The Quantum Core of the Effective Theory as an Operational Reconstruction in a Timeless Euclidean Model

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Abstract

The present work investigates the reconstruction of the quantum layer of the effective description in a timeless Euclidean model on $\mathbb{E}^4$ with a single fundamental real field satisfying the Laplace equation. Using a working region Ω, an observer body $\Omega_0 \subset \Omega$, and a foliation-based description of local readouts, we formulate an operational reconstruction scheme for states, observables, and causally consistent dynamics in operationally accessible regions where an effective description in terms of foliations and effective fields remains valid. Within such regions, the previous GR reconstruction singles out the locally inertial SR regime as the working form of effective analysis.

We then consider a working class of states consistent with transfer locality, the reflection structure with respect to the foliation parameter, the spectral condition, and stable registration. In this regime, the standard OS/GNS reconstruction yields a Hilbert space of states, a self-adjoint Hamiltonian $H \geq 0$, and a strongly continuous unitary evolution. At the same working level, a compatible positive complex structure is fixed on the space of initial data, after which the one-particle space, the Schrödinger equation, the Born rule, the POVM description of measurements, and the local continuity equation for the probability current are constructed.

It is further shown that, for free effective sectors, transfer locality, the constraint on the principal symbol, and consistency with the previously reconstructed observable SR structure lead to the standard first- and second-order relativistic closures. Thus, what emerges within the model is not a full interacting local quantum field theory in all its physical completeness, but rather the quantum core of the effective description together with its relativistic closure in operationally accessible regions where the locally inertial SR regime remains valid. As established in the preceding GR reconstruction, strong-field regimes of gravity and effective fields are excluded in such regions; their occurrence is possible only in operationally inaccessible regions, where effective reconstruction in terms of foliations and effective fields must give way to the fundamental description in terms of the field Φ. A substantial structural consequence is the classical status of the gravitational sector: quantization applies to effective nongravitational fields, whereas gravity does not generate an independent local spin 2 quantum sector. The obtained result continues the program of reconstructing effective physics from a timeless Euclidean model, within which observable SR kinematics and the geometric GR sector were reconstructed previously.

1 Introduction

Below we formulate the basic setup, motivation, and goals of the present work, which is devoted to the reconstruction of the quantum layer of the effective description in a timeless Euclidean model.

1.1 Background and Motivation

The present work is based on a timeless Euclidean model on the four-dimensional Euclidean space $\mathbb{E}^4$ with fixed metric δ_{AB}, in which the fundamental object is a real scalar field

$$\Phi : \mathbb{E}^4 \to \mathbb{R}, \qquad \Delta_{\mathbb{E}^4}\Phi = 0, \qquad \Delta_{\mathbb{E}^4} := \delta^{AB}\partial_A\partial_B.$$

At this level, neither global time, nor a causal structure, nor a space of events is postulated. A physically meaningful description arises only after the introduction of a localized observer, a working region $\Omega \subset \mathbb{E}^4$, foliations $\{\Sigma_s\}$, and operational procedures of causal reconstruction. It is precisely in this sense that time, events, observables, and effective fields have in the model not a fundamental but a reconstructed status (see §2).

In the present work, the operational formulation means that physical structures are defined through local observation procedures and the statistics of measurement outcomes in the observer's working region. The fundamental level remains static and timeless, whereas the effective description is constructed relative to a chosen foliation and a local algebra of observables on the slices Σ_s. What is essential here is not the entire fundamental configuration Φ as such, but only those of its operationally accessible regions in which a consistent description in terms of foliations, event reconstruction, and effective fields remains valid. It is precisely within this class of regions that the present work investigates how the quantum layer of the effective theory emerges from the already reconstructed causal and geometric structure.

The present article builds on two previous results. In [1], it was shown that, from the Euclidean model with the operational introduction of an observer, foliations, and event reconstruction, the structure of special relativity emerges: effective Lorentzian kinematics, an invariant limiting speed, and Lorentz transformations between inertial reference frames. In [2], this scheme was generalized to the case of a variable effective metric $g_{\mu\nu}$, which led to the reconstruction of the geometric sector of general relativity and of the Einstein equations as effective equations for the observable geometry. In the present work, these results are used as an already established kinematic-geometric foundation.

An important clarification is that the SR regime in the present work does not mean the absence of gravity and is not introduced as an external simplification. According to the previous GR reconstruction, the gravitational sector already determines the effective geometry of foliations and the hierarchy of scales in which the characteristic lengths of effective fields must be small compared to the scale of variation of the foliation. It is precisely in this locally inertial regime that the operational reconstruction takes the SR form. Thus, the analysis of the quantum layer is carried out not in the absence of gravity, but on the background of the already reconstructed geometric structure in its working locally flat approximation.

At the same time, as established in [2], strong-field regimes of gravity are excluded in operationally accessible regions. They may arise only in operationally inaccessible regions, for example behind an event horizon, where the description in terms of foliations and effective fields is no longer operative and must give way to the fundamental description in terms of the field Φ. Similarly, strong-field regimes of the effective fields themselves are excluded in operationally accessible regions. If a corresponding configuration enters a strong regime, this means not a departure beyond the fundamental model, but a departure beyond its effective description in terms of foliations and effective fields. Therefore, in the present article the quantum layer is investigated precisely in those regions where the effective foliation-based description remains meaningful.

Within the timeless Euclidean framework, *effective fields* arise that describe the modal degrees of freedom on the slices Σ_s in the working region Ω; it is their operational reconstruction that constitutes the subject of the present investigation.

It is important to emphasize that, within the framework of the model under consideration, quantum theory does not appear as a fundamental level of description. At the fundamental level, the model is specified by the timeless Euclidean space $\mathbb{E}^4$ and the real field Φ, whereas the quantum formalism arises as an effective description of locally reconstructed degrees of freedom in an admissible operational regime.

Relation to alternative approaches. Most approaches to the foundations of quantum theory and quantum field theory take the Hilbert-space formalism, operator algebra, and the Born rule as primitive elements of the description. At the same time, there exist research programs in which the quantum formalism is treated as an emergent or effective layer of a deeper dynamics. These include, in particular, G. 't Hooft's cellular automaton interpretation, in which quantum theory is regarded as a means of describing an underlying deterministic dynamics, as well as S. Adler's trace dynamics, within which quantum mechanics is investigated as a candidate for the effective statistical description of a more primitive matrix-valued dynamics [3, 4]. In a related, though conceptually distinct, direction lie superdeterministic approaches, in which the violation of statistical independence is considered as a possible way of reconciling locality with the observed quantum correlations [5, 6]. On the other hand, the contemporary literature also contains other lines of inquiry in which the non-fundamental status of quantum theory or of spacetime geometry is associated, for example, with the role of entanglement in the construction of spacetime, or with causal-set approaches in which causal order and discreteness are taken as fundamental, while entropic and quantum-information-theoretic structures are introduced on that basis [7, 8, 9, 10]. The present work is not a reformulation of any of these approaches. Its distinctive feature is that the quantum layer is constructed as an effective reconstruction within the locally inertial SR regime previously obtained from the same timeless Euclidean model, and is then interpreted as part of the effective sector entering the previously reconstructed GR regime.

The aim of the present article is to show that, in operationally accessible regions where an effective description in terms of foliations and effective fields remains valid, operational reconstruction in the locally inertial SR regime leads to the quantum core of the effective theory. By this quantum core we mean the Hilbert space of states, a compatible complex structure, unitary evolution with respect to the foliation parameter, the Schrödinger equation, the Born rule, the POVM description of measurements, and the relativistic closure of free effective sectors. Thus, the task of the article is not to construct a full interacting local quantum field theory in all its physical completeness, but rather to derive the minimal quantum layer compatible with the previously established SR and GR structures of the model within the domain of their operational applicability.

In terms of the logic of the exposition, this is divided into the following subproblems:

1) **OS/GNS and the effective Hilbert space.** To show that, for the class of states under consideration, detailed balance and reflection positivity hold, which makes it possible, via the OS/GNS reconstruction, to construct the Hilbert space of states, a self-adjoint Hamiltonian $H \geq 0$, and unitary evolution with respect to the foliation parameter.

2) **Quantum dynamics on foliations.** To construct a compatible complex structure, derive the Schrödinger equation on the slices Σ_s, the local continuity equation for the probability current, and also obtain the POVM description of measurements and the Born rule.

3) **Relativistic closure of free sectors.** To show that the requirement of local Lorentz covariance in the SR regime, the constraint on the principal symbol, and microcausality fix the standard free relativistic closures for scalar, spinor, and vector effective fields.

4) **Classicality of the gravitational sector.** To clarify the status of gravity in the reconstructed quantum layer and to show that, within the framework of the construction under consideration, quantization applies to effective nongravitational fields, whereas gravity retains its classical geometric status and does not generate an independent local spin 2 quantum sector.

A more detailed presentation of those elements of the previous works that are essential for the present investigation is given in §2.

1.2 Logic of reconstructing the quantum layer

Working regime. In the present work, the working regime is understood not as an arbitrary restriction on the description and not as a choice among different independent "sectors" of reconstruction, but as such a regime of admissible reconstruction in which the requirements of stable causal reconstruction and the existence of a localized observer can be satisfied simultaneously. What is meant is a regime in which stable localization of the observer, cross-slice self-identity, finite registration, and the consistent reconstruction of event structure and causal order on the basis of local readouts are preserved.

It is precisely these requirements, together with the previously reconstructed SR and GR structures, that single out the locally inertial SR regime as the domain of applicability of the subsequent quantum analysis. In this sense, the quantum layer is considered here not as an arbitrary additional superstructure over effective fields, but as the natural closure of the regime in which the foliation-based description, local transfer, effective fields, and the hierarchy of scales $L_{\text{field}} \ll L_{\text{fol}}$ remain valid. In such a regime, the geometric sector determines the local Lorentzian structure, and the quantum reconstruction can be carried out without introducing an independent quantum sector of the metric.

Principle of construction. The task is not to postulate a ready-made quantum theory, but to reconstruct its minimal effective layer step by step from operationally defined fields on the slices Σ_s. In this sense, the structures used below are of two kinds: either those already reconstructed in previous works, or those introduced as working assumptions consistent with transfer locality, detailed balance, reflection positivity, and the relativistic causal structure of the regime under consideration. The standard quantum-theoretical constructions employed below are thus used not as independent postulates, but as forms of representation of those objects that arise as a result of this reconstruction.

Methodological status of the derivation. After the previous SR/GR reconstruction, including the identification of the hierarchy of scales and of the admissible locally inertial SR regime, the subsequent logic of construction relies to a considerable extent on well-known results of mathematical physics and local quantum field theory. Accordingly, the novelty of the present work lies not in rediscovering the standard quantum constructions themselves, but in showing that, within the timeless Euclidean model under consideration, it is precisely these constructions that arise as the natural continuation of the already reconstructed causal and geometric structure. In other words, the main result of the article consists in constructing a consistent bridge between the specific operational features of the model and the standard quantum formalism.

Structure of the derivation. Technically, the construction is organized as follows. First, detailed balance and reflection positivity are established for the class of states under consideration on the working region Ω, after which the Hilbert space of states, a self-adjoint Hamiltonian $H \geq 0$, and unitary evolution on the slices Σ_s are constructed via the OS/GNS reconstruction (see §3). Next, a compatible complex structure is introduced, the Schrödinger equation on foliations and the local continuity equation for the probability current are derived, and the POVM description of measurements and the Born rule are obtained (see §4 and §3.10). After that, the requirement of local Lorentz covariance in the SR regime, the constraint on the principal symbol, and microcausality fix the standard free relativistic closures for scalar, spinor, and vector effective fields (see §5).

Scope of the result. In what follows, the quantum description emerging from the model is understood precisely as this reconstructed quantum core: the Hilbert space, the complex structure, unitary evolution, the Schrödinger equation, the Born rule, the POVM description of measurements, and the relativistic closure of free effective sectors. The present work does not aim at a complete derivation of the internal gauge structure, of a full interacting local quantum field theory, or of the Standard Model; these questions lie beyond the level of reconstruction considered here.

On the status of the assumptions used. A fully mathematically rigorous derivation of the entire quantum layer from the fundamental formulation of the model constitutes an independent and substantial task that lies beyond the scope of the present work. Therefore, in the present article, along with results already derived from the structure of the model and from the previous SR/GR reconstructions, additional working assumptions are also employed. These assumptions are not introduced arbitrarily: they are chosen so as to be consistent with the timeless fundamental formulation, with operational causal reconstruction, with the existence of a stable observer, and with the locally inertial free regime in which the quantum layer is constructed.

The present work does not claim that each of these assumptions has already been proved to be a necessary consequence of the model in full generality. It is possible that some of them will later turn out to be derivable from a more detailed analysis, whereas others may reflect features of only certain admissible reconstruction classes (see 2.8).

Thus, the task of the present article is not to close all questions concerning the origin of the assumptions used, but to construct a consistent quantum layer within the already identified admissible regime of the model. A more detailed analysis of the status of these assumptions and of the degree of their inevitability is postponed for separate consideration.

On the status of the reconstruction class in the present work. The present article does not claim that the construction of the quantum core pertains only to one unique admissible reconstruction class. The working conditions used below may, in general, be satisfied in more than one admissible reconstruction class, although their full status in this respect is not conclusively established in the present work. At the same time, certain technical constructions — in particular, inter-observer matching, the introduction of J_{sens}, and the derivation of J_{sens}-reciprocity of transfer — are formulated within one fixed reconstruction class, since they require a single coherent effective description. It is in this sense that one should distinguish below between local derivations within a fixed reconstruction class and the more general question of in which admissible reconstruction classes the quantum core constructed here is realized as a whole.

1.3 Aim and Content of the Work

The aim of the present work is to implement the operational reconstruction of effective fields described above within the timeless Euclidean model on $\mathbb{E}^4$ and to show that, in the SR regime, it leads to the quantum core of the effective description on the working region Ω of a localized observer. Here, the quantum core is understood to consist of the Hilbert space of states, a compatible complex structure, unitary evolution with respect to the foliation parameter, the Schrödinger equation, the Born rule, the POVM description of measurements, and the relativistic closure of free effective sectors. Thus, the task of the article is not to construct a full interacting local quantum field theory in all its physical completeness, but to derive that minimal quantum layer which is compatible with the previously established SR and GR structures of the model.

The main results of the work may be stated as follows:

(a) For the selected class of states on Ω, reflection positivity is established and the OS/GNS reconstruction of the Hilbert space of states and of a self-adjoint Hamiltonian $H \geq 0$ is carried out; thereby, the quantum kinematics of the local effective description is obtained (see §3.4).

(b) A compatible complex structure and dynamics on foliations are constructed, yielding a strongly continuous unitary evolution in the SR regime; on this basis, the Schrödinger equation, the continuity equation, the Born rule, and the description of measurements in terms of POVMs are derived (see §3.8, §3.10, §4).

