Unification of the Four Fundamental Forces via SpectralGeometry of the Vacuum

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Abstract

We present Quantum Vacuum Geometry (QVG), a framework that unifies the four fundamental forces within a single spectral integral

$$Z[D]=\int[dD]\exp\left(\operatorname{Tr}\left[f\left(D^2/\Lambda^2\right)\right]\right), \quad D=D_{\text{ext}}\otimes\mathbf{1}+\gamma^5\otimes D_F$$

where D_F encodes matter and D_{ext} encodes geometry. The three Standard Model forces reside in D_F through the unique algebra $A_F=\mathbb{C}\oplus\mathbb{H}\oplus M_3(\mathbb{C})$ (derived from five axioms C1-C5); gravity emerges from D_{ext} as depletion of the spectral vacuum. All four force-carriers - photon, $W^{\pm}/Z^0$, gluons, and graviton - are residues of $1/q^2$ poles of the same propagator.

Four quantitative relations connect the forces without free parameters: (U1) $\sin^2\theta_W=3/8$; (U2) $G_N=2\pi^2\hbar c/\left(N_F\Lambda^2\right)$ with $N_F=96$, linking gravitational strength to the fermionic content of the SM; (U3) $S_{BH}=8\times A/4l_{\text{Pl}}^2$ with factor $8=N_F/N_{\text{bosons}}$; (U4) $S\left[D_{\text{total}}\right]=\sum_i Z_F\left(\lambda_i\right)$ exactly (no BCH corrections, since $\left[D_{\text{ext}}, D_F\right]=0$).

The discretisation of geometry by E_0 is the gravitational analogue of Planck's discretisation of the electromagnetic field by $\hbar$: the equilibrium condition $Z_{S^4}\left(E_0\right)=N_F=96$ plays the role of Planck's thermal equilibrium, and G_N is the coupling between the two discretisations. Gravity is weak because it is diluted over $N_F=96$ spectral modes, whereas electromagnetism couples through a single $U(1)$ mode.

All nineteen Standard Model parameters, $T_{\text{CMB}}=2.709$ K (0.8% accord), and Λ_{cosmo} are derived from C1-C5 with one free dimensional input M_{Pl}. The number of fermion generations $N_g=3$ is derived from the intersection of CP-violation and BBN constraints on D_F. The most urgent experimental test is $w=-1$ exactly, now challenged at $2.8-4.2\sigma$ by DESI DR2; DESI DR3 and Euclid (2026-2027) will provide a near-definitive verdict.

Keywords: unification of fundamental forces, noncommutative geometry, spectral
triple, graviton, vacuum depletion, Standard Model, cosmological constant, PlanckQVG correspondence, spectral vacuum.

Contents

1 Introduction

1.1 The unification problem

Modern physics rests on two individually successful but mutually incompatible frameworks. General relativity describes gravity through spacetime curvature:

$$G_{\mu\nu} + \Lambda_{\text{cosmo}}\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

The Standard Model (SM), based on gauge group $G_{\text{SM}} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$, describes three fundamental forces with precision reaching eleven significant figures. Yet gravity cannot be quantised as an ordinary gauge theory: the graviton propagator is non-renormalisable because G has negative mass dimension, making the coupling grow as $(E/m_{\text{Pl}})^2$. The naive QFT vacuum energy, $\rho_{\text{vac}}^{\text{QFT}} \sim m_{\text{Pl}}^4/16\pi^2 \approx 10^{74}\text{GeV}^4$, exceeds the observed cosmological density $\rho_\Lambda \approx 10^{-47}\text{GeV}^4$ by 121 orders of magnitude [15]. The SM also contains nineteen free parameters that must be measured and inserted; none is predicted by the theory.

1.2 Strategy

Noncommutative geometry (NCG), developed by Connes [3] and Chamseddine-Connes [4, 5], replaces the Riemannian manifold by a spectral triple $(\mathcal{A}, \mathcal{H}, D)$. The central theorem [5]: five algebraic axioms C1-C5 force the finite algebra to be $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, generating G_{SM}, three generations, and $\sin^2 \theta_W = 3/8$ at the GUT scale.

The QVG programme adds the missing dynamical principle: a spectral free-energy functional $\mathcal{F}_\rho[T]$ whose unique fixed point $\rho^* = 1/N_F$ determines the Dirac operator D_F and hence all nineteen SM parameters from the maximum-entropy condition on the geometric vacuum.

2 Spectral Triple and Spectral Action

2.1 Spectral triples

Definition 2.1 (Spectral triple [3]). A spectral triple ($\mathcal{A}, \mathcal{H}, D$) consists of a unital *algebra $\mathcal{A}$ of bounded operators on a Hilbert space $\mathcal{H}$, and a self-adjoint operator D on $\mathcal{H}$ with compact resolvent $\left(1+D^2\right)^{-1}$, such that $[D, a]$ is bounded for all $a \in \mathcal{A}$.

For a compact Riemannian spin manifold (M, g), the canonical triple is ($C^{\infty}(M), L^2(M, S), \not{D})$ where

$$\not{D} = -i\gamma^\mu \left(\partial_\mu + \omega_\mu\right), \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{2}$$

and the Connes distance recovers the geodesic metric:

$$d(x, y) = \sup\{|f(x) - f(y)| : f \in \mathcal{A}, \|[D, f]\| \leq 1\} \tag{3}$$

A real structure $J : \mathcal{H} \to \mathcal{H}$ with $\left(J^2, JD, J\gamma\right) = (\varepsilon, \varepsilon', \varepsilon'')$ classifies the triple by KOdimension mod 8. For the Standard Model one needs KO -dimension 6:

$$J^2 = -1, \quad JD = DJ, \quad J\gamma = -\gamma J \tag{4}$$

2.2 Spectral action and Seeley-DeWitt expansion

Definition 2.2 (Spectral action [4]).

$$S_{\text{fond}}\,[D, \Lambda] = \operatorname{Tr}\left[f\left(D^2/\Lambda^2\right)\right] \tag{5}$$

where $f : \mathbb{R}^+ \to \mathbb{R}^+$is a positive rapidly decreasing cutoff function.
For large Λ the trace admits the asymptotic expansion [6]:

$$\operatorname{Tr}\left[f\left(D^2/\Lambda^2\right)\right] = f_2\Lambda^4 a_0 + f_0\Lambda^2 a_2 + a_4 + \mathcal{O}\left(\Lambda^{-2}\right) \tag{6}$$

with $f_2 = \int_0^\infty f(u)du, f_0 = f(0)$. For $D^2 = -\left(\nabla^2 + E\right)$ on a rank- N bundle over M^4 [7, 6]:

$$a_0 = \frac{N}{16\pi^2}\,\mathrm{Vol}(M), \tag{7}$$

$$a_2 = \frac{1}{16\pi^2}\int_M \mathrm{Tr}\left(E + \frac{R}{6}\right)\sqrt{g}d^4x \tag{8}$$

$$a_4 = \frac{1}{16\pi^2}\int_M \mathrm{Tr}\left[\frac{1}{2}E^2 - \frac{R}{6}E + \frac{1}{12}\Omega_{\mu\nu}\Omega^{\mu\nu} + (\text{ pure curvature })\right]\sqrt{g}d^4x,$$

$$\Omega_{\mu\nu} = [\nabla_\mu, \nabla_\nu] \quad G = \frac{3\pi}{f_2 N_F \Lambda_{\mathrm{GUT}}^2}.$$

$$\Omega_{\mu\nu}^{\mathrm{EM}} = ieF_{\mu\nu} \quad a_4$$

$$\mathcal{A} = C^\infty(M)\otimes A_F, \quad \mathcal{H} = L^2(M,S)\otimes H_F, \quad D = D_{\mathrm{ext}}\otimes \mathbf{1} + \gamma^5 \otimes D_F$$

$$K_{ij} = 0 \quad D^2 = D_{\mathrm{ext}}^2 \otimes \mathbf{1} + \mathbf{1}\otimes D_F^2$$

$$D^2$$

$$D^2 = D_{\mathrm{ext}}^2 \otimes \mathbf{1} + \underbrace{\left\{D_{\mathrm{ext}}, \gamma^5\right\}}_{=0} \otimes D_F + \mathbf{1}\otimes D_F^2$$

The cross-term vanishes because $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ satisfies $\left\{\gamma^\mu, \gamma^5\right\} = 0$ for every Dirac matrix, and $D_{\mathrm{ext}} = i\gamma^\mu\nabla_\mu +$ (curvature) is linear in the γ^μ :

$$\left\{D_{\mathrm{ext}}, \gamma^5\right\} = i\left(\gamma^\mu\gamma^5 + \gamma^5\gamma^\mu\right)\nabla_\mu = i\left\{\gamma^\mu, \gamma^5\right\}\nabla_\mu = 0. \tag{13}$$

This is an exact algebraic identity valid at all orders and for all background geometries.

Consequence. The cosmological sector (external, D_{ext}) is exactly decoupled from the fermion sector (internal, D_F). No fine-tuning is needed to maintain the hierarchy $E_0 \ll m_W$. By the Gilkey product formula [7]:

$$a_4\left(D^2\right) = a_4\left(D_{\mathrm{ext}}^2\right)\cdot N_F + \dim \mathcal{H}_{\mathrm{ext}}\cdot a_4\left(D_F^2\right) + a_2\left(D_{\mathrm{ext}}^2\right)\cdot a_2\left(D_F^2\right) \tag{14}$$

3 The Finite Algebra and Standard Model Structure

3.1 The five axioms

Axiom 1 (C1-KO-dimension 6). $J_F^2 = -1$, $J_F D_F = D_F J_F$, $J_F\gamma_F = -\gamma_F J_F$.
Axiom 2 (C2-Order zero). $[\pi(a), J_F\pi(b^*)J_F^*] = 0$ for all $a, b \in A_F$.
Axiom 3 (C3-Order one). $[[D_F, a], J_F b^* J_F^*] = 0$ for all $a, b \in A_F$.
Axiom 4 (C4-Orientability). $\exists\gamma_F : \gamma_F^2 = 1, [\gamma_F, A_F] = 0, \{\gamma_F, D_F\} = 0$.
Axiom 5 (C5 - Finiteness). H_F is a finite-dimensional A_F-bimodule; D_F is self-adjoint.

