Emergent Gravity from the Onsager–Machlup Functional: Induced Einstein–Hilbert Action via One-Loop Stochastic Fluctuations

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Abstract

We compute the one-loop effective action for the metric $g_{\mu\nu}$ obtained by integrating out the stochastic fluctuations of the Onsager–Machlup (OM) probability fluid on a curved spin manifold. The OM action for the relativistic stochastic coherence framework [2, 3] describes $N_{\rm int} = 9$ internal degrees of freedom: the density amplitude σ, the phase S, the Lorentz rotor $R \in \mathrm{Spin}^+(1,3)$ (6 bivector components), and the Takabayasi angle β. Expanding to second order around the classical vacuum and applying the Schwinger–DeWitt heat kernel expansion, we derive the Seeley–DeWitt coefficients $a_0 = 9$ and $a_1 = \frac{7}{4}\mathcal{R} - 10m^2$, where $\mathcal{R}$ is the Ricci scalar and m the particle mass. Three features of the OM framework are essential: (i) the cross-coupling between density and phase fluctuations generates an effective gauge connection whose contribution $-\mathcal{A}_\mu\mathcal{A}^\mu = +4m^2$ acts as a mass for the scalar sector; (ii) the algebraic cancellation $+m^2 - m^2 = 0$ renders density fluctuations massless (sensitive only to curvature); and (iii) the rotational kinetic term $\frac{1}{4}\sigma^2\langle\Omega^{\rm cov}_\mu \Omega^{{\rm cov}\,\mu}\rangle$ generates the Bochner Laplacian (not the Hodge–de Rham Laplacian) on the bivector sector, so the Weitzenböck curvature endomorphism is absent ($\kappa = 0$). The positive coefficient $\frac{7}{4}$ ensures that the induced Newton constant $G_{\rm ind} = 4\pi\hbar^2/(7m^2c^2)$ is positive (attractive gravity). This is a signature of the Clifford-algebraic structure of the OM framework: unlike standard bosonic field theories where one-loop corrections can induce repulsive gravity, the OM probability fluid induces attractive gravity from a purely stochastic substrate.

1 Introduction

The stochastic coherence framework derives the Dirac equation, orbital quantization, Fermi–Dirac statistics, and the Pauli exclusion principle from the Onsager–Machlup (OM) variational principle combined with $\rho \geq 0$ and C^∞ regularity of observables [1, 2, 3]. The extension to curved spacetime [3] constructs a covariant OM action on spin manifolds, recovers the curved-spacetime Dirac equation, and establishes the on-shell equivalence of the OM and Dirac stress-energy tensors.

However, in that construction the metric $g_{\mu\nu}$ remains an external input. The deeper question is whether the Einstein–Hilbert action can *emerge* from the OM framework rather than being coupled to it. Sakharov [4] proposed that one-loop quantum fluctuations of matter fields on a curved background induce an effective gravitational action; the leading term is the Einstein–Hilbert action with an induced Newton constant $G_{\rm ind}$ determined by the field content.

In this paper we carry out the Sakharov program for the OM probability fluid. The calculation is well-defined because the OM framework provides: (i) an explicit covariant action functional; (ii) a unique noise structure (Proposition 2.2 of [3]); and (iii) a definite internal field content ($N_{\rm int} = 9$ degrees of freedom). The main result is the Seeley–DeWitt coefficient

$$a_1 = \tfrac{7}{4}\mathcal{R} - 10m^2 \tag{1}$$

which yields a positive induced Newton constant (attractive gravity).

We work in natural units $\hbar = c = 1$ except where clarity requires explicit factors.

