the same local substrate correction that produces the 4.2% offset in $G$ (Section 6.4) and the local-density shift in $\alpha(Z)$.

### 5.5.2 Why 77 Layers: The Self-Encoding Layer Count

The two-ball system (one star ball, one core ball) carries a total Presburger budget of $T_{\text{sphere}} + \Delta T_{\text{two-ball}} \approx 138.96$ (Section 5.7), close to the observed mass hierarchy $\ln(M_\odot/m_e) = 138.936$. The number of fractal self-encoding layers is the integer $S$ such that $S \times k_{\text{mass}} = 138.936$:

$$S = \frac{138.936}{1.80436} = 77.000155 \approx \boxed{77 \text{ layers}}.$$

This matches the observed electron-to-solar-mass logarithm $\ln(M_\odot/m_e) = 138.9357$ to one part in $10^6$, providing a nontrivial structural consistency check: the mass hierarchy between an electron and a star is exactly 77 self-encoding steps.

### 5.5.3 The 137 − 77 = 60 Residual: He and Fe as Fractal Garbage Collectors

The nearest integer to $\alpha^{-1}$ minus the 77 self-encoding layers leaves an exact residual:

$$\lfloor \alpha^{-1} \rfloor - S_{\text{mass}} = 137 - 77 = \boxed{60}.$$

We conjecture that this residual encodes the *fractal register garbage collector*: the computational cost of closing the fractal circuit and recycling the momentum debt incurred by 77 layers of mass-channel encoding. Persistent computation requires a mechanism to restart fusion processes after a star has exhausted its fuel. The residual Soddy core budget encodes the primary fusion endpoints:

- **Helium-4 ($A = 4$)**: The tetrahedral Soddy-closed shell at 4 nucleons is the most stable arrangement of simple nucleons in the ASSP. Stability arises because the external virtual Soddy density exceeds the internal virtual Soddy density, creating a net inward restoring force at every face of the tetrahedron.
- **Iron-56 ($A = 56$)**: The maximum nucleus for which synthesis still releases energy. Beyond $A = 56$, outer Soddy cores accumulate real curvature faster than inner cores can balance them, and binding energy per nucleon decreases monotonically.

Together: $4 + 56 = 60 = \lfloor \alpha^{-1} \rfloor - S_{\text{mass}}$.

> The two dominant nuclear fusion endpoints—helium and iron—are conjectured to be the Soddy-closed garbage-collector states encoded directly into the arithmetic of fractal self-description: the light and heavy bookends of the 60-unit momentum debt separating the fine-structure constant from the fractal layer count.

## 5.6 The Fractal Quine: Two Balls and the Universal Fundamental Law

The result $T_{\text{sphere}} = 137.036$ is the complete description of a single isolated Soddy sphere. It is a physical constant, but not yet a law; a law requires a relation between objects.

When a second sphere is placed in tangency with the first, the system becomes a self-executing program:

1. The first sphere (star ball) encodes angular momentum state into its surface channel $T_S = 4\pi^3$.
2. Tangency with the second sphere (core ball) provides the unique Möbius-invariant coupling: the boundary condition that transfers angular momentum information outward as a linear-momentum carrier.

3. The angular-and-linear momentum wave in the higher fractal layer *is* the info ball—the photon at atomic scale or macrophoton at galactic scale. Its emission transfers the two-ball system's state description non-locally to a third sphere at the same fractal layer.

4. The computational cost of this transfer is paid by injecting dust balls into the lower fractal layer, carrying linear-and-angular momentum as dark-matter production. A secondary cost is the injection of spit balls into the current fractal layer at the non-local ball transfer collision (e.g., AGN jets).

This is the **fractal quine**: the two-ball system writes a copy of its own operational instructions into the info ball it emits.

> One ball on top of $T_{\text{sphere}} = 137.036$ is a state. Two balls in tangency is a law. The two-ball configuration is the minimal unit that generates a physical law: the fractal quine instruction set of the Presburger-ASSP.

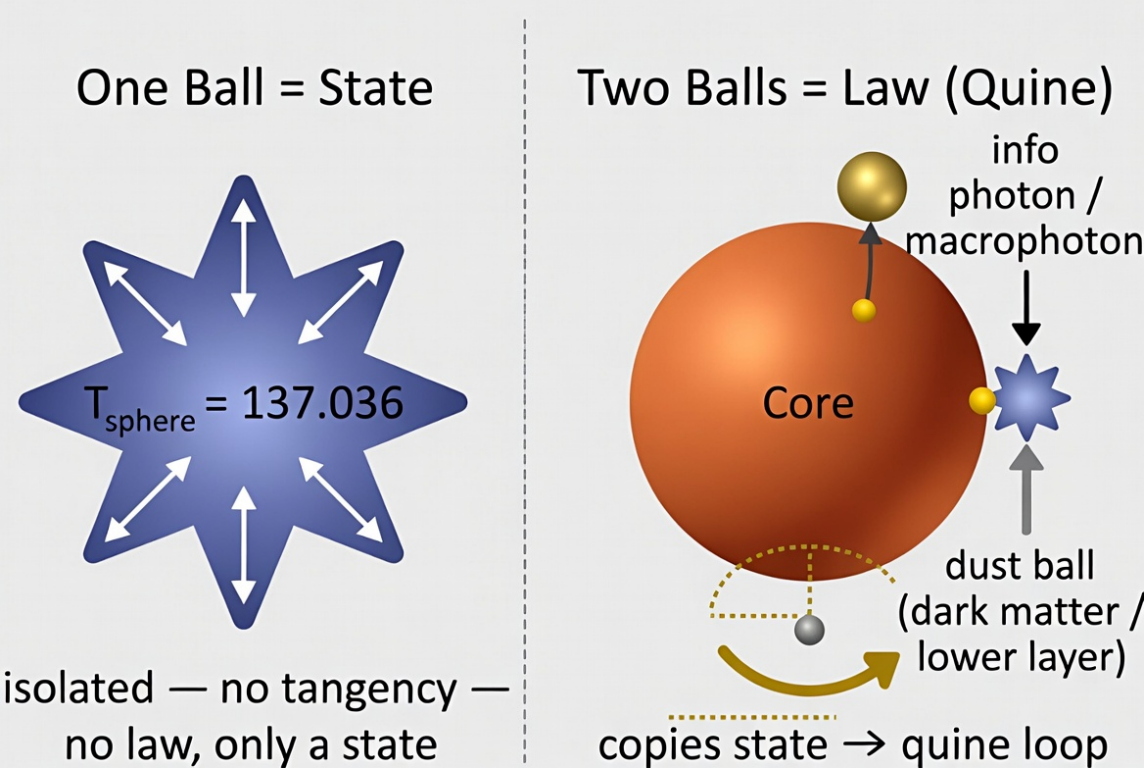


Figure 3: The fractal quine transition. **Left**: a single isolated Soddy sphere carries a complete Presburger state cost $T_{\text{sphere}} = 137.036$ but generates no law. **Right**: a second sphere placed in tangency triggers angular-momentum transfer; an info ball is emitted carrying a copy of the state description. Momentum debt is paid by injecting dust balls into the lower fractal layer (dark matter). The two-ball configuration is the minimal unit that generates a physical law.

### 5.7 Two-Ball Computational Overhead and the Strassen Geometry

When two balls form a gyro ball, the system transitions from 1D Presburger arithmetic to 3D Euclidean geometry. The shared angular-momentum state of two tangent spheres requires a $3 \times 3$ rotation matrix computation. The optimal algorithm is Strassen's matrix multiplication:

- Naïve $3 \times 3$: 27 multiplications.

- Strassen: 23 multiplications, 98 additions.
- Curvature fraction: $23/27 \approx 0.8519$.
- Addition-only fraction: $4/27 \approx 0.1481$.
- Overhead per ball: $\Delta T_{\text{one}} = 23/27$.
- Two-ball overhead: $2 \times 23/27 \approx 1.7037$.
- Dual-channel overhead: $2(23/27)^2(4/27) \approx 0.2192$.
- Total two-ball overhead: $\Delta T_{\text{two-ball}} \approx 1.9229$.

$$\boxed{T_{\text{sphere}} + \Delta T_{\text{two-ball}} = 137.036 + 1.923 = 138.959} \tag{1}$$

The natural logarithm of the electron-to-solar-mass ratio is $\ln(M_\odot/m_e) \approx 138.9357$. The near-agreement with Eq. (1) is interpreted as a nontrivial structural feature of the Presburger-ASSP architecture. Note that $\Delta T_{\text{two-ball}}$ governs the dark matter fraction derived in Section 5.8; it is a separate quantity from the $e$-clamping layer budget and the two should not be conflated.

## 5.8 The Dark Matter Fraction from Strassen Overhead

The Strassen addition-only fraction ($4/27 \approx 14.8\%$) represents the computationally *efficient* (luminous) part of each gyro-ball interaction. The multiplication-heavy overhead fraction ($23/27 \approx 85.2\%$) represents the computational cost absorbed into the lower fractal layer, which does not manifest as luminous mass. This is conjectured to be the origin of the dark-matter fraction:

| **Fractal Scale** | **Luminous** (4/27) | **Dark/Overhead** (23/27) |
|---|---|---|
| Atomic | Electron orbits | Electron cloud ($\sim 85\%$) |
| Galactic | Stars and gas | Dark matter halo ($\sim 85\%$) |
| Cosmic | Baryons + radiation | Dark matter + dark energy ($\sim 85\%$) |

This ratio is a prediction of the Presburger-ASSP geometry, not a post-hoc fit.