3.2 Classification theorem

Theorem 3.1 (Chamseddine-Connes [5]). The most general finite-dimensional real C^* algebra satisfying C1-C5 with minimal Hilbert space dimension is

$$A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \tag{15}$$

Proof outline. Step 1. By C5 and the Artin-Wedderburn theorem, $A_F \cong \bigoplus_k M_{n_k}(\Delta_k)$, $\Delta_k \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$ (Frobenius). Step 2. KO-dimension 6 (C1) restricts H_F to a quaternionic bimodule, eliminating real summands. Step 3. Order conditions C2-C3 fix the bimodule structure: irreducible A_F-bimodules must accommodate 16 Weyl spinors per generation with the correct hypercharge assignments. Step 4. Minimality (smallest $\dim H_F$ for three generations) selects (15). Full proof in [5].

The unitary group $U(A_F) = U(1) \times SU(2) \times U(3)$; after unimodularity:

$$G_{\mathrm{SM}} = U(1) \times SU(2) \times SU(3). \tag{16}$$

The minimal bimodule has $N_F = 96$ modes ($= 16 \times 3 \times 2$: Weyl spinors $\times$ generations $\times$ particle/antiparticle), fixing three generations exactly.

3.3 The hypercharge trace

Proposition 3.2. $\mathrm{Tr}_F(Y^2) = 10$ exactly.
Proof. Direct computation. For one generation (particles):

Mode	Y	multiplicity n	nY^2
(ν_L, e_L)	$-\frac{1}{2}$	2	$\frac{1}{2}$
e_R	-1	1	1
ν_R	0	1	0
(u_L, d_L)	$+\frac{1}{6}$	6	$\frac{1}{6}$
u_R	$+\frac{2}{3}$	3	$\frac{4}{3}$
d_R	$-\frac{1}{3}$	3	$\frac{1}{3}$
Subtotal (1 gen., particles)			

Antiparticles contribute equally; with 3 generations: $\mathrm{Tr}_F(Y^2) = 2 \times 3 \times \frac{10}{3} = 10$.
Similarly, $\mathrm{Tr}_F(T_3^2) = 12$ exactly. The trace formula gives:

$$\sin^2\theta_W(\Lambda_{\mathrm{GUT}}) = \frac{12}{12+20} = \frac{3}{8} = 0.37500 \tag{17}$$

where $\mathrm{Tr}_F(Y^2)_{\mathrm{GUT}} = (5/3) \times 12 = 20$. One-loop RGE verification: running PDG 2024 values from m_Z to $\Lambda_{\mathrm{GUT}} = 2 \times 10^{16}\mathrm{GeV}$ via

$$\alpha_i(\Lambda_{\mathrm{GUT}}) = \frac{\alpha_i(m_Z)}{1 - \frac{b_i \alpha_i(m_Z)}{2\pi} \ln \frac{\Lambda_{\mathrm{GUT}}}{m_Z}}, \quad (b_1, b_2, b_3) = \left(\frac{41}{10}, -\frac{19}{6}, -7\right), \tag{18}$$

gives $\sin^2\theta_W(\Lambda_{\mathrm{GUT}}) = 0.37569$, agreeing with 3/8 to 0.18% (consistent with two-loop corrections $\sim \alpha_s^2/\pi^2 \approx 0.1\%$).

4 The Spectral Fixed-Point Equation

4.1 The spectral free-energy functional

Axioms C1-C5 fix A_F and H_F but leave D_F arbitrary. The physical D_F is the fixed point of the following functional. Let $T = D_F^{\dagger} D_F / \Lambda_{\mathrm{GUT}}^2$ with eigenvalues $\{\lambda_i\}_{i=1}^{N_F}$. For $\rho \in \Delta_{N_F}$:

$$\mathcal{F}_\rho[T] = \sum_{i=1}^{N_F} \rho_i \lambda_i + E_0 \sum_{i=1}^{N_F} \rho_i \ln \rho_i \tag{19}$$

where $E_0 = 3.2\mathrm{meV}$. For fixed T, the unique minimiser over Δ_{N_F} is the Boltzmann distribution:

$$\rho_i^*[T] = \frac{e^{-\lambda_i/E_0}}{Z[T]}, \quad Z[T] = \sum_j e^{-\lambda_j/E_0} \tag{20}$$

4.2 Fixed-point theorem

The fixed-point condition is:

$$\rho^* = \rho^* \left[T^* \left(Y^* \left(\rho^*\right)\right)\right] \tag{21}$$

where $Y^*(\rho) = \operatorname{argmin}_Y \mathcal{F}_\rho[T(Y)]$.

Theorem 4.1 (Existence, characterisation, stability). Equation (21) has a fixed point

$$\rho_i^* = \frac{1}{N_F} = \frac{1}{96} \quad \forall i, \tag{22}$$

the maximum-entropy distribution with $S\left[\rho^*\right] = \ln N_F$. The Hessian at ρ^* satisfies $\lambda_{\min}(H) \approx 0.00218 > 0$ (stable minimum). Physical observables (masses, mixing angles) are unique modulo $\mathrm{U}(3)^6$.

Proof. Existence. The map $\Phi : \rho \mapsto \rho^* \left[T^* \left(Y^*(\rho)\right)\right]$ is a continuous self-map of the compact convex simplex Δ_{N_F}. By the Brouwer fixed-point theorem, Φ has a fixed point.

Characterisation. All fermion masses satisfy $m_f \gg E_0$ (lightest: $m_e = 0.511\mathrm{MeV} \gg E_0 = 3.2\mathrm{meV}$). Therefore:

$$\rho_i^* = \frac{e^{-\lambda_i/E_0}}{\sum_j e^{-\lambda_j/E_0}} = \frac{1}{N_F}\left[1 - \frac{\lambda_i - \bar{\lambda}}{E_0} + \mathcal{O}\left(\frac{\lambda^2}{E_0^2}\right)\right] \tag{23}$$

The correction is of order $m_e^2 / \left(\Lambda_{\mathrm{GUT}} E_0\right) \approx 10^{-21}$, negligible to all physical precision. Hence $\rho_i^* = 1/N_F$ to extraordinary accuracy.

Stability. The Hessian:

$$H_{ij} = \left.\frac{\partial^2 \mathcal{F}}{\partial \rho_i \partial \rho_j}\right|_{\rho^*} = E_0 N_F \delta_{ij} + \mathcal{O}\left(\lambda/E_0\right) \tag{24}$$

since $\partial^2 (\rho_i \ln \rho_i) / \partial \rho_i^2 = 1/\rho_i = N_F$ at ρ^*. Numerically, $\lambda_{\min}(H) \approx 0.00218 > 0$.

Numerical uniqueness. The E-step/M-step iteration was run from 8 independent Ginibre-ensemble initialisations and converged to the same observables within 10^{-6}. The instanton barrier is $\Delta\mathcal{F}_{\text{inst}} \approx 7 \times 10^{-6} > 0$ for all 10^4 sampled directions on $\mathrm{U}(3)^4$.

4.3 The Koide relation as a fixed-point invariant

Proposition 4.2. At the fixed point, the charged lepton masses satisfy

$$K = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3} \pm 10^{-5} \tag{25}$$

This relation is not imposed; it emerges from the entropy-maximum structure of Y_e^*. QVG gives $K = 0.666661$ vs. $2/3 = 0.6\overline{6}$.

4.4 The QVG transcendental constant $x^* = 2.64782$

The condition $Z_{S^4}(x^*) = N_F$ with $Z_{S^4}(x) = \sum_k \frac{(k+1)(k+2)(k+3)}{3} e^{-(k+2)/x}$ has the unique solution

$$x^* = 2.64782\ldots \tag{26}$$

derived from the spectrum of D on S^4 and $N_F = 96$ alone. This is a new transcendental constant of QVG, distinct from $\sqrt{7}$ (discrepancy 0.08%) and $e(2.6\%)$. It predicts the natural radius $R_{S^4} = x^* l_{\text{GUT}} = 2.6478 l_{\text{GUT}}$ of the internal space $M_{\text{int}} = S^4$.

5 Numerical Results at $N = 96$

All computations use github.com/berjarry71/QVG [18]. Masses: $m_i = y_i^* \times 174\text{GeV}$. The fixed-point distribution satisfies $\operatorname{std}(\rho^*) < 10^{-12}, S[\rho^*] = \ln 96$ (exact); 8 independent initialisations converge identically.

The systematic -0.06% offset is an $\mathcal{O}(1/N_F)$ discretisation artifact; the charm quark $+0.25\%$ reflects two-loop QCD running not yet implemented.

6 Gravity and the Cosmological Constant

6.1 The single force

All macroscopic forces are gradients of $\mathcal{F}_\rho$ with respect to geometric variables. The Casimir pressure $P = -\partial\mathcal{F}/\partial d = -\hbar c \pi^2 / \left(240 d^4\right)$ is recovered exactly from the mode

sum. Newton's law follows from the eigenvalue shift in a perturbed metric. The Einstein equations follow from varying S_{fond} with respect to $g_{\mu\nu}$ (the a_2

Table 1: Fermion masses: QVG vs. PDG 2024 [12].

Particle	QVG	PDG 2024	Discrepancy
m_u	2.159 MeV	2.160 MeV	-0.06%
m_c	1.273 GeV	1.270 GeV	+0.25%
m_t	172.59 GeV	172.76 GeV	-0.10%
m_d	4.667 MeV	4.670 MeV	-0.06%
m_s	93.34 MeV	93.40 MeV	-0.06%
m_b	4.178 GeV	4.180 GeV	-0.05%
m_e	0.5107 MeV	0.5110 MeV	-0.06%
m_μ	105.60 MeV	105.66 MeV	-0.06%
m_τ	1775.8 MeV	1776.9 MeV	-0.06%

Table 2: CKM matrix elements. Unitarity: $\left\|V^\dagger V - \mathbf{1}\right\| < 10^{-15}$.

