A modification of the Sundaram sieve, and the count of twin primes

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Abstract. A modification of the Sundaram sieve of twin primes is introduced. A twin prime pair is obtained from the set D which consists of elements n such that n not representable as the forms 2ij + i + j and 2ij + i + j -1 for positive integers i, j. An element n is in D if and only if (2n+1, 2n+3) is a twin prime pair. This sieve algorithm can find the number of twin primes below a certain value.

By definition [1], let D be a set of n such that n not representable as n= 2ij+i+j and n= 2ij+i+j -1 for positive integers i, j. Let f(n) is a number of elements of D <= $3n^2$ +1.

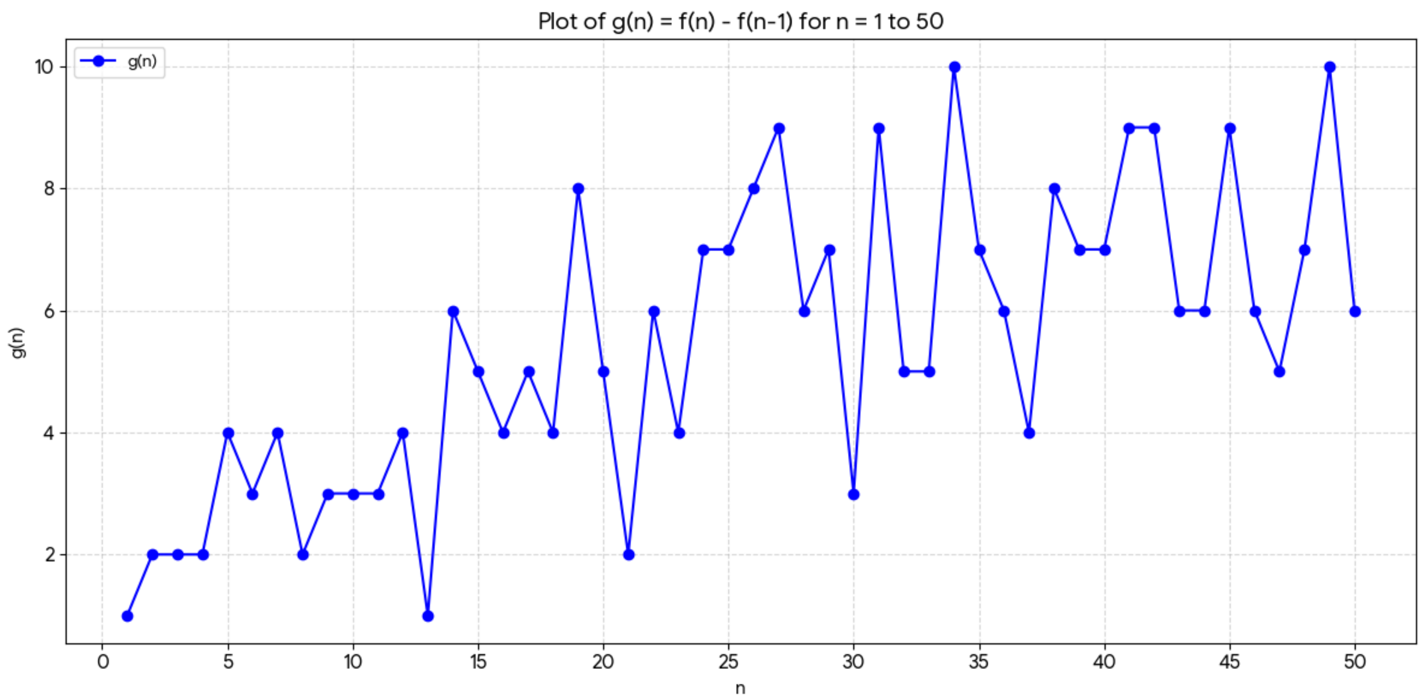
As an example, elements of D <= 433 are:

D = [1, 2, 5, 8, 14, 20, 29, 35, 50, 53, 68, 74, 89, 95, 98, 113, 119, 134, 140, 155, 173, 209, 215, 230, 260, 284, 299, 308, 320, 329, 404, 410, 413, 428].

Let g(n) = f(n) - f(n-1). The values of g(n) are calculated based on the number of elements in the set D that fall within the interval (3(n-1)^2+1, 3n^2+1]. An integer n is in D if and only if (2n+1, 2n+3) is a twin prime pair.

As an example, g(n) for n = 1 to 50 are:

g(1:50) = [1, 2, 2, 2, 4, 3, 4, 2, 3, 3, 3, 4, 1, 6, 5, 4, 5, 4, 8, 5, 2, 6, 4, 7, 7, 8, 9, 6, 7, 3, 9, 5, 5, 10, 7, 6, 4, 8, 7, 7, 9, 9, 6, 6, 9, 6, 5, 7, 10, 6].



The plot shows the values of g(n) for n=1 to 50.

As established, $g(n) = f(n) - f(n-1)$ counts the number of integers k in set D such that $3(n-1)^2+1 < k \leq 3n^2 + 1$. The plot illustrates how many twin prime pairs are indexed in each subsequent quadratic interval. Because the length of the interval $3n^2 + 1 - (3(n-1)^2 + 1) = 6n - 3$ grows linearly with n, there is a general upward trend in the values of g(n), despite the inherent fluctuations associated with the distribution of prime numbers.

We have a conjecture that $g(n) \geq 1$ for all n. It effectively states that within every interval of the form $(3(n-1)^2 +1, 3n^2+1]$ there is at least one index k such that $(2k+1, 2k+3)$ is a twin prime pair.

Reference

[1] Homsup, Wiroj, A sieve of Sundaram for twin prime, ai.viXra: 2603.2029.