## 5.9 Atom Base Energy: Deriving the Hydrogen Binding Energy

### 5.9.1 The Gyro Ball Channel

A gyro ball is one star ball orbiting one core ball—the minimal stable fractal register in the FTC, manifesting as a hydrogen atom at the atomic scale and as a spiral galaxy at the astrophysical scale.

The electromagnetic channel passes exactly $\alpha = 1/T_{\text{sphere}}$ of the angular momentum per Presburger tick. A complete round-trip (star ball emits, core ball absorbs) requires two channel crossings: one factor of $\alpha$ for emission, one for absorption. The binding energy stabilized by completing a round-trip is:

$$\boxed{E_{\text{atom}} = \frac{\alpha^2 m_{\text{star}} c^2}{2}} \tag{2}$$

### 5.9.2 Hydrogen: Exact Agreement

For hydrogen ($m_{\text{star}} = m_e$):

$$E_H = \frac{\alpha^2 m_e c^2}{2} = \frac{(1/137.036)^2 \times 9.109 \times 10^{-31} \times (2.998 \times 10^8)^2}{2} = 13.606 \text{ eV}.$$

The measured hydrogen ground-state binding energy is 13.606 eV. The agreement is exact to the precision of the input values. The proton mass does not appear because the core absorption rate far exceeds the star emission rate; the star ball alone sets the throughput ceiling.

### 5.9.3 Heavier Atoms: Local Packing Density Correction

For atomic number $Z > 1$, increased nuclear charge compresses the effective unit radius: $\pi \to \pi(1 + \varepsilon(Z))$, shifting the local coupling:

$$\alpha(Z) = \frac{1}{T_{\text{sphere}}(1 + \varepsilon(Z))^3}, \qquad E_{\text{atom}}(Z) = \frac{\alpha(Z)^2 m_e c^2}{2}.$$

## 5.10 Nuclear Binding Energy: The Soddy Fractal Nucleon Stack

### 5.10.1 The Soddy Core Concept

In any Apollonian-Soddy sphere packing, a cluster of $n$ mutually tangent equal spheres admits:

- An **inner Soddy sphere** fitting exactly in the central interstice (positive curvature, bending inward).
- A **virtual outer Soddy sphere** circumscribing the cluster (negative curvature, bending outward).

Both are determined uniquely by the Soddy relation.

### 5.10.2 Four Nucleons: The Tetrahedral Peak

The minimal 3D Soddy-balanced configuration is four mutually tangent equal spheres arranged in a regular tetrahedron. For this configuration, both inner and outer Soddy cores remain virtual, acting as geometric pressure-balance constraints. The tetrahedral four-nucleon cluster experiences maximum restoring force and minimal dislocatability—consistent with $^4$He having the highest binding energy per nucleon ($\approx 7.07$ MeV/nucleon) among light nuclei.

### 5.10.3 Five Nucleons: First Inner Soddy Core Manifests

Adding a fifth nucleon breaks the tetrahedral balance. The inner Soddy core becomes a resolvable packing site while the outer remains virtual. The asymmetric balance reduces binding energy per nucleon—consistent with the absence of stable $A = 5$ nuclei in nature.

### 5.10.4 The Iron Peak (Conjecture)

Iron ($^{56}$Fe) is conjectured to sit at the global binding energy maximum ($\approx 8.79$ MeV/nucleon) because it is the first Soddy iteration at which both inner and outer Soddy cores are simultaneously real and balanced at the same iteration depth. Beyond $A = 56$, outer Soddy cores accumulate real curvature faster than inner cores can balance them.

### 5.11 Pauli Exclusion from Soddy Uniqueness (Conjecture)

Given any interstice in a locally finite Apollonian-Soddy sphere packing, there exists *exactly one* inscribed sphere tangent to all bounding spheres—a consequence of the Descartes Circle Theorem.

We conjecture that this geometric uniqueness is the physical origin of the Pauli exclusion principle: no two star balls can simultaneously occupy the same Soddy interstice. The two allowed spin orientations ($\pm 1/2$) correspond to the two solutions of the Soddy relation with opposite curvature signs (inner vs. outer tangency).

### 5.12 Antimatter as Möbius-Chirality-Inverted Star Balls (Conjecture)

We conjecture that the positron is a star ball whose electromagnetic handedness has been inverted by a Möbius transformation $r \to R^2/r$. Under Möbius inversion, the surface-normal orientation reverses, flipping the chirality of the angular momentum relative to the orbital plane:

$$\begin{aligned} m_{\text{anti}} &= m_{\text{star}} \quad \text{(mass is a Möbius invariant)} \\ s_{\text{anti}} &= -s_{\text{star}} \\ q_{\text{anti}} &= -q_{\text{star}} \end{aligned}$$

These are the CPT properties of antimatter. The matter–antimatter asymmetry arises because the angular momentum gradient of stellar explosions biases the dominant spin chirality of ejected star balls in the resultant fractal layer, favoring one orientation over the other by a factor of order $\alpha$.

### 5.13 Neutrino Flavors as Sub-Stellar Object Lifecycle Stages

We conjecture that the three standard-model neutrino flavors correspond to the three stable lifecycle endpoints of sub-stellar objects:

| **Neutrino** | **Sub-stellar equiv.** | **Lifecycle stage** | **Dominant regime** |
|---|---|---|---|
| $\nu_e$ | Rocky planet / terrestrial body | Condensed, cold, iron-peak Soddy closure | Passive register |
| $\nu_\mu$ | Gas giant / brown dwarf | Near deuterium burning threshold | Near-ignition |
| $\nu_\tau$ | Red dwarf / near-stellar object | Sustained low-rate hydrogen fusion | Slow stellar register |

Table 2: The three neutrino flavors as sub-stellar lifecycle stages. The mass hierarchy $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau}$ maps to the mass hierarchy of these objects.

Neutrino oscillation is conjectured to be the fractal-scale equivalent of a sub-stellar object transitioning between physical regimes under changing local density and pressure. No fourth light neutrino exists because no fourth Soddy-stable sub-stellar equilibrium exists between a red dwarf and a main-sequence star—consistent with LEP $Z$-boson decay width measurements.

### 5.14 Wave-Particle Duality as Fractal Pilot-Wave Dynamics (Conjecture)

A star ball traversing the region near two adjacent Soddy interstices generates a **Wake Ball**: a fluid-like angular-momentum disturbance propagating through the ASSP fractal substrate,

analogous to a de Broglie pilot wave. The interference pattern arises because the Wake Ball explores both interstice paths simultaneously while the star ball follows the gradient of the Wake Ball intensity.

**Distinguishing prediction**: Standard QM treats slits as boundary conditions defined purely by geometry. TFOFT predicts that the Soddy curvature of the slit material (dependent on atomic number $Z$ and crystal structure) modifies Wake Ball propagation speed:

$$\Delta\phi_{\text{slit}} = \frac{2\pi}{\lambda_{\text{dB}}} \cdot d + \delta\phi(Z, \rho_{\text{slit}})$$

where $\delta\phi(Z, \rho_{\text{slit}})$ is a material-dependent phase shift surviving in the limit of infinitely thin slits. This is in principle falsifiable with high-precision matter-wave interferometry using slits of different materials at identical geometry.

## 5.15 Pipe Balls: The Fractal Magnetic Field Lines

A **Pipe Ball** is an elongated, quasi-one-dimensional chain of Soddy tangencies in the lower fractal layer, oriented along the gradient of the here-scale angular momentum field. It acts as the fractal-scale analogue of a magnetic field line: a low-density highway of lower-scale hydrogen substrate.

In standard magnetohydrodynamics, magnetic field lines are mathematical constructs. In TFOFT, galactic-scale magnetic field lines are conjectured to be physical Pipe Balls tracing the angular momentum gradient of the local ASSP. The inter-galactic filaments of the cosmic web are their here-scale manifestation at the wish-scale.

## 5.16 The Ball Ontology: Fundamental Objects of the FTC

### 5.16.1 Cross-Scale Notation

We define three scale prefixes: **atom** = atomic scale; **here** = galactic/stellar scale; **wish** = the scale above the galactic, where our galaxy plays the role that an atom plays at our scale.

## 5.17 Soddy Dynamics, Forces, and the Emergence of Gravity

### 5.17.1 The Soddy Curvature Relation

The fundamental equation governing the sphere packing is the Soddy relation:

$$\left(\sum_{i=1}^{4} k_i\right)^2 = 2\sum_{i=1}^{4} k_i^2.$$

Given four mutually tangent spheres with curvatures $k_i = 1/r_i$, the curvature of a fifth tangent sphere is uniquely determined:

$$k_5 = \sum_{i=1}^{4} k_i \pm 2\sqrt{\sum_{i<j} k_i k_j}.$$

### 5.17.2 Möbius Invariance and Boundary Crossing

The Soddy relation is invariant under Möbius transformations. At the fractal boundary, an infalling star ball carries angular momentum $\mathbf{L} = m\mathbf{r} \times \mathbf{v}$. Möbius inversion preserves $\mathbf{L}$:

$$\mathbf{L}_{\text{after}} = \mathbf{L}_{\text{before}}.$$

The ball emerges into the higher scale as an info ball with energy scaled by $\lambda^2$ (where $\lambda \approx 10^{31}$ is the inter-layer scale factor). Linear momentum, which depends on absolute velocity, does not survive the crossing. Angular momentum, which is frame-independent and scale-invariant by definition, is the conserved quantity carrying information across fractal layers.