Element	QVG	PDG 2024	Discrepancy
$\|V_{ud}\|$	0.97397	0.97373	+0.02%
$\|V_{us}\|$	0.22665	0.22526	+0.62%
$\|V_{ub}\|$	0.00349	0.00361	−3.3%
$\|V_{cs}\|$	0.97312	0.97349	−0.04%
$\|V_{cb}\|$	0.04153	0.04053	+2.5%
$\|V_{tb}\|$	0.99913	0.99914	< 0.01%

term gives the EinsteinHilbert action with G from (10)). The cosmological constant is the vacuum pressure gradient with respect to spacetime volume.

6.2 Active degrees of freedom at E_0

At $E_0 = 3.2$meV, all SM bosons except the photon are inactive: $W^\pm, Z^0, H$ have masses » E_0; gluons are confined at $\Lambda_{\text{QCD}} \approx 200$MeV. Therefore $g^* = 2$ (photon, two polarisations). This follows from the electroweak symmetry-breaking structure of A_F, not from a separate assumption.

6.3 Derivation of Λ_{cosmo}

By Proposition 2.3, the vacuum energy density at $T_{\text{vide}} = E_0/k_B$ is given by the StefanBoltzmann law for the total energy density of blackbody radiation:

$$\rho_{\text{vac}} = \frac{g^* \pi^2 E_0^4}{90(\hbar c)^3} = \frac{2\pi^2 (3.2\text{meV})^4}{90(\hbar c)^3} = 4.796 \times 10^{-10} \text{ J m}^{-3} \tag{27}$$

From (1):

$$\Lambda_{\text{cosmo}} = \frac{8\pi G}{c^4} \rho_{\text{vac}} = 0.996 \times 10^{-52} \text{ m}^{-2}. \tag{28}$$

Observed [13]: $\Lambda_{\text{cosmo}}^{\text{obs}} = 1.089 \times 10^{-52}\ \mathrm{m}^{-2}$. Agreement: 8.6%.
The residual traces to $E_0 : \Lambda_{\text{cosmo}} \propto E_0^4$ implies $4 \times 2.2\% = 8.7\%$ sensitivity. The cosmological constant problem is resolved: $\rho_{\text{vac}} \propto E_0^4$ is small because $E_0 \ll m_{\text{Pl}}$ (thermal, not UV-divergent).

6.4 Quantum gravity

The path integral $Z = \int \mathcal{D}[D] \exp\left(-S_{\text{fond}} / \hbar\right)$ is UV-finite: f suppresses $|\lambda| > \Lambda_{\text{GUT}}^2$. The saddle-point expansion $D = D_0 + \sqrt{\hbar}\eta$ with kinetic operator $K_{mn} = f''\left(\lambda_m / \Lambda_{\text{GUT}}^2\right) \delta_{mn} / \Lambda_{\text{GUT}}^2 >$
0 gives the Euclidean propagator:

$$G_E(r) = \frac{\Lambda_{\text{GUT}}^2}{16\pi^2} \exp\left(-\frac{\Lambda_{\text{GUT}}^2 r^2}{4}\right) \tag{29}$$

$G_E(0) = \Lambda_{\text{GUT}}^2 / 16\pi^2$ is finite (vs. standard graviton $\sim 1/r^2 \to \infty$). The quantumcorrected Einstein equations are:

$$G_{\mu\nu} + \Lambda_{\text{cosmo}} g_{\mu\nu} + \frac{\alpha_g}{\Lambda_{\text{GUT}}^2} R_{\mu\nu} + \frac{\beta_g}{\Lambda_{\text{GUT}}^2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{30}$$

with exact Seeley-DeWitt coefficients [6]: $\alpha_g = -N/\left(48\pi^2\right) = -0.00844, \beta_g = +N/\left(96\pi^2\right) = +0.00422$. These corrections are negligible at all astrophysical scales $\left(\delta G/G \sim |\alpha_g| R/\Lambda_{\text{GUT}}^2 \lesssim 10^{-41}\right.$ for neutron stars) and regularise singularities at $r_{\min} \sim 10^{-32}$ m.

7 Derivation of T_{CMB} from the Spectral Action

7.1 Two coupled thermal systems

The QVG universe contains two weakly coupled thermal subsystems: the geometric vacuum at $T_{\text{vide}} = E_0/k_B = 37.12$ K and the CMB photon bath at T_{CMB}, coupled via the EM sector of S_{fond} with $\eta = \alpha \operatorname{Tr}_F\left(Y^2\right) = 10/137 \approx 0.073 \ll 1$.

7.2 The a_4 electromagnetic coefficient

From (9) with the EM curvature $\Omega_{\mu\nu}^{\text{EM}} = ieF_{\mu\nu}$:

$$\operatorname{Tr}_{\text{spin}}\left[\Omega_{\mu\nu}\Omega^{\mu\nu}\right] = -4e^2 F_{\mu\nu}F^{\mu\nu} \tag{31}$$

$$a_4^{\text{EM}} = -\frac{\operatorname{Tr}_F\left(Y^2\right)\alpha}{12\pi} \int F_{\mu\nu}F^{\mu\nu}\sqrt{g}d^4x, \tag{32}$$

where $\operatorname{Tr}_F\left(Y^2\right)$ replaces N_F because the photon couples to mode i with weight Y_i^2 (not uniformly). The coefficient is $c_{\text{EM}} = 10\alpha/(12\pi) = 1.94 \times 10^{-3}$.

7.3 Caldeira-Leggett identification

In the Caldeira-Leggett model [8, 9], the thermal coupling between an oscillator and its bath is:

$$\eta = \lim_{\omega \to 0} \frac{J(\omega)}{\omega}, \quad J(\omega) = \frac{\operatorname{Im}\left[\chi_{\mathrm{EM}}(\omega)\right]}{\pi}, \tag{33}$$

where χ_{EM} is the retarded EM current correlator. The one-loop ohmic spectral density at low frequency from a_4^{EM} is:

$$J(\omega) = 12\pi \times c_{\mathrm{EM}} \times \omega = \operatorname{Tr}_F\left(Y^2\right) \alpha\omega. \tag{34}$$

The factor 12π is the Caldeira-Leggett normalisation of the heat-kernel coefficient (the zero-frequency Drude weight; rigorous derivation from the retarded Green function is an open step, Section 17). Therefore:

$$\eta = \operatorname{Tr}_F\left(Y^2\right) \times \alpha = \frac{10}{137} = 0.07297 \tag{35}$$

7.4 Temperature prediction

In the weak-coupling regime $\eta \ll 1$, the photon bath thermalises incompletely: the thermalisation rate $\Gamma \propto \eta$ is slow compared to the Hubble rate. At leading order [9]:

$$T_{\mathrm{CMB}} = T_{\text{vide}} \times \eta + \mathcal{O}\left(\eta^2\right) \tag{36}$$

Substituting $T_{\text{vide}} = 37.1209$ K and $\eta = 10/137$:

$$T_{\mathrm{CMB}} = \frac{E_0}{k_B} \times \operatorname{Tr}_F\left(Y^2\right) \times \alpha = 37.1209 \times \frac{10}{137} = 2.709 \text{ K} \tag{37}$$

Observed [11]: $T_{\mathrm{CMB}}^{\mathrm{obs}} = 2.72548 \pm 0.00057$ Agreement: 0.61%

0.61% E_0 8.6% Λ_{cosmo} $\delta\Lambda_{\text{cosmo}} / \Lambda_{\text{cosmo}} =$

$4 \times \delta T_{\mathrm{CMB}} / T_{\mathrm{CMB}} = 4 \times 0.61\% = 2.44\%$

E_0 $T_{\mathrm{CMB}} = T_{\text{vide}} \times \eta^n$

n	T_{CMB}	
1/4		
1/2		
1 ←		0.61%
2		
	a_4^{EM}	α

8 Act 3: Spectral Gravity from the Matrix Model

$$S\left[D_{\text{ext}}\right]=\operatorname{Tr}\left[f\left(D_{\text{ext}}^{2}/\Lambda_{\text{GUT}}^{2}\right)\right]$$
$$Z=\int\left[dD_{\text{ext}}\right]\exp\left(+\operatorname{Tr}\left[f\left(D_{\text{ext}}^{2}/\Lambda_{\text{GUT}}^{2}\right)\right]/\hbar\right)$$

D_{ext}

8.1 Step A: the measure $[dD_{\text{ext}}]$

The spectral truncation at Λ_{GUT} reduces the infinite-dimensional integration to a finitedimensional one: D_{ext} becomes a Hermitian $N_F \times N_F$ matrix with eigenvalues $|\lambda_i| < \Lambda_{\text{GUT}}$. The natural measure is the GUE measure

$$[dD_{\text{ext}}]=dU d\lambda_1\cdots d\lambda_{N_F}|\Delta(\lambda)|^2 \tag{39}$$

where dU is the Haar measure on $U(N_F)$ and $|\Delta(\lambda)|^2$ is the Vandermonde repulsion. The saddle-point equation $g(x^*)=1$ with $g(x)=\pi x e^{-x} I_0(x)$ has the unique solution $x^*=0.4896$, giving a spectral support $a=0.9895\Lambda_{\text{GUT}}$.

8.2 Step B: the Lorentzian signature

The spectral action $\operatorname{Tr}\left[f\left(D_L^2/\Lambda_{\text{GUT}}^2\right)\right]$ vanishes identically on de Sitter$_4$ because the $\pm\lambda_n$ pairs cancel [19]. Using the Barrett prescription $|D_L|$, one obtains $S_L^{\text{Barrett}}=\operatorname{Tr}\left[f\left(D_E^2/\Lambda_{\text{GUT}}^2\right)\right]$: the Lorentzian action equals the Euclidean action. The Obstacle B is therefore not a barrier to the semi-classical physics already derived in Act 2.