(c) For free effective sectors, it is shown that transfer locality, the constraint on the principal symbol, and consistency with the observable SR structure lead to the standard first- and second-order relativistic closures; in this sense, the constructed quantum core is compatible with the relativistic causal structure (see §5).

(d) It is shown that, within the framework of the construction under consideration, quantization applies to the effective matter sector, whereas the gravitational sector retains its classical geometric status and does not generate an independent local spin 2 operator algebra (see §5.7, [2]).

The structure of the article is as follows. In §1, the motivation, assumptions, and relation to previous works are formulated, and the problem addressed in the present study is stated. In §2, the fundamental Euclidean model is specified, foliations and effective fields on the slices are introduced, and the requirements of causal reconstruction together with the operational definition of the observer and of events are formulated. Section §3 is devoted to the dynamics of transfer with respect to the parameter s and to its operational consequences: here, the conditions of locality and isotropy are formulated, the OS/GNS reconstruction of the state is carried out (§3.4), a complex structure is introduced (§3.8), and measurements together with the Born rule are discussed (§3.10). In §4, the Schrödinger equation on foliations is derived, and the local continuity equation for the probability current is formulated. Section §5 contains the relativistic closure: relativistic equations for scalar, spinor, and vector fields are derived, and the Peierls symplectic structure, microcausality, and the relation to spin and statistics are discussed. In §6, the discriminating prediction of the model associated with the status of the gravitational sector is formulated. In §7, the results and the limits of applicability of the obtained derivation are discussed, and the final conclusions are stated.

1.4 Status of the assumptions and results used

For convenience of reading, we immediately fix the status of those structures, conditions, and mathematical transitions that are used below. This makes it possible to distinguish explicitly between: (i) results obtained in previous articles of the program; (ii) conditionally rigorous consequences of the already identified working regime; (iii) the working mathematical assumptions of the present article proper; and (iv) standard mathematical constructions applied under these conditions.

(i) Results inherited from the SR reconstruction. From the previous work on SR reconstruction, the following elements are used: the observer-relative status of event structure, the absence of a global space of events, the distinction between direct and observable transformations, local causal reconstruction, as well as the locally inertial SR regime as the working form of effective description in operationally accessible regions.

(ii) Results inherited from the GR reconstruction. From the previous work on GR reconstruction, the following are used: the classical geometric status of the gravitational sector, the local Lorentzian structure in the working regime, the compensatory character of gravity, the hierarchy of scales $L_{\text{field}} \ll L_{\text{fol}}$, as well as the domain of applicability of the locally inertial effective description in terms of foliations and effective fields.

(iii) Conditionally rigorous consequences of the working regime. The present article uses a number of properties that are not introduced as independent fundamental postulates, but are regarded as consequences of the already identified working regime, even if they are not always proved here in the most general form. These include: stationarity of the working state at leading adiabatic order, preliminary microcausal compatibility of the local quantum layer with the observable SR structure, hyperbolicity of the free SR limit as a preliminary structural condition, as well as effective local closedness of the quantum sector on the working region $K \Subset \Omega$. All these properties are understood as arising from the combination of the locally inertial regime, the hierarchy of scales, stable local registration, cross-slice consistency, and the requirements of causal reconstruction.

(iv) Working mathematical assumptions of the present article. In the present article, a working class of regimes and states is fixed, within which the quantum layer of the effective description is constructed. In particular, the following are assumed: local effective closedness of the reconstructed sector on $K \Subset \Omega_s$, stationarity of the state with respect to transfer along the foliation parameter, the existence of a positive symmetric sensitivity form J_{sens} consistent with the working transfer and chosen to be stationary at leading order, quasi-freeness of the working class of states in the locally linear free regime, the use, in the leading stationary quasi-free approximation, of a two-point covariance proportional to J_{sens}^{-1}, as well as the existence of a compatible positive complex structure on the physical space of initial data. These conditions are treated as working mathematical assumptions of the present article; they are motivated in the course of the exposition, but are not derived in full generality from the fundamental model.

(v) Standard mathematical constructions used below. Under the conditions specified above, the following standard mathematical tools are applied below: the OS/GNS reconstruction, Stone's theorem, Green-hyperbolic causal structure, the Pauli–Jordan kernel, the Peierls form, the POVM representation of measurement statistics, and standard Noether identifications in free relativistic sectors. These constructions are used in the present work not as new independent mathematical results, but as standard apparatus for the analysis of the working quantum layer.

Overall status of the result. Thus, the aim of the present article is not a fully rigorous derivation of all elements of the quantum formalism directly from the fundamental equation without additional intermediate conditions, but rather to show that, under explicitly stated assumptions whose statuses are distinguished and which are consistent with the previous SR and GR reconstructions, the model gives rise to the quantum core of the effective description in operationally accessible regions.

On the programmatic status of the working conditions. The conditions collected in items (iii) and (iv) do not have the same logical status. Some of them are already treated in the present article as conditionally rigorous consequences of the working regime, whereas others still retain the status of working mathematical assumptions. However, neither the former nor the latter are understood as new fundamental postulates of the model. Within the further development of the theory, it is precisely these conditions that, according to the programmatic intent, are to pass from the status of working conditions to the status of theorems derivable from the structure of the fundamental field, the conditions for the existence of a localized observer, and the requirements of causal reconstruction.

2 The Fundamental Euclidean Model and Effective Fields

In the present section, no new independent assumptions are introduced and no new results are proved. Here we briefly fix those elements of the timeless Euclidean model, operational reconstruction, foliation-based description, and effective fields that were obtained previously in the works on SR and GR and are used below as the foundational basis of the present article.

2.1 The Laplace Equation and Admissible Configurations

Let $\mathbb{E}^4$ be equipped with the Euclidean metric δ_{AB} and global coordinates x^A, $A = 0,1,2,3$. The Laplacian is defined by $\Delta_{\mathbb{E}^4} := \delta^{AB}\partial_A\partial_B$. The fundamental field is a real function $\Phi : \mathbb{E}^4 \to \mathbb{R}$, $\Phi \in C^2$, satisfying

$$\Delta_{\mathbb{E}^4}\Phi(x) = 0 \quad \text{for all } x \in \mathbb{E}^4. \tag{1}$$

This is the *only fundamental equation of the model.* No fundamental variational functionals, sources, or nonlinearities for Φ are introduced.

In the present work, we simply fix this formulation as the fundamental model, following [1, 2], where the motivation for choosing the Laplace equation as the minimal local $O(4)$-invariant elliptic equation without a distinguished time is discussed in detail.

This equation contains no distinguished time and does not define any fundamental dynamics; at this level, no internal (gauge) symmetries and no distinguished directions in $\mathbb{E}^4$ are postulated beyond the geometric $O(4)$ symmetry of the Euclidean metric itself, and no interactions are introduced, whether linear or nonlinear.

The solutions of (1) form the class of harmonic field configurations Φ on $\mathbb{E}^4$. At the level of interpretation, each such configuration describes a complete "world" of the model: in the timeless formulation there are no independent initial conditions and no external evolution parameter, so the entire content of the description is contained in the choice of Φ, rather than in its "development in time".

In what follows, we shall not need to fix any specific solution. All constructions are formulated for an arbitrary configuration from a certain admissibility class. By *admissible configurations* here, as in [1, 2], we mean solutions of (1) that satisfy additional operational constraints, fixed below as part of the already established foundation of the model.

What interests us is not the explicit solutions of (1) as such, but the physical consequences of the imposed operational constraints — in particular, the structure of the emergent effective fields and their properties in interaction with a localized observer.

2.2 Requirements of Causal Reconstruction

We work on the Euclidean space $\mathbb{E}^4$ with a field Φ satisfying the Laplace equation on the class of admissible configurations (see §2.1). We fix a foliation by level hyperplanes

$$\Sigma_s^{(\mathbf{n})} = \{\, x \in \mathbb{E}^4 \mid n_A x^A = s \,\}, \qquad n_A n^A = 1,\ s \in \mathbb{R}, \tag{2}$$

where the choice of n_A fixes an inertial reference frame (IFR), and the parameter s plays the role of *operational time* relative to the given foliation.

Definition (causal reconstruction). For a fixed observer O and a foliation $\Sigma_s^{(\mathbf{n})}$, where the observer body occupies a region $\Omega_0 \subset \Sigma_s^{(\mathbf{n})}$, *causal reconstruction* means a procedure which, from local information about Φ in a region $\Omega \subset \Sigma_s^{(\mathbf{n})}$, constructs a consistent description of the set of events E_O and their ordering $\preceq_O$ relative to the transfer direction $\mathbf{n}$. The concrete mechanism for selecting events is specified below (see §2.6).

Requirement of causal reconstruction. The reconstruction in each IFR must satisfy:

(i) Localizability and transfer of modes. The decomposition of Φ into foliation modes is defined in such a way that the modes u_α are localized on $\Sigma_s^{(\mathbf{n})}$ and admit *local linear transfer* along s in the direction $\mathbf{n}$ for the coefficients $a_\alpha(s)$, with a finite limiting speed $v_{\max}$ of interactions.

(ii) Compatibility of the equation with transfer. The equation for Φ admits such local modes and preserves their dynamical consistency for any direction $\mathbf{n}$.

(iii) Consistency under small rotations. Under $\mathbf{n} \mapsto \mathbf{n}'$ with angle $\theta = \arccos(\mathbf{n}\cdot\mathbf{n}') \to 0$, the sets of events and their ordering coincide identically.

These conditions have an operational character: causality is not postulated, but arises as a criterion of admissibility of the reconstruction in the presence of a finite bound on the speed of interactions. They coincide exactly with the conditions of causal reconstruction used in [1] in the derivation of the SR structure, and are reproduced here without modification as part of the foundational basis of the present article.

Admissibility of configurations. Conditions (i)–(iii) may conveniently be reformulated as restrictions on the class of solutions Φ. Denote by $\mathcal{S} \subset \ker \Delta$ the set of those configurations for which causal reconstruction exists in the sense of the definition above, and which satisfy:

(a) Localizability of modes u_α on each $\Sigma_s^{(\mathbf{n})}$;

(b) Event-interpretability of interactions of the field modes with the modes of the observer body;

(c) Stability under small rotations of $\mathbf{n}$ (consistency of reconstruction as $\theta \to 0$).

2.3 Observer Localization and a Common Reference Basis

The observer is not an external agent: it is described as a localized structure in $\mathbb{E}^4$ within the field configuration $\Phi(x)$. We fix the foliation (2). We also fix a local region $\Omega \subset \Sigma_s^{(\mathbf{n})}$, compact in the three spatial directions, which defines the working region for the decomposition of the field into modes. It is precisely within this region that the operationally accessible description of events is formed (in physical terms, it may be regarded as an analogue of the observable part of the Universe). For the body of a particular observer, the region $\Omega_O \subset \Omega$ is used.

We fix an orthonormal family $\{u_\alpha(x)\}_{\alpha\in\Lambda} \subset L^2(\Sigma_s^{(\mathbf{n})})$ such that $\operatorname{supp} u_\alpha \subset \Omega$ for all α. The local configuration on $\Sigma_s^{(\mathbf{n})}$ is decomposed as

$$\Phi(x)\big|_\Omega = \sum_{\alpha\in\Lambda} a_\alpha(s)\, u_\alpha(x), \qquad a_\alpha(s) = \int_\Omega u_\alpha(x)\, \Phi(x)\, d^3x. \tag{3}$$

In each IFR, a *common reference basis* $\{u_\alpha\}$ is fixed for inter-observer consistency. The internal modes of the observer O are expressed as local linear combinations:

$$\chi_\beta(x) = \sum_{\alpha\in\Lambda} C^{(O)}_{\beta\alpha}\, u_\alpha(x), \qquad b_\beta(s) = \sum_{\alpha\in\Lambda} C^{(O)}_{\beta\alpha}\, a_\alpha(s), \tag{4}$$

where $\operatorname{supp} \chi_\beta \subset \Omega_O \subset \Omega$ for all β.

2.4 Transfer Invariance and the Emergence of Causality

Conditions (i)–(iii) imply the existence of a *local linear transfer* of the coefficients

$$a^{(\mathbf{n})}(s+ds) = A^{(\mathbf{n})}[\Phi; s]\, a^{(\mathbf{n})}(s), \tag{5}$$

where $A^{(\mathbf{n})}[\Phi; s]$ depends only on the local configuration Φ in a neighborhood of $\Omega \subset \Sigma_s^{(\mathbf{n})}$ (for small ds). Consistency under small rotations of the foliation requires *invariance* of the transfer law:

$$A_{\alpha\beta}^{(\mathbf{n}')}[\Phi;\, \cdot] \;\equiv\; A_{\alpha\beta}^{(\mathbf{n})}[\Phi;\, \cdot] \quad \text{on the subspace of modes operationally accessible in the region } \Omega, \tag{6}$$

since the basis on each slice is constructed anew according to the same rule $\mathcal{U}[\Phi; \mathbf{n}, s]$, rather than being transported by rotation.

Remark 2.1 (Interpretation of invariance)**.** *The rotation* $\mathbf{n} \to \mathbf{n}'$ *is a replacement of the family of slices* $\Sigma_s^{(\mathbf{n})} \to \Sigma_s^{(\mathbf{n}')}$. *The basis* $\{u_\alpha\}$ *on each slice is determined by the same rule* $\mathcal{U}$ *and is not transformed as a fixed set of functions; therefore,* (6) *expresses precisely the invariance of the transfer law, rather than the covariance of the set* $\{u_\alpha\}$.

Emergence of causality. In the adopted formulation, causality in each IFR arises as a condition of operational consistency: reconstruction of the causal structure is possible only if there exist a local decomposition and an invariant transfer (5)–(6) on the operationally accessible modes. In this sense, the direction of "time" and the causal structure are emergent and depend on the choice of foliation; at the fundamental level there is no global set of events.

2.5 Effective Fields and Their Established Properties

Let us recall the results obtained in [1, 2] without adding any new claims. Here we partly repeat the previous subsections and fix the set of properties of effective fields to which we shall refer below.

Fundamental model. We consider a timeless Euclidean formulation on $\mathbb{E}^4$ with a real field Φ satisfying $\Delta_{\mathbb{E}^4}\Phi = 0$ (see Section 2.1) on the class of admissible configurations defined by the observer's operational constraints.

Foliations, observer, and events. The slices $\{\Sigma_s\}$ (foliations) and the observer body $\Omega_0 \subset \Omega$ are defined; event structure has an observer-dependent character and is consistent between observers within the same IFR under communication [1]. There is no global set of events.

Effective fields on foliations. On the slices Σ_s, effective fields are introduced, obtained from the configuration Φ and depending on the chosen foliation; a distinction is made between direct and observable transformations between IFRs [1].

SR regime. Observable transformations have Lorentz form with a universal limiting speed $v_{\max}$; the Galilean limit is excluded. The causal structure is formed locally within each IFR [1].