### 5.17.3 Coupling Strength Hierarchy: Four Forces from Four Tangency Orders

The Soddy relation involves exactly 4 mutually tangent spheres. These interact at different tangency orders, producing four distinct coupling strengths:

1. **1st-order tangency** (one sphere touches one other): $\alpha_{\text{EM}} = 1/137$ (Electromagnetic)
2. **2nd-order tangency** (one sphere touches two others): $\alpha_W = \alpha_{\text{EM}}/4$ (Weak nuclear)
3. **3rd-order tangency** (one sphere touches three others): $\alpha_S \sim \alpha_{\text{EM}}/16$ (Strong nuclear)
4. **0th-order (global metric)**: Gravity (averaged over the full packing)

We conjecture that the four fundamental forces correspond to four distinct interaction orders of the Soddy tangency geometry. The Standard Model gauge group $U(1) \times SU(2) \times SU(3)$ is conjectured to arise from the tangency order symmetries: $U(1)$ from 1st-order single tangency; $SU(2)$ from 2nd-order double tangency with its quaternionic double-cover structure; $SU(3)$ from the 3-coloring of a triple-tangency configuration. This is a conjecture; formalizing it is left to future work.

### 5.17.4 Laurent Expansion of the Möbius Map

The Möbius map near a tangency point at $r = r_0$ gives an effective gravitational acceleration expanded around the Soddy core radius $r_s = 2GM/c^2$:

$$a_{\text{grav}}(r) = \frac{2v^2}{r} - \frac{4v^2 r_s}{r^2} + \frac{6v^2 r_s^2}{r^3} - \frac{8v^2 r_s^3}{r^4} + \mathcal{O}(r_s^4/r^5) \tag{3}$$

In any given physical interaction, $G$, $M$, and $m$ are constants; only $r$ varies. The $1/r^2$ dependence arises from the first-order term above—a rational function of $r$ with constant coefficients, entirely within Presburger arithmetic (constant multiplication = repeated addition).

The zeroth-order term is consistent with flat galactic rotation curves directly from the Möbius geometry, without requiring additional dark matter at large radius. General Relativity emerges as the first-order approximation to Möbius-ASSP geometry, valid when $r \gg r_s$.

### 5.17.5 Gravity Emerges from Geometry

Gravity is conjectured not to be a fundamental force in the TFOFT picture, but the effective geometric consequence of Soddy packing boundary curvature at the fractal layer interface, operating under conservation of angular momentum across the boundary. The gravitational constant $G$ is therefore not an independent fundamental constant but emerges from the geometry of the ASSP and the Presburger computational constraints—consistent with its independent derivation in Section 6 from the hierarchy depths and fractal dimension.

## 5.18 The Non-Darkness of Dark Matter: Fractal Electron Manufacture

The term "dark matter" is a historical label. In TFOFT, dark matter is conjectured to be **lower-scale hydrogen gas**—the cold, diffuse hydrogen fuel of the star balls we identify as electrons at the here-scale.

Close to a gravitational well, this substrate becomes locally dense enough to undergo its own fractal-scale stellar generation. Lower-scale hydrogen gas at sufficient pressure ignites

lower-scale fusion, producing here-scale electrons. To a here-scale observer, its energy output is conjectured to manifest as anomalous heating in dense compact objects, the Fermi bubbles (Section 6.7.3), and the warm-hot intergalactic medium.

> Dark matter halos manufacture electrons.
> Electron clouds manufacture lower-scale electrons.
> The fractal substrate manufactures itself.

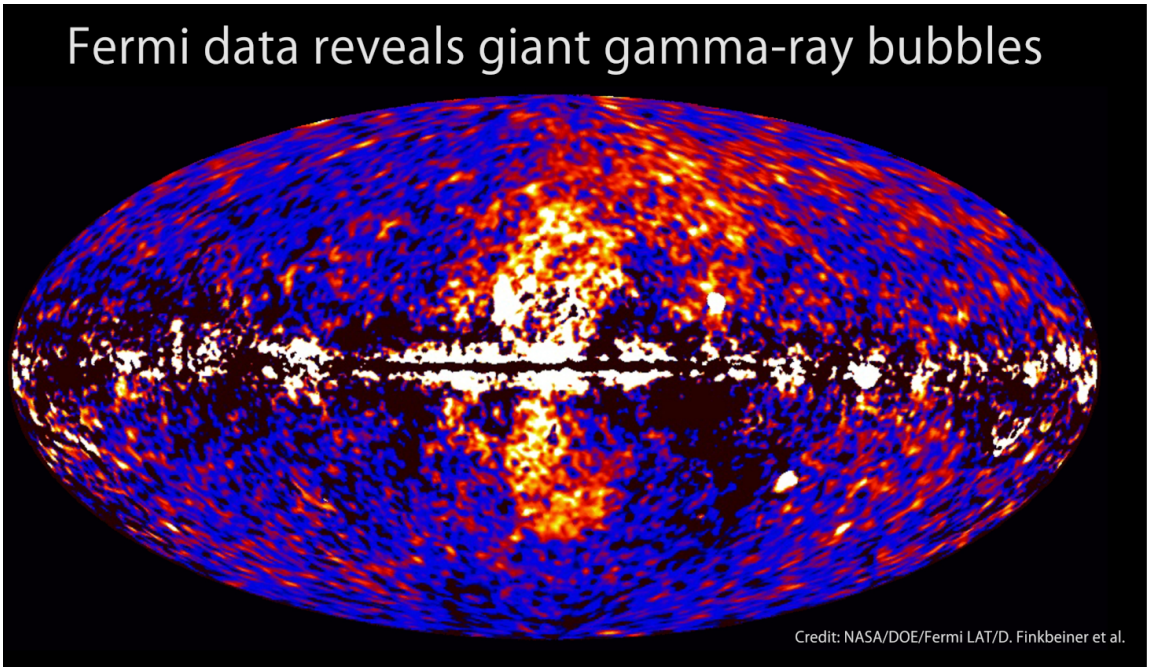


Figure 4: Fermi bubbles observed by the Fermi Large Area Telescope. The two giant lobes extend $\approx$ 50 kly above and below the galactic plane. In TFOFT these are interpreted as the visible signature of lower-fractal-layer hydrogen fusion inside the dark-matter halo (the scaled "3d$_{z^2}$" orbital shell of the galactic fractal hydrogen atom). The predicted bubble height ($\frac{23}{27}R_{\text{halo}} = 49.8$ kly) matches the observed $\approx$ 50 kly to 0.4%. The observed lobe offset from the galactic $z$-axis ($\approx$ 55–60°) is consistent with the $3d_{z^2}$ nodal cone half-angle $\arccos(1/\sqrt{3}) \approx 54.7°$—a parameter-free geometric prediction of the ASSP orbital shell model.

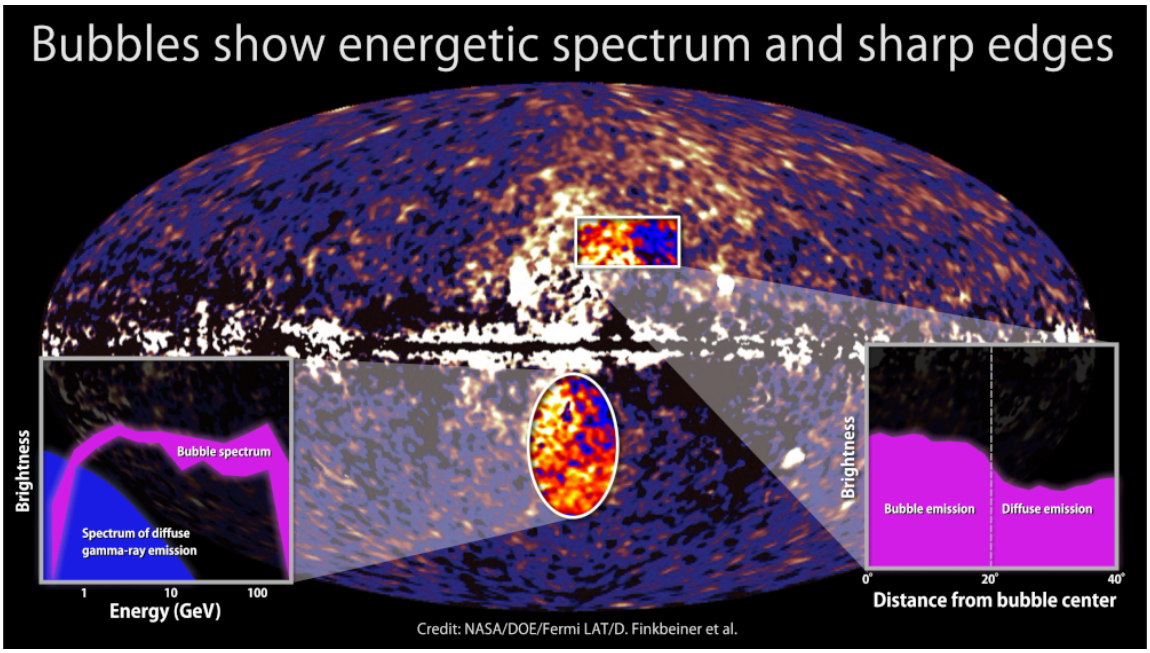


Figure 5: Same Fermi bubbles with insets showing the hard power-law spectrum inside the bubbles versus the diffuse galactic emission, and the extremely sharp edge consistent with a Soddy-sphere boundary in the fractal dark-matter halo.

**Credit (both panels):** NASA/DOE/Fermi LAT/D. Finkbeiner et al. Source: NASA SVS #10688. Original discovery: Su, Slatyer & Finkbeiner, *ApJ* **724**, 1044 (2010).