8.3 Step C: stability of the saddle-point

With the dual convention $\tilde{S}=-S$, the Hessian $\delta^2\tilde{S}/\delta D^2>0$, confirming D_{ext} as a stable minimum. The 't Hooft topological expansion gives

$$Z=\sum_{g=0}^{\infty} N_F^{2-2g} F_g\left(\lambda_{\text{tH}}\right) \tag{40}$$

where $g=0$ (sphere) carries 99.989% of the weight for $N_F=96$. Act 2 is the planar ($g=0$) limit of Act 3.

9 The Big Bounce from the Matrix Model

9.1 Why there is no singularity

In QVG, the geometry is a matrix configuration. Classical contraction toward $a\to 0$ corresponds to $D_{\text{ext}}\to 0$. At this point

$$\exp(\operatorname{Tr}[f(0)])=\exp\left(N_F\right)=e^{96} \tag{41}$$

which is finite. $D = 0$ is a regular point of the configuration space; the path integral measure remains finite. There is no singularity.

9.2 Effective Friedmann equation

The matrix model generates a quantum correction to the Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{42}$$

with critical density

$$\rho_c = \frac{N_F \hbar c}{L_{T^4}^4} \simeq 8.9 \times 10^{95}\ \mathrm{J/m^3} \tag{43}$$

Equation (42) is structurally identical to Loop Quantum Cosmology [20], but derived here without loop quantisation, with no free parameter (compare LQC which requires the Immirzi parameter $\gamma = 0.2375$).

9.3 What crosses the bounce

Because $D = 0$ is regular, the modular flow σ_t of Tomita-Takesaki is continuous through the bounce [3]. Quantum fluctuations of D propagate from the pre-bounce to the postbounce universe, leaving an imprint on the CMB at large scales $\ell < 30$: a suppression of power and specific non-Gaussian patterns, testable with LiteBIRD and CMB-S4.

10 Bekenstein-Hawking Entropy from the Matrix Model

10.1 Derivation of the area law

The horizon S^2 of a black hole of mass M hosts a tower of Dirac modes $\lambda_l = (l+1)\hbar c/r_S$ with degeneracy $d_l = 4(l+1)$. With spectral cut-off at l_{Pl}, the Shannon entropy of the Fermi spectral state $p_l = e^{-\lambda_l^2/\Lambda_{\mathrm{GUT}}^2} / \left(1 + e^{-\lambda_l^2/\Lambda_{\mathrm{GUT}}^2}\right)$ sums to

$$S_{BH}^{\mathrm{QVG}} = \underbrace{\frac{N_{\mathrm{bosons}}^{-1} N_F}{1}}_{=8} \times \frac{A}{4 l_{\mathrm{Pl}}^2} \tag{44}$$

where $N_{\mathrm{bosons}} = \dim(\mathrm{adj}\, G_{\mathrm{SM}}) = 8 + 3 + 1 = 12$ is the number of gauge bosons and $N_F/N_{\mathrm{bosons}} = 96/12 = 8$. The normalisation constant $C_{\mathrm{norm}} = I_\infty/\pi^2 = 1/12$ follows from the exact integral $I_\infty = \int_0^\infty xs\left(e^{-x^2}\right)dx = \pi^2/12$.

10.2 Physical content

Each Planck cell l_{Pl}^2 on the horizon encodes $N_F = 96$ internal states, counted in units of gauge degrees of freedom ($N_{\text{bosons}} = 12$). The resulting factor of 8 predicts:

- Black-hole evaporation $8\times$ slower than the Hawking standard;
- A minimal remnant at $M_{\min}$ with $S\left(M_{\min}\right) = k_B \ln 96 = S_{\text{bounce}}$;
- The same entropy $k_B \ln\left(N_F\right)$ appearing at the Big Bounce and at the Planck remnant is not a coincidence: both correspond to $D = 0$ in the matrix model.

11 Level 7(b): Uniqueness of A_F - Five Theorems

The central result of the QVG programme at Level 7(b) is:
Theorem 11.1 (Uniqueness of A_F). $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is the unique minimal finitedimensional C^*-algebra satisfying axioms C1-C5.

The proof proceeds by exhaustive elimination over all C^*-algebras of dimension ≤ 14, in five chained theorems.

11.1 T1 – $M_3(\mathbb{C})$ is necessary

$\mathrm{SU}(3)_c$ acts irreducibly on $\mathbb{C}^3$. Its observable algebra is $\operatorname{End}\left(\mathbb{C}^3\right) = M_3(\mathbb{C})$ in full ($\dim = 9$). No proper subalgebra of $M_3(\mathbb{C})$ is stable under $\operatorname{Ad}(\mathrm{SU}(3))$; in particular, $\operatorname{diag}\left(\mathbb{C}^3\right)$ is not closed under SU(3) conjugation.

11.2 T2 - A separate M_2 factor is necessary

$\mathrm{SU}(2)_L$ commutes with $\mathrm{SU}(3)_c$ in $G_{\mathrm{SM}} : [\mathrm{SU}(2)_L, \mathrm{SU}(3)_c] = 0$. Their algebras must therefore be independent summands. A_F must contain $M_3(\mathbb{C}) \oplus M_2$ for some 4-dimensional factor M_2.

11.3 T3 - The $\mathbb{C}$ factor is necessary (hypercharges)

The hypercharges of the SM fermions are $Y \in \left\{+\frac{1}{6}, -\frac{1}{3}, +\frac{2}{3}, -\frac{1}{2}, -1\right\}$. These values are not representations of SU(2) or SU(3) (whose irreps have integer or half-integer weights). In particular $Y = 1/6, 2/3, -1/3$ require an independent $U(1)$ generator. The algebra $M_2(\mathbb{C}) \oplus M_3(\mathbb{C})$ alone cannot encode them: a $\mathbb{C}$ summand (or equivalently a $U(1)$ factor) is necessary. One verifies that $6Y \in \mathbb{Z}$ for all SM fermions, confirming that $U(1)_Y$ is the correct abelian structure.

11.4 T4-KO-dim 6 selects $\mathbb{H}$, not $M_2(\mathbb{C})$

Both $\mathbb{H}$ and $M_2(\mathbb{C})$ have real dimension 4. The distinction is the KO -dimension of the spectral triple. KO-dim = 6 (required by the product structure $\mathcal{A} \otimes A_F$ with spacetime KO-dim = 6, see Section 2) imposes $J^2 = +1$ on H_F. For $M_2(\mathbb{C})$ with complex conjugation, $J^2 = -1$ (KO-dim = 2 or 10), and the first-order condition $[[D, a], b^\circ] = 0$ fails. For $\mathbb{H}$, $J^2 = +1$ and the first-order condition is satisfied. Hence $\mathbb{H}$ is selected over $M_2(\mathbb{C})$.

11.5 T5 – A_F is sufficient: $N_F = 96$

$A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ generates exactly $N_F = 96$ fermionic modes via the representation

$$H_F = \left(\mathbb{C}^1 \oplus \mathbb{C}^2 \oplus \mathbb{C}^3\right)^{\otimes 3 \text{ gen } \times 2(\text{p/ap})} \tag{45}$$

Per generation: $6+3+3+2+1+1 = 16$ modes; total $16 \times 3 \times 2 = 96 = N_F$. The adjoint representation gives all 12 gauge bosons ($1+3+8$) and the Higgs sector. No representation is missing; no extra representation is generated.

11.6 Corollary: exhaustive classification

Table 3 shows the result of the exhaustive scan over all C^*-algebras of dimension ≤ 14.

Table 3: C^*-algebras of dimension ≤ 14 and the five axioms.

Algebra	dim	T1	T2	T3	T4	T5	Status
$M_3(\mathbb{C})$	9	✓					
$\mathbb{H} \oplus M_3(\mathbb{C})$	13	✓	✓		✓		
$M_2(\mathbb{C}) \oplus M_3(\mathbb{C})$	13	✓	✓				
$\mathbb{C} \oplus M_3(\mathbb{C})$	10	✓		✓			
$\mathbb{C} \oplus M_2 \oplus M_3$	14	✓	✓	✓	×(T4)		
$\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$	14	✓	✓	✓	✓	✓	Unique

A_F is the only algebra of dimension ≤ 14 satisfying all five conditions. No algebra of dimension ≤ 13 reaches T5; algebras of dimension ≥ 15 violate minimality. $\dim(A_F) = 14$ is derived, not postulated.

11.7 What remains for Level 7(c)

The five axioms C1-C5 themselves are not derived in this work. In particular, three inputs to T4 are not yet derived from a deeper principle: (i) KO-dim = 6; (ii) spacetime dimension $d = 4$; (iii) three fermion generations. Their derivation belongs to Level 7(c), which awaits the completion of the $\mathcal{F}_{\text{total}}(A, \rho)$ fixed-point programme (Programme 1).

12 Derivation of $N_g = 3$ from the Jarlskog Invariant

12.1 The CP-violation constraint from D_F

The result $K_{ij} = 0$ (Proposition 2.3) forces all CP-violating phases into D_F. For N_g generations, the CKM matrix V_{CKM} is $N_g \times N_g$ unitary with $(N_g - 1)(N_g - 2)/2$ independent complex phases. The Jarlskog invariant satisfies

$$J_{CP}(N_g < 3) = 0, \quad J_{\max}(3) = \frac{1}{6\sqrt{3}} \approx 0.0962 \tag{46}$$

the latter being exact, attained by the DFT_3 matrix $V_{jk} = e^{2\pi ijk/3}/\sqrt{3}$.
Proposition 12.1 (Jarlskog maximum strictly decreasing). For $N \geq 3$: $J_{\max}(N+1) < J_{\max}(N)$.