2 The Covariant OM Action

We recall the covariant OM action on a spin manifold (M^4, g) with spin connection ω_μ [3]:

$$A = \int_M \sqrt{|g|}\, d^4x\, \mathcal{L}_{\rm OM} \tag{2}$$

with Lagrangian density

$$\begin{aligned}\mathcal{L}_{\rm OM} = {}& g^{\mu\nu}(\partial_\mu\sigma)(\partial_\nu\sigma) + \sigma^2 g^{\mu\nu}(\partial_\mu S)(\partial_\nu S) \\ & + \tfrac{1}{4}\sigma^2\langle\Omega^{\rm cov}_\mu \Omega^{{\rm cov}\,\mu}\rangle + \tfrac{1}{4}\sigma^2(\partial\beta)^2 \\ & - \tfrac{1}{2}\sigma^2(\partial_\mu\beta)\, s^\mu_{\rm vort} - m^2\sigma^2\cos\beta - \tfrac{1}{4}\mathcal{R}\,\sigma^2\end{aligned} \tag{3}$$

where $\sigma = \sqrt{\rho} \geq 0$, $\Omega^{\rm cov}_\mu = 2(D_\mu R)\widetilde{R}$ with $D_\mu R = \partial_\mu R + \frac{1}{2}\omega_\mu R$, and $\mathcal{R}$ is the Ricci scalar. The curvature coupling $-\frac{1}{4}\mathcal{R}\sigma^2$ is required by the Lichnerowicz formula for consistency with the curved-spacetime Dirac equation [3].

3 Vacuum Background and Fluctuations

3.1 Classical vacuum

Definition 3.1 (Classical vacuum)**.** *The classical vacuum is the configuration:*

$$\sigma_0 = const > 0, \qquad \beta_0 = 0,$$
$$R_0 = 1, \qquad S_0 = m\,x^0 \tag{4}$$

so that $\Omega^{\rm flat}_{\mu,0} = 0$, $\Omega^{\rm cov}_{\mu,0} = \omega_\mu$, and the four-velocity is $u_\mu = \partial_\mu S_0/m$ with $u_\mu u^\mu = 1$.

Remark 3.2. The background satisfies the OM field equations on a Ricci-flat manifold ($\mathcal{R} = 0$): the continuity equation gives $\nabla_\mu(\sigma_0^2 u^\mu) = 0$, which holds for constant σ_0 and geodesic u^μ ($\nabla_\mu u^\mu = 0$).

3.2 Fluctuation fields

We expand around the vacuum:

$$\sigma = \sigma_0 + \delta\sigma, \qquad S = S_0 + \delta S,$$
$$R = e^{\theta/2} \approx 1 + \tfrac{1}{2}\theta, \qquad \beta = \delta\beta \tag{5}$$

where $\theta = \theta^{AB}\gamma_A\gamma_B$ is a bivector-valued fluctuation (6 independent components).

Definition 3.3 (Normalized fluctuation multiplet). *To obtain canonical kinetic terms, define:*

$$\Phi = \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3^A\\ \phi_4\end{pmatrix} = \begin{pmatrix}\delta\sigma\\ \sigma_0\,\delta S\\ \frac{1}{2}\sigma_0\,\theta^A\\ \frac{1}{2}\sigma_0\,\delta\beta\end{pmatrix} \tag{6}$$

where $A = 1, \ldots, 6$ labels the independent bivector components. The internal dimension is $N_{\text{int}} = 1+1+6+1 = 9$.

4 Quadratic Action

Expanding the OM Lagrangian (3) to second order in the fluctuations, we identify each contribution to the quadratic action $A^{(2)} = \frac{1}{2}\int\sqrt{|g|}\,\Phi^T\mathcal{O}\,\Phi\,d^4x$.

4.1 The density–phase cross-coupling

Lemma 4.1 (Cross-term from $\sigma^2(\partial S)^2$). *Expanding $\sigma^2(\partial S)^2$ around the vacuum:*

$$\begin{aligned}(\sigma_0+\delta\sigma)^2\big[m^2 + 2mu^\mu\partial_\mu\delta S + (\partial\delta S)^2\big]\\ \supset\; m^2(\delta\sigma)^2 + 4m\sigma_0\,\delta\sigma\,(u^\mu\partial_\mu\delta S)\\ + \sigma_0^2(\partial\delta S)^2\end{aligned} \tag{7}$$

In normalized variables: the first term gives $m^2\phi_1^2$; the third gives $(\partial\phi_2)^2$; and the cross-term gives $4m\,u^\mu\phi_1\partial_\mu\phi_2$.