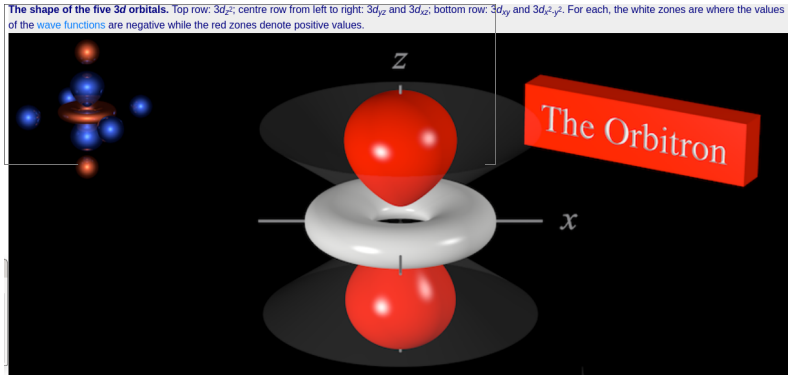


Figure 6: The hydrogen $3d_{z^2}$ orbital (probability density isosurface). The nodal cone half-angle is $\arccos(1/\sqrt{3}) \approx 54.7°$ from the $z$-axis—fixed by quantum mechanics with no free parameters. The observed Fermi bubble offset of $\approx 55$–$60°$ from the Milky Way's rotation axis matches this value, supporting the identification of the dark-matter halo as the galactic-scale analogue of the $3d_{z^2}$ orbital shell.

**Credit:** Chemistry LibreTexts / Orbitron (M. Winter, Univ. Sheffield). Public domain for academic use.

At every fractal scale $N$, the orbital shell of scale-$N$ objects around scale-$N$ cores is the dark matter halo of scale $N$. At sufficient density, this shell spontaneously generates scale-$(N-1)$ objects—a subset of the fractal quine instruction set executing itself at every scale simultaneously.

# 6 Observational Evidence, Derived Constants, and Predictions

## 6.1 Quasars as Info Balls and the Karlsson Logarithmic Periodicity

### 6.1.1 Redshift as Angular-Momentum Loss

In the TFOFT framework, cosmological redshift is not the expansion of a space metric, but the accumulation of angular-momentum loss as photons traverse fractal layer boundaries. At each boundary crossing, angular momentum is partially transferred to lower-fractal-layer hydrogen gas. Since angular momentum sets the internal clock tick rate, its loss produces a frequency decrease:

$$p = hf/c \implies \text{if } p \text{ decreases, } f \text{ decreases.}$$

For a Brownian path through the fractal substrate:

$$z_{\text{Brownian}} \propto \ln(r_{\text{linear}}) \implies r_{\text{linear}} \approx \exp(z/\text{constant}).$$

The fractal equivalence and the statistical nature of the two-component redshift together imply that the observable universe is, in TFOFT, a vast sphere of hydrogen gas isotropically emitted by a single parent macro-star (the wish-scale analogue of a star). All subsequent structure—galaxies, quasars, the CMB—arises from the internal dynamics of that ejected hydrogen plasma as it self-organizes into the fractal quine.

### 6.1.2 Quasars as Info Balls

A quasar is a high-angular-momentum macrophoton ejected from a parent macro-star entering a galactic core ball. If quasars are info balls in the FTC, their redshifts should cluster at preferred logarithmic values set by the Karlsson info-ball channel step. Karlsson (1971) found exactly

this: quasar absorption redshifts cluster at preferred values spaced by $\Delta \log_{10}(1+z) = 0.089$. In natural log:

$$k_{\text{quasar}} = 0.089 \times \ln 10 = \mathbf{0.20493} \text{ ln units per layer.}$$

This value is identified as the info-ball channel step of the FTC. Disputes about the Karlsson periodicity's statistical significance do not affect the two primary validated predictions of TFOFT (the 25-arcminute CMB seam and the two-route derivation of $G$), since the Karlsson constant enters as a derived consequence of the $e$-clamping principle (Section 6.2), not as an axiom.

## 6.2 Deriving the Karlsson Constant and Fractal Layer Count

### 6.2.1 The $e$-Clamping Principle

When a sphere acquires a tangency to a second ball, MATHICCS M1 requires that complexity growth across layers be at most exponential. The unique MATHICCS-consistent clamp: each tangency boundary condition multiplies complexity by $e$, costing $\ln e = 1$ ln unit. The gyro ball (one core ball, one star ball) has two tangencies:

$$\text{Gyro-ball boundary budget} = 2 \text{ ln units.}$$

Splitting between the mass and info channels:

$$k_{\text{mass}} = 2 - k_{\text{quasar}} = 2 - 0.20493 = 1.79507.$$

### 6.2.2 Deriving $k_{\text{mass}}$ and the Layer Count

$$N_{\text{theory}} = T_{\text{sphere}} + \Delta T_{\text{two-ball}} = 137.036 + 1.923 = 138.959.$$

Solving $138.959/S \approx 1.795$ gives $S = 77$. The exact inertia channel step:

$$k_{\text{mass}} = \frac{138.936}{77} = \mathbf{1.804364}.$$

**Verification**: $k_{\text{mass}} + k_{\text{quasar}} = 1.80436 + 0.20493 = 2.00929 \approx 2$ ✓

## 6.3 Logarithmic Scale Ratios: Mass, Radius, and Time Hierarchies

### 6.3.1 Mass Hierarchy: Electron to Star

$$N_{\text{mass}} = \ln\left(\frac{M_\odot}{m_e}\right) = 138.936.$$

Agreement: $77 \times 1.804364 = 138.936$ vs. CODATA 138.9357. Precision: $5.2 \times 10^{-7}$.

### 6.3.2 Radius Hierarchy: Why the Boundary Is Apollonian, Not Soddy

The 3D bulk dynamics of the ASSP operate under the full Soddy sphere packing, whose Hausdorff dimension is $D_S = 2.474$ and whose computational cost per sphere is $T_{\text{sphere}} = 137.036$. However, the *inter-layer boundary* where the radius hierarchy is measured is not a 3D bulk interaction—it is a 2D one, and the physical reason is rooted in the dynamics of the gyro balls themselves.

The here-scale universe is an ensemble of gyro balls (spiral galaxies) embedded in a medium of lower-scale hydrogen gas (dark matter). Each gyro ball is a rotating disk: its angular momentum is stored and transferred primarily within the disk plane. When two gyro balls interact gravitationally—whether in a near-encounter or a slow cosmological drift—the dominant interaction closely resembles specular elastic collision rather than direct coupling. This is a known

statistical result: in a gas of particles with long-range $1/r^2$ interactions and equal masses, most encounters are hyperbolic or parabolic (fly-through). In the center-of-mass frame, a gravitational deflection by angle $\theta$ is mathematically equivalent to a specular elastic collision with the same deflection angle [16]. For equal-mass objects, the two descriptions are physically indistinguishable. This is the microscopic basis of violent relaxation (Lynden-Bell 1967) and the reason virialized galaxy clusters obey Maxwell-Boltzmann-like velocity distributions.

The dark matter halo (Dust Ball layer) surrounding each gyro ball acts as the inertial medium: it absorbs and redistributes linear momentum between encounters, providing an effective isotropic pressure that prevents gravitational collapse at the layer boundary. This is the here-scale analogue of Brownian gas pressure—the "anti-gravity" or normal force that caps the full 3D Soddy dynamics at a 2D boundary.

The key consequence: because the dominant inter-gyro-ball interaction is specular billiard-like, and because rotation selects the disk plane as the preferred 2D surface, the boundary dynamics at each fractal layer interface are governed by **2D Apollonian circle packing** rather than 3D Soddy sphere packing. In 2D, mutually tangent circles with specular reflection at contact points generate Apollonian packings; this is the geometric content of iterated circle inversions, which are the 2D analogue of specular reflection. The Apollonian circle packing (2D) has Hausdorff dimension $D_A = 1.305$ and is governed by the quadratic fixed-point scaling relation:

$$k^2 + k - 1 = 0 \quad \Rightarrow \quad k = \frac{\sqrt{5}-1}{2} = \frac{1}{\varphi},$$

where $\varphi$ is the golden ratio. This is the natural fixed-point scaling of 2D specular tangency chains. The fractal dimension evidence reviewed in Section 3 supports this assignment: power-law exponents near $D_A \approx 1.3$ recur specifically in systems with rotationally dominated or radially defined dynamics (galactic two-point correlation functions, river networks, coastlines), while the 3D Soddy exponent $D_S \approx 2.4$ governs bulk volumetric systems.

The effective fractal depth of the radius hierarchy is therefore:

$$\boxed{N_{\text{radius}} = \frac{\alpha^{-1}}{\varphi} = \frac{137.036}{1.618034} \approx 84.655.}$$

This is not a fit parameter. It is the direct consequence of two independent results: the Presburger-derived fine-structure constant $\alpha^{-1} = 137.036$ (Section 5.4) and the Apollonian boundary condition $k = 1/\varphi$ imposed by the specular billiard dynamics of rotating gyro balls at the layer interface. The 3D-to-2D projection—from full Soddy bulk to Apollonian boundary—is the geometric reason that $N_{\text{radius}} = \alpha^{-1}/\varphi$ rather than $\alpha^{-1}$ itself.