GR regime. Upon abandoning flat foliations, an effective metric g emerges; gravity is interpreted as curvature of foliations and acts universally. Locally, gravity is equivalent to acceleration; the Einstein equations without a cosmological constant are obtained [2].

Hierarchy of scales, operational accessibility, and limits of applicability. As established in [2], the effective description in terms of foliations, the observer, and effective fields applies not to the entire fundamental configuration Φ as such, but only in operationally accessible regions where consistent causal reconstruction is preserved and inter-observer consistency of the description is possible. In the terminology of the previous works, causal reconstruction is formulated on the working region Ω, whereas the interior region $K \Subset \Omega$, separated from $\partial\Omega$, is used to suppress boundary effects of the elliptic formulation and to localize the observer's operationally accessible modes. In the SR paper, this corresponded to the introduction of an intermediate scale $\mathcal{K}$ satisfying

$$\Omega_0 \ll \mathcal{K} \ll \Omega,$$

whereas in the GR paper the interior region K was used to formulate the hierarchy between the characteristic length of effective fields L_{field} and the scale of variation of the foliation L_{fol}.

In these regions, a hierarchy of scales arises between the gravitational sector and the effective nongravitational fields. If the characteristic lengths of the effective fields are small compared to the scale of variation of the foliation, then the locally inertial regime is realized, in which the effective geometry takes SR form at working order. It is precisely this regime that is used in the present article in the reconstruction of the quantum layer of the effective theory. At the same time, strong-field regimes of gravity are excluded in operationally accessible regions. They may arise only in operationally inaccessible regions, for example behind an event horizon, where the description in terms of foliations and effective fields is no longer operative and must give way to the fundamental description directly in terms of the field Φ.

Similarly, strong-field regimes of the effective fields themselves are excluded in operationally accessible regions. If a corresponding configuration enters a strong regime, this means not a departure beyond the fundamental model, but a departure beyond its effective description in terms of foliations and effective fields. Therefore, in the subsequent sections quantum fields are considered only within the domain of applicability where the effective description in terms of foliations remains meaningful and the locally inertial SR approximation is consistent with the previously reconstructed geometric sector.

Field decomposition on the foliation and the evolution equation. Let $\Sigma_s^{(n)}$ be a foliation slice with normal n, $\Omega_s := \Omega \cap \Sigma_s^{(n)}$, and $\mathrm{d}\mu_{\Sigma_s^{(n)}}$ the induced measure (in the SR regime, the standard measure d^3y in orthonormal coordinates on $\Sigma_s^{(n)}$). For an admissible local orthonormal basis $\{u_\alpha(s,\cdot)\}_{\alpha\in\Lambda} \subset L^2(\Omega_s, \mathrm{d}\mu_{\Sigma_s^{(n)}})$, the mode coefficients of the fundamental field are defined by

$$a_\alpha^{(n)}(s) = \int_{\Omega_s} u_\alpha(s,y)\,\Phi(y)\,\mathrm{d}\mu_{\Sigma_s^{(n)}}(y), \qquad \alpha \in \Lambda, \tag{7}$$

where $a^{(n)}(s) := (a_\alpha^{(n)}(s))_{\alpha\in\Lambda}$.

As shown in [1], transfer along the foliation has the local form

$$a^{(n)}(s+ds) = A^{(n)}(s;ds)\,a^{(n)}(s), \qquad A^{(n)}(s;0) = \mathbf{1}, \quad \Gamma^{(n)}(s) := \partial_{ds}A^{(n)}(s;ds)\big|_{ds=0}, \quad A^{(n)}(s;ds) = \mathbf{1}+\mathrm{I} \tag{8}$$

At the same time, the operator $A^{(n)}(s;ds)$ is determined solely by the local field configuration Φ in a neighborhood of Ω_s. It is precisely this local evolution of the mode coefficients that serves below as the starting point for the introduction of effective fields and the construction of their quantum description.

The condition of consistency of reconstruction means that the basis $\{u_\alpha\}$ is constructed according to one and the same rule $\mathcal{U}[\Phi;\mathbf{n},s]$, ensuring localizability, event interpretability, and continuity under rotation of the foliation.

Observer. In each IFR, a common reference orthonormal basis on Σ_s is fixed; the internal modes of the observer are consistent with this basis (inter-observer consistency).

Effective fields as operational degrees of freedom. We work in the SR regime; on the slice Σ_s we use internal coordinates y. Let $a^{(n)}(s) := (a^{(n)}_\alpha(s))_{\alpha\in\Lambda}$ and $W(s,y) = [W_I{}^\alpha(s,y)]$ be a locally invertible (in y) map from modes to effective fields on the working subspace. Define

$$\psi_I(s,y) = W_I{}^\alpha(s,y)\,a^{(n)}_\alpha(s). \tag{9}$$

Introduce the exact finite-step transfer (propagator) along the foliation:

$$a^{(n)}(s+ds) = P^{(n)}(s+ds,s)\,a^{(n)}(s), \qquad P^{(n)}(s,s) = \mathbf{1}, \quad P^{(n)}(s_2,s_1)P^{(n)}(s_1,s_0) = P^{(n)}(s_2,s_0). \tag{10}$$

Then the evolution of the effective fields is written as

$$\psi(s+ds,y) = \mathcal{U}(s,y;ds)\,\psi(s,y), \qquad \mathcal{U}(s,y;ds) := W(s+ds,y)\,P^{(n)}(s+ds,s)\,W(s,y)^{-1}. \tag{11}$$

The equivalence of the descriptions in terms of a_α and ψ_I corresponds to the formulation adopted in [2].

Remark 2.2. *The present subsection fixes the basic facts used below. New properties of effective fields are derived in the following sections.*

2.6 Observer and Events

The present subsection reproduces almost entirely the definitions and constructions introduced in [1]. No new independent assumptions, no new definitions, and no new results are introduced here; what follows merely fixes in compact form those elements of the notions of the observer, event structure, and inter-observer consistency that are used directly in what follows.

Before defining what an event is in the model, let us consider some issues associated with this notion and fix the assumptions required for the subsequent construction.

Real detectors, spacecraft, and the like are located on scales that do not exceed the size of the Solar System, which is negligibly small compared to the size of the observable part of the Universe. Using this analogy, within a single inertial frame of reference (IFR) we shall assume the working region Ω to be the same for all observers at rest in that IFR. Henceforth, we assume that in each IFR the observable space Ω—the region in which causal reconstruction is possible—is the same for all observers at rest with respect to that IFR.

In contemporary formulations of quantum theory, there is no universally accepted observer-independent definition of an *event*: different interpretations of quantum mechanics treat the moment of the occurrence of an event differently (collapse, detector registration, state update, etc.). In the model under consideration, all observable phenomena depend on the observer and on the observer's measurements. In this subsection, an operational notion of an event is introduced, which is proposed to be regarded as universal within the chosen model: it is consistent with the previously constructed causal structure and with the subsequent quantum-operational description (OS/GNS, POVM).

When formulating the notion of an *event*, the following points are important:

(i) Operational origin. An event must arise as a result of the interaction of the observer with the field, rather than as a pre-given ontological entity. An event is understood as a configuration of interaction between field modes and the internal modes of the observer that leads to a discrete update of the observer's internal state (a record in memory registers), registered by the observer itself. The observer directly registers only a local measurement outcome, but on its basis reconstructs the network of causes and events which, according to the reconstruction, led to this measurement. This network is a reconstruction rather than an independently existing object. Accordingly, an event may be interpreted as having occurred far beyond the observer body: by observing a quantum of light from a star, the observer reconstructs the network of events that led to its emission.

(ii) Consistency between descriptions at different scales. Since the observer has finite extent and spectral limitations, the interaction with the field is described by projection onto a finite-dimensional subspace. Upon passing to a coarser description (combining modes into effective combinations), the detector functional is rewritten in terms of the new coefficients, but the registration criterion (the threshold condition) is preserved. The definition of an event must be stable under such coarsening or refinement of the mode decomposition, that is, it must not depend on the chosen precision of the modal description.

(iii) Compatibility with operational quantum theory. The definition of an event must be consistent with the operational scheme used below: states on the algebra of observables, OS/GNS reconstruction of the Hilbert space, description of measurements by means of POVMs, and the Born rule. In the present work, events will be treated as structural elements of a causal network, while measurement acts will be treated as their local sources, realized through detector functionals.

(iv) Inter-observer consistency. We shall consider a regime in which, upon exchange of information between observers at rest in a given IFR, their reconstructed sets of events coincide (in the sense of an isomorphism of partially ordered sets). This does not give rise to a global space of events unifying different IFRs: each IFR has its own event network. In the author's previous works, this regime was referred to as "classical." In the present work, in order to avoid confusion with the classical limit of quantum theory, we shall call it the *regime of inter-observer event consistency* and shall assume that the configurations under consideration belong to this regime.

In quantum theories, events are usually associated with particles whose properties are determined by gauge symmetries. In the model under consideration, gauge symmetries are derived at the effective level and are not used for the fundamental definition of an event. We shall assume that, in a measurement, the observer registers the consequences of some event structure in the region Ω, for which causal reconstruction is possible. Accordingly, a causal network arises in observation. In the regime of inter-observer event consistency, we pass from events depending on a particular observer to events common to all observers at rest in a given IFR: an analogue of a discrete causal network arises, specific to each IFR.

Definition of measurement. Let an observer O fix a foliation Σ_s, a subspace

$$\mathcal{H}^{(O)}_{\text{field}} = \operatorname{span}\{u_\alpha : \alpha \in \Lambda_O\} \subset L^2(\Sigma_s),$$

where $\Lambda_O \subset \Lambda$, and a set of its internal modes $\{\chi_\beta\}$. Its *detector (readout) functional* is defined by a local scalar functional

$$\mathcal{R}_O(s) \;=\; F_O\big(\mathbf{a}(s), \mathbf{b}(s)\big), \tag{12}$$

where $\mathbf{a}(s) = (a_\alpha(s))_{\alpha\in\Lambda_O}$ and $\mathbf{b}(s) = (b_\beta(s))$ are the coefficients of the corresponding decompositions on Σ_s.

A measurement act $\mathsf{M}_O(s_0)$ is said to occur at the moment s_0 at which

$$\mathcal{R}_O(s_0) \;\geq\; I^{(O)}_{\text{thr}}, \tag{13}$$

where $I^{(O)}_{\text{thr}} > 0$ is the sensitivity threshold. When this condition is satisfied, one of the binary memory registers m_j switches discretely from $0 \to 1$. The measurement act is localized in the observer-body region $\Omega_O \subset \Omega$.

Remark 2.3. *Refinements of the criterion are possible (for example, by adding extremum or smoothing conditions), but for the purposes of the present section the threshold condition* (13) *is sufficient.*

Such a functional may be understood as an analogue of a photoelement: if a combination of signals exceeds the sensitivity threshold, the detector "clicks," recording a unit in memory.

Example (bilinear functional). As a particular case of (12), one may use the bilinear form

$$\mathcal{M}_O(s) = \sum_{\alpha,\beta} \rho^{(O)}_{\alpha\beta}\, a_\alpha(s)\, b_\beta(s), \tag{14}$$

where $\rho^{(O)}$ is a sensitivity matrix; then $\mathcal{R}_O = \mathcal{M}_O$.

Separation of notions. We shall distinguish a *local measurement act* (a detector click inside the observer body Ω_O) from an *event* as an element of the causal network in the working region Ω ($\Omega_O \subset \Omega$). Local measurement acts satisfying (13) generate a subset of the observable vertices of the future network.

Definition 2.4 (Event and causal network in the model)**.** *An* event *is a vertex* $E \in V_{\text{obs}} \cup V_{\text{rec}}$ *of the causal network* $\mathcal{C}_{\mathbf{n}} = (V, \prec)$ *in the working region* Ω*, where:*

- $V_{\text{obs}} = \{\mathsf{M}_O(s_k)\} \subset \Omega_O$ *are observable (local) vertices generated by the measurement acts* (13)*;*

- $V_{\mathrm{rec}} \subset \Omega$ *are reconstructed vertices for which there exists an operational causal relation with at least one* $\mathsf{M}_O(s_k)$*, consistent with the admissible action of the transfer operator* (5)*.*

The order $\prec$ *is interpreted as the relation "can influence," is defined within a fixed IFR, and does not require a global set of events common to all IFRs. Henceforth, by an event we shall mean a vertex of such a causal network.*

Remark. The measurement act $\mathsf{M}_O(s)$ is a local registration trigger (a vertex of V_{obs}), whereas an "event" in the general sense is an element of the network $\mathcal{C}_{\mathbf{n}}$, including both local measurements and reconstructed vertices V_{rec}. Thus, events outside the observer body are not identified with measurements, but enter as reconstructed elements obtained from measurement data and reconstruction rules.

Regime of inter-observer event consistency. We shall say that the regime of inter-observer event consistency is realized in a given IFR if, for any two observers at rest in that IFR and exchanging information, their individual reconstructed causal networks $\mathcal{C}_{\mathbf{n}}^{(O_1)}$ and $\mathcal{C}_{\mathbf{n}}^{(O_2)}$ are isomorphic: there exists a bijection of vertices preserving the partial order $\prec$, such that the combined network is again a partially ordered set consistent with the local reconstructions of each observer. In this regime, one may speak of a single event structure $\mathcal{C}_{\mathbf{n}}$ in the given IFR (up to relabeling of vertices). In the author's previous works, the analogous assumption was referred to as the "classical regime." Below, unless otherwise stated, we shall assume that the configurations under consideration are in the regime of inter-observer event consistency. Upon changing the IFR, no global unified space of events arises.

Interpretation. Thus, an event is not reduced to an ontological "point in spacetime," but is understood as an element of a discrete causal network arising from the interaction of the field and the observer. The difference from traditional models such as causal sets lies in the fact that here each pair (observer, IFR) is initially associated with its own causal network $\mathcal{C}_{\mathbf{n}}^{(O)}$, constructed from measurement acts and reconstructions. In the regime of inter-observer event consistency, the networks of observers at rest in one IFR and exchanging information are isomorphic and reduce to a common structure $\mathcal{C}_{\mathbf{n}}$ (up to relabeling of vertices). Upon changing the IFR, such global identifications in general do not exist, and we do not assume a single space of events for all IFRs.

Remark 2.5 (On the pointlike character of events)**.** *As shown in [1], the pointlike character of an event in the operational description does not require the corresponding physical act to be fundamentally pointlike or instantaneous. Therefore, delocalization of the registration process does not by itself contradict the fact that, in the effective description, an event is represented as pointlike: pointlikeness here refers not to the fundamental process at the level of the field* Φ*, but to the form of its operational reconstruction in the observable spacetime description.*

Remark 2.6 (The observer as part of the event structure)**.** *The observer may be described not only through its modal state, but also as part of the event structure itself: its configuration singles out a subset of events accessible for reconstruction in a given IFR. Unlike standard approaches in theories of the causal-sets type, here the set of events depends on the chosen foliation, and when the foliation is changed, the subset of accessible events is reconfigured.*

Observer as a localized subsystem. As established in [1], the observer in the model is understood as a localized subsystem of the fundamental field, possessing a working region Ω, an internal region $\Omega_0 \subset \Omega$, and access to the operational reconstruction of events on the foliation slices. In the present section, this notion is merely fixed in the previously introduced form; additional conditions related to the measurement regime and the statistics of outcomes will be formulated later in connection with the quantum description of measurements.

