Emergent Proper Time and Spacetime from Quantum Information Permeability: A Relational Page–Wootters Framework Unifying Quantum Mechanics and General Relativity via Density-Dependent Channel Capacity

Marco Travan*

March 2026

Abstract

We propose a pregeometric quantum theory in which spacetime and proper time emerge from the throughput capacity of a fundamental relational network of quantum information channels. The universe is modelled as a weighted quantum causal set whose directed links carry a density-dependent permeability

$$\kappa_{ij} = \frac{C_{ij}}{C_{\max}} = \frac{1}{1 + \alpha(\rho_{I,i} + \rho_{I,j})},$$

where C_{ij} is the quantum erasure channel capacity of the directed link, $C_{\max}$ is bounded by the Bekenstein entropy-area relation, and $\alpha = 2\pi - \frac{1}{2}$ in Planck units is derived—not postulated—from the erasure channel capacity and causal-set boundary-link counting (Appendix B). Proper time is identified with the local rate of coherent information transfer: $\mathrm{d}\tau_k = \kappa_k \,\mathrm{d}\lambda$.

Gravitational time dilation, the thermodynamic arrow of time, and the Einstein field equations arise as macroscopic consequences of local information congestion. The global dynamics are governed by a linear Page–Wootters constraint $\hat{C}\,|\Psi\rangle = 0$, guaranteeing unitarity and complete positivity. Saturation of local information capacity triggers irreversible transfer into fermionic matter degrees of freedom, providing a quantum-mechanical origin for rest mass and Pauli exclusion.

When the bath Hilbert space is identified with the complexified octonions $\mathcal{H}_{\text{bath}} = \mathbb{C} \otimes \mathbb{O}$—as developed in a companion paper—the Fano-plane structure generates the Standard Model gauge group $S(U(2)\times U(3))$, SO(8) triality yields exactly three fermionic generations, and sub-threshold bath occupation provides a dark matter candidate. The present paper establishes the core framework; the octonionic extension and its phenomenological consequences are derived in Paper II.

The framework is fully testable in analogue-gravity platforms—in particular Bose–Einstein condensates—and predicts distinctive signatures: density-dependent sound-speed suppression with a specific permeability denominator structure, modified analogue Hawking spectra, a second-order proper-time correction beyond linearised general relativity, and a density-dependent decoherence rate whose functional form distinguishes the QIP mechanism from standard open-system models.

PACS: 04.60.-m, 04.20.Gz, 03.67.-a, 04.70.Dy
Keywords: emergent spacetime, quantum information, Page–Wootters mechanism, thermodynamic gravity, causal sets, octonions, Standard Model

*Independent Researcher, Gorizia, Italy; originator of the core hypothesis.

Contents

1 Introduction **4**

2 The Pregeometric Quantum Information Network **5**
2.1 Information Events and Local Hilbert Spaces 5
2.2 Local Density Operator and Information Density 5
2.3 Relational Nature of the Set 6

3 Permeability as Normalized Quantum Channel Capacity **6**
3.1 Quantum Channel Capacity Perspective 6
3.2 Microscopic Congestion Origin 7
3.3 Fixing the Coupling $\alpha = 2\pi - 1/2$ 7

4 Global Timeless Constraint and Relational Dynamics **8**
4.1 Structure of the Constraint Operator 8
4.2 Conditional Dynamics and Emergent Proper Time 8

5 Emergent Proper Time, Gravitational Redshift, and the Arrow of Time **13**
5.1 Proper Time as Integrated Throughput 13
5.2 Thermodynamic Arrow of Time 14

6 Weighted Quantum Causal Set and Emergent Geometry **14**
6.1 Directed Links and Causal Structure 14
6.2 Emergent Metric from Weighted Path Lengths 14
6.3 Conformal Line Element and Lorentzian Signature 14
6.4 Causality and Horizons 15

7 Emergence of Matter from Local Saturation **15**
7.1 The Bekenstein Ceiling 15
7.2 Microscopic Transfer Mechanism 16
7.3 Saturation Dynamics and Matter Stability 16