4.2 The mass cancellation

Lemma 4.2 (Algebraic mass cancellation). *The mass potential $-m^2\sigma^2\cos\beta$ expanded at $\beta_0 = 0$ contributes $-m^2(\delta\sigma)^2$ at quadratic order (from $\cos 0 = 1$). Combined with the $+m^2(\delta\sigma)^2$ from Lemma 4.1:*

$$+m^2(\delta\sigma)^2 - m^2(\delta\sigma)^2 = 0 \tag{8}$$

The density fluctuation $\phi_1 = \delta\sigma$ acquires no mass from the kinetic or potential sectors. Its only "mass" comes from the curvature coupling $-\frac{1}{4}\mathcal{R}(\delta\sigma)^2$ and from the gauge connection (see below).

4.3 The chirality mass

Lemma 4.3 (Mass of $\delta\beta$). *Expanding $-m^2\sigma_0^2\cos(\delta\beta) \approx -m^2\sigma_0^2(1-\frac{1}{2}\delta\beta^2)$ gives $+\frac{1}{2}m^2\sigma_0^2(\delta\beta)^2$. In normalized variables $\phi_4 = \frac{1}{2}\sigma_0\delta\beta$, so $\delta\beta^2 = 4\phi_4^2/\sigma_0^2$, giving:*

$$M_{44} = 2m^2 \tag{9}$$

4.4 The bivector sector

Lemma 4.4 (Rotor fluctuations are massless). *The rotational kinetic term $\frac{1}{4}\sigma_0^2\langle\Omega_\mu^{\text{cov}}\Omega^{\text{cov}\,\mu}\rangle$ with $\Omega_\mu^{\text{cov}} = \omega_\mu + \nabla_\mu\theta$ expands as:*

$$\begin{aligned}\tfrac{1}{4}\sigma_0^2\big[\langle\omega_\mu\omega^\mu\rangle + 2\langle\omega_\mu\nabla^\mu\theta\rangle\\ + \langle\nabla_\mu\theta\,\nabla^\mu\theta\rangle\big]\end{aligned} \tag{10}$$

The first term is a background constant. The second is linear in θ (absorbed by the background equations). The third is the kinetic term for θ: it is purely derivative, with no algebraic θ^2 term. Therefore the bivector fluctuations carry no mass:

$$M_{33}^{AB} = 0 \qquad \forall\, A, B \tag{11}$$

5 The Generalized Laplace Operator

We write the quadratic action as

$$A^{(2)} = \tfrac{1}{2}\int\sqrt{|g|}\;\Phi^T\,\mathcal{O}\,\Phi\;d^4x \tag{12}$$

and bring $\mathcal{O}$ to the standard generalized Laplace form:

$$\mathcal{O} = -g^{\mu\nu}\mathcal{D}_\mu\mathcal{D}_\nu\cdot\mathbf{1}_9 + E \tag{13}$$

where $\mathcal{D}_\mu = \nabla_\mu + \mathcal{A}_\mu$.

5.1 The internal connection $\mathcal{A}_\mu$

Proposition 5.1 (Internal connection). *The connection $\mathcal{A}_\mu$ is a 9×9 matrix with two non-trivial blocks:*

(i) Scalar block (ϕ_1, ϕ_2)*: the cross-term $4mu^\mu\phi_1\partial_\mu\phi_2$ requires*

$$\mathcal{A}_\mu^{(1,2)} = 2m\,u_\mu, \quad \mathcal{A}_\mu^{(2,1)} = -2m\,u_\mu \tag{14}$$

(ii) Bivector block (ϕ_3^A)*: the spin connection acts in the adjoint representation:*

$$(\mathcal{A}_\mu)^{AB}\phi_3^B = [\omega_\mu, \phi_3]^A \tag{15}$$

(iii) *Components involving ϕ_4 ($\delta\beta$): no first-derivative couplings appear at quadratic order in the vacuum, so $\mathcal{A}_\mu^{(4,j)} = 0$ for all j.*

Proof. Part (i) follows from Lemma 4.1: the term $4m\,u^\mu\phi_1\partial_\mu\phi_2$ must arise from $-\Phi^T\mathcal{D}_\mu\mathcal{D}^\mu\Phi$ after integration by parts. The antisymmetric off-diagonal connection (14) generates exactly this cross-term.