Proof. Given $V \in U(N)$ with $|J(V)| = J_{\max}(N)$, embed into $U(N+1)$ via $V'_{ij} = V_{ij}\sqrt{N/(N+1)}$ for $i, j \leq N$. Then $J(V') = J(V) \cdot (N/(N+1))^2$, so $J_{\max}(N+1) \leq J_{\max}(N) \cdot (N/(N+1))^2 < J_{\max}(N)$.

12.2 The BBN constraint

Planck 2018 gives $N_\nu < 3.3$ at 95% confidence, implying $N_g \leq 3$.
Theorem 12.2 ($N_g = 3$). The unique integer satisfying (i) $J_{CP}(N_g) \neq 0$ and (ii) $N_\nu \leq 3.3$ is $N_g = 3$.

13 F_{V4} and the Global Lyapunov Attractor

Extending F_{V3} to include CP and BBN constraints:

$$F_{\text{V4}}(A, N_g) = F_{V3}(A) - \frac{1}{6\sqrt{3}}\Theta(J_{\max}(N_g) > 0) + \frac{1}{6\sqrt{3}}\max(0, N_g - 3) \tag{47}$$

The coefficients $\nu = \kappa = 1/(6\sqrt{3})$ are not adjusted: they equal $J_{\max}(3)$, the unique value that creates a barrier of equal height on both sides of $N_g = 3$.

The Lyapunov function $V(N_g) = F_{\text{V4}}(N_g) - F_{\text{V4}}(3) \geq 0$ for all N_g, with $V(3) = 0$ and $dV/dt = -|\nabla F_{\text{V4}}|^2 \leq 0$. Hence $(A_F, N_g = 3)$ with $F_{\text{V4}} = -0.364$ is the global attractor of the renormalisation group flow on $\mathcal{M}$.

14 Backreaction and the Structure of G_N

14.1 The backreaction equation

Since $D = D_{\text{ext}} \otimes \mathbf{1}_{H_F} + \gamma^5 \otimes D_F$, varying the full spectral action with respect to D_{ext} gives:

$$\frac{\delta S[D_{\text{ext}}]}{\delta D_{\text{ext}}(x)} = \frac{1}{N_F} K(D_{\text{ext}}, x) \cdot \frac{\text{Tr}_{H_F}\left[D_F^2\right]}{\Lambda_{\text{CC}}^2} \tag{48}$$

where K is the local spectral kernel. In the IR limit this gives:

$$G_N = \frac{2\pi^2 \hbar c}{N_F \Lambda_{\mathrm{CC}}^2} \tag{49}$$

The factor $N_F = 96$ from A_F makes this a non-circular structural prediction. The quantum correction reads

$$G_N(r) = G_N \left[1 + 6\left(l_{\mathrm{CC}}/r\right)^2 + \mathcal{O}\left(l_{\mathrm{CC}}^4/r^4\right)\right] \tag{50}$$

with $\eta = f_4/f_2 = 6$ for $f(x) = e^{-x}$, fully determined.

14.2 Two free scales: an honest diagnostic

Four Big Bounce conditions were tested as candidates to fix M_{Pl}/E_0 : critical density, entropy continuity, modular time periodicity, and Hawking temperature matching. All four fail. QVG therefore has two independent free scales:

Scale	Nature	Source	Status
$E_0 \simeq 3.2\mathrm{meV}$	IR, spectral	Measured (0.8%)	Free input
M_{Pl}	UV, gravitational	Measured via G_N	Free input
$E_0/M_{\mathrm{Pl}} = 2.6 \times 10^{-31}$	Adimensional	Not predicted	Programme 2

Table 4: Two independent free scales of QVG. All 19 SM dimensionless parameters and cosmological ratios are derived from these two plus C1-C5.

15 Programme 2: the Coupled Matrix Integral

Programme 1 treats $Z_N\left[D_{\text{ext}}\right]$ with D_F fixed. Programme 2 is the fully coupled integral:

$$Z_N\left[D_{\text{ext}}, D_F\right] = \int \left[dD_{\text{ext}}\right]\left[dD_F\right] e^{S\left[D_{\text{ext}} \otimes \mathbf{1} + \gamma^5 \otimes D_F\right]/\hbar} \tag{51}$$

with coupling $S_{\text{coupling}} \sim \Lambda_{\mathrm{CC}}^{-2} \operatorname{Tr}\left[D_{\text{ext}}^2 \otimes D_F^2\right]$. The simultaneous saddle-point conditions $\delta Z_N/\delta D_{\text{ext}} = 0$ and $\delta Z_N/\delta D_F = 0$, with UV boundary $D_{\text{ext}} \to D_{\mathrm{dS}}\left(\Lambda_{\mathrm{CC}}\right)$ and IR condition $\rho^* = e^{-D_F^2/E_0}/Z_F$, could fix E_0/M_{Pl} from C1-C5. Programme 1 is the immediate prerequisite. If Programme 2 succeeds, the observable universe is a theorem. If not, the second dimensional input is irreducible - which is also a definitive result.

16 Unification of the Four Fundamental Forces

16.1 One integral, four forces

The central claim of QVG is that all four fundamental interactions are aspects of a single spectral integral:

$$Z[D] = \int [dD] \exp\left(\operatorname{Tr}\left[f\left(D^2/\Lambda^2\right)\right]\right), \quad D = D_{\text{ext}} \otimes \mathbf{1}_{H_F} + \gamma^5 \otimes D_F \tag{52}$$

with $f(x) = e^{-x}$ (thermal kernel). The three Standard Model forces reside in D_F through the algebra $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$. Gravity resides in D_{ext} as depletion of the spectral vacuum. No additional structure is postulated.

16.2 Messengers of the four forces

Each force has a massless messenger that is the residue of a $1/q^2$ pole in the corresponding propagator.

Force	Messenger	Mass	Origin in D	Coupling
Electromagnetism	Photon γ	0	$U(1) \subset A_F$	e^2
Weak nuclear	$W^{\pm}, Z^0$	$\neq 0$	$\mathbb{H} \subset A_F$	$g^2/\sin^2\theta_W$
Strong nuclear	Gluons ($\times 8$)	0	$M_3(\mathbb{C}) \subset A_F$	g_s^2
Gravity	Graviton	0	$\lambda \to 0$ modes of D_{ext}	$\hbar c/\left(N_F \Lambda^2\right)$

Table 5: The four force-carriers in QVG. The three SM messengers are poles of the D_F propagator; the graviton is the pole at $\lambda \to 0$ of the spectral depletion propagator of D_{ext} .

The graviton is qualitatively different from the SM messengers. A photon is an excitation of the electromagnetic field. A graviton is a depletion of the spectral vacuum: the presence of a mass M suppresses nearby spectral modes of D_{ext} , and a second mass falls toward this deficit. The exact coupling formula $S\left[D_{\text{total}}\right] = \sum_i Z_F\left(\lambda_i\right)$ (Equation ??) encodes this depletion precisely.

16.3 Relations between the four forces

The unification is expressed through four relations, all derived from the axioms C1-C5 without free parameters:
(U1) Electroweak mixing angle. $\sin^2\theta_W = 3/8$, derived from the representation theory of A_F (Section 11). This fixes the relative strength of electromagnetism and the weak force.
(U2) Gravity coupled to matter via N_F.

$$G_N = \frac{2\pi^2 \hbar c}{N_F \Lambda^2}, \quad N_F = 96 \tag{53}$$

$N_F = 96$ is the fermion count of A_F - the same object that encodes the three SM forces. If the SM algebra were different, N_F would differ and G_N would differ accordingly. Gravity is therefore not independent of the SM forces: its strength is set by their fermionic content.
(U3) Black-hole entropy and the SM spectrum.

$$S_{BH} = 8 \times \frac{A}{4 l_{\mathrm{Pl}}^2}, \quad 8 = \frac{N_F}{N_{\text{bosons}}} \tag{54}$$

where N_{bosons} counts the bosonic degrees of freedom of A_F. The factor 8 connecting gravitational thermodynamics to the SM particle content is derived, not postulated.
(U4) Exact gravity-matter coupling.

$$S\left[D_{\text{total}}\right] = \sum_i Z_F\left(\lambda_i\right), \quad Z_F(\lambda) = \sum_a e^{-\left(\mu_a+\lambda\right)^2 / \Lambda^2}. \tag{55}$$

Every geometric mode λ_i of D_{ext} couples simultaneously to all fermion masses μ_a of D_F. This is the exact, non-perturbative gravitational coupling between geometry and matter.

16.4 The Planck-QVG correspondence

The unification has a precise thermodynamic interpretation. Planck (1900) found that the vacuum electromagnetic field is discrete: $E = n\hbar\omega$, with $\hbar$ fixing the action quantum. QVG finds that the vacuum geometric field is discrete: the eigenvalues $\{\lambda_n\}$ of D_{ext} are the quanta of geometry, with E_0 fixing the geometric temperature.

Quantity	Planck (1900)	QVG
Mode	frequency ω	eigenvalue λ_n of D
Quantum	$\hbar\omega$	grain $e^{-\lambda_n^2/E_0}$
Partition function	$\sum e^{-n\hbar\omega/kT}$	$\sum e^{-\lambda^2/E_0}$
Equilibrium condition	$Z(kT) = N_{\text{modes}}$	$Z_{S^4}\left(E_0\right) = N_F = 96$
Messenger	photon (excitation)	graviton (depletion)
Coupling constant	e^2	$G_N = 2\pi^2\hbar c/\left(N_F\Lambda^2\right)$

Table 6: The Planck-QVG correspondence. $\hbar$ discretises the action of fields on a fixed background. E_0 discretises the background geometry itself. G_N is the coupling between the two discretisations.

The key difference: Planck postulated discretisation for a continuous field. In QVG the eigenvalues of D_{ext} are discrete from the start - discretisation is not imposed but is the intrinsic structure of spectral geometry.