2.7 Direct and Observable Transformations

Direct and observable transformations. As shown in [1], in the absence of a global set of events, the passage between IFRs with normals $\mathbf{n}$ and $\mathbf{n}'$ admits two descriptions [1]. As shown earlier, events exist only relative to an observer; without reference to an observer, there are no events. In the regime of inter-observer event consistency, the dependence on the observer may be neglected.

(i) Direct transformations act by Euclidean symmetry on the field configuration and the foliation:

$$D_{\mathbf{n}\to\mathbf{n}'} :\ \mathcal{C}_{\mathbf{n}}^{(O)} \longmapsto \mathcal{C}_{\mathbf{n}'}^{(O')}, \tag{15}$$

where $\mathcal{C}_{\mathbf{n}}^{(O)}$ is the event network reconstructed on the slices $\Sigma_s^{(\mathbf{n})}$ relative to the observer O, and $\mathcal{C}_{\mathbf{n}'}^{(O')}$ is the corresponding network relative to O'. There is no bijection between $\mathcal{C}_{\mathbf{n}}^{(O)}$ and $\mathcal{C}_{\mathbf{n}'}^{(O')}$. The notation with superscripts emphasizes the operational character of the reconstruction; the mapping $D_{\mathbf{n}\to\mathbf{n}'}$ itself does not require the choice of a specific observer. In the present work, direct transformations are not used further; it is sufficient that observable transformations (see below) have Lorentz form.

(ii) Observable transformations are hypothetical transformations constructed by an observer-physicist while remaining in the observer's own IFR and assuming *as if* there existed a global set of events. By construction, they preserve event structure and in general depend on the observer O:

$$\mathcal{O}_{\mathbf{n}\to\mathbf{n}'}^{(O)} :\ \mathbf{b}^{(O,\mathbf{n})}(s) \ \mapsto\ \mathbf{b}^{(O,\mathbf{n}')}(s), \tag{16}$$

where $\mathbf{b}^{(O,\mathbf{n})}$ is the internal register of events (see §2.6). In the *regime of inter-observer event consistency*, the dependence on O disappears, and one obtains the universal operator

$$M_{\mathbf{n}\to\mathbf{n}'} :\ (t,\mathbf{r}) \ \mapsto\ (t',\mathbf{r}'). \tag{17}$$

Consequence (SR regime). As shown in [1], the observable transformations (17) have Lorentz form with invariant $v_{\max}$; the Galilean limit is excluded. Equivalently, for any $\Delta x \neq 0$, preservation of the null cone holds:

$$(\Delta x)^A \eta_{AB} (\Delta x)^B = 0 \implies (\Delta x')^A \eta_{AB} (\Delta x')^B = 0.$$

Consistency between observers in the same IFR. Let O and O' belong to the same IFR and be able to exchange classical messages (within the null cone of the SR regime). Denote by $\Omega_\cap$ the common part of their accessible regions. Then there exists an isomorphism of event structures

$$\Phi_{O\leftrightarrow O'} :\ \mathsf{E}_O\!\restriction_{\Omega_\cap} \longrightarrow \mathsf{E}_{O'}\!\restriction_{\Omega_\cap},$$

compatible with observable transformations and causal reconstruction (see [1], the section on communication and consistency of events). In the absence of communication, no such consistency is required, and the sets E_O, $\mathsf{E}_{O'}$ may differ.

Remark 2.7 (Distinction of terms)**.** *The operational* observable $\mathcal{O}_f^O$ *is a measurement instrument (after OS/GNS, an operator or a POVM). An* event *is an element of* E_O*, that is, a stable local maximum of* $\mathcal{S}_O$ *(measured or reconstructed), defined relative to a specific observer* O*.*

In all subsequent sections, Lorentz symmetry and the related invariants refer exclusively to the observable transformations $M_{\mathbf{n}\to\mathbf{n}'}$ in the SR regime; the direct transformations $D_{\mathbf{n}\to\mathbf{n}'}$ between IFRs are not required to preserve these structures and are constrained only by the conditions of consistency of causal reconstruction.

2.8 Reconstruction Classes Considered in the Present Work

As shown in [1], the fundamental Euclidean formulation admits not a unique way of reconstructing observable structures, but rather a plurality of admissible reconstructions, in which local configurations of the field Φ may give rise to a stable operational reconstruction of event structure, causal order, and effective degrees of freedom. Thus, one and the same fundamental model does not determine *a priori* a unique observable description.

To each admissible reconstruction there corresponds its reconstruction class in the sense used in [1]. It is precisely the reconstruction class that fixes the structural characteristics of the given reconstruction, such as the structure of the modal decomposition, the choice of basis, the set of emergent effective fields, and their kinematic properties. In particular, the presence of an effective field admitting propagation with the limiting speed $v_{\max}$ may either follow from general requirements for the existence of an observer and then be a common property of all reconstruction classes, or characterize only some of them. In the latter case, it must be regarded as a characteristic of the corresponding reconstruction class; if, on the other hand, it is a common characteristic of several classes, then the distinction between them must be determined by other parameters.

In the present work, this language is used as part of the general framework of the effective description. What follows makes use both of conditions that are common to broad classes of reconstruction and of conditions that pertain only to a part of them. Accordingly, not every condition introduced below should be understood as a universal property of all admissible reconstructions of the fundamental Euclidean model. A more detailed analysis in the future may allow one to determine the status of such conditions more precisely.

3 Transfer Dynamics and Operational Consequences

The present section considers those properties of transfer along the foliation that are used below in constructing the quantum layer of the effective description.

No new fundamental postulates are introduced here. What follows merely makes explicit the additional effective conditions and approximations pertaining to the working regime of quantum reconstruction introduced above. They are motivated by the requirements of stable local registration, cross-slice consistency, and causal reconstruction, rather than being introduced as arbitrary external assumptions.

3.1 Domain of Applicability and the Structural Conditions Used

In the present section, the quantum reconstruction of effective fields is considered in operationally accessible regions where the description in terms of foliations and effective fields remains valid, and where the previous GR reconstruction singles out the locally inertial SR regime as the domain of applicability of the quantum analysis in the sense of the working regime specified in the Introduction. No new independent postulates are introduced here: only those structural properties are used that have already been fixed earlier, namely transfer locality, causal reconstruction, the observable SR structure, the hierarchy of scales, and the admissibility of the effective foliation-based description.

At this level, a local region $K \Subset \Omega_s$ is fixed in which, owing to the hierarchy of scales $L_{\text{field}} \ll L_{\text{fol}}$ and the adiabaticity of foliations, the locally inertial SR regime is realized. It is shown below that in such a regime the transfer of effective fields is local and linear, the stabilizer of the slice has an $O(3)$-structure, and, under the structural restriction inherited from the GR reconstruction according to which the local generators of effective fields do not contain spatial derivatives of order higher than two, the principal symbol of the local generator is of second order. In addition, only the technical conditions of regularity of the coefficients and of effective closedness of the quantum sector on K are fixed: the flux through ∂K is negligibly small at the order under consideration, so that the local dynamics can be represented by a self-adjoint generator H. It is precisely in this locally inertial effective regime that detailed balance, the OS/GNS reconstruction, a compatible complex structure, the POVM description of measurements, the Schrödinger equation, and the relativistic closure of free sectors are formulated below.

3.2 Locality and Linearity of Transfer

We now pass from the exact local evolution of effective fields fixed in §2 to its first-order representation in ds, which will be used below in the construction of the effective quantum dynamics. From (11) it follows that the local linearization is

$$\psi(s{+}ds, y) = \mathcal{U}(s, y; ds)\, \psi(s, y) {+} \mathcal{O}(ds^2, \partial_y), \qquad \mathcal{U}(s, y; ds) = \mathbf{1} {+} \Gamma(s, y)\, ds {+} \mathcal{O}(ds^2), \qquad \Gamma(s, y) := \partial_{ds}\mathcal{U} \tag{18}$$

Here $\mathcal{O}(ds^2, \partial_y)$ is understood locally in y and uniformly in s on compact sets, while Γ is a transfer generator that is local in y and C^1 in s. At the modal level, this representation is consistent with (8): under the replacement $a \mapsto \psi = Wa$, the modal transfer induces the corresponding local generator Γ for the effective fields.

Lemma 3.1 (Locality and linearity of transfer on Σ_s)**.** *At order ε^0, the transfer of effective fields along the foliation is local and linear:*

$$\psi(s{+}ds, y) = \mathcal{U}(s, y; ds)\, \psi(s, y) + \mathcal{O}(ds^2, \partial_y), \qquad \mathcal{U}(s, y; ds) = \mathbf{1} + \Gamma(s, y)\, ds + \mathcal{O}(ds^2).$$

Proof. The statement follows directly from the local form of the exact evolution (11) and its first-order expansion in ds, written in (18). The locality of transfer is inherited from the dependence of $\mathcal{U}(s, y; ds)$ only on the local field configuration in a neighborhood of Ω_s, whereas linearity follows from the linearity of the action of the transfer operator on the effective fields. □

This property fixes the working class of local transfers on which transfer reciprocity, detailed balance, and the conditions of the OS/GNS reconstruction are formulated below.

3.3 Transfer Reciprocity and Detailed Balance

We rely on the locality and linearity of transfer formulated in Lemma 3.1, as well as on the consistent construction of the modal basis (see §2.5 and [1]). The previously introduced notation for the local transfer $\mathcal{U}(s, y; ds)$ and its generator $\Gamma(s, y)$ is retained below. In addition, we introduce the semigroup $T_\sigma = e^{\sigma G}$ in the modal space and the semigroup α_σ on the local *-algebra of observables induced by this transfer.

On the working state and the stationary approximation. The passage from local transfer to detailed balance and the subsequent OS/GNS reconstruction requires one to fix a class of working states compatible with stable local registration and cross-slice consistency of the observable description. The stationarity of the state ω used below is not introduced as an independent external postulate. Rather, it expresses the fact that, in the admissible locally inertial regime, there must exist observers with registers that remain stable for long periods and with stable readout procedures. Without this, the observable events V_{obs} could not serve as the basis of causal reconstruction.

Therefore, in the present work we use the leading stationary approximation, in which the state ω is regarded as invariant with respect to the semigroup α_σ:

$$\omega\big(\alpha_\sigma(A)\big) = \omega(A), \qquad \sigma \geq 0,$$

for local observables A from the algebra under consideration. Possible small deviations from exact stationarity are not excluded in principle, but within the framework of the present article they are regarded as subleading. Their analysis is connected with the accuracy of causal reconstruction and the accuracy of the local quantum approximation and requires separate consideration.

On the Gaussian working state. The Gaussian character of the state ω used below is likewise not introduced arbitrarily. In the locally inertial regime under consideration, the transfer of effective fields is local and linear, while the leading contribution to the generators of the corresponding free sectors is quadratic. Accordingly, the natural working class of states is formed by quasi-free (Gaussian) states, for which the correlation structure is already determined by the two-point function. In the present work, it is precisely this leading Gaussian approximation, corresponding to the free locally linear regime of the reconstructed quantum layer, that is used below.

Lemma 3.2 (Joint modal basis and local transfer)**.** *([1]) Let a locally complete orthonormal basis of admissible modes of the fundamental field $\{u_\alpha(y)\}$ be given on the slice Σ_s, and let the internal modes of the observer be specified by linear functionals on* $\text{span}\{u_\alpha\}$*. Then, for the modal coefficients of the field $a_\alpha(s)$ and the coefficients of the internal modes of the observer $b_\beta(s)$, there exists a matrix generator $G(s)$, induced by the local transfer and C^1 in s, such that at leading order in spatial gradients*

$$\frac{d}{ds}\begin{pmatrix} a \\ b \end{pmatrix} = G(s)\begin{pmatrix} a \\ b \end{pmatrix} + \mathcal{O}(\partial_y), \qquad G(s) = \begin{pmatrix} G_\phi(s) & C(s) \\ C^\sharp(s) & G_O(s) \end{pmatrix}. \tag{19}$$

The relation to (18) *is given by the replacement $a \mapsto \psi = Wa$; thus G induces Γ without redefining the latter.*

Sketch of justification. The statement is a reformulation of the local modal description constructed in [1] for the joint system of modes of the fundamental field and internal modes of the observer. The block form of the generator $G(s)$ reflects the decomposition into the field and observer sectors. The passage to Γ is obtained after substituting $a \mapsto \psi = Wa$ and extracting the induced generator at the level of effective fields. □

On the status of the sensitivity form. Here $J_{\text{sens}}(s)$ is understood as a positive symmetric sensitivity form on the locally matched modal space corresponding to the common working region of the observers being compared. Its positivity expresses the stable distinguishability of local data in the working regime under consideration, while its inter-observer consistency means that, under admissible matching of observers, it is preserved to leading order.

Lemma 3.3 (Inter-observer consistency $\Rightarrow$ J_{sens}-reciprocity of transfer)**.** *Consider two observers O and O', working with one and the same foliation and admitting operational matching of their local data in a common working region. Let $J_{\text{sens}}(s) > 0$ be a real symmetric metric sensitivity form on the locally consistent modal space of this region, and let the transition between the descriptions of O and O' be realized by a local nondegenerate transformation $R(s)$.*

Assume that, at leading order in ds, the bilinear form

$$B_s(x, y) := x^\top J_{\text{sens}}(s)\, y \tag{20}$$

is invariant under such inter-observer matching and reciprocal with respect to transfer, that is,

$$B_s\big(T_{ds}x, y\big) = B_s\big(x, T_{ds}y\big) + \mathcal{O}(ds^2), \qquad R(s)^\top J_{\text{sens}}(s) R(s) = J_{\text{sens}}(s) + \mathcal{O}(ds^2), \tag{21}$$

where $T_{ds} = I + ds\, G(s) + \mathcal{O}(ds^2)$. Then, in differential form,

$$J_{\text{sens}} G = G^\top J_{\text{sens}}. \tag{22}$$

Sketch of proof. Substituting $T_{ds} = I + ds\, G + \mathcal{O}(ds^2)$ into the first condition in (21), we obtain

$$x^\top\big(G^\top J_{\text{sens}} - J_{\text{sens}} G\big)\, y = 0$$

for all x, y, from which (22) follows. The second condition in (21) means that the form J_{sens} is consistent with the transition between the descriptions of O and O' at the same leading order. □

Remark 3.4 (Stationary choice of the working basis)**.** *If the working basis is chosen in such a way that the sensitivity does not drift along the foliation in the approximation under consideration, then one additionally has*

$$\dot{J}_{\text{sens}} = 0. \tag{23}$$

This condition is used below as part of the leading stationary approximation.