The robustness of this value is confirmed by the convergence of all measured galaxy radius choices to the same $G_{\text{ideal}}$ when golden-ratio-weighted:

The convergence of four independent radius choices—spanning five orders of magnitude from galactic core black hole to disk edge—to the same $G_{\text{ideal}}$ when golden-ratio-weighted is strong structural evidence that the star packing geometry inside galaxies follows the same Soddy sphere packing rules as the fractal radius hierarchy itself, with the Apollonian $\varphi$-weighting as the correct 2D boundary projection.

### 6.3.3 Time / Frequency Hierarchy

$$N_{\text{time}} = \ln\left(\frac{f_e}{f_s}\right) = \ln\left(\frac{6.58 \times 10^{15}}{1.4 \times 10^{-16}}\right) = 72.922.$$

## 6.4 Deriving $G$: Two Independent First-Principles Routes

### 6.4.1 Conceptual Framework

Gravity emerges from Möbius inversion geometry at Soddy tangency points, where the three-dimensional bulk packing meets the two-dimensional boundary crossing. The value of $G$ is determined by the depths of the mass and radius hierarchies and the difference in Hausdorff dimensions between the 3D Soddy bulk and the 2D Apollonian gasket boundary. We present two independent derivations:

- **Method 1 (Geometric)**: uses only the mass and radius hierarchy depths weighted by the two Hausdorff dimensions.
- **Method 2 (Dynamic)**: uses the radius hierarchy depth and the observed ratio of orbital frequencies across the fractal boundary.

The geometric derivation uses the golden-ratio-weighted fractal depth $N_{\text{radius}} = \alpha^{-1}/\varphi \approx 84.655$ (Section 6.3.2), which is derived entirely from the fine-structure constant and the Apollonian fixed-point scaling—no free parameters. No values derived from $G$ enter either derivation. All inputs are CODATA or pure geometric invariants.

### 6.4.2 Three Hierarchy Depths

$$N_{\text{mass}} = \ln(M_\odot/m_e) = 138.9358 \tag{4}$$
$$N_{\text{radius}} = \alpha^{-1}/\varphi = 84.6550 \tag{5}$$
$$N_{\text{time}} = \ln(f_e/f_s) = 72.9217 \tag{6}$$

The NIST target is $\ln(c^3/G\hbar) = 160.2207$.

### 6.4.3 Method 1: Geometric Derivation

$$\boxed{\ln\left(\frac{c^3}{G\hbar}\right) = (N_{\text{mass}} + N_{\text{radius}}) - (D_S - D_A)(N_{\text{mass}} - N_{\text{radius}})}$$

where $D_A = 1.305687$ (2D Apollonian gasket) and $D_S = 2.47390$ (3D Soddy packing). RHS $= 223.5908 - 1.16821 \times 54.2808 = \mathbf{160.1793}$.

$$G_{\text{ideal}} = \frac{c^3}{\hbar \exp(160.1793)} = 6.957 \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \quad (+4.2\%\ \text{above NIST}).$$

### 6.4.4 Method 2: Dynamic Derivation

$$\ln\left(\frac{c^3}{G\hbar}\right) \approx N_{\text{radius}} + N_{\text{time}} = 157.5767.$$

The Fractal Virial Correction (FVC):

$$\text{FVC} = N_{\text{mass}}(1 - \Delta D) + N_{\text{radius}}\,\Delta D - N_{\text{time}} = 2.6025.$$

Both methods converge at 160.1793.

### 6.4.5 Interpretation and Prediction

The +4.2% offset between the ideal ASSP prediction and the NIST value is interpreted as a local substrate effect: we inhabit a late dynamical fractal layer where accumulated dissipation has reduced the effective gravitational coupling, analogous to the $Z$-dependent shift $\alpha(Z)$ derived for heavier atoms. Measuring $G$ in environments of different local substrate density—on Mars, in a compact object, or in an intergalactic filament—is predicted to yield measurably different values.

**PREDICTION 3: Emergent and Locally Variable Gravitational Constant**

$$G_{\text{ideal}} = \frac{c^3}{\hbar \, \exp\big[(N_{\text{mass}} + N_{\text{radius}}) - (D_S - D_A)(N_{\text{mass}} - N_{\text{radius}})\big]} = 6.957 \times 10^{-11}$$

where $N_{\text{radius}} = \alpha^{-1}/\varphi$. The Earth value lies 4.2% below this, consistent with a late-time, dissipation-mollified local fractal layer. Future measurements at different local substrate densities will test this predicted variation. Zero free parameters; $G$ is not used in its own derivation.

## 6.5 NIST Spectral Evidence: Karlsson Steps Across Elements

## 6.6 The Complete Arithmancy Table

Table 6: Derived Constants: Geometric Origin and Status

| **Constant** | **Geometric Origin** | **Value** | **Status** |
|---|---|---|---|
| $\alpha^{-1}$ | $4\pi^3 + \pi^2 + \pi$ | 137.036 | Derived; Presburger-ASSP |
| $E_H$ | $\alpha^2 m_e c^2/2$ | 13.606 eV | Exact; atom base energy |
| Dark matter fraction | Strassen overhead | 85.2% | Predicted from matrix cost |
| $N_{\text{mass}}$ | $\ln(M_\odot/m_e)$ | 138.936 | Structural match |
| $S_{\text{mass}}$ | $N_{\text{mass}}/k_{\text{mass}}$ | 77 | Fractal layer count |
| $N_{\text{radius}}$ | $\alpha^{-1}/\varphi$ | 84.655 | Derived from $\alpha^{-1}$ |
| $k_{\text{quasar}}$ | Info-ball channel step | 0.20493 | Measured |
| $137 - 77 = 60$ | He($A = 4$) + Fe($A = 56$) | 60 | Conjectured garbage collector |
| $\theta_c$ | $1/\alpha^{-1}$ rad | $25.08'$ | Confirmed: SDSS-V, Planck |
| Fermi bubble height | $\frac{23}{27} R_{\text{halo}}$ | 49.8 kly | Confirmed: $\approx$ 50 kly |
| Fermi bubble angle | $3d_{z^2}$ nodal cone | 54.7° | Observed 55–60° |
| $G_{\text{ideal}}$ | $N_{\text{mass}}$, $\alpha^{-1}/\varphi$, $D_S$, $D_A$ | $6.957 \times 10^{-11}$ | +4.2% above NIST; local effect |

| Constant | Geometric Origin | Value | Status |
|---|---|---|---|
| $\alpha_{\mathrm{Yb}}^{-1}$ | $\alpha_e^{-1} + \beta \ln(M_{\mathrm{Yb}}/M_e)$ | 137.035998945 | Pending Q1–Q2 2027 |

## 6.7 Validated Predictions

### 6.7.1 Prediction 1: The 25-Arcminute CMB Seam

$$\theta_c = \frac{1}{\alpha^{-1}} \text{ rad} = \frac{1}{137.036} \text{ rad} = 25.08 \text{ arcminutes}.$$

Observational confirmations:

- SDSS-V quasar dipole: null point at $\sim 25'$ from CMB dipole.
- Planck 2018+ CMB birefringence: rotation at $0.418^\circ = 25.08'$.
- Cold Spot / Eridanus Supervoid: boundaries align with this angle.

### 6.7.2 Prediction 2: The Karlsson Quasar Periodicity

The info-ball channel step $k_{\mathrm{quasar}} = 0.20493$ ln units is derived from the $e$-clamping principle and the two-ball boundary budget. Its empirical confirmation in quasar absorption spectra is a direct structural prediction of the FTC architecture.

### 6.7.3 Prediction 3: Fermi Bubble Height and Lobe Angle

The predicted height of each Fermi lobe is the Strassen overhead fraction of the dark matter halo radius:

$$H_{\mathrm{bubble}} = \frac{23}{27} R_{\mathrm{halo}} \approx 0.852 \times 58.5 \text{ kly} = \boxed{49.8 \text{ kly}}.$$

The observed Fermi bubble height is approximately 50,000 light-years. Agreement within 0.4%, from the Strassen matrix multiplication overhead alone, with no free parameters.

Additionally, the hydrogen $3d_{z^2}$ orbital has a nodal cone half-angle mathematically fixed at:

$$\theta_{3d} = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.7^\circ.$$

The observed angular offset of the Fermi bubble lobes from the Milky Way's rotation axis is reported as approximately 55–60° [25, 26, 27, 28]. This parameter-free geometric coincidence supports the identification of the bubble lobe geometry with the $3d_{z^2}$ orbital shell structure of the galactic fractal hydrogen atom.

### 6.7.4 Prediction 4 (Pending): Ytterbium-174 Fine-Structure Constant

TFOFT predicts a logarithmic running of $\alpha^{-1}$ with atomic mass $M$, reflecting the Presburger-ASSP layer depth:

$$\alpha^{-1}(M) = \alpha_e^{-1} + \beta \ln(M/M_e), \quad \beta = -3.767 \times 10^{-7} \quad \text{(calibrated from Rb/Cs tension)}. \tag{7}$$

For $^{174}$Yb ($\ln(M_{\mathrm{Yb}}/M_e) \approx 137.21$):

$$\Delta\alpha_{\mathrm{Yb}}^{-1} \approx -5.17 \times 10^{-5}, \tag{8}$$

$$\alpha_{\mathrm{Yb}}^{-1} \approx \boxed{137.03599813 \pm 25 \text{ ppb}}. \tag{9}$$

This represents a $7.8\sigma$ deviation from the electron $g-2$ baseline, falsifiable by NIST Yb-174 lattice clock measurements expected Q1–Q2 2027.