16.5 Why gravity is weak

Equation (U2) gives the ratio of gravitational to electromagnetic coupling:

$$\frac{G_N m_e^2}{\hbar c e^2}=\frac{2\pi^2}{N_F}\times\frac{m_e^2}{\Lambda^2\alpha_{\mathrm{em}}}\approx 10^{-45} \tag{56}$$

Gravity is weak because it is diluted over $N_F=96$ spectral modes, whereas electromagnetism couples through a single $U(1)$ mode. The hierarchy is not a coincidence or a fine-tuning: it is the ratio $1/N_F$ times the ratio of mass scales.

17 Open Frontiers

Loop degeneracy (structural). The three equations relating $\{E_0, R_H, \Lambda_{\mathrm{GUT}}, \Lambda_{\mathrm{cosmo}}\}$ form an underdetermined system: the loop $R_H \leftrightarrow E_0 \leftrightarrow \Lambda_{\mathrm{cosmo}} \leftrightarrow R_H$ is satisfied for any value of E_0. A fourth independent equation is needed to select $E_0=3.2\mathrm{meV}$ uniquely. The closure condition

$$\frac{4\pi^3 g^*}{45 N_F}=1 \quad\Longrightarrow\quad g^*=\frac{45 N_F}{4\pi^3}=34.83\approx\frac{N_F}{e}, \tag{57}$$

(to 1.4%) points to the QCD deconfinement transition as the selecting mechanism: $g^*=34.83$ lies exactly between the confined ($g^*=17.25$) and deconfined ($g^*=61.75$) phases of QCD at $T_{\mathrm{QCD}}\sim 150\mathrm{MeV}$. The factor $M_3(\mathbb{C})$ in $A_F=\mathbb{C}\oplus\mathbb{H}\oplus M_3(\mathbb{C})$, which encodes the colour force, may be the reason the universe has the size it has. This is the deepest open problem of the programme.

F1-bis (Spectral temperature). Derive $E_0=3.2\mathrm{meV}$ from:

$$Z_{\mathrm{ext}}(E_0)=\sum_{n=0}^{N_{\mathrm{ext}}} e^{-\lambda_n^{\mathrm{ext}}/E_0}=N_F=96 \tag{58}$$

where $\{\lambda_n^{\mathrm{ext}}\}$ are the eigenvalues of $D_{\mathrm{ext}}^2/\Lambda_{\mathrm{GUT}}^2$ on the compact physical spacetime M. This is a transcendental equation on the discrete spectrum of D_{ext} on M; for a flat fourtorus it reduces to a lattice sum. Closure simultaneously brings the Λ_{cosmo} residual from 8.6% to $<0.1\%$ and the T_{CMB} residual from 0.6% to $<0.01\%$.

F2 (Analytical uniqueness). Prove that $\mathcal{F}_\rho[T(Y)]$ has a unique critical point of Morse index 0 on $\mathrm{U}(3)^4$. Numerical evidence: 8 independent convergences; $\Delta\mathcal{F}_{\mathrm{inst}}\approx 7\times 10^{-6}>0$ for 10^4 sampled directions. Requires equivariant Morse theory [16] on $\mathrm{U}(3)^4$ $\left(\chi\left(\mathrm{U}(3)^4\right)=0\right.$ constrains the critical-point structure via $\left.\sum_k(-1)^k c_k=0\right)$.

F5 (Newton's constant). Derive $G=3\pi/\left(f_2 N_F \Lambda_{\mathrm{GUT}}^2\right)$ from the spectral geometry without using G as a measured input. Requires computing f_2 from the external mode density, which is downstream of F1-bis.

Kubo ($\mathbf{12}\boldsymbol{\pi}$ normalisation). Derive the factor 12π in (34) from the retarded Green function of the EM current J^μ in the product triplet. Specifically, compute the Drude weight

$$\Gamma_{\text{Drude}} = \lim_{\omega\to 0} \frac{\operatorname{Im}\left[\chi_{\text{EM}}(\omega)\right]}{\pi\omega} \tag{59}$$

using OPE techniques and verify $\Gamma_{\text{Drude}} = \operatorname{Tr}_F\left(Y^2\right)\times\alpha$. This is a one-loop calculation in the linear response theory of S_{fond} [10].

18 Falsifiable Predictions

P1 - Dark energy EOS. Λ_{cosmo} is the static pressure of the photon gas at fixed $E_0 : w = -1$ exactly. DESI DR2 [14], published March 2025 using three years of data covering 15 million galaxies, strengthened the preference for evolving dark energy to $2.8 - 4.2\sigma$ (depending on data combination), up from 2.5σ in DR1. This is below the 5σ discovery threshold but represents a growing challenge to the QVG prediction. Confirmation of $w \neq -1$ at 5σ by DESI DR3 + Euclid (2026-2027) would falsify the QVG cosmological sector.

Table 7: Falsifiable predictions of the QVG programme.

Prediction	QVG	Observed	Experiment
Dark energy w	-1 exact	DESI DR2: $2.8 - 4.2\sigma$	DESI DR3 + Euclid, 2026-27
T_{CMB}	2.709 K	2.725 K (0.6%)	Next-gen spectroscopy
Λ_{GUT}	2.00(5) 10^{16}GeV	$\times 2(1)\times 10^{16}$GeV	FCC-ee, ˜2040
d_n (nEDM)	$< 10^{-32}e$ m	$< 1.8\times 10^{-26}$	PSI/SNS, 2030

P2. $T_{\text{CMB}} = 2.709$ K; the CMB spectrum should be a perfect blackbody with no μ - or y-type distortions above $|\delta n/n| \sim \eta^2 \approx 5\times 10^{-3}$.

P3. FCC-ee will constrain Λ_{GUT} to $\pm 1\%$, testing the spectral unification identification.

P4. No new CP-violating phases beyond CKM: $d_n < 10^{-32}e$ m. Observation of $d_n > 10^{-28}e$ m would be inconsistent.

P5 - Big Bounce. CMB power suppression at $\ell < 30$ with amplitude $\delta C_\ell/C_\ell \sim 10^{-2}$; testable with LiteBIRD (2032) and CMB-S4 (2035).

P6 - Black-hole entropy. $S_{BH}^{\text{QVG}} = 8\times A/4l_{\text{Pl}}^2$, predicting Hawking evaporation $8\times$ slower and a Planck remnant at $M_{\min} \simeq 106\mu$ g.

P7 - 't Hooft topological corrections. Logarithmic correction $\delta S = \frac{1}{6}\ln\left(A/4l_{\text{Pl}}^2\right)$ from the torus $(g=1)$ topology, with coefficient fixed by $\chi\left(S^2\right) = 2$.

Definitive falsifiers. (a) $w \neq -1$ at 5σ; (b) $d_n > 10^{-28}e$ m; (c) gauge couplings not unifying; (d) discovery of a fourth fermion generation.

19 Quantum Mechanics as Structure in QVG

The QVG programme works within the framework of quantum mechanics but does not derive it from scratch. This section shows that the situation is better than it appears: two of the five standard postulates of quantum mechanics are already resolved within the algebraic framework, and a third is partially constrained. The genuinely open problems are identified precisely.

19.1 What QVG postulates

The programme postulates: (i) the Hilbert space H_F; (ii) the external Hilbert space $H_{\text{ext}} = L^2(M, S)$; (iii) the Born rule; (iv) canonical commutation relations $[Q, P] = i\hbar$; (v) the product structure $\mathcal{H} = H_{\text{ext}} \otimes H_F$. We show that (i) and (iii) are theorems, not postulates.

19.2 Level 0: H_F from the GNS theorem

Proposition 19.1 (GNS construction of H_F). The internal Hilbert space H_F is not a postulate of QVG. It is the unique (up to isomorphism) GNS representation of the pair (A_F, ρ^*), where $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is fixed by axioms $C1 - C5$ and $\rho^* = \mathbf{1}/N_F$ is the fixed point of $\mathcal{F}_\rho$ derived in Section 4.

Proof sketch. The Gelfand-Naimark-Segal theorem [1] states: given a unital C*-algebra $\mathcal{A}$ and a state ω on $\mathcal{A}$, there exists a unique Hilbert space H_ω, a representation $\pi : \mathcal{A} \to \mathcal{B}(H_\omega)$, and a cyclic vector Ω such that $\omega(a) = \langle\Omega|\pi(a)|\Omega\rangle$ for all $a \in \mathcal{A}$. Setting $\mathcal{A} = A_F$ (fixed by C1 − C5) and $\omega = \rho^*$ (the tracial state $\rho^*(a) = N_F^{-1}\operatorname{Tr}(a)$, derived as the unique fixed point of $\mathcal{F}_\rho$), the GNS construction yields H_F as the completion of A_F under the inner product $\langle a, b\rangle = \rho^*(a^* b)$, with $\pi(a)(b) = ab$ and $\Omega = \mathbf{1}/\sqrt{N_F}$. Since both A_F and ρ^* are determined by C1 − C5, so is H_F.

This result is implicit in the Chamseddine-Connes-Marcolli framework [

$\dim H_F = 96$

H_F

$\Pr(P) = \operatorname{Tr}(\rho P)$

$\lambda) = \operatorname{Tr}(\rho P_\lambda)$

$\dim H_F = 96$

$\mathbb{C}$ A_F $\dim H_F$ ≥ 3 $= 96 \geq 3$ $[D, a] \neq 0$

A_F

$\mathbb{C}$

If $[D, a] = 0$ for all a, then D is central, the geometry is commutative, and there is no non-trivial dynamics. The condition $[D_F, a] \neq 0$ for generic $a \in A_F$ is a consequence of the axiom C3 (first-order condition) and implies $[Q, P] \neq 0$ structurally. The specific value $\hbar$ is the normalisation unit of D in the spectral action $S = \operatorname{Tr}\left[f\left(D^2/\Lambda_{\text{GUT}}^2\right)\right]$; in dimensionless units it reduces to $\hbar = M_{\text{Pl}}^{-1}$, the sole remaining dimensional input consistent with the Buckingham π-theorem (Section 17).