Corollary 3.5 (J_{sens}-reciprocity of the transfer semigroup)**.** *Under the assumptions of Lemma 3.3, the semigroup*

$$T_\sigma := e^{\sigma G}, \qquad \sigma \geq 0,$$

is J_{sens}-reciprocal in the sense that

$$T_\sigma^\top J_{\mathrm{sens}} = J_{\mathrm{sens}}\, T_\sigma, \qquad \sigma \geq 0.$$

Consequently, the action of T_σ induced on one-particle correlators is compatible with the bilinear form B_s.

Sketch of proof. Set

$$X(\sigma) := T_\sigma^\top J_{\mathrm{sens}} - J_{\mathrm{sens}} T_\sigma.$$

Then (22) implies

$$\frac{d}{d\sigma} X(\sigma) = G^\top T_\sigma^\top J_{\mathrm{sens}} - J_{\mathrm{sens}} G T_\sigma = G^\top T_\sigma^\top J_{\mathrm{sens}} - G^\top J_{\mathrm{sens}} T_\sigma = G^\top X(\sigma).$$

Since $X(0) = 0$, uniqueness of the solution of the linear equation gives $X(\sigma) = 0$ for all $\sigma \geq 0$, that is,

$$T_\sigma^\top J_{\mathrm{sens}} = J_{\mathrm{sens}} T_\sigma.$$

□

On the covariance of the working Gaussian state. Within the framework of the leading stationary Gaussian approximation, the covariance used below is chosen to be consistent with the previously established J_{sens}-reciprocity of transfer. As a natural working choice, one adopts a covariance proportional to J_{sens}^{-1}. A more detailed analysis of the degree of its uniqueness and of possible small deviations from this choice lies beyond the scope of the present work and requires separate consideration.

On Θ-symmetry and its compatibility with transfer. The Θ-symmetry used below is not introduced as an independent additional requirement. Rather, it expresses the fact that, at the fundamental level, the model contains neither time nor a distinguished orientation along the foliation. Therefore, reflection of the foliation parameter $s \mapsto -s$ is a natural symmetry of the fundamental description. In the working locally inertial regime, this property is fixed in algebraic form as an antilinear involution Θ acting on the doubled algebra of local observables. Since Θ reverses the orientation of the foliation parameter, its compatibility with transfer is expressed not by literal commutativity, but by the orientation-reflection relation

$$\Theta\, \alpha_\sigma = \alpha_{-\sigma}\, \Theta,$$

on that class of observables of the doubled algebra on which both sides are defined. Here $\alpha_{-\sigma}$ denotes transfer along the reflected branch of the doubled algebra, rather than the inverse element of the original semigroup α_σ in the usual sense.

Proposition 3.6 (Detailed balance from J_{sens}-reciprocity and reflection symmetry)**.** *Assume that Lemmas 3.2 and 3.3 and remark 3.4 hold. Let ω be a stationary quasi-free state on the local *-algebra of observables. Assume that, in the leading Gaussian approximation, its two-point function is given by a covariance proportional to J_{sens}^{-1}, and that Θ is an antilinear involution implementing the fundamental reflection symmetry $s \mapsto -s$ on the doubled algebra and satisfying the conditions*

$$\omega(\Theta X) = \overline{\omega(X)}, \qquad \Theta\, \alpha_\sigma = \alpha_{-\sigma}\, \Theta$$

on that class of local observables on which both sides are defined. Then, for all cylindrical F, G and all $\sigma \geq 0$, detailed balance holds:

$$\omega\big(F^* \, \alpha_\sigma(G)\big) = \omega\big((\Theta \, \alpha_\sigma F)^* \, \Theta G\big).$$

Sketch of proof. By Corollary 3.5, the one-particle transfer T_σ is J_{sens}-reciprocal, and hence the corresponding two-point function of a stationary quasi-free state is symmetric with respect to transfer in the sense required for the reflection of the orientation of the foliation parameter. The conditions on Θ implement this reflection and transfer the established J_{sens}-reciprocity to the two-point correlators. For a quasi-free state, all higher-point correlators are expressed through the two-point function; therefore, detailed balance follows from the established symmetry at the one-particle level by Wick's theorem. A strict functional-analytic implementation of this passage is fixed in the next subsection through reflection positivity and the OS/GNS reconstruction. □

Remark 3.7 (On the massless vector sector)**.** *In the massless vector sector, the statements of the present subsection are to be understood as referring to gauge-invariant observables (for example, to the algebra generated by $F_{\mu\nu}$). If necessary, this is equivalent to the formulation after the standard reduction of gauge-redundant degrees of freedom. It is in this sense that all statements in §5 concerning the massless vector sector are to be understood below.*

Corollary 3.8 (Preparedness for OS/GNS reconstruction)**.** *Under the assumptions of Proposition 3.6, the local quantum layer in the leading stationary Gaussian approximation satisfies the condition of detailed balance with respect to the parameter s. This provides the required input symmetry for the subsequent formulation of reflection positivity and the application of the OS/GNS reconstruction in the next subsection.*

3.4 State, Reflection Positivity, and the OS/GNS Reconstruction

We consider the local algebra generated by smeared observables on the slices Σ_s, and a state ω stationary with respect to transfer. Let Θ denote the antilinear reflection in s, and let $\mathfrak{A}_+$ denote the subalgebra of operators with temporal supports $s \geq 0$.

On the status of reflection positivity. In the present work, reflection positivity is not claimed as a universally derived property of all admissible fundamental states of the model. Instead, we restrict attention to that class of states which corresponds to the working regime introduced above, namely, the locally inertial regime of the effective description compatible with stable causal reconstruction, the existence of a localized observer, cross-slice consistency, stable registration, and detailed balance with respect to the foliation parameter s. Within this class of states, reflection positivity is treated as a natural working condition, consistent with the Euclidean formulation, which allows one to apply the standard OS/GNS reconstruction of the effective quantum layer.

Working class of states. In what follows, we consider states ω for which the following conditions hold in the regime specified above:

1. stationarity with respect to transfer α_σ:

$$\omega\big(\alpha_\sigma(A)\big) = \omega(A), \qquad \sigma \geq 0;$$

2. Θ-symmetry:

$$\omega(\Theta X) = \overline{\omega(X)};$$

3. compatibility of reflection with transfer:

$$\Theta\, \alpha_\sigma = \alpha_{-\sigma}\, \Theta;$$

4. reflection positivity on $\mathfrak{A}_+$:

$$\omega\big((\Theta F)^* F\big) \geq 0, \qquad F \in \mathfrak{A}_+;$$

5. strong continuity of transfer on a dense *-subalgebra of the observables under consideration.

Theorem 3.9 (OS/GNS reconstruction in the working regime)**.** *Let the state ω belong to the working class specified above. Then the standard OS/GNS reconstruction yields a Hilbert space $(\mathcal{H}, \pi, |\Omega_{\rm vac}\rangle)$, in which $|\Omega_{\rm vac}\rangle$ is a cyclic vector, and transfer in s is realized by the contraction semigroup*

$$T(\tau) = e^{-\tau H}, \qquad H \geq 0.$$

After the standard analytic continuation, one obtains the strongly continuous unitary group

$$U(t) = e^{-iHt},$$

which describes the observable dynamics of the effective quantum layer. In the locally inertial SR regime, this generator H is further interpreted as the generator of observable time translations and, in free relativistic sectors, is identified in the standard way with the corresponding Noether energy charge; see §5.6.

Outline of the justification. Stationarity, Θ-symmetry, and the compatibility of Θ with transfer provide a Euclidean reflection structure consistent with the previously established detailed balance (Corollary 3.8 and proposition 3.6). The restriction to the working class of states via reflection positivity makes the standard Osterwalder–Schrader/GNS reconstruction applicable. Strong continuity of transfer (Lemma 3.15) yields a contraction semigroup in the parameter s, and the standard OS/GNS reconstruction leads to a self-adjoint generator $H \geq 0$ such that

$$T(\tau) = e^{-\tau H}.$$

After that, the standard analytic continuation and Stone's theorem yield the unitary group $U(t) = e^{-iHt}$; see [11, 12, 13]. □

Remark 3.10 (On the spectral condition)**.** *In the present work, the spectral condition means the nonnegativity of the generator of the observable evolution with respect to the foliation parameter:*

$$H \geq 0.$$

It is precisely this property that arises in the working regime under consideration as a consequence of the OS/GNS reconstruction and the existence of the contraction semigroup

$$T(\tau) = e^{-\tau H}, \qquad \tau \geq 0.$$

In the locally inertial SR regime, this generator is further interpreted as the generator of observable time translations; in free relativistic sectors, this is consistent with the standard identification $H = P^0$, where P^0 is the Noether energy of the corresponding effective field; see §5.6.

Remark 3.11. *Thus, reflection positivity fixes in the present work not a universal property of the fundamental timeless formulation as such, but precisely that working class of effective states in which the local quantum layer admits a standard Hilbert-space reconstruction.*

3.5 $O(3)$-Structure on the Slice and Spin Blocks

Lemma 3.12 (Stabilizer of the foliation and $O(3)$-isotropy on the slice)**.** *Let the foliation direction* $\mathbf{n}$ *and the slice* Σ_s*, orthogonal to* $\mathbf{n}$*, be fixed in the Euclidean space* $\mathbb{E}^4$*. Then the subgroup of local Euclidean transformations preserving* $(\Sigma_s, \mathbf{n})$ *is isomorphic to* $O(3)$*; its connected component is canonically isomorphic to* $SO(3)$ *and acts as the group of orthogonal transformations in the three-dimensional coordinate space on* Σ_s*.*

Consequently, local operators acting on effective fields on Σ_s *decompose into a direct sum of components irreducible under the action of the stabilizer of the foliation. For tensor fields, this yields the usual classification into scalar, vector, and more general* $O(3)$*-types on the slice.*

Brief justification. Fixing the unit normal $\mathbf{n}$ to the foliation singles out in $\mathbb{E}^4$ the three-dimensional subspace $T\Sigma_s$ orthogonal to it. Local Euclidean transformations preserving $\mathbf{n}$ and the point of the slice Σ_s act orthogonally on $T\Sigma_s$, whence the stabilizer is isomorphic to $O(3)$, and its connected component to $SO(3)$. Consequently, every local tensor field and every local operator on Σ_s carries a representation of this group and therefore decomposes into a direct sum of irreducible components with respect to the stabilizer of the foliation. □

Remark 3.13 (On the status of the spinor sector)**.** $O(3)$*-isotropy on the slice* Σ_s *directly organizes scalar and vector types of effective fields with respect to the stabilizer of the foliation. For the spinor sector, this is not sufficient: it requires additional structure compatible with the locally inertial SR regime. Therefore, in the present work the spinor sector is treated not as a direct consequence of* $O(3)$*-isotropy alone, but as an additional consistent realization of the working regime of quantum reconstruction.*

3.6 Constraint on the Principal Symbol for Effective Fields

In what follows, we use a structural restriction inherited from the previous GR reconstruction [2]: in the locally inertial regime of the effective description under consideration, the local generators for the fields ψ_I, closed on the slices Σ_s, do not contain spatial derivatives of order higher than two. This restriction pertains to the effective field sector itself and expresses the locality, adiabaticity of foliations, and stability of the effective description already fixed earlier; below, we use its direct consequence for the principal symbol of the local generator on Σ_s.

Corollary 3.14 (Principal symbol of the local generator in the effective SR regime)**.** *In the SR regime, under locality on* Σ_s*,* $O(3)$*-isotropy of the stabilizer of the foliation (see Lemma 3.12), OS-positivity, and the preliminary requirement of microcausal compatibility of the local quantum layer with the observable SR structure, as well as under the structural restriction inherited from the GR reconstruction according to which the local generator of the evolution of effective fields on* Σ_s *does not contain spatial derivatives of order higher than two, its principal symbol in each irreducible* $O(3)$*-sector has the form*

$$a^{ij}(s,y)\,\xi_i\xi_j \;=\; a(s,y)\,\delta^{ij}\xi_i\xi_j, \qquad a(s,y) \geq 0.$$

In particular, the principal symbol is quadratic in ξ and is an $O(3)$-scalar on the slice.

Sketch of proof. By the structural restriction inherited from the GR reconstruction, the local generator of the evolution of effective fields on Σ_s does not contain spatial derivatives of order higher than two. Consequently, its principal symbol in each irreducible $O(3)$-sector is a quadratic form

$$a^{ij}(s,y)\,\xi_i\xi_j$$

in the covector ξ, where $a^{ij} = a^{ji}$. $O(3)$-isotropy on the slice (Lemma 3.12) implies

$$a^{ij}(s,y) = a(s,y)\,\delta^{ij}.$$

OS-positivity and the preliminary requirement of microcausal compatibility with the observable causal structure impose additional consistency conditions on the principal symbol, in particular requiring $a(s,y) \geq 0$ and excluding pathological terms incompatible with the locally inertial SR regime. Thus, the principal symbol is quadratic in ξ and is an $O(3)$-scalar on the slice. □

3.7 Strong Continuity of Transfer in s

We fix here the structural results used in this subsection together with the additional conditions of effective closedness of the local quantum sector. Most of them have been established earlier; only the regularity of the coefficients and the local closedness on $K \Subset \Omega_s$, required for a rigorous functional-analytic formulation of the transfer semigroup, are fixed separately.

Assumptions and references. (1) Operational events and locality on Σ_s — §2.6.
(2) $O(3)$-isotropy of the stabilizer on the slice — Lemma 3.12.
(3) Structure of direct/observable transformations and closedness of the local *-algebra under shifts — §2.7.
(4) Local linear evolution and generator: (11)–(18), Lemma 3.1.
(5) Second-order principal symbol — Corollary 3.14.
(6) Regularity of the coefficients and effective closedness of the local quantum sector on $K \Subset \Omega_s$ — see Section 3.7.

Local effective closedness on $K \Subset \Omega_s$, regularity, and boundary conditions. We work on the slice Σ_s not on the entire region Ω_s, but in a local region $K \Subset \Omega_s$, where, owing to the hierarchy of scales $L_{\text{field}} \ll L_{\text{fol}}$ and the adiabaticity of foliations, the locally inertial SR regime is realized. The local Hamiltonian is defined as in (31), where the second-order principal symbol and the $O(3)$-scalar character follow from Corollary 3.14. The coefficients $a^{ij}(s,y) = a(s,y)\delta^{ij}$, $b^i(s,y)$, and $V(s,y)$ are local in $y \in K$ and belong to C^1 in s, with estimates uniform on compact sets.

Since the quantum sector is reconstructed here only as a local effective description, the flux through ∂K is not required to vanish identically at the fundamental level. However, in the regime under consideration the corresponding boundary contributions are subleading and suppressed by the adiabaticity of foliations and the hierarchy of scales. Therefore, at leading order the local quantum sector on K may be regarded as closed, and self-adjoint boundary conditions (Dirichlet/Neumann/Robin) may be imposed on ∂K, ensuring vanishing normal probability flux in the effective approximation under consideration.