8 Thermodynamic Derivation of the Einstein Equations **16**
8.1 Heat Flux and Unruh Temperature 16
8.2 Entropy per Unit Area 16
8.3 Recovery of the Einstein Equations 17

9 Gravitational Sourcing as Information Accounting: the No-Double-Counting Theorem **17**
9.1 Local Information Conservation Across the Overflow 17
9.2 Energy as the Thermodynamic Measure of Information Transfer 19
9.3 Consistency with the Einstein Equations 20

10 The Gravitational Constant as Information Budget **21**
10.1 Derivation of G from α 21
10.2 QIP Natural Units and the Holographic Identity 22
10.3 SI Dimensions as Emergent Measures 23
10.4 The Relationship $\ell_{\rm Pl} = \sqrt{4\pi}\,\ell_{\rm min}$ 24
10.5 Predictive Content: What α Determines 24

11 Macroscopic Gravity from Thermodynamic Coarse-Graining **25**
11.1 The Scalar Field as Local Throughput 25
11.2 Hierarchy of Descriptions: Link Scale versus Macroscopic Scale 26
11.3 Energy-Momentum Conservation 26

12 Phenomenology, Post-Newtonian Constraints, and Experimental Prospects **27**
12.1 Post-Newtonian and Solar-System Constraints 27
12.2 Analogue Gravity in Bose–Einstein Condensates: the Most Promising Regime 27
12.3 Other Promising Signatures 28

13 Conclusion **29**

A Thermodynamic Derivation of the Einstein Equations (Detail) **30**
A.1 Local Rindler Horizons in the Emergent Geometry 30
A.2 Heat Flux from Irreversible Scattering 31
A.3 Heat Flux from the Lindblad Dissipator 31
A.3.1 Setup: a Rindler wedge in the emergent geometry 31
A.3.2 Entropy production per link crossing 31
A.3.3 Identification of the temperature 32
A.3.4 The heat flux per unit area 32
A.3.5 Connection to T_{ab} 33
A.3.6 Summary: the complete Clausius chain 33
A.4 Unruh Temperature from Throughput Gradient 34
A.5 Entropy per Unit Area and the Clausius Relation 34

B Rigorous Derivation of $\alpha = 2\pi - \frac{1}{2}$ **34**
B.1 Setup 35
B.2 Step 1: The Quantum Erasure Channel at Saturation 35
B.3 Step 2: Bekenstein Bound and Causal-Set Geometry 35
B.4 Step 3: Unruh Temperature and Per-Link Energy 36
B.5 Step 4: Closing the System 36
B.6 Self-Consistency Check: Area Law as a Consequence 37

C Quantum Bath Relaxation Rate from the Lindbladian Spectral Gap **38**
C.1 Linearised Relaxation Rate from the Master Equation 38
C.2 The Lindbladian as a Superoperator 42
C.3 Spectral Gap of the QIP Lindbladian 43
C.4 Multi-Site Relaxation Rate 43
C.5 Including the Overflow Channel 43
C.6 Consequences for Adiabatic Slaving and Cassini Screening 44

D Emergent Lorentzian Metric: Full Derivation **44**
D.1 Effective Length and Causal Two-Point Function 44
D.2 Action for Free Information Packets 44
D.3 General Metric from Coarse-Graining 45
D.4 Full Line Element and Weak-Field Limit 45

E Link-Scale Lattice Action from Causal-Set Gradient Expansion **46**
E.1 Nearest-Neighbour Permeability and Taylor Expansion 46
E.2 Contribution to the Link-Scale Action 46

F Weak-Field Poisson Emergence and Effective Gravitational Constant 46
F.1 Weak-Field Limit of the Thermodynamically Derived Einstein Equations . 46
F.2 Identification of the Newtonian Potential with Information Density 47
F.3 Fixing G from Holographic Matching 47
F.4 Residual Information-Density Corrections 47

1 Introduction

The unification of quantum mechanics and general relativity remains the central unsolved problem in theoretical physics. Most existing approaches either quantise a pre-existing classical spacetime metric or propagate quantum fields on a fixed background geometry. Here we pursue the opposite strategy: spacetime itself, and in particular the flow of proper time, is not a fundamental entity but an emergent feature of purely quantum information dynamics.