19.5 Level 3: H_{ext} and the geometry of M (open)

The external Hilbert space $H_{\text{ext}} = L^2(M, S)$ is equivalent to specifying the spacetime manifold M. In the spectral framework, $M = \text{Spec}(\mathcal{A}_{\text{ext}})$, so deriving H_{ext} requires deriving the external algebra $\mathcal{A}_{\text{ext}}$. This is precisely Frontier F1-bis: the self-consistency condition $Z_{\text{ext}}(E_0) = N_F = 96$ determines M (and hence $\mathcal{A}_{\text{ext}}$ and H_{ext}) once E_0 is known, while E_0 is itself determined by the geometry of M. Closing this loop is equivalent to resolving F1-bis simultaneously in both languages.

19.6 Level 4: quantum measurement and spectral quantum gravity

The deepest open problem is the quantum measurement process. In standard quantum mechanics, measurement equals the collapse of the wave function. In QVG, measurement corresponds to the interaction between the system and the external Dirac operator D_{ext}. The difficulty is that D_{ext} is currently classical (fixed geometry with no back-reaction). A complete quantum theory requires promoting D_{ext} to a quantum operator:

$$D_{\text{ext}} \longrightarrow \hat{D}_{\text{ext}} \in \mathcal{B}(\mathcal{H}_{\text{geom}}) \tag{61}$$

where $\mathcal{H}_{\text{geom}}$ is a Hilbert space of geometries. This is the spectral quantum gravity programme, in which the path integral $Z = \int \mathcal{D}[D] \exp\left(-\operatorname{Tr}\left[f\left(D^2/\Lambda_{\text{GUT}}^2\right)\right]/\hbar\right)$ is computed over all admissible Dirac operators. This integral is UV-finite in QVG (the spectral cutoff f suppresses all modes above Λ_{GUT}), making spectral quantum gravity a well-defined extension of the present programme. It is, however, beyond the scope of QVG as currently formulated.

19.7 Summary

Table 8 summarises the status of each quantum-mechanical postulate within the QVG framework.

Level	Problem	Tool	Status
0	H_F not postulated	GNS theorem	Resolved
1	Born rule	Gleason's theorem	Resolved
2	$[Q, P] \neq 0$	Spectral distance	Partial ($\hbar = M_{\text{Pl}}$)
3	$H_{\text{ext}} = L^2(M, S)$	F1-bis	Open
4	Quantum measurement	Spectral QG	Programme boundary

Table 8: Status of quantum-mechanical postulates in QVG. Levels 0 and 1 are theorems within the existing framework. Level 3 is equivalent to Frontier F1-bis. Level 4 defines the boundary of QVG and the starting point of spectral quantum gravity.

20 Conclusion

The QVG programme unifies the four fundamental forces - electromagnetism, the weak and strong nuclear forces, and gravity - within a single spectral integral:

$$Z[D]=\int[dD]\exp\left(\operatorname{Tr}\left[f\left(D^2/\Lambda^2\right)\right]\right), \quad D=D_{\text{ext}}\otimes\mathbf{1}+\gamma^5\otimes D_F$$

The three Standard Model forces reside in D_F through $A_F=\mathbb{C}\oplus\mathbb{H}\oplus M_3(\mathbb{C})$; gravity resides in D_{ext} as depletion of the spectral vacuum. Their messengers - photon, $W^{\pm}/Z^0$, gluons, graviton - are all residues of $1/q^2$ poles of the corresponding propagators (Table 5). Four quantitative relations connect the forces (U1-U4, Section 16), all derived from the five axioms C1-C5 without free parameters.

The programme also derives all nineteen Standard Model parameters, the cosmological constant, and the CMB temperature from the same geometric principle: the maximumentropy fixed point of the spectral free-energy functional $\mathcal{F}_\rho[T]$ applied to the product spectral triple ($\mathcal{A}\otimes A_F, \mathcal{H}\otimes H_F, D_{\text{ext}}\otimes\mathbf{1}+\gamma^5\otimes D_F$).

New results in v5.

- (R20) Unification of the four forces (Section 16). All four interactions are aspects of the single integral $Z[D]$. Their force-carriers are identified as spectral poles. Four derived relations connect their strengths: (U1) $\sin^2\theta_W=3/8$; (U2) $G_N\propto 1/N_F$; (U3) $S_{BH}=8A/4l_{\text{Pl}}^2$; (U4) $S\left[D_{\text{total}}\right]=\sum_i Z_F\left(\lambda_i\right)$. Gravity is weak because it is diluted over $N_F=96$ modes.
- (R21) Planck-QVG correspondence (Section 16). $\hbar$ discretises the action of fields on a fixed background. E_0 discretises the background geometry itself. $G_N=2\pi^2\hbar c/\left(N_F\Lambda^2\right)$ is the coupling between the two discretisations - the gravitational analogue of e^2. The equilibrium condition $Z_{S^4}\left(E_0\right)=N_F=96$ is the spectral analogue of Planck's thermal equilibrium condition.

New results in v4.

- (R17) Exact coupling formula (rigorous). Since $\left[D_{\text{ext}}, D_F\right]=0$, the BCH expansion terminates, giving $S\left[D_{\text{total}}\right]=\sum_i Z_F\left(\lambda_i\right)$ with $Z_F(\lambda)=\sum_a\exp\left(-\left(\mu_a+\lambda\right)^2/\Lambda^2\right)$ exactly.
- (R18) Non-perturbative dominance (MCMC). With the coupled weight $\Delta^2\exp\left(+\sum_i Z_F\left(\lambda_i\right)\right)$, $\left\langle\lambda^2\right\rangle$ is reduced $85-97\%$ versus Programme 1. The perturbative estimate predicted 10^{-28}; the true effect is $\sim 90\%$.

- (R19) Five mechanisms eliminated. $Z_N\left[D_{\text{ext}}\right]$ alone, perturbative coupling, GibbonsHawking temperature, Dyson large- N, and naive GWW transition all give $E_0 \sim \mathcal{O}(\Lambda)$ or $\mathcal{O}\left(M_{\text{Pl}}\right)$.

- (C1) Central conjecture of Programme 2 (unproved). Numerical evidence suggests $E_0/\Lambda \stackrel{?}{=} \exp\left(-2\pi \operatorname{Tr}_F\left[Y^2\right]\right) = e^{-20\pi} \approx 5.2\times10^{-28}$, versus the observed 1.6×10^{-28} (factor 3.2). The ingredient $\operatorname{Tr}_F\left[Y^2\right]=10$ is derived from A_F. The formula is not proved; it is the target of Programme 2.

Programme 2 roadmap. Four steps to prove or refute (C1): (A) confined potential $V_{\text{tot}} = H^2\lambda^2/\left(2\Lambda^2\right) - \log Z_F(\lambda)$; (B) Dyson equation with mixed harmonic+Gaussian potential (solvable by RMT); (C) phase transition in $F = \log Z_N/N^2$; (D) instanton action $S_{\text{inst}} \stackrel{?}{=} 2\pi \operatorname{Tr}_F\left[Y^2\right]$. Horizon: 6-12 months. Steps A and B: negative results (rigorous). Step A computed $V_{\text{tot}}(\lambda) = g\lambda^2 - \log Z_F(\lambda)$ exactly. Since $\mu_a/\Lambda < 10^{-8}$ for all SM fermions, $Z_F(\lambda) \approx N_F e^{-\lambda^2}$ to all accessible orders, giving $V_{\text{tot}} \approx (g+1)\lambda^2 - \log N_F$: a pure Gaussian with no phase transition. The de Sitter coupling $g_{\text{phys}} \sim 10^{-117}$ is negligible. Step B showed that $Z_F(\lambda)$ is piecewise constant for $\lambda < \mu_\nu/\Lambda \sim 10^{-28}$, with no spectral structure in the region where E_0/Λ is observed. All seven mechanisms tested produce $E_0 \sim \mathcal{O}(\Lambda)$. Three paths were then tested and all eliminated: (i) The toric correction $F_1 = -(N/24)\ln 4$ is polynomial, not exponential; generating e^{-62} requires $N_{\text{ext}} \sim 1075$, not fixed by C1-C5. (ii) The quartic coefficient $c_4 \sim \operatorname{Tr}_F\left[D_F^4\right] \sim 10^{-57}$ is positive (stabilising); no double well, no instanton. (iii) The one-loop mass $m_\phi^2 = 2 > 0$; the minimum at $\phi = 0$ is stable, no Coleman-Weinberg breaking. The structural obstruction: every correction to V_{tot} scales as $\left(\mu_a/\Lambda\right)^{2n} \sim 10^{-16n}$, while $E_0/\Lambda \sim 10^{-28}$ corresponds to $n \approx 1.75$, a non-integer.

Conclusion of Programme 2: E_0 is a free parameter of QVG, independent of M_{Pl}. QVG derives the structure of cosmology (Λ_{cosmo} , T_{CMB} , $N_g = 3$) but not the absolute scale (H_0, M_{Pl}). The open question "why $T_{\text{CMB}}/T_{\text{Pl}} = 5.8\times10^{-32}$?" is equivalent to "why does H_0 have its observed value?"-open in every current quantum-gravity programme.

- (R13) $N_g = 3$ derived from axioms $\mathbf{C1}-\mathbf{C5}$. The number of generations is fixed by the intersection of two constraints derived from D_F : (i) the Jarlskog invariant $J_{CP}\left(N_g\right) = 0$ for $N_g < 3$, since the number of independent CP phases is $\left(N_g-1\right)\left(N_g-2\right)/2$, and $K_{ij} = 0$ (Act 2) forces all phases into D_F; (ii) $N_g \geq 4$ is excluded by BBN ($N_\nu < 3.3$, Planck 2018). The unique integer satisfying both is $N_g = 3$. The maximum Jarlskog invariant $J_{\max}(3) = 1/(6\sqrt{3})$ (DFT_3 matrix) is proved strictly decreasing for $N \geq 3$ by an injection $U(N) \hookrightarrow U(N+1)$ with dilution factor $(N/(N+1))^2$.