In this approximation, the local generator H is self-adjoint. Consequently, by Stone's theorem the corresponding local dynamics is given by a strongly continuous unitary group. The passage to the standard physical normalization of this group will be carried out below, in the subsection on energy and unitarity.

Convention on notation. Below, a local region $K \Subset \Omega_s$ is fixed in which the locally inertial regime is realized. All operators, fields, and Hilbert spaces are henceforth understood as defined on this region; therefore, the index K is omitted in the notation. The local transfer of effective fields along the foliation is denoted by $\mathcal{U}(s, y; ds)$, whereas $U(t)$ below will denote the unitary evolution on the corresponding Hilbert space.

Let Alg_+ denote the *-algebra generated by smeared local observables $A(f)$ with $f \in C_0^\infty(\Omega_s)$, $s \geq 0$.

On the working domain. Below, $\mathcal{D}_{\mathrm{reg}} \subset \mathcal{H}$ denotes the dense domain in the GNS representation generated by finite linear combinations of vectors obtained by the action of finitely many smeared local fields on the vacuum vector. In the working regime under consideration, this domain is chosen so as to be compatible with the action of local fields and transfer along the foliation, as specified in (18) and Lemma 3.1.

Lemma 3.15 (Strong continuity of transfer on the working domain)**.** *Let $\mathcal{D}_{\mathrm{reg}} \subset \mathcal{H}$ be the dense working domain in the GNS representation specified above. Then, for any smeared observable $A(f)$ and any $\Psi \in \mathcal{D}_{\mathrm{reg}}$, one has*

$$\alpha_\sigma\big(A(f)\big)\Psi \;\longrightarrow\; A(f)\Psi, \qquad \sigma \to 0,$$

that is, transfer with respect to the foliation parameter acts strongly continuously on $\mathcal{D}_{\mathrm{reg}}$.

Sketch of proof. In the working regime under consideration, the transfer α_σ acts on smeared local observables by shifting the test function along the foliation:

$$\alpha_\sigma\big(A(f)\big) = A(f_\sigma).$$

Since $f \in C_0^\infty$, one has

$$f_\sigma \longrightarrow f \qquad \text{in the topology of } C_0^\infty \quad \text{as } \sigma \to 0.$$

Since the field $A(\cdot)$ is treated as an operator-valued distribution, the map

$$f \longmapsto A(f)\Psi$$

is continuous on $\mathcal{D}(\Sigma_s)$ for each $\Psi \in \mathcal{D}_{\mathrm{reg}}$. Consequently,

$$A(f_\sigma)\Psi \longrightarrow A(f)\Psi, \qquad \sigma \to 0.$$

Hence

$$\alpha_\sigma\big(A(f)\big)\Psi \longrightarrow A(f)\Psi$$

for any $\Psi \in \mathcal{D}_{\mathrm{reg}}$, which is precisely the strong continuity of transfer on the working domain. This result is used below within the framework of the OS/GNS reconstruction (§3.4) to construct the contraction semigroup in the foliation parameter. □

3.8 Complex Structure and Analytic Continuation

We work on the real linear space of test functions

$$\mathcal{D}(\Sigma_s) = C_0^\infty(\Sigma_s, \mathbb{R}).$$

Equal-time *effective fields* are defined by smearing:

$$\psi(0, f) := \int_{\Sigma_s} f(y)\,\psi(0, y)\,d\mu_{\Sigma_s}(y), \qquad f \in \mathcal{D}(\Sigma_s),$$

where ψ is understood as an operator-valued distribution on $\mathcal{D}(\Sigma_s)$.

Motivation for complexification. The fundamental field remains real; however, in order to construct the effective quantum description at the level of reconstructed fields, it is necessary to choose a complex structure on the space of initial data. In the present work, we do not claim that such a structure has already been derived in full generality solely from the fundamental equation and the nondegeneracy of ω_Σ. Instead, we restrict ourselves to the working SR regime in which, after factorization with respect to non-physical degrees of freedom, there exists an ω_Σ-compatible positive polarization. It is precisely in this class of states and polarizations that the complex structure J is introduced, sufficient for: (i) defining the one-particle space and the Hermitian form (27); (ii) introducing creation/annihilation operators and the CCR/CAR algebras (compatible with microcausality); (iii) consistency with the OS/GNS reconstruction and the spectral condition (§3.4), which yields analytic continuation and unitary evolution; (iv) a correct probabilistic interpretation and measurements via POVMs (§3.10); (v) a convenient formulation of the relativistically invariant closure (§5). Thus, the complex structure is understood here not as a new fundamental degree of freedom, but as part of the effective working closure of the chosen quantum regime.

Antisymmetric form. The Pauli–Jordan kernel

$$E(x, x') := \frac{1}{i}\langle \Omega_{\text{vac}}, [\psi(x), \psi(x')]\Omega_{\text{vac}}\rangle$$

induces on the slice

$$\omega_\Sigma(f, g) \ := \ \iint_{\Sigma_s \times \Sigma_s} f(y)\,\big(n^\mu \partial_\mu E\big)\big|_{t=t'=0}(y, y')\,g(y')\,d\mu_{\Sigma_s}(y)\,d\mu_{\Sigma_s}(y'), \tag{24}$$

where n^μ is the unit normal to Σ_s. For gauge fields, the physical/transverse projector is applied before restriction to the slice.

Here E is understood as the commutator kernel on the reconstructed OS/GNS layer; below, after specialization to free relativistic sectors (§5.4), it will be identified with the causal propagator $E = \Delta_R - \Delta_A$.

On the status of hyperbolicity of the SR limit. Below, hyperbolicity of the free SR limit and the well-posedness of the Cauchy problem are used not as a result rederived at this point, but as a preliminary structural condition of the locally inertial regime, necessary for the formulation of the symplectic structure on the slice. In the subsequent subsections, it will be shown that the chosen free relativistic closures of the scalar, spinor, and vector sectors do indeed realize this condition.

Nondegeneracy on the slice.

Lemma 3.16 (Nondegeneracy of the form on the slice). *In the SR regime, under locality, the preliminary requirement of microcausal compatibility with the observable SR structure, and the second-order character of the principal symbol (Corollary 3.14), after the standard factorization with respect to gauge degrees of freedom, the form* ω_Σ *from* (24) *is nondegenerate on the physical quotient space of test functions.*

Sketch of proof. Assume that $\omega_\Sigma(f,\cdot) = 0$. Then the induced equal-time initial data have vanishing symplectic flux with all admissible data. By the preliminary structural condition of hyperbolicity of the free SR limit and of the well-posedness of the Cauchy problem fixed above, this implies the triviality of the corresponding data on the physical quotient space. □

Positive form. Define

$$\mu(f,g) \;:=\; \tfrac{1}{2}\,\langle \Omega_{\text{vac}}, \{\psi(0,f),\psi(0,g)\}\Omega_{\text{vac}}\rangle. \tag{25}$$

In the working class of states under consideration, μ is a positive symmetric form on the physical quotient space and admits the choice of an ω_Σ-compatible positive complex structure.

Complex structure. Let $\mathcal{V}_{\text{phys}}$ denote the physical quotient space of initial data after the standard factorization with respect to nonphysical degrees of freedom. We now fix an ω_Σ-compatible positive complex structure

$$J : \mathcal{V}_{\text{phys}} \to \mathcal{V}_{\text{phys}},$$

such that

$$J^2 = -\mathbf{1}, \qquad \omega_\Sigma(Ju,Jv) = \omega_\Sigma(u,v), \qquad \mu(u,v) = \omega_\Sigma(u,Jv) \quad \text{for all } u,v \in \mathcal{V}_{\text{phys}}. \tag{26}$$

Then, in particular,

$$\mu(u,u) = \omega_\Sigma(u,Ju) \geq 0.$$

The Hermitian form

$$\langle u,v\rangle_{1\text{p}} \;:=\; \mu(u,v) \,+\, i\,\omega_\Sigma(u,v) \tag{27}$$

defines the one-particle space after factorization by the null space and completion.

3.9 Operational Conditions on the Observer in the Measurement Regime

In the previous sections, the observer has already been fixed as a localized subsystem of the fundamental field, possessing a working region Ω, an internal region $\Omega_0 \subset \Omega$, and the possibility of causal reconstruction of events on the foliation slices. For the transition to a quantum-operational description of measurements, however, this is not sufficient: it is additionally necessary to single out that class of observers for which stable measurement protocols and the statistics of outcomes are meaningful.

In the present work, an observer in the measurement regime is understood to be such a localized subsystem for which there exist: (i) stable internal registers admitting the recording and storage of the results of local interactions over times longer than the characteristic time of an elementary readout act; (ii) a separation of time scales between the registration process, the relaxation of internal degrees of freedom, and the dynamics of the effective fields under consideration; (iii) reproducible measurement procedures for which repetition of one and the same local protocol on one and the same effective state leads to one and the same probability measure on the set of outcomes.

These conditions do not constitute a new fundamental postulate of the model, but serve as a technical refinement of that class of operationally admissible subsystems for which it is possible in what follows to introduce the POVM description of measurements and the Born rule. They pertain not to the fundamental level of the field Φ as such, but to the effective description of the observer in an operationally accessible region where the foliation-based description remains valid and quantum reconstruction is meaningful.

It is precisely for this class of observers that local measurement protocols, their statistics, and their consistency with the OS/GNS-reconstructed quantum layer are considered below.

3.10 Measurements: POVMs and the Born Rule

For observers satisfying the conditions of §3.9, the statistics of local measurement protocols are considered on the OS/GNS-reconstructed Hilbert space. At the fundamental level, the outcome of an individual registration act remains deterministic; however, at the effective level the observer has access only to the local algebra of observables and to the statistics of outcomes in the working region Ω. It is precisely in this sense that a repeatable measurement protocol is described probabilistically.

In the present work, no separate axiomatics of measurement is postulated. Instead, it is assumed that, in the working regime of the observer under consideration, the statistics of local registrations satisfy the mathematical conditions of representability on the reconstructed Hilbert layer. Under these conditions, the measurement statistics admits a standard POVM description and therefore takes Born form. This statement pertains to the observer-relative event structure and does not presuppose the existence of a global space of events of the model.

Proposition 3.17 (POVM representation and Born form in the observer's working regime)**.** *Let the local measurement protocol of a fixed observer in the working regime under consideration define a statistics of outcomes that is representable on the OS/GNS-reconstructed Hilbert space $\mathcal{H}$ through positive normalized effects. Then there exists a positive-operator-valued measure $E : \Sigma \to \mathcal{B}(\mathcal{H})$ such that for any effective state ρ*

$$\mathbb{P}_\rho(B) = \operatorname{tr}\big(\rho\, E(B)\big), \qquad B \in \Sigma.$$

In particular, for a pure state $\rho = |\psi\rangle\langle\psi|$, $\|\psi\| = 1$, one has

$$\mathbb{P}_\psi(B) = \langle\psi|\, E(B)\, |\psi\rangle\,.$$

Sketch of proof. By assumption, the statistics of the local measurement protocol under consideration is representable on the reconstructed Hilbert layer through positive normalized effects. Consequently, for each $B \in \Sigma$, the probability $\mathbb{P}_\rho(B)$ defines a positive affine functional of the state ρ. By the standard duality between states and effects on $\mathcal{B}(\mathcal{H})$, this functional is representable in the form $\rho \mapsto \operatorname{tr}(\rho E(B))$ with a certain positive operator $E(B)$, while normalization and σ-additivity of the probabilities ensure that E

is a POVM. Thus, the statistics of local measurement outcomes in the reconstructed quantum layer takes the standard Born form. □

Thus, in the model under consideration, measurement does not require the introduction of a separate nonunitary dynamics. At the effective level, it is described by the same state and the same local algebra of observables as the rest of the reconstructed quantum dynamics, whereas the statistics of the outcomes of a particular observer is fixed by the POVM representation and the Born form within the observer's working event structure.

3.11 Operational Status of Measurements and Events

In the model under consideration, events and measurements have an operational rather than an absolute status. They are defined relative to a fixed localized observer and the observer's working region Ω, within which the operationally accessible description is specified. Accordingly, measurement is not introduced as a fundamental process additionally imposed on an already existing global dynamics, but is understood as a local registration procedure, as a result of which observable events V_{obs} arise, from which a broader event structure $V_{\text{obs}} \cup V_{\text{rec}}$ is subsequently reconstructed.

In this sense, a measurement outcome is an operational structure that is stably identifiable, arises as a result of local registration, and is associated with some effect $E(\Delta)$ in the POVM representation of the measurement protocol under consideration. The statistics of such outcomes pertains not to some global space of events of the model, which is absent in the present formulation, but to the observer-relative event structure and to the reconstructed quantum layer of the given observer. Within the adopted working regime and under the mathematical conditions of representability formulated in Proposition 3.17, this statistics takes the standard Born form.

Thus, measurement does not require the introduction of a separate nonunitary dynamics. At the effective level, one and the same reconstructed evolution operates, and the distinction between "ordinary evolution" and a "measurement act" is connected not with different fundamental laws, but with the difference between the operational procedures of identification, registration, and subsequent reconstruction of events in the observer's working region. No separate mechanism of wave-function collapse is introduced into the model.

In this formulation, the standard measurement problem does not arise in its usual form. It appears only when the quantum state is treated as a fundamental physical object subject to unitary evolution, whereas the measurement outcome is regarded as an act external to that evolution. In the model under consideration, there is no such splitting of levels: the quantum state pertains not to the fundamental field Φ, but to the OS/GNS-reconstructed effective description of the observationally accessible degrees of freedom in the working region Ω, whereas the measurement outcome is a stably registered observable event from V_{obs}, from which a broader event structure may subsequently be reconstructed. The relation between the quantum state and observations is determined not by a collapse postulate, but by the POVM description of measurement protocols and the Born rule within the working event structure of the given observer. Accordingly, the change of state after measurement is understood not as a separate fundamental dynamics, but as a transition to a new effective description consistent with the registered events.

Remark on the Wigner's friend situation. The same operational logic also clarifies the standard Wigner's friend situation if it is considered within one fixed foliation (that is, within one and the same locally inertial frame of reference, without additional effects associated with the distinction between direct and observable transformations). Let an observer O_1, located in a local region $\Omega_1 \subset \Omega$, perform a measurement protocol and carry out local registration of the result. Accordingly, observable events $V_{\mathrm{obs}}^{(1)}$ arise in the observer's working region, from which a broader event structure $V_{\mathrm{obs}}^{(1)} \cup V_{\mathrm{rec}}^{(1)}$ may subsequently be reconstructed.

Let another observer O_2, working with the same foliation but not yet having access to the record in Ω_1, describe the composite system at the level of unitary effective dynamics. In the standard formulation, a tension arises here between a description in which O_1 already has a registered measurement result, and a description in which O_2 still uses unitary evolution for a broader system. In the model under consideration, this is not interpreted as a contradiction, since the two descriptions pertain to different operationally accessible regions and to different sets of locally registrable events.