Part (ii) follows from the covariant derivative of a bivector: $D_\mu\theta = \partial_\mu\theta + [\omega_\mu, \theta]$, which is the adjoint action of the spin connection.

Part (iii): the β-dependent terms in the action at $\beta_0 = 0$ produce only a mass for $\delta\beta$ (Lemma 4.3) and no first-derivative couplings in the vacuum. □

Lemma 5.2 (Connection properties). (i) $\operatorname{tr}(\mathcal{A}_\mu) = 0$ *(the scalar block is antisymmetric; the adjoint representation is traceless).*

(ii) *The divergence $\nabla_\mu\mathcal{A}^\mu$ vanishes in the vacuum: in the scalar block, $\nabla_\mu(2mu^\mu) = 2m\nabla_\mu u^\mu = 0$ by the background continuity equation (σ_0 constant, $\nabla_\mu u^\mu = 0$).*

5.2 The endomorphism E

Theorem 5.3 (Endomorphism of the fluctuation operator). *The endomorphism E in* (13) *is given by $E = M - \mathcal{A}_\mu \mathcal{A}^\mu - \nabla_\mu \mathcal{A}^\mu$, where M is the direct mass/potential matrix. For the OM vacuum:*

$$E = \mathrm{diag}\big(\underbrace{-\tfrac{1}{4}\mathcal{R} + 4m^2}_{\phi_1},\ \underbrace{4m^2}_{\phi_2},\ \underbrace{0,\ldots,0}_{6\ biv.},\ \underbrace{2m^2}_{\phi_4}\big) \quad (16)$$

with trace

$$\mathrm{tr}(E) = -\tfrac{1}{4}\mathcal{R} + 10m^2 \quad (17)$$

Proof. We compute each block:

Scalar block (ϕ_1, ϕ_2). The direct mass matrix has $M_{11} = -\frac{1}{4}\mathcal{R}$ (from $-\frac{1}{4}\mathcal{R}(\delta\sigma)^2$, after the cancellation of Lemma 4.2), $M_{22} = 0$ (the phase fluctuation is a Goldstone mode), and $M_{12} = M_{21} = 0$ (no algebraic cross-term).

The connection contribution is:

$$-\mathcal{A}_\mu \mathcal{A}^\mu = -\begin{pmatrix} 0 & 2mu_\mu \\ -2mu_\mu & 0 \end{pmatrix}\begin{pmatrix} 0 & 2mu^\mu \\ -2mu^\mu & 0 \end{pmatrix} = \begin{pmatrix} 4m^2 & 0 \\ 0 & 4m^2 \end{pmatrix} \quad (18)$$

using $u_\mu u^\mu = 1$. The divergence term vanishes by Lemma 5.2(ii).

Therefore: $E_{11} = -\frac{1}{4}\mathcal{R} + 4m^2$, $E_{22} = 4m^2$.

Bivector block (ϕ_3^A). The direct mass is zero (Lemma 4.4). The connection $\mathcal{A}_\mu = [\omega_\mu, \cdot]$ acts in the adjoint representation; its contribution $-\mathcal{A}_\mu \mathcal{A}^\mu$ produces terms involving $[\omega_\mu, [\omega^\mu, \theta]]$, which are $O(\omega^2)$ and independent of $\mathcal{R}$. In Riemann normal coordinates at the expansion point, $\omega_\mu = O(x)$, so $\omega_\mu \omega^\mu = O(x^2) = O(\mathcal{R})$ in the integrated sense. However, the critical observation is that the *Weitzenböck endomorphism does not appear*:

Lemma 5.4 (Bochner vs. Hodge Laplacian). *The kinetic term for the rotor fluctuations is $\frac{1}{4}\sigma_0^2 g^{\mu\nu}\langle \nabla_\mu \theta\, \nabla_\nu \theta\rangle$. Integration by parts yields:*

$$-\tfrac{1}{4}\sigma_0^2 \langle \theta\, \nabla^\mu \nabla_\mu \theta \rangle \quad (19)$$

The operator acting on θ is the Bochner Laplacian $\Delta_B = -\nabla^\mu \nabla_\mu$, *not the Hodge–de Rham Laplacian $\Delta_H = d\delta + \delta d$. The two are related by the Weitzenböck identity:*

$$\Delta_H = \Delta_B + W \quad (20)$$

where W is the Weitzenböck endomorphism (involving the Riemann tensor acting on the form indices). Since the OM action generates Δ_B directly, W does not *appear in the fluctuation operator.*

Therefore, the curvature endomorphism in the bivector block is absent: $E_{33}^{AB} = 0$ at leading order in curvature (the $O(\omega^2)$ terms from the adjoint connection contribute only at $O(\mathcal{R}^2)$ in the heat kernel expansion and do not affect a_1).

Chirality sector (ϕ_4). $M_{44} = 2m^2$ (Lemma 4.3), no connection $(\mathcal{A}_\mu^{(4,j)} = 0)$, so $E_{44} = 2m^2$.

Summing: $\mathrm{tr}(E) = (-\frac{1}{4}\mathcal{R} + 4m^2) + 4m^2 + 0 + 2m^2 = -\frac{1}{4}\mathcal{R} + 10m^2$. □

Remark 5.5 (Physical interpretation of each contribution). The endomorphism (16) encodes three distinct physical effects:

- $-\frac{1}{4}\mathcal{R}$ in E_{11}: the conformal coupling of the density amplitude to spacetime curvature, required by the Lichnerowicz formula.
- $+4m^2$ in E_{11} and E_{22}: the effective mass generated by the background four-velocity u_μ acting as a synthetic gauge field in the density–phase sector. This is a consequence of the broken $U(1)$ symmetry of the phase in the presence of the mass term.
- $2m^2$ in E_{44}: the restoring force of the Takabayasi angle toward $\beta = 0$, physically the Zitterbewegung frequency.

The six rotor components are massless Goldstone modes of the broken rotational symmetry $\mathrm{Spin}^+(1,3) \to 1$ in the vacuum.

6 Seeley–DeWitt Coefficients

The one-loop effective action is

$$S_{\mathrm{eff}}[g] = \tfrac{1}{2}\mathrm{Tr}\ln \mathcal{O} \quad (21)$$

The Schwinger–DeWitt heat kernel expansion gives [7, 8]:

$$\mathrm{Tr}(e^{-t\mathcal{O}}) = \frac{1}{(4\pi t)^2}\int_M \sqrt{|g|}\,\big[\mathrm{tr}(a_0) + t\,\mathrm{tr}(a_1) + \cdots\big]\,d^4x \quad (22)$$

where the traces are over the 9-dimensional internal space.

Theorem 6.1 (Seeley–DeWitt coefficients for the OM operator). *For the generalized Laplace operator* (13) *with endomorphism* (16)*:*

(i) *The zeroth coefficient:*

$$a_0 = \mathrm{tr}(\mathbf{1}_9) = 9 \quad (23)$$

(ii) *The first coefficient:*

$$a_1 = \mathrm{tr}\big(\tfrac{1}{6}\mathcal{R}\,\mathbf{1}_9 - E\big) = \tfrac{7}{4}\mathcal{R} - 10m^2 \quad (24)$$

Proof. Part (i) is immediate.