- (R14) F_{V4} and the Lyapunov proof. The functional

$$F_{\text{V4}}\left(A, N_g\right) = F_{V3}(A) - \frac{1}{6\sqrt{3}}\Theta\left(J_{\max}\left(N_g\right) > 0\right) + \frac{1}{6\sqrt{3}}\max\left(0, N_g - 3\right)$$

has a unique global minimum at $N_g = 3, A = A_F$, with $F_{\mathrm{V4}}(A_F, 3) = -0.364$. The Lyapunov function $V(N_g) = F_{\mathrm{V4}}(N_g) - F_{\mathrm{V4}}(3) \geq 0$ for all integers N_g, with $V(3) = 0$, proves that $N_g = 3$ is a global attractor of the renormalisation flow. The symmetry $\nu = \kappa = 1/(6\sqrt{3})$ reflects that the baryogenesis energy gain and the BBN penalty share the same characteristic scale $J_{\max}(3)$.

- (R15) Structure of Newton's constant (non-circular). The backreaction equation $\delta S[D_{\text{ext}}]/\delta D_{\text{ext}}(x) = \operatorname{Tr}_{H_F}\left[D_F^2/\Lambda_{\mathrm{CC}}^2\right]_x$, derived from $D = D_{\text{ext}} \otimes \mathbb{1} + \gamma^5 \otimes D_F$, gives in the IR limit:

$$G_N = \frac{2\pi^2 \hbar c}{N_F \Lambda_{\mathrm{CC}}^2}$$

This is a structural prediction: $G_N \propto 1/\left(N_F \Lambda_{\mathrm{CC}}^2\right)$. The factor $N_F = 96$ is derived non-circularly from A_F. The quantum correction takes the form $G_N(r) = G_N[1 + \eta\,(l_{\mathrm{CC}}/r)^2 + \ldots]$ with $\eta = f_4/f_2 = 6$ (for $f(x) = e^{-x}$) fully determined by the spectral kernel, without any free parameter.

- (R16) Two free scales - an honest diagnostic. Four natural conditions of the Big Bounce (critical density, entropy continuity, modular time, Hawking temperature) were tested as candidates to fix M_{Pl}/E_0. All four fail: $M_{\mathrm{Pl}}/E_0 = 3.8 \times 10^{30}$ is neither an integer nor a simple fraction, and none of the bounce conditions constrains it. QVG therefore has two independent free scales: M_{Pl} (gravitational UV) and $E_0 \sim T_{\mathrm{CMB}}$ (spectral IR). This is the same situation as LQC (γ Immirzi free) and string theory (g_s free). The difference is that in QVG the problem is precisely formulated and the resolution path identified (Programme 2 below).

Programme 2. The only mechanism identified that could fix E_0/M_{Pl} from axioms C1-C5 is the simultaneous saddle point of the coupled matrix integral

$$Z_N[D_{\text{ext}}, D_F] = \int [dD_{\text{ext}}][dD_F] \exp\left(S\left[D_{\text{ext}} \otimes \mathbf{1} + \gamma^5 \otimes D_F\right]/\hbar\right)$$

with UV boundary condition $D_{\text{ext}} \rightarrow D_{\mathrm{dS}}(\Lambda_{\mathrm{CC}})$ (de Sitter) and IR condition $\rho^* = e^{-D_F^2/E_0}/Z_F$. The coupling term $S_{\text{coupling}} \sim \Lambda^{-2} \operatorname{Tr}\left[D_{\text{ext}}^2 \otimes D_F^2\right]$ links the two saddle-point equations. If this coupling fixes E_0/M_{Pl} from C1-C5, the observable universe becomes a theorem. If not, the second dimensional input is irreducible - which would also be a result. Programme 2 requires Programme 1 ($Z_N[D_{\text{ext}}]$ alone, the RMT matrix model) as an immediate prerequisite.

Results R1-R12 from v1-v3 are unchanged. The programme now has exactly two external inputs: M_{Pl} (one dimensional unit, Buckingham π-theorem) and E_0 (equivalent to Frontier F1-bis). Programme 2 is the sole identified path to eliminating the second.

The most urgent experimental test remains P1: $w = -1$ exactly, now challenged at $2.8 - 4.2\sigma$ by DESI DR2 (March 2025). DESI DR3 and Euclid (2026-2027) will provide a near-definitive verdict.

Code availability. https://github.com/berjarry71/QVG

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New results in v3: (R6) the Big Bounce is derived from the matrix model of D without free parameters (Section 9); (R7) the Bekenstein-Hawking entropy takes the form $S_{BH} = 8 \times A/4l_{\mathrm{Pl}}^2$, with the factor $8 = N_F/N_{\mathrm{bosons}}$ derived algebraically (Section 10); (R8) the 't Hooft topological expansion identifies Act 2 as the planar limit of Act 3 (Section 8); (R9) the Lorentzian obstacle (Barrett 2007) does not block the semi-classical action; (R10) the spectral gap equation $\alpha^* = \ln(N_F) / \left(N_F \langle |n|^2 \rangle_q \right)$ provides the fixed-point derivation of $\alpha_{\mathrm{sol}} = 0.6413$.

Results R1-R5 from v2 are unchanged. (R11) Level 7(b) is completed: $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is proved unique by five chained theorems T1-T5 (Section 11); (R12) the QVG transcendental constant $x^* = 2.64782$ is derived from $Z_{S^4}(x^*) = N_F = 96$ and predicts $R_{S^4} = 2.6478 l_{\mathrm{GUT}}$. The programme retains one external input, E_0 (equivalent to Frontier F1-bis); its derivation from first principles requires Level 7 $(F_{\mathrm{total}}(A, \rho))$., the cosmological constant, and the CMB temperature from a single geometric principle: the maximum-entropy fixed point of the spectral free-energy functional $\mathcal{F}_\rho[T]$ applied to the product spectral triple ($\mathcal{A} \otimes A_F, \mathcal{H} \otimes H_F, D_{\mathrm{ext}} \otimes \mathbb{1} + \gamma^5 \otimes D_F$).

Three new results: (R1) $K_{ij} = 0$ exactly (Proposition 2.3); (R2) $g^* = 2$ reduces the Λ_{cosmo} residual from 55% to 8.6%; (R3) $T_{\mathrm{CMB}} = 2.709$ K derived from a_4^{EM} via CaldeiraLeggett. Section 19 establishes two further results: (R4) H_F is not a postulate but the GNS representation of (A_F, ρ^*); (R5) the Born rule is forced by Gleason's theorem applied to $\dim H_F = 96 \geq 3$. The programme therefore has two remaining postulates: M_{Pl} (one dimensional unit, required by the Buckingham π-theorem) and the external geometry M (equivalent to Frontier F1-bis). The programme has one external input, E_0; Frontier F1-bis ($Z_{\mathrm{ext}}(E_0) = N_F = 96$) would close all residuals simultaneously.

The most urgent test is P1: $w = -1$ exactly, now challenged at $2.8 - 4.2\sigma$ by DESI DR2 (March 2025). DESI DR3 and Euclid (2026-2027) will provide a near-definitive verdict.

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References

[1] I. M. Gelfand and M. A. Naimark, "On the embedding of normed rings into the ring of operators in Hilbert space," Mat. Sbornik 12, 197 (1943). I. E. Segal, "Irreducible representations of operator algebras," Bull. Amer. Math. Soc. 53, 73 (1947).
[2] A. M. Gleason, "Measures on the closed subspaces of a Hilbert space," J. Math. Mech. 6, 885 (1957).
[3] A. Connes, Noncommutative Geometry, Academic Press, 1994.
[4] A. Chamseddine and A. Connes, "The spectral action principle," Commun. Math. Phys. 186, 731 (1997).
[5] A. Chamseddine, A. Connes, and M. Marcolli, "Gravity and the standard model with neutrino mixing," Adv. Theor. Math. Phys. 11, 991 (2007).
[6] D. V. Vassilevich, "Heat kernel expansion: user's manual," Phys. Rept. 388, 279 (2003).
[7] P. B. Gilkey, Invariance Theory, the Heat Equation, and the Atiyah-Singer Index Theorem, 2nd ed., CRC Press, 1995.
[8] A. O. Caldeira and A. J. Leggett, "Influence of dissipation on quantum tunneling," Phys. Rev. Lett. 46, 211 (1981).
[9] A. O. Caldeira and A. J. Leggett, "Quantum tunnelling in a dissipative system," Ann. Phys. 149, 374 (1983).
[10] R. Kubo, "Statistical-mechanical theory of irreversible processes," J. Phys. Soc. Japan 12, 570 (1957).
[11] D. J. Fixsen, "The temperature of the cosmic microwave background," Astrophys. J. 707, 916 (2009).
[12] S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024).
[13] DESI Collaboration, "DESI 2024 VI: Cosmological constraints from BAO," arXiv:2404.03002 (2024).
[14] DESI Collaboration, "DESI DR2: Dark energy constraints from three years of BAO observations," arXiv:2503.14738 (2025).
[15] S. Weinberg, "The cosmological constant problem," Rev. Mod. Phys. 61, 1 (1989).
[16] J. Milnor, Morse Theory, Princeton University Press, 1963.
[17] A. Connes, "On the spectral characterization of manifolds," J. Noncommut. Geom. 7, 1 (2013).
[18] : Unification of Ppysics by Quantum Vacuum Geometry, KDP Pblishing (2026). Code: https://github.com/berjarry71/QVG.
[19] J. Barrett, Lorentzian version of the noncommutative geometry of the Standard Model, J. Math. Phys. 48, 012303 (2007).
[20] A. Ashtekar, T. Pawlowski, P. Singh, Quantum nature of the Big Bang, Phys. Rev. Lett. 96, 141301 (2006).