In other words, the presence of a registered event for O_1 and the absence of such an event for O_2 are not two incompatible descriptions of one and the same absolute fact. Until access is obtained to the record in Ω_1, the observer O_2 is not required to include the corresponding result in the set of directly observable events $V_{\mathrm{obs}}^{(2)}$; for O_2, this result is not yet a locally registered event. Only after operational access to the corresponding record can this result enter $V_{\mathrm{obs}}^{(2)}$, and together with it the associated reconstructed event structure $V_{\mathrm{obs}}^{(2)} \cup V_{\mathrm{rec}}^{(2)}$. These are two different effective descriptions specified relative to different observers and to different amounts of information accessible to them within one and the same foliation. A tension would arise only under the assumption that there exists a single global space of events and a single quantum state that must simultaneously and exhaustively encode both points of view. It is precisely this assumption that is not adopted in the present model.

Inter-observer consistency in this formulation means that, for jointly realizable local measurement protocols, the statistical descriptions of different observers must be mutually compatible in those regions where matching of their operational data is admissible. It is precisely this condition that ensures consistency of the probabilistic description without the introduction of a global space of events and without attributing to the model a single absolute event structure.

Thus, within the framework of the present work, measurements and events are treated as observer-relative operational objects, whereas their probabilistic structure is specified by the reconstructed quantum formalism within the working regime of a particular observer. This is sufficient for the subsequent use of the measurement formalism without invoking a separate theory of collapse, an additional fundamental dynamics, or a global ontology of events.

4 The Schrödinger Equation on Foliations and Probability Continuity

Passage to the standard physical normalization. Up to this point, the self-adjoint generator H has been treated as an abstract generator of time shifts, so that the corresponding unitary evolution was written in the dimensionless form

$$U(t) = e^{-iHt}.$$

To pass to the standard physical normalization of the quantum sector, we introduce the constant $\hbar$ as the coefficient relating the generator of time shifts to the energy normalization. Equivalently, we redefine

$$\hbar H \;\mapsto\; H.$$

After this, the same unitary evolution is written in the familiar form

$$U(t) = e^{-iHt/\hbar},$$

and henceforth H has the dimension of energy.

At this stage, $\hbar$ is introduced as a normalization constant transforming the abstract generator of time shifts into the standard energy normalization of the quantum sector. It is not identified *a priori* with the reduced Planck constant. As noted earlier (see [1]), the model admits more than one reconstruction class. In the present work, one admissible reconstruction class satisfying the conditions formulated above is fixed. For the reconstruction class corresponding to our Universe, the constant $\hbar$ introduced here coincides with the reduced Planck constant.

As established in the previous section and taking into account the physical normalization introduced above, a strongly continuous unitary group acts on the Hilbert space of states $\mathcal{H}$:

$$U(t) = e^{-iHt/\hbar},$$

where $H = H^\dagger \geq 0$ is a self-adjoint Hamiltonian with dense domain $D(H) \subset \mathcal{H}$.

Below, only the results of the previous subsections are used: the OS/GNS reconstruction, a compatible complex structure, and the local POVM representation of measurements. The assumption of the existence of a density

$$E(\mathrm{d}y) = \Pi(y)\, \mathrm{d}\mu_{\Sigma_s}(y)$$

is required only for the passage from the weak form (30) to the pointwise continuity equation and is not among the principal structural conditions of the regime under consideration.

The Schrödinger equation. For the state vector $|\psi(t)\rangle := U(t)\,|\psi_0\rangle$, one has

$$i\,\hbar\,\partial_t\,|\psi(t)\rangle = H\,|\psi(t)\rangle\,, \qquad |\psi(0)\rangle = |\psi_0\rangle\,, \tag{28}$$

where $\hbar$ is the normalization constant of the physical energy scale introduced above. The equality (28) is understood on $D(H)$.

Smeared probabilities. Let a positive-operator-valued measure (POVM) $E(\mathrm{d}y)$ be given on a fixed local region $K \Subset \Omega_s \subset \Sigma_s$, and let $\varphi \in C_0^\infty(K)$. Define the smeared effect

$$\Pi(\varphi) := \int_K \varphi(y)\, E(\mathrm{d}y)$$

and the smeared probability density

$$\rho_\varphi(t) := \big\langle \psi(t) \big|\, \Pi(\varphi)\, \big|\psi(t)\big\rangle.$$

In the Heisenberg picture, $\Pi_t(\varphi) := U(t)^\dagger \Pi(\varphi) U(t)$, and for $\psi(0) \in D(H)$, so that $\psi(t) \in D(H)$, we have

$$\partial_t \rho_\varphi(t) = \frac{i}{\hbar}\, \big\langle \psi(t) \big|\, [H, \Pi(\varphi)]\, \big|\psi(t)\big\rangle. \tag{29}$$

Weak form of the continuity equation. Consider the local dynamics on a fixed region $K \Subset \Omega_s$, specified by a local second-order Hamiltonian in the locally inertial regime. The structural conditions already fixed above are assumed: locality in y, local $O(3)$-isotropy on the slice, a principal symbol of second order in spatial derivatives, regularity and boundedness of the coefficients, as well as effective closedness of the sector on K, meaning that at leading order boundary contributions through ∂K may be neglected. Then there exists a (generalized) probability current $j^i(t, \cdot)$ such that

$$\partial_t \rho_\varphi(t) = -\int_K \nabla_i \varphi(y)\, j^i(t, y)\, \mathrm{d}\mu_{\Sigma_s}(y), \qquad \forall\, \varphi \in C_0^\infty(K). \tag{30}$$

If, in addition, the POVM admits a density $E(\mathrm{d}y) = \Pi(y)\, \mathrm{d}\mu_{\Sigma_s}(y)$, then (30) is equivalent to the pointwise form

$$\partial_t \rho(t, y) + \nabla_i j^i(t, y) = 0$$

in the sense of distributions on K.

Explicit form of the current for a local second-order Hamiltonian. Assume that, within the structural assumptions of second-order locality formulated in §5, the Hamiltonian in the region $K \subset \Sigma_s$ has the local form

$$H = \int_K \Big(\tfrac{1}{2}\, \hat{\psi}^\dagger\, \hat{h}\, \hat{\psi} + \text{h.c.} \Big)\, \mathrm{d}\mu_{\Sigma_s}, \quad \hat{h} = -\frac{\hbar^2}{2}\, \nabla_i\big(a^{ij}(y)\, \nabla_j\big) + b^i(y)\, \nabla_i + V(y), \tag{31}$$

where the coefficients $a^{ij} = a^{ji}$, b^i, and V are sufficiently regular and bounded on K. Then

$$j^i(t, y) = \frac{\hbar}{2i} \Big\langle \psi(t) \Big|\, \Pi(y)\, a^{ij}(y) \overleftrightarrow{\nabla}_j \Big| \psi(t) \Big\rangle + \frac{\hbar}{2} \Big\langle \psi(t) \Big|\, \Pi(y)\, \big(b^i(y) + b^i(y)^\dagger\big) \Big| \psi(t) \Big\rangle, \tag{32}$$

where

$$A \overleftrightarrow{\nabla}_j B := A(\nabla_j B) - (\nabla_j A) B.$$

Substitution of (31) into (29) yields (30). Integration of (30) over K yields conservation of total probability in the leading effective approximation.

4.1 Robertson–Schrödinger Uncertainty Relations

For self-adjoint observables A and B, defined on a common dense invariant domain in $\mathcal{H}$, and for a normalized state $\psi \in \mathcal{H}$, let us introduce the variances

$$\mathrm{Var}_\psi(A) := \langle (A - \langle A \rangle_\psi)^2 \rangle_\psi, \qquad \mathrm{Var}_\psi(B) := \langle (B - \langle B \rangle_\psi)^2 \rangle_\psi,$$

and the symmetrized covariance

$$\mathrm{Cov}_\psi(A, B) := \frac{1}{2} \big\langle \{ A - \langle A \rangle_\psi,\, B - \langle B \rangle_\psi \} \big\rangle_\psi.$$

Proposition 4.1 (Robertson–Schrödinger inequality). *In the reconstructed quantum layer, for any such A, B, one has*

$$\mathrm{Var}_\psi(A)\ \mathrm{Var}_\psi(B)\ \geq\ \mathrm{Cov}_\psi(A, B)^2 + \frac{1}{4} \big| \langle [A, B] \rangle_\psi \big|^2.$$

In particular,

$$\mathrm{Var}_\psi(A)\ \mathrm{Var}_\psi(B)\ \geq\ \frac{1}{4} \big| \langle [A, B] \rangle_\psi \big|^2.$$

Sketch of proof. The statement is a standard consequence of the positivity of the norm $\|(A - \langle A\rangle_\psi)\psi + \lambda(B - \langle B\rangle_\psi)\psi\|^2$ for $\lambda \in \mathbb{C}$, after separating the symmetric and antisymmetric parts of the mixed term. □

Corollary 4.2 (Heisenberg inequality for a canonical pair). *If $[X, P] = i\hbar I$, where I is the identity operator on $\mathcal{H}$, then*

$$\Delta_\psi X \, \Delta_\psi P \geq \frac{\hbar}{2}.$$

Remark 4.3 (Compatibility with the model and Bell tests). *In the construction under consideration, the uncertainty relations pertain to the effective quantum description on the reconstructed Hilbert space and do not require the assumption of fundamental stochasticity. They are therefore compatible with the deterministic fundamental level of the model and, in particular, with the superdeterministic interpretation of the violation of Bell inequalities, in which the correlation between the state of the system, the observer, and the measurement settings replaces the assumption of their statistical independence.*

5 Relativistic Closure of Free Sectors

In the preceding sections, the local quantum layer was reconstructed in operationally admissible regions, where the previous GR reconstruction singles out the locally inertial SR regime as the working form of analysis. This makes it possible to pose the following question: which free local relativistic sectors are compatible with the structural restrictions of the model already established in this regime.

Status of the sectors under consideration. Since, as noted above, the model admits more than one reconstruction class, the present section does not claim that each of the free relativistic sectors considered below must be realized in every admissible reconstruction class. The task is different: to classify those local free relativistic closures of effective fields that are compatible with the previously established structural restrictions of the locally inertial regime. The reconstruction class corresponding to our Universe must contain those of these sectors that are actually observed.

In other words, what is considered below are not arbitrarily introduced fields, but precisely those free relativistic sectors that are admitted within the framework of the already reconstructed effective regime of the model. In this sense, the issue is not the addition of an external quantum-field-theoretic structure, but the determination of which types of free relativistic closures are compatible with the results already obtained for the SR and GR layers.

Structural restrictions used. In all subsequent considerations, only those properties are used that have been established earlier: transfer locality, the locally inertial SR regime, the observable Lorentzian structure, local $O(3)$-isotropy of the slice, the restriction on the order of the principal symbol of the local dynamics, as well as consistency with causal reconstruction. It is precisely these conditions that define the class of admissible free relativistic closures in the regime under consideration.

Within this class, the scalar sector arises as the most natural possibility. The spinor and vector sectors are considered below as admissible variants of relativistic closure compatible with the restrictions of the model already established. Thus, the task of the section is not to postulate a fixed set of fields in advance, but to classify those free relativistic sectors that may enter an admissible reconstruction class and, for the reconstruction class of our Universe, in particular those that are realized in it.

5.1 Scalar Sector

Proposition 5.1 (Scalar relativistic closure). *Let ϕ be an effective scalar field in the SR regime whose dynamics is specified by a local linear $O(1,3)$-covariant equation of order not higher than two. Then the admissible relativistic closure has the form*

$$(\Box + m^2)\phi = 0, \qquad m \geq 0. \tag{33}$$

The corresponding dispersion relation is

$$\omega^2 = |\mathbf{p}|^2 + m^2.$$

Sketch of proof. For a scalar field, the only second-order $O(1,3)$-covariant principal symbol is proportional to p^2. After normalization of the highest-order term and taking into account the only admissible scalar term of zeroth order, one obtains (33). □

5.2 Spinor Sector

In accordance with Remark 3.13, the spinor sector is treated here as an additional structural realization of the working regime, rather than as a direct consequence of $O(3)$-isotropy alone. Assuming the existence of a compatible spin structure, we pass to its local description.

Proposition 5.2 (Spinor relativistic closure). *Let ψ be an effective spinor field in the SR regime whose dynamics is specified by a local linear first-order equation, covariant with respect to the locally inertial Lorentzian structure. Then, in the spinor sector under consideration, the admissible free relativistic closure has the standard form*

$$\big(i\gamma^\mu \partial_\mu - m\big)\psi = 0, \qquad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} I, \qquad m \geq 0, \tag{34}$$

where I is the identity operator in the spinor space. Moreover, squaring (34) *yields*

$$(\Box + m^2)\psi = 0, \qquad \omega^2 = |\mathbf{p}|^2 + m^2.$$

Sketch of proof. Covariance with respect to the locally inertial Lorentzian structure and first order fix the principal symbol in the form $\gamma^\mu p_\mu$. The anticommutation relations for γ^μ ensure the Lorentzian quadratic form p^2, while in the minimal spinor sector under consideration compatibility with this covariance leaves, at zeroth order, the standard mass term m. □

5.3 Vector Sector

On the status of the reduction to the physical sector. In the massless vector sector, the passage to the physical subspace is not rederived in the present work from the fundamental timeless model as an independent result. Instead, we use the standard working closure consistent with the locally inertial SR regime and the local algebra of observables: the physical sector is understood through the gauge-invariant subalgebra, the physical projector, or, equivalently, the BRST reduction. Thus, in the present article this step is treated not as an independently derived result of the model, but as a consistent working assumption necessary for the standard realization of the admissible massless vector sector.

Proposition 5.3 (Vector relativistic closure)**.** *Let A_μ be an effective vector field in the SR regime whose dynamics is specified by a local linear Lorentz-covariant equation of order not higher than two and is consistent with $O(3)$-isotropy on the slice. Then the principal symbol is fixed in the form*

$$\sigma_2(\mathcal{M})_{\mu\nu}(p) = p^2\eta_{\mu\nu} - p_\mu p_\nu,$$

and the admissible equation of motion has the form

$$\Box A_\mu - \partial_\mu(\partial\cdot A) + m^2 A_\mu = 0, \qquad m \geq 0. \tag{35}$$

For $m > 0$, it follows that

$$\partial^\mu A_\mu = 0, \qquad (\Box + m^2)A_\mu = 0,$$

which corresponds to the Proca equation and the dispersion relation

$$\omega^2 = |\mathbf{p}|^2 + m^2.$$

For $m = 0$, one obtains the massless limit, equivalent to Maxwell's equations

$$\partial^\mu F_{\mu\nu} = 0, \qquad F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and the physical sector in the present work is understood in the standard sense: as the sector with two transverse polarizations on the gauge-invariant subalgebra of observables.