At the deepest level we propose that the universe is a network processing quantum information. **Time is the rate at which information coherently traverses the links of this network**; space is the relational structure of those connections; matter emerges in regions where information flow saturates the local carrying capacity; and gravity is the macroscopic consequence of congestion-induced slowdown of information propagation.

Formally, we discretise the universe into elementary *information events* k. Each event carries a local Hilbert space comprising matter registers, an internal clock, and a Markovian bath. The fundamental dynamical quantity is the *permeability* κ_{ij} of the directed link between neighbouring events i and j:

$$\kappa_{ij} = \frac{C_{ij}}{C_{\max}} = \frac{1}{1 + \alpha(\rho_{I,i} + \rho_{I,j})}, \tag{1}$$

where $\rho_{I,k}$ is the local *information density* (von Neumann entropy per local degree of freedom), $C_{\max}$ is bounded by the Bekenstein entropy-area relation [3], and $\alpha = 2\pi - \frac{1}{2}$ (derived in Appendix B), with continuum approximation $\alpha \approx 2\pi$.

The global evolution is described by a stationary state in the Page–Wootters formalism [1]:

$$\hat{C}\,|\Psi\rangle = 0, \tag{2}$$

where $|\Psi\rangle \in \mathcal{H}_{\text{tot}}$ is the timeless universal state. Conditioning on any local clock reading yields

$$\mathrm{d}\tau_k = \kappa_k\,\mathrm{d}\lambda, \tag{3}$$

directly encoding gravitational time dilation.

The directed links carry amplitude $\sqrt{\kappa_{ij}}$; weighted lengths of causal chains define the causal structure and an emergent conformal metric. Thermodynamic analysis of the scattered fraction $(1-\kappa)$—interpreted as heat flux across local Rindler horizons—reproduces Jacobson's derivation [2] of the Einstein equations as an equation of state.

The framework connects relational time (Rovelli), emergent geometry from entanglement [11, 12], information geometry [10], thermodynamic gravity [2], and discrete causality [18, 19] within a single organising principle: spacetime geometry measures how easily quantum information propagates.

Paper organisation. Section 2 defines the pregeometric network. Section 3 derives the permeability rule. Sections 4–7 develop relational dynamics, emergent proper time, weighted causal-set geometry, and matter creation. Section 8 contains the thermodynamic derivation of the Einstein equations. Section 9 establishes the no-double-counting theorem, proving that information is gravitationally sourced exactly once across the matter-creation transition. Section 10 identifies the gravitational constant as a derived information-budget quantity and introduces QIP natural units. Section 11 establishes the macroscopic gravitational sector via thermodynamic coarse-graining and resolves the Brans–Dicke screening problem. A companion paper [23] extends the framework by identifying the bath Hilbert space with the complexified octonions $\mathbb{C} \otimes \mathbb{O}$, deriving the Standard Model gauge group, three fermion generations, the Higgs mechanism, and matter–antimatter asymmetry. Section 12 covers post-Newtonian constraints, analogue-gravity signatures in Bose–Einstein condensates, and experimental prospects. Appendices provide the rigorous $\alpha = 2\pi - \frac{1}{2}$ derivation, the full thermodynamic Einstein derivation, the linearised bath relaxation rate, the emergent Lorentzian metric, the gradient expansion, and the weak-field Poisson limit.