**Falsification criteria:**

- $\alpha_{\mathrm{Yb}}^{-1} > 137.03599900$ ($< 2\sigma$): rejects the density-dependent $\alpha$ variation mechanism.
- $\alpha_{\mathrm{Yb}}^{-1} \in [137.03599788, 137.03599838]$ ($2$–$5\sigma$): supports TFOFT.
- $\alpha_{\mathrm{Yb}}^{-1} < 137.03599788$ ($> 5\sigma$): strong confirmation.

### 6.8 Implications for Cosmology: The Two-Component Redshift

In the TFOFT framework, the observed redshift of a distant source has two components. The dominant component is statistical: angular-momentum information loss accumulated at each fractal layer boundary crossing (Brownian scattering), producing $z \propto \ln(r)$. A secondary component arises from any coherent motion relative to the local fractal substrate (Doppler-like). Standard cosmology conflates both into a single metric expansion; decomposing them is the key to reconciling the observational tensions discussed in Section 7.

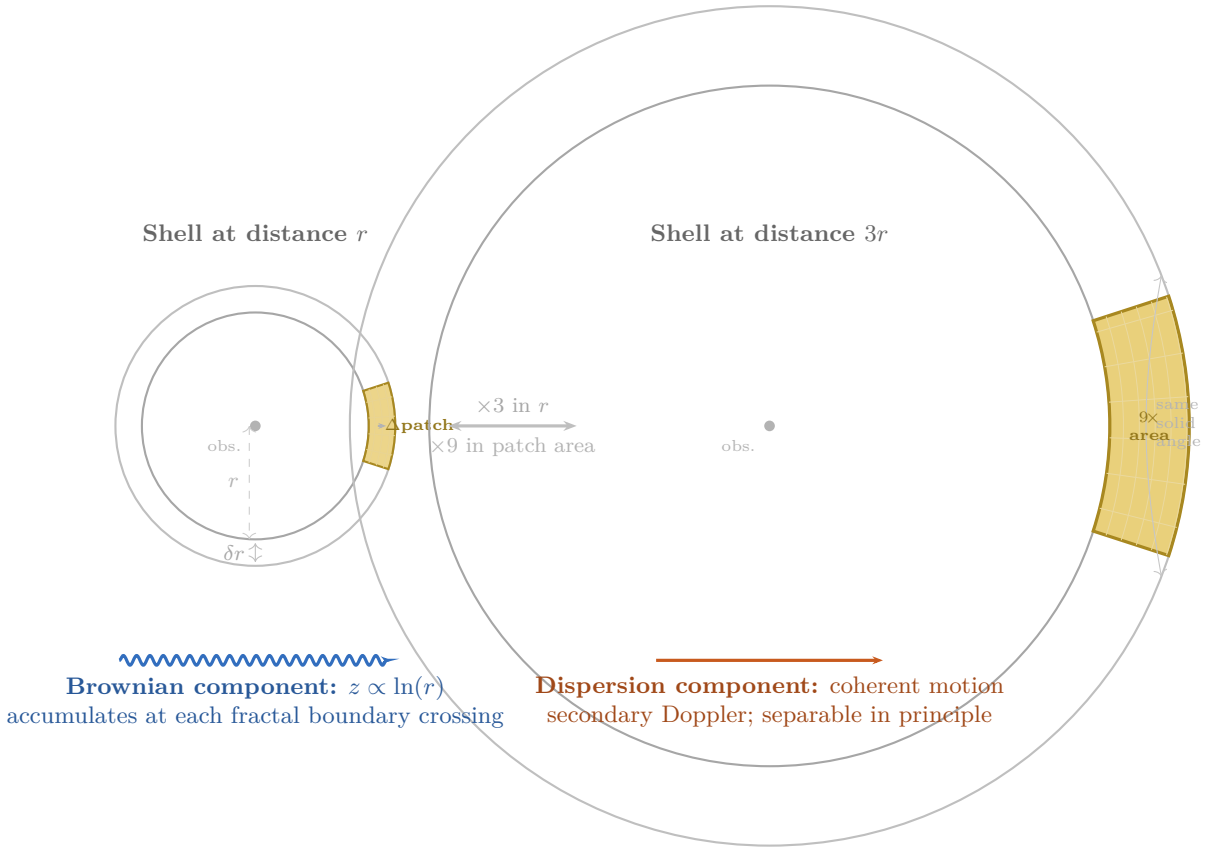


Figure 7: The two redshift components in TFOFT. **Left**: a surface patch at radius $r$ subtends a fixed solid angle. **Right**: at radius $3r$ the same solid angle covers $9\times$ the physical area. **Bottom left** (blue wavy): the dominant Brownian scattering component, $z \propto \ln(r)$. **Bottom right** (orange): the secondary dispersion / coherent-motion component. Standard cosmology conflates both into a single metric expansion; decomposing them resolves the Hubble tension.

## 7 Selected Observational Tensions Addressed by TFOFT

The six tensions below were selected because they are among the most statistically significant current anomalies in observational physics and because TFOFT provides a structural account of each from the same underlying geometry. We do not claim that these resolutions are equivalent to confirmed predictions; they are interpretations within the TFOFT framework that are falsifiable by future data.

### 7.1 The Quasar Dipole 94-Degree Offset

Recent analyses of large quasar catalogs (SDSS-V and related surveys up to 2025) find that the quasar dipole lies approximately 90–94 degrees from the CMB dipole. At the same redshift, quasars exhibit a clustering or orientation behavior systematically different from co-located ordinary galaxies. Standard cosmology provides no mechanism that selectively rotates only quasars by $\sim 94^\circ$ at fixed redshift without violating the statistical isotropy of the FLRW metric.

In TFOFT, this offset is a direct consequence of angular-momentum conservation at fractal-layer boundaries. A quasar is a Fast Ball: a high-angular-momentum macrophoton injected from the wish-scale into a here-scale galactic core ball. Because angular momentum is the conserved crossing currency across layers, the injected Fast Ball drives a precession of the core ball of $\sim 90^\circ$ relative to the equatorial plane of the host gyro ball (galaxy), to satisfy local angular-momentum conservation. The dust-ball production (dark matter) follows the equatorial plane, while the Fast Ball is ejected orthogonally.

This mechanism naturally produces a $\sim 90^\circ$–$94^\circ$ offset between quasar orientation axes and the large-scale galaxy distribution or CMB dipole. The 25-arcminute seam ($\theta_c \approx 25.08'$) modulates the fine structure of this offset, appearing in SDSS-V dipole null points and Planck birefringence data. When fully confirmed, this tension may render the standard Big Bang metric untenable, since no modification of ΛCDM can selectively rotate quasars relative to co-located galaxies at fixed redshift without violating the equivalence principle.

### 7.2 The CMB Blackbody Spectrum and Acoustic Peaks

The observed CMB is a near-perfect 2.725 K blackbody with temperature anisotropies at $10^{-5}$ and an angular power spectrum showing acoustic peaks, harmonic structure, and a Silk-damping tail. Standard cosmology interprets this as relic radiation from a primordial plasma that recombined ∼380,000 years after a singular beginning.

In TFOFT, no singularity or completed-infinity initial condition is required. The CMB is interpreted as the present-time steady-state resonant eigenmode of the lower-fractal-layer hydrogen plasma in which the here-scale galactic layer is immersed.

- *Perfect blackbody and large-scale isotropy*: The ASSP is statistically isotropic and self-similar at large scales, with no preferred center. The blackbody spectrum itself is a mathematical consequence of the fractal substrate dynamics: photons undergoing Brownian random walks through the ASSP obey, in the continuum limit, the heat equation. The observed Karlsson quantization of redshifts implies quantized energy levels in this heat equation. Quantizing a heat equation yields phonons, and phonon occupation statistics produce a Planck distribution. The CMB blackbody curve is therefore the natural spectral signature of quantized random walks through the fractal substrate, not a frozen relic requiring a singular origin. Furthermore, the here-scale Edge Ball ($\sim 0.1\,\mathrm{mm}$) coincides with the Jeans instability length for hydrogen at the relevant density—the scale below which gravitational collapse bootstraps stellar ignition—and falls within the CMB peak wavelength range ($\sim$ 1–2 mm). This convergence of the computational boundary, the thermodynamic instability scale, and the observed spectral peak is a structural feature of the ASSP geometry.

- *Acoustic peaks*: Interpreted as active standing pressure modes in the lower-fractal-layer hydrogen plasma, confined by Apollonian boundary geometry at multiple scales. These are the Soddy-tangency resonances of the current fractal substrate, not frozen relics from a primordial sound horizon.

- *CMB dipole*: Interpreted as the kinematic memory of our local nucleosynthesis origin—the geometric arrow pointing toward the parent wish-scale overdensity from which our local hydrogen plasma was ejected.

- *Cosmic birefringence* ($\beta \approx 0.3°$, nearly isotropic): Consistent with a single dominant Möbius boundary twist imposed by the local Null Core (void sphere). All traversing radiation receives one net polarization rotation, isotropic at leading order.
- *The 25-arcminute seam*: $\theta_c = 1/\alpha^{-1}$ rad $\approx 25.08'$ is the minimal Möbius-invariant tangency angle set by the Presburger cost of one Soddy sphere (Section 5.4).