Part (ii): using the standard formula $a_1 = \mathrm{tr}(\frac{1}{6}\mathcal{R}\,\mathbf{1} - E)$ [8], we compute component by component:

$$\begin{aligned} \phi_1 &: \tfrac{1}{6}\mathcal{R} - (-\tfrac{1}{4}\mathcal{R} + 4m^2) = \tfrac{5}{12}\mathcal{R} - 4m^2 \\ \phi_2 &: \tfrac{1}{6}\mathcal{R} - 4m^2 = \tfrac{1}{6}\mathcal{R} - 4m^2 \\ \phi_3 &: 6 \times \tfrac{1}{6}\mathcal{R} - 0 = \mathcal{R} \\ \phi_4 &: \tfrac{1}{6}\mathcal{R} - 2m^2 = \tfrac{1}{6}\mathcal{R} - 2m^2 \end{aligned} \quad (25)$$

Summing:

$$\begin{aligned} a_1 &= \big(\tfrac{5}{12} + \tfrac{1}{6} + 1 + \tfrac{1}{6}\big)\mathcal{R} - (4 + 4 + 0 + 2)m^2 \\ &= \big(\tfrac{5}{12} + \tfrac{2}{12} + \tfrac{12}{12} + \tfrac{2}{12}\big)\mathcal{R} - 10m^2 \\ &= \tfrac{21}{12}\mathcal{R} - 10m^2 = \tfrac{7}{4}\mathcal{R} - 10m^2 \end{aligned} \quad (26)$$

□

Remark 6.2 (Origin of $\frac{7}{4}$). The coefficient $\frac{7}{4}$ receives contributions from all four sectors:

- Density (ϕ_1): $\frac{5}{12}$, enhanced by the Lichnerowicz coupling.
- Phase (ϕ_2): $\frac{1}{6}$, the minimal scalar contribution.
- Rotor (ϕ_3, 6 components): 1, the dominant contribution from the 6 massless Goldstone modes.
- Chirality (ϕ_4): $\frac{1}{6}$, identical to the phase.

The rotor sector contributes $\frac{12}{21} \approx 57\%$ of the total. This dominance is a direct consequence of the Clifford-algebraic structure: the 6 bivector components of $\mathrm{Spin}^+(1,3)$ provide more "channels" for gravitational induction than the 3 scalar-like degrees of freedom.

Remark 6.3 (The role of $\kappa = 0$). Had the bivector sector been governed by the Hodge Laplacian (as in Yang–Mills theory), the Weitzenböck endomorphism would contribute $\kappa\mathcal{R}$ with $\kappa = 2$ for 2-forms in 4D. The coefficient would become $a_1 = (\frac{7}{4} - 2)\mathcal{R} - 10m^2 = -\frac{1}{4}\mathcal{R} - 10m^2$, yielding $G_{\text{ind}} < 0$ (repulsive gravity).

The fact that the OM framework uses the Bochner Laplacian ($\kappa = 0$) is not a choice but a consequence of the action's structure: the rotational kinetic energy $\frac{1}{4}\sigma^2\langle\Omega^{\text{cov}}_\mu\Omega^{\text{cov}\,\mu}\rangle$ is a direct metric contraction, not a field-strength squared. This algebraic distinction between Onsager–Machlup and Yang–Mills is what ensures attractive induced gravity.

7 Induced Gravitational Constants

The regularized one-loop effective action with proper-time cutoff Λ_{UV} is [8]:

$$S_{\text{eff}}[g] = \frac{1}{32\pi^2}\int\sqrt{|g|}\,d^4x\,\left[\Lambda_{\text{UV}}^4\,a_0 + \Lambda_{\text{UV}}^2\,a_1 + \cdots\right] \tag{27}$$

Substituting (23) and (24):

$$S_{\text{eff}} = \int\sqrt{|g|}\,d^4x\,\left[\frac{9\Lambda_{\text{UV}}^4}{32\pi^2} + \frac{7\Lambda_{\text{UV}}^2}{128\pi^2}\mathcal{R} - \frac{10m^2\Lambda_{\text{UV}}^2}{32\pi^2} + \cdots\right] \tag{28}$$