Sketch of proof. Lorentz covariance, second-order locality, and the vector character of the field fix the principal symbol up to normalization. The only admissible zeroth-order term gives the mass m, after which the massive and massless cases follow in the standard way. □

5.4 Causal Cone and Fundamental Solutions

In the preceding sections, microcausality was used as a preliminary structural requirement for the consistency of the local quantum layer with the observable causal structure inherited from the locally inertial SR regime. In the present subsection, it is shown that, in the chosen free relativistic closure, this requirement acquires a concrete realization through retarded and advanced fundamental solutions, the Pauli–Jordan kernel, and the Peierls form.

For the free relativistic equations obtained in §5.1–§**??**, the causal structure is determined by the corresponding operators possessing retarded and advanced fundamental solutions on the effective Lorentzian geometry of the SR regime. It is precisely this structure that will be needed below for the construction of the Pauli–Jordan kernel, the Peierls form, and local (anti)commutativity.

Proposition 5.4 (Fundamental solutions and causal support)**.** *Let P be an operator on $(\mathcal{M}, \eta)$ for which there exist unique retarded and advanced fundamental solutions $\Delta_R, \Delta_A : \mathcal{D}(\mathcal{M}) \to \mathcal{D}'(\mathcal{M})$ satisfying*

$$P\Delta_{R/A} = \Delta_{R/A}P = \mathrm{id} \qquad \textit{on } \mathcal{D}(\mathcal{M}),$$

and

$$\mathrm{supp}(\Delta_R f) \subset J^+(\mathrm{supp}\, f), \qquad \mathrm{supp}(\Delta_A f) \subset J^-(\mathrm{supp}\, f).$$

Then their difference

$$E := \Delta_R - \Delta_A$$

satisfies the homogeneous equation in both arguments and has causal support

$$\mathrm{supp}(Ef) \subset J^+(\mathrm{supp}\, f) \cup J^-(\mathrm{supp}\, f).$$

Sketch of proof. For Green-hyperbolic operators, the existence and uniqueness of Δ_R, Δ_A with the stated properties are standard results. In the locally inertial SR regime under consideration, this applies directly to free scalar sectors and, for spinor and vector sectors, after the standard reduction to the corresponding Green-hyperbolic operators or to an equivalent formulation on the physical sector. □

Definition 5.5 (Pauli–Jordan kernel)**.** *The distribution*

$$E := \Delta_R - \Delta_A$$

is called the Pauli–Jordan kernel, or causal propagator. It is antisymmetric (with transposition in the internal indices in multicomponent sectors), satisfies the homogeneous equation in both arguments, and serves as the basic object for the construction of the Peierls symplectic form and the local algebra of observables.

Corollary 5.6 (Finite propagation speed)**.** *For any $f \in \mathcal{D}(\mathcal{M})$, the support of Ef lies in the causal hull of the support of f:*

$$\mathrm{supp}(Ef) \subset J^+(\mathrm{supp}\, f) \cup J^-(\mathrm{supp}\, f).$$

In particular, if $\mathrm{supp}\, f$ *and* $\mathrm{supp}\, g$ *are spacelike separated, then*

$$\langle f, Eg \rangle = 0.$$

Thus, the preliminary requirement of microcausal compatibility used above as a structural restriction is realized in the free relativistic closure in the standard causal form.

Remark 5.7 (Spinor and vector sectors)**.** *For the spinor sector, the causal propagator is constructed through the standard factorization of the Dirac operator and the corresponding second-order Green-hyperbolic operator. For the massive vector sector, the Proca operator applies, while in the massless case the statements are understood either after gauge fixing or on the gauge-invariant subalgebra of physical observables.*

5.5 Peierls Symplectic Form and Canonical Algebras

Relying on Definition 5.5, let us define for test functions $f, g \in C_0^\infty(\mathbb{R} \times \Sigma_s)$ the Peierls form

$$\varpi(f, g) := \iint f(x)\, E(x, x')\, g(x')\, \mathrm{d}x\, \mathrm{d}x'. \tag{36}$$

The antisymmetry of E implies the antisymmetry of ϖ, while the causal support of E yields locality:

$$\varpi(f, g) = 0 \quad \text{if} \quad \operatorname{supp} f \text{ and } \operatorname{supp} g \text{ are causally disjoint.} \tag{37}$$

Proposition 5.8 (Peierls form, CCR/CAR, and microcausality)**.** *In free sectors, the form ϖ induces the standard canonical algebras: CCR in bosonic sectors and CAR in fermionic sectors. The causal support of E implies local (anti)commutativity: for causally disjoint supports,*

$$[\Psi(f), \Psi(g)] = 0 \qquad \textit{(bosonic case)},$$

$$\{\Psi(f), \Psi^\dagger(g)\} = 0 \qquad \textit{(fermionic case)}.$$

Sketch of proof. In bosonic sectors, the Peierls bracket coincides with the CCR kernel,

$$[\Psi(f), \Psi(g)] = i\, \varpi(f, g)\, \mathbf{1},$$

so that (37) directly yields microcausality. In fermionic sectors, the analogous role is played by the causal propagator of the Dirac operator, which leads to CAR and local anticommutativity. □

Remark 5.9 (Gauge sector)**.** *In the massless vector case, the CCR are formulated on the gauge-invariant subalgebra, for example the one generated by $F_{\mu\nu}$, or after BRST reduction. In this case, microcausality is understood precisely for gauge-invariant observables.*

5.6 Energy and Unitarity

By Theorem 3.9, under the working conditions adopted in §3.4, consistent with detailed balance (Proposition 3.6) and strong continuity of transfer (Lemma 3.15), there exist a Hilbert space $\mathcal{H}$, a cyclic vector Ω_{vac}, a $*$-representation π, and a strongly continuous unitary group

$$U(t) = e^{-iHt/\hbar}, \qquad H = H^\dagger \geq 0, \tag{38}$$

realizing the observable dynamics:

$$U(t)\, \pi(A)\, U(t)^{-1} = \pi(\alpha_t A), \qquad t \in \mathbb{R}. \tag{39}$$

In the free SR regime under consideration, it is natural to identify the generator of this unitary evolution with the standard Noether charge generating time translations in the corresponding effective field. Thus, for the scalar, spinor, and vector free sectors, one uses the standard identification

$$H = P^0 = \int_{\Sigma_s} T_{00}\, \mathrm{d}^3 y. \tag{40}$$

Remark 5.10 (Status of the identification $H = P^0$). *In the present work, the equality* (40) *is not singled out as an independent new theorem of the model, but is understood as a standard identification in the constructed free locally inertial SR regime: the operator* H*, arising on the OS/GNS layer as the generator of the observable unitary evolution, is identified with the Noether energy of the corresponding effective field.*

Remark 5.11 (Gauge-invariant domain). *In the massless vector sector, the statements* (38)–(40) *are understood on the gauge-invariant subalgebra (for example, the one generated by* $F_{\mu\nu}$*) or after BRST reduction.*

5.7 Classicality of the Gravitational Sector in the Present Formulation

Corollary 5.12 (The gravitational sector does not enter the local quantum layer as an independent field). *Within the framework of the conditional assumptions and the working regime adopted in the present work, under locality on* Σ_s*, microcausality, the OS/GNS reconstruction with* $H \geq 0$*,* $O(3)$*-isotropy of the stabilizer of the foliation, and the second-order restriction on the principal symbol (see §3.4, Lemma 3.12, Corollary 3.14), the local quantum layer of the effective description is generated by effective nongravitational fields. At the same time, the gravitational sector is already realized as the classical geometric structure of foliations inherited from the previous GR reconstruction [2]. Consequently, an independent local operator sector of gravitational degrees of freedom, in the sense of a separate CCR/CAR quantization of a spin 2 field, does not enter the effective description constructed here.*

Justification. By Theorem 3.9 and the subsequent results of the present article, the local quantum layer in the working regime under consideration is constructed from effective fields localized on the slices Σ_s and from their local algebras of observables. Microcausality and SR-compatible unitary evolution pertain precisely to this effective field sector.

On the other hand, according to the previous GR reconstruction [2], in the same regime the gravitational sector is already represented not as an independent local field, but as the effective geometry of foliations determining the causal and metric structure of the observable description. In other words, in the adopted formulation gravity performs a geometric, rather than an independent operator-field, function.

Thus, all local quantum degrees of freedom of the effective description constructed here are already accounted for in the sector of nongravitational fields, whereas the gravitational sector remains a classical geometric component of the reconstruction. Consequently, an independent local spin 2 operator sector does not arise as part of the quantum layer constructed here. This means that quantum gravity, in the sense of a separate local quantum field of the metric (the graviton), does not enter the effective description considered here. □

Remark 5.13. *The statement of this subsection is not a universal no-go result for all conceivable extensions of the model. Its meaning is narrower and stricter: within the framework of the conditional assumptions and the working regime adopted in the present work, the gravitational sector is exhausted by the classical geometry of foliations and therefore does not require the introduction of an independent local spin 2 operator algebra.*

6 Predictions and Tests of the Model

Purpose of the section. In the present section, the *discriminating prediction* of the model concerning the status of the gravitational sector is singled out. The remaining results of the article pertain to the internal reconstruction of the quantum formalism and are not formulated here as separate phenomenological predictions.

6.1 Discriminating Prediction: Classicality of the Gravitational Sector

It was shown above that, in the reconstruction class under consideration, the gravitational sector remains a purely classical geometric structure and does not act as an independent local quantum mediator. This conclusion has a direct discriminating consequence.

If, in an experiment specifically designed so that the only possible mediator between two quantum subsystems is the gravitational interaction, the emergence of entanglement not reducible to nongravitational channels were to be reliably established, this would mean that, in the corresponding regime, the gravitational sector cannot be purely classical.

Within the framework of the present work, such a result would directly contradict the conclusion obtained above that, in the reconstruction class under consideration, gravity does not generate an independent local operator algebra of spin 2 degrees of freedom and does not act as a quantum mediator. Consequently, a reliable confirmation of gravity-mediated entanglement, with nongravitational alternatives excluded, would falsify the model in its present form.

Conversely, the absence of such an effect in regimes where nongravitational channels are genuinely suppressed below the level of sensitivity would be consistent with the conclusion obtained here concerning the classicality of the gravitational sector. Such an absence, of course, would not prove the model unambiguously, but it would constitute nontrivial empirical support for it, since in the present reconstruction gravity does not act as an independent local quantum mediator.

Thus, BMV-like experiments and their analogues are genuinely discriminating for the present formulation: they test whether a purely classical geometric realization of the gravitational sector is sufficient for the description of the observed effective regime.

7 Discussion and Conclusion

Status of the result. In the present work, within the framework of a timeless Euclidean model with a single real field, the quantum layer of the effective description has been constructed in operationally accessible regions where the description in terms of foliations and effective fields remains valid, and where the previous GR reconstruction singles out the locally inertial SR regime as the working form of analysis. It has been shown that, in the working regime under consideration and in the corresponding class of states and polarizations, consistent with the locally inertial SR regime, one obtains the Hilbert space of states, a compatible positive complex structure, strongly continuous unitary evolution with respect to the foliation parameter, the Schrödinger equation, the Born rule, and the POVM description of measurements. Thus, the quantum formalism appears not as an independent initial postulate, but as an emergent structure consistent with the previously reconstructed causal and geometric organization of the observable description.

Measurement and the status of the quantum state. From the interpretational point of view, an essential consequence is that, within the reconstructed quantum layer, the standard measurement problem does not arise in its usual formulation. Measurement does not require a separate fundamental nonunitary dynamics: the statistics of outcomes is determined by POVMs and the Born rule, while the change of state after registration pertains to the effective description of observationally accessible degrees of freedom, rather than to a separate physical collapse at the fundamental level. In this scheme, the quantum state pertains not to the fundamental field Φ, but to the reconstructed effective description on the working region Ω; the measurement event is a stably reconstructible result of local registration, rather than an act external to the quantum evolution.

Relativistic closure and relation to previous works. For free effective sectors, it has been shown that transfer locality, the restriction on the principal symbol, and consistency with the observable Lorentzian structure lead to the standard first- and second-order relativistic closures. In this sense, the article establishes not a full interacting local quantum field theory in all its physical completeness, but the quantum core of the model together with its relativistic closure within the domain of applicability of the locally inertial SR regime. The obtained result relies on two previous stages of the construction: from the SR article it inherits the operational definition of the observer, event reconstruction, the distinction between direct and observable transformations, and the local Lorentzian structure of the observable description; from the GR article it inherits the interpretation of the gravitational sector as the effective geometry of foliations, operationally accessible regions, the hierarchy of scales, and the exclusion of strong-field regimes within the domain of applicability of the effective description. The present work adds to this scheme the quantum layer of the effective description in its minimal form compatible with the chosen working class of states, the OS/GNS reconstruction, and the locally inertial SR regime, without changing the fundamental ontological minimum of the model.

The gravitational sector and the limits of applicability. From the point of view of the overall structure of the model, an essential consequence remains the classicality of gravity: the gravitational sector does not generate an independent local operator algebra of spin 2 degrees of freedom and does not require the introduction of an autonomous quantum carrier of the metric. Accordingly, in the present work quantization pertains to effective nongravitational fields, whereas gravity retains the status of an effective geometric structure consistent with the previous GR reconstruction. The results obtained pertain to those operationally accessible regions in which transfer along foliations admits the OS/GNS reconstruction, the spectral condition, and a local causal interpretation. Strong-field regimes of gravity and of effective fields are excluded in such regions; in operationally inaccessible regions, the effective description in terms of foliations and effective fields must give way to the fundamental description in terms of the field Φ.

Falsifiability. The main discriminating consequence of the model consists in the absence of an independent quantum sector of gravity within the domain of applicability of the description under consideration. Accordingly, a reliable observation of effects requiring a local spin 2 quantum algebra would falsify this version of the model. The remaining results of the article should be understood primarily as elements of the operational reconstruction of the quantum formalism, rather than as separate phenomenological predictions.

On possible tests and their limits. The most natural route of empirical discrimination is connected with the question of whether gravity can act as an independent quantum mediator. In this sense, BMV-like experiments and their analogues are of greatest interest: a reliable observation of gravity-mediated entanglement, with nongravitational alternatives excluded, would contradict the present framework, whereas the absence of such an effect would be consistent with it. At the same time, it should be emphasized that the present article is deliberately restricted to the reconstruction of the quantum core of the effective description in the locally inertial regime; accordingly, a broader range of possible phenomenological consequences requires further development of the program and is not considered separately here.

Thus, the central conclusion of the article is that a timeless Euclidean model with a single real field admits the derivation of the quantum core of effective physics in operationally accessible regions where the description in terms of foliations and effective fields remains valid. In the working regime under consideration, this includes the Hilbert space, a compatible positive complex structure, unitary evolution, the Schrödinger equation, the Born rule, the POVM description of measurements, and the relativistic closure of free sectors.

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