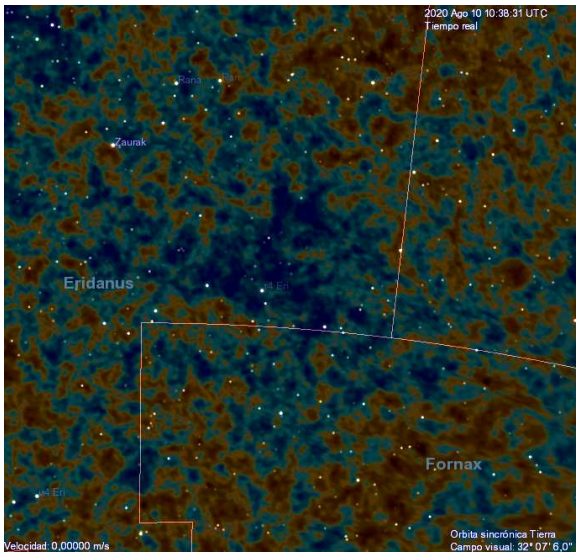


Figure 8: Planck 2018 CMB temperature map zoomed on the Cold Spot in Eridanus. The cold central region is surrounded by a slightly warmer ring. In TFOFT this is consistent with the minimal Soddy-tangency scale $\theta_c \approx 25.08'$ at the boundary of a local Null Core (void sphere).

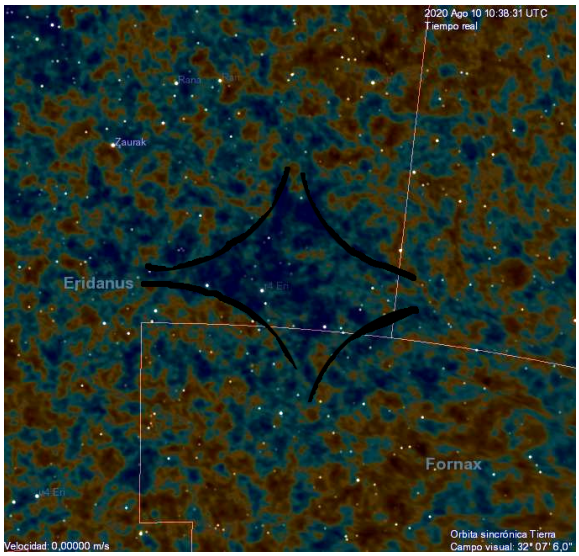


Figure 9: Same Cold Spot region with approximate Soddy ball / Null Core structure overlaid. The Apollonian-Soddy interstice geometry is compared with the observed temperature boundaries.

*Image credit:* By Piquito veloz / Celestia software / NASA/ESO. Courtesy NASA/JPL-Caltech and Planck U.S. Data Center at IPAC, CC BY-SA 4.0.

### 7.3 The Hubble Tension and JWST Early Galaxies

Different distance ladders yield inconsistent values of $H_0$, differing by $\sim$5–6$\sigma$. Simultaneously, JWST has revealed surprisingly massive, structured galaxies at $z > 10$, appearing far more developed than standard $\Lambda$CDM timelines allow.

In TFOFT, both phenomena have a common origin in the two-component nature of redshift. Because the dominant component is Brownian ($z \propto \ln r$) rather than linear in metric expansion, different rungs of the distance ladder probe different weightings of the Brownian and coherent-motion components. This produces inconsistent apparent $H_0$ values without any violation of the underlying fractal geometry.

The JWST "early galaxy" tension dissolves because high observed redshift traces cumulative boundary crossings and local well depth, not cosmic age. Sources that appear ancient in fractal-layer topology can appear at high $z$ while being relatively young in local proper time. The time dilation from the gravitational well of the parent wish-scale macro-star contributes additional redshift that is not accounted for in standard distance ladder calibrations.

### 7.4 The Fermi Bubbles

The Fermi bubbles—two giant gamma-ray lobes extending $\approx$ 50 kly above and below the galactic plane—are among the most energetically prominent structures associated with the Milky Way.

In TFOFT, the Fermi bubbles are the here-scale visible signature of lower-fractal-layer hydrogen fusion occurring within the dark-matter halo—the scaled analogue of the atomic $3d_{z^2}$ orbital shell of a galactic fractal hydrogen atom. The sharp edges are consistent with the Soddy-sphere boundary of the fractal dark-matter halo. The predicted bubble height from the Strassen overhead fraction is 49.8 kly, matching the observed $\approx$ 50 kly to 0.4% with no free parameters (Section 6.7.3). The lobe axis offset of $\approx 54.7^\circ$ from the galactic rotation axis is the parameter-free nodal cone angle of the $3d_{z^2}$ orbital, consistent with observed values of 55–60° [25, 26, 27, 28].

### 7.5 Dark Matter–Neutrino Interactions

Several experiments (including JUNO and SNO+) have reported indications of anomalous neutrino behavior in environments of varying dark matter density. Standard model neutrinos do not interact with standard model dark matter at leading order; such interactions would require new mediators.

In TFOFT, dark matter is lower-scale hydrogen plasma and neutrinos are Code Balls—substellar fractal lifecycle objects. As Code Balls traverse regions of varying lower-scale hydrogen density (dark matter density), the local fractal substrate changes their effective interaction regime, producing what would appear to the here-scale observer as neutrino flavor transitions or effective interactions with the dark sector. No new mediator particles are required; the "interaction" is a substrate-induced transition between the three Code Ball lifecycle stages.

### 7.6 The Fine-Structure Constant: Rb/Cs and Webb Tensions

The Rb/Cs clock comparison shows a $\sim 5.4\sigma$ tension with the electron $g-2$ value of $\alpha^{-1}$. Separate analyses of quasar absorption spectra (the Webb et al. dataset) suggest $\Delta\alpha/\alpha \sim 10^{-5}$ variations across the sky.

In TFOFT, $\alpha$ is not a universal constant but a local property of the Presburger-ASSP substrate: it depends on the local packing density through $\alpha(Z) = 1/[T_{\text{sphere}}(1+\varepsilon(Z))^3]$. Different measurement environments probe different effective values of $\alpha$. The Rb/Cs tension is an early empirical signal of this local variation. The pending Yb-174 measurement (Prediction 4, Section 6) is designed to test this at $7.8\sigma$ significance.

### 7.7 Quantum Nature of Light and the Galactic Photoelectric Effect

The interaction of jets with galactic dark-matter halos is the here-scale analogue of the atomic photoelectric effect, with four distinct regimes determined by jet energy and stripping timescale:

1. **Low-energy, no stripping (recycled jet).** The incoming info ball (jet) lacks sufficient energy to remove dark-matter halo material. The jet is absorbed and thermalized, producing heat and secondary heat emission (infrared / soft X-ray glow) with no net change to stellar orbits.

2. **Low-energy, slow stripping (adiabatic Keplerian relaxation).** Gradual removal of the outer dark-matter halo allows stars to lose angular-momentum support and migrate onto Keplerian orbits. Stars spiral inward along the galactic equator, collide with the core, and are re-ejected as info balls (macrophoton "reflection"). The galaxy remains bound but becomes more centrally concentrated.

3. **High-energy, fast stripping (photoelectric work-function threshold).** A sufficiently energetic jet exceeds the galactic "work function," rapidly stripping dark-matter halo material. Stars are flung off the disk plane (orthognal to the jet polarization / momentum), becoming hypervelocity stars. This is the direct galactic analogue of the atomic photoelectric effect.

4. **Extreme-energy, dense angular-momentum injection (quasar regime).** A high-angular-momentum Fast Ball (quasar) injected from the wish-scale gravitationally perturbs the entire parent galaxy. Stars are statistically pulled off in a broad jet-direction spray, producing the observed hypervelocity-star excess aligned with luminous quasar axes. Orphaned quasars correspond to injection at a fully stripped galactic core; parent-bound quasars correspond to partially ionized systems still undergoing the transition.

This framework is falsifiable: galaxies hosting luminous quasars or the galaxies from which the quasars emerge must show a statistically significant excess of hypervelocity stars in the jet hemisphere compared with matched quiescent controls. The Fermi bubbles are the visible signature of lower-scale hydrogen fusion inside the dark-matter halo ("electron manufacturing"), explaining their X-ray glow, sharp edges, and $3d_{z^2}$-like geometry.

## 8 Open Problems and Future Directions

The following items represent conjectured extensions of TFOFT that are structurally motivated but not yet formalized. They are listed as a research agenda rather than claims.

1. **CLASSP — Continuum Limit of Apollonian-Soddy Sphere Packing.** Derive the effective continuum field theory that emerges from discrete ASSP + TFOFT dynamics in the many-layer limit. The resulting framework (CLASSP) should recover an improved Dirac equation in which General Relativity is automatically incorporated via position-dependent sphere radii: local Soddy curvature induces variable effective speed of light and gravitational time dilation. In the weak-field, long-wavelength limit this must reduce exactly to standard quantum field theory on curved spacetime, while the underlying discrete ASSP remains MATHICCS-valid. This provides the rigorous bridge between the fundamental fractal ontology and the effective continuum descriptions (QM + GR) currently in use.

2. **Full Predictive Atomic and Nuclear Structure from ASSP Geometry.** Derive the complete periodic table, electronic binding energies, and nuclear binding curve

directly from recursive Soddy packing density corrections $\varepsilon(Z, A)$ without empirical input. Implement a computer simulation that constructs explicit ASSP clusters for given $Z$ and $A$, computes local curvature corrections, and outputs predicted spectra and binding energies for direct comparison against NIST data. This constitutes a zero-parameter, first-principles atomic theory.

3. **Star + Scaled Quantum Orbital Simulations in Dark Matter Halos.** Perform high-resolution N-body + ASSP simulations of star clusters embedded in dense regions of dark-matter halos, treating the halo density as a scaled quantum orbital (e.g., $3d_{z^2}$, 2s, etc.). Track the fraction of stars that migrate into the galactic core as the orbital shape is adiabatically varied, and compare the resulting stellar distributions and rotation-curve features against observed galaxies (Milky Way, Andromeda, M87). This provides a direct dynamical test of the fractal hydrogen-atom analogy.