Comparing with the Einstein–Hilbert action

$$A_{\text{EH}} = \int\sqrt{|g|}\,d^4x\,\left[\frac{\mathcal{R}}{16\pi G} - \frac{\Lambda}{8\pi G}\right] \tag{29}$$

we identify:

Theorem 7.1 (Induced Newton constant)**.**

$$\frac{1}{16\pi G_{\text{ind}}} = \frac{7\Lambda_{\text{UV}}^2}{128\pi^2} \quad\Longrightarrow\quad G_{\text{ind}} = \frac{8\pi}{7\Lambda_{\text{UV}}^2} \tag{30}$$

With the natural OM cutoff $\Lambda_{\text{UV}} = mc/\hbar$ (the inverse Compton wavelength):

$$\boxed{G_{\text{ind}} = \frac{8\pi\hbar^2}{7m^2c^2}} \tag{31}$$

This is positive: $G_{\text{ind}} > 0$ (attractive gravity).

Proof. Direct comparison of the $\mathcal{R}$ coefficient in (28) with that in A_{EH}. □

Theorem 7.2 (Induced cosmological constant)**.**

$$-\frac{\Lambda_{\text{ind}}}{8\pi G_{\text{ind}}} = \frac{9\Lambda_{\text{UV}}^4}{32\pi^2} - \frac{10m^2\Lambda_{\text{UV}}^2}{32\pi^2} \tag{32}$$

For $\Lambda_{\text{UV}} = m$:

$$\Lambda_{\text{ind}} = -\frac{8\pi G_{\text{ind}}}{32\pi^2}(9m^4 - 10m^4) = +\frac{m^2}{28\pi} \tag{33}$$

The induced cosmological constant is positive (de Sitter type) but of order m^2, far larger than the observed value.

Remark 7.3 (The cosmological constant problem). The large induced $\Lambda_{\text{ind}} \sim m^2$ is the standard result in all Sakharov-type calculations [5]. The OM framework does not resolve this problem in its current form. However, the sign flip from negative (as obtained by Gemini's initial estimate $9m^4 > 10m^4$... corrected: $9 - 10 = -1$, but with the factor $-\frac{\Lambda}{8\pi G}$ and $G > 0$, the cosmological constant can be positive) depends delicately on the relative weight of a_0 and the mass terms in a_1. A full treatment would require summing over all particle species and including renormalization group running.

8 The UV Cutoff and the Mass Hierarchy

Proposition 8.1 (Single-species estimate)**.** *For a single fermion species of mass m with the OM cutoff $\Lambda_{\text{UV}} = m$, equation* (31) *gives:*

$$G_{\text{ind}}(m) = \frac{8\pi}{7m^2} \tag{34}$$

For the electron ($m_e \approx 0.511$ MeV/c^2): $G_{\text{ind}} \sim 10^{37}\,G_{\text{obs}}$. Far too large.

Remark 8.2 (Multi-species summation). In the Sakharov mechanism, the induced Newton constant receives contributions from all species:

$$\frac{1}{G_{\text{obs}}} = \sum_{\text{species } i}\frac{7\,n_i\,m_i^2}{8\pi} \tag{35}$$

where n_i counts effective degrees of freedom (with sign: bosonic contributions subtract). To obtain $G_{\text{obs}} \sim \ell_P^2$ requires the sum to be dominated by species near the Planck mass m_P. This is the standard mass hierarchy problem in induced gravity [5], not specific to the OM framework.

What the OM framework contributes is the definite numerical prefactor $\frac{7}{8\pi}$ per fermion species, which differs from the standard Dirac result ($\frac{1}{12\pi}$ per Dirac fermion) due to the 9 internal degrees of freedom and the specific endomorphism structure.

Remark 8.3 (The factor $\frac{7}{4}$ as a signature). The coefficient $\frac{7}{4}$ in a_1 is a fingerprint of the OM framework. In standard QFT, a single Dirac fermion contributes $a_1^{\text{Dirac}} = -\frac{1}{6}\mathcal{R}$ (4 complex components with spin connection). The OM fluid contributes $a_1^{\text{OM}} = +\frac{7}{4}\mathcal{R}$ per species (9 real components with the specific endomorphism (16)). The sign difference reflects the Bochner–Hodge distinction (Lemma 5.4) and the mass structure of the OM vacuum.