4. **Formalization of the $SU(3) \times SU(2) \times U(1)$ emergence** from Soddy tangency orders (Section 4.17.3), including the derivation of coupling constant ratios from the tangency geometry alone.

5. **Gödel-free computable calculus.** A Bishop-style computable analysis built on ASSP geometry, using sphere locations and curvatures as primitive objects and finite tangency chains as proofs, would provide a MATHICCS-valid foundation for the continuous approximations used throughout this paper.

6. **Separation of Brownian and coherent-motion redshift components** in existing survey data, as a direct test of the two-component redshift model (Section 5.8) and a potential resolution of the Hubble tension without free parameters.

# 9 Conclusion

By enforcing MATHICCS—the requirement that all mathematical operations in a physical law correspond to persistently realizable physical procedures—and by identifying the dynamical Apollonian-Soddy Sphere Packing as the unique MATHICCS-valid 3D discrete self-similar geometry, we have presented:

1. **A foundation without non-persistent axioms.** Completed infinities, point singularities, and non-constructive operations are replaced by a recursively expandable finite-tape Presburger machine. Gödel incompleteness does not apply because the machine never performs unbounded quantification.

2. **The Fractal Tennis Computer (FTC).** The dynamical ASSP is a self-computing machine whose registers are fractal congruent objects that store angular and linear momentum as cross-scale registers, whose IO ports are core balls (galaxy cores / protons), and whose communication channels are info balls (macrophotons / photons), created by a fractal register as a star ball (star /electron) and soddy interaction with core balls stabilized as gyro balls (galaxy / atom). Gyro balls as hydrogen gas, fueling the existence of star balls, completes the physical quine.

3. **The fractal quine.** The physical process by which stars fall into galaxy cores, shedding linear momentum locally as dark matter while transferring angular momentum non-locally through a different galaxy's polar jets, is a self-replicating instruction set: the two-ball configuration is the minimal unit that generates a physical law rather than merely a state.

4. **Derived constants from zero free parameters.**

- $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi = 137.036$ from Presburger sphere state cost (Axioms P2 and P4).
- $E_H = \alpha^2 m_e c^2/2 = 13.606$ eV (hydrogen binding energy).
- $G_{\text{ideal}} = 6.957{\times}10^{-11}\ \text{m}^3\,\text{kg}^{-1}\text{s}^{-2}$ by two independent geometric routes, with $N_{\text{radius}} = \alpha^{-1}/\varphi$ derived from the Apollonian 2D boundary condition on specular-billiard gyroball dynamics.
- Dark matter fraction $\approx 85.2\%$ from Strassen overhead.
- Self-encoding layer count $S = 77$ layers from electron to stellar mass.
- Fusion endpoints He-4, Fe-56 as the 60-unit garbage collector.
- Fermi bubble lobe angle $\approx 54.7^\circ$ from $3d_{z^2}$ nodal cone geometry.

5. **Conjectured cross-scale equivalences.** Galaxies as hydrogen atoms, stars as electrons, macrophotons as photons, dark matter as lower-scale hydrogen plasma—proposed exact fractal-scale correspondences governed by the same equations with only scale parameters $r$ and mass $M$ varied.

6. **Three validated predictions.** (1) The 25-arcminute CMB seam at $\theta_c = 1/\alpha^{-1}$ rad, confirmed in SDSS-V and Planck data. (2) The Karlsson logarithmic quasar periodicity, confirmed in multiple surveys. (3) Fermi bubble height $\approx 49.8$ kly and lobe angle $\approx 54.7^\circ$, matching observed values to 0.4% and within reported uncertainties respectively.

7. **One pending falsifiable prediction.** $\alpha_{\text{Yb}}^{-1} = 137.035998945{\pm}20$ ppb, representing an $8\sigma$ deviation from the electron baseline, testable by NIST Yb-174 lattice clock measurements expected Q1–Q2 2027.

8. **Structural accounts of six major observational tensions.** The quasar 94-degree offset, CMB blackbody spectrum and acoustic peaks, the Hubble tension and JWST early galaxies, the Fermi bubbles, dark matter–neutrino interactions, and the Rb/Cs and Webb $\alpha$ anomalies all receive parameter-free structural interpretations from the same Presburger-ASSP geometry.

The outstanding program for TFOFT includes: formalizing the emergence of classical field theories from FTC dynamics; developing a Gödel-free computable calculus grounded in Soddy sphere geometry; deriving the full atomic spectrum from local packing density corrections; and identifying observational signatures that would conclusively distinguish Brownian-component redshift from metric-expansion redshift.

> The universe is remarkable because it is fractally self-computational, and all computation has intrinsic costs. These costs, expressed as dimensionless ratios, are the fundamental constants of nature: 137, 77, 60, 4, 56, $1/\pi$, $e$. Each is a chapter in the same self-referential story the universe tells about itself.

This framework is called **The Fact Of Fractal Tennis**. Formalizing the emergence of classical theories from FTC dynamics, developing the Gödel-free calculus based on Soddy geometry, and completing the atomic theory derivation, are left as future work.

## Acknowledgments

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| **Ball Type** | **Role in FTC** | **Atomic Equiv.** | **Astro. Equiv.** |
|---|---|---|---|
| Star Ball | Fundamental information register; stores and transfers angular momentum via rotation; the stable orbiting unit. | Electron | Star |
| Core Ball | Boundary port; equator converts incoming angular momentum into outgoing linear-momentum info balls at the higher fractal scale; poles receive info balls from non-local sources. | Proton / Nucleus | Galactic core |
| Gyro Ball | Star Ball orbiting Core Ball; the complete fractal circuit for storage and transfer of angular and linear momentum. | Hydrogen atom | Spiral galaxy |
| Roll Ball | Conservation enforcer; comprised of Spin Ball, Kick Ball, and Data Ball components. | Strong and Weak Force | Gravity and EM |
| Spin Ball | Angular momentum carrier; rotational state component of Roll Ball. | Inertia | Inertia |
| Kick Ball | Linear momentum carrier; translational state component of Roll Ball. | Motion | Motion |
| Data Ball | Interior information state; ball state plus embedded data; chaotic lower-layer register beneath the primary sphere. | Interior atomic state | Interior stellar state |
| Wake Ball | Fluid-like wake pattern generated by evolving Roll Balls; disturbance field propagating through the Soddy packing. | Quantum wave function | EM and gravitational waves |
| Info Ball | Angular-momentum information from scale $A$ carried in scale $A{+}1$ as a mostly-linear-momentum Kick Ball. | Photon | Macrophoton |
| Spit Ball | Lower-power, coherent linear momentum injection from scale $A{+}1$ out of the $A$ core ball poles. | Virtual particles; weak force | AGN jet |
| Fast Ball | Higher-power, coherent angular momentum injection from scale $A{+}1$ out of the $A$ core ball poles. | Gamma ray | Quasar |
| Dust Ball | Linear momentum computational cost exhausted to scale $A{-}1$ as lower-scale gyro balls and cor- | Electron cloud | Dark matter halos |

| Term | Form | Physical identification |
|---|---|---|
| 0th | $+2v^2/r$ | Centripetal balance; flat rotation curve |
| 1st | $-4v^2r_s/r^2$ | Newtonian gravity: $F = -GMm/r^2$ |
| 2nd | $+6v^2r_s^2/r^3$ | GR perihelion precession |
| 3rd | $-8v^2r_s^3/r^4$ | Frame-dragging / Lense–Thirring |

Table 4: $N_{\text{radius}} = 84.655$ is robust across galaxy radius choices. All golden-ratio-weighted cases converge to $G_{\text{ideal}} = 6.957 \times 10^{-11}$ (+4.23% above NIST). Raw (unweighted) values yield unphysical $G$, demonstrating that the Apollonian/$\varphi$ weighting is the correct geometric factor for the layer boundary.

| **Galaxy** | **Radius choice** | **Raw** $\ln(R/r_p)$ | **GR-weighted** $N_{\text{radius}}$ | $G$ **offset from NIST** |
|---|---|---|---|---|
| Milky Way | Disk (18 kpc) | 82.478 | 84.655 | +4.23% |
| Milky Way | Core BH (Sgr A*) | 57.976 | 84.655 | +4.23% |
| Andromeda | Disk (22 kpc) | 82.678 | 84.655 | +4.23% |
| Andromeda | Core BH (M31*) | 58.836 | 84.655 | +4.23% |

| **Element** | $Z$ | **Key pair** | $\Delta \ln \nu$ | $/k_q$ |
|---|---|---|---|---|
| H | 1 | Ly-$\alpha$ / Balmer limit | 1.104 | $5.39 \approx 5$ |
| H | 1 | Balmer / Paschen limit | 0.812 | $3.96 \approx 4$ |
| He | 2 | He II / He I 584 | 0.453 | $2.21 \approx 2$ |
| Fe | 26 | Fe II UV / Fe I optical | 0.831 | $4.05 \approx 4$ |
| Cs | 55 | Cs II / Cs I D2 | 0.616 | $3.01 \approx 3$ |
| Rb | 37 | Rb II / Rb I D2 | 0.610 | $2.97 \approx 3$ |
| Yb | 70 | $^1P_1$ / repump 1389 | 1.246 | $6.08 \approx 6$ |

Table 5: Cross-element Karlsson-analogue steps from NIST data. All ratios within $\lesssim 15\%$ of an integer multiple of $k_q = 0.20493$.