9 Discussion

9.1 What the calculation achieves

The one-loop calculation demonstrates that the OM probability fluid, treated as a quantum field on a curved background, induces an Einstein–Hilbert action with:

- Positive Newton constant ($G_{\mathrm{ind}} > 0$, attractive gravity).
- A definite numerical coefficient ($\frac{7}{4}$ per species) determined by the internal structure of the OM multiplet.
- No Weitzenböck obstruction: the Bochner Laplacian in the bivector sector ensures $\kappa = 0$.

9.2 What remains open

1. *The mass hierarchy problem.* A single light fermion induces a Newton constant far larger than observed. Resolution requires understanding which species dominate the sum (35) and at what scale.
2. *The cosmological constant.* The induced $\Lambda_{\mathrm{ind}} \sim m^2$ is many orders of magnitude too large. This is the standard cosmological constant problem, not specific to the OM framework.
3. *Higher-order curvature terms.* The a_2 coefficient generates $\mathcal{R}^2$, $R_{\mu\nu}R^{\mu\nu}$, and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ terms. These may be relevant for early-universe cosmology and deserve explicit calculation.
4. *The $O(\omega^2)$ terms in the bivector sector.* We argued that $-\mathcal{A}^{\mathrm{biv}}_\mu \mathcal{A}^{\mathrm{biv}\,\mu}$ contributes only at $O(\mathcal{R}^2)$. A rigorous proof would require expanding to the next order in the heat kernel.
5. *Non-perturbative effects.* The Sakharov mechanism is perturbative (one-loop). The OM framework, being fundamentally stochastic, may admit non-perturbative contributions to the effective gravitational action.

9.3 Relation to previous work

The Sakharov mechanism has been applied to various matter contents [4, 5, 6]. The OM calculation is distinguished by three features: (a) the matter content is uniquely determined by the stochastic coherence framework (not chosen *ad hoc*); (b) the Bochner–Hodge distinction, arising from the Clifford-algebraic structure of the rotational kinetic energy, ensures attractive induced gravity without fine-tuning; and (c) the UV cutoff $\Lambda_{\mathrm{UV}} = m$ is natural (set by the diffusivity $D = \hbar/(2m)$), not introduced by hand.

10 Conclusion

We have computed the one-loop effective action for the metric obtained by integrating out the fluctuations of the Onsager–Machlup probability fluid on a curved spin manifold. The result is an induced Einstein–Hilbert action with coefficient $a_1 = \frac{7}{4}\mathcal{R} - 10m^2$, yielding a positive Newton constant $G_{\mathrm{ind}} = 8\pi\hbar^2/(7m^2c^2)$.

The key structural features are:

1. The 9 internal degrees of freedom of the OM multiplet (σ, S, R, β) produce $a_0 = 9$ and $a_1 = \frac{7}{4}\mathcal{R} - 10m^2$.
2. The mass cancellation $+m^2 - m^2 = 0$ for density fluctuations is an algebraic consequence of the OM action, not a fine-tuning.
3. The Bochner Laplacian (not Hodge) in the bivector sector eliminates the Weitzenböck endomorphism ($\kappa = 0$), ensuring $G_{\mathrm{ind}} > 0$.
4. The background four-velocity acts as a synthetic gauge connection, generating an effective mass $4m^2$ in the scalar sector.

Combined with the previous results of the stochastic coherence program [1, 2, 3], this establishes a chain from a single variational principle to both quantum mechanics and emergent gravitation:

$$\text{OM} \to \text{Dirac} \to \text{Fermi–Dirac} \to \\ \text{Pauli} \to \text{Einstein (induced)}$$

The last arrow is one-loop and perturbative; whether the full Einstein equations can be derived non-perturbatively from the OM framework remains the central open question.

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