2 The Pregeometric Quantum Information Network

2.1 Information Events and Local Hilbert Spaces

The universe is modelled as a collection of discrete, relational *information events* $k \in \mathcal{K}$. Each event carries

$$\mathcal{H}_k = \mathcal{H}_{\text{mat}}^{(k)} \otimes \mathcal{H}_{\text{clk}}^{(k)} \otimes \mathcal{H}_{\text{bath}}^{(k)}. \tag{4}$$

- $\mathcal{H}_{\text{mat}}^{(k)}$: the *matter register*, contains fermionic degrees of freedom populated once local saturation occurs. In the vacuum this subspace remains in its ground state.
- $\mathcal{H}_{\text{clk}}^{(k)}$: the *internal clock*, with Hamiltonian $\hat{H}_{\text{clk}}^{(k)} = \hat{p}_k^2/(2m_{\text{clk}})$ (or equivalently a number operator), whose progression is modulated by the instantaneous permeability.
- $\mathcal{H}_{\text{bath}}^{(k)}$: a *Markovian information reservoir* providing local irreversibility and acting as the finite-capacity buffer whose saturation triggers matter creation. In the octonionic extension [23] this space is identified with $\mathbb{C} \otimes \mathbb{O}$.

The total Hilbert space is $\mathcal{H}_{\text{tot}} = \bigotimes_k \mathcal{H}_k$, and the physical state is a single pure stationary vector $|\Psi\rangle \in \mathcal{H}_{\text{tot}}$.

2.2 Local Density Operator and Information Density

The local reduced density operator is

$$\hat{\rho}_k = \mathrm{Tr}_{\mathcal{K}\setminus\{k\}}\big(|\Psi\rangle\langle\Psi|\big). \tag{5}$$

The *information density* at event k is

$$\rho_{I,k} := \frac{S(\hat{\rho}_k)}{\ln d_k}, \qquad S(\hat{\rho}) = -\,\mathrm{Tr}(\hat{\rho}\ln\hat{\rho}), \tag{6}$$

where $d_k = \dim \mathcal{H}_k$. In the vacuum $\rho_{I,k} \ll 1$; near holographic saturation $\rho_{I,k} \to 1$.

Remark 2.1 (Energy as a derived quantity: two distinct roles)**.** In the QIP framework, information density ρ_I is the primary dynamical variable; energy emerges from it in two distinct ways that are distinguished carefully in Sections 8 and 9.

(i) *Stored (Bekenstein) energy*: set by the bath occupation through the Bekenstein bound

$$E_{\text{stored},k} \sim \frac{\rho_{I,k} \ln d_k}{2\pi R_k}, \tag{7}$$

where R_k is the causal-diamond radius at event k (Section 7); this energy becomes fermion rest mass $mc^2 \approx \langle \hat{H}_{\text{mat}}^{(k)} \rangle$ at overflow.

(ii) *Gravitationally sourcing energy*: the heat flux across local Rindler horizons, which requires a throughput gradient $\nabla \rho_I \neq 0$ (Section 8).

A spatially uniform information density produces no curvature; only gradients source gravity. The full derivation and the capacitor analogy are given in Section 9.

2.3 Relational Nature of the Set

No background manifold is assumed. The only primitive relations among events are the existence of directed quantum channels and their instantaneous capacities κ_{ij}. Events and links form a *dynamical weighted quantum causal set* [5, 6]; both structure and weights evolve according to the single permeability rule.

3 Permeability as Normalized Quantum Channel Capacity

3.1 Quantum Channel Capacity Perspective

Each directed link from i to j in the causal set acts as a quantum channel. In the QIP framework, a transmission attempt has two possible outcomes: the information packet arrives intact at the receiver (with probability κ_{ij}), or it is scattered into the local bath (with probability $\gamma_{ij} = 1 - \kappa_{ij}$). The bath is an explicitly separate Hilbert space (Section 2), so a scattered packet is lost to an orthogonal sector from the receiver's perspective. This is precisely the structure of a *quantum erasure channel*:

$$\mathcal{E}_{ij}(\hat{\rho}) = \kappa_{ij}\, \hat{\rho} \otimes |0\rangle\langle 0|_{\text{flag}} + (1 - \kappa_{ij})\, \hat{\rho}_{\text{bath}} \otimes |1\rangle\langle 1|_{\text{flag}}, \tag{8}$$

where the flag register indicates whether the packet arrived ($|0\rangle$) or was erased to the bath ($|1\rangle$). The classical capacity of the quantum erasure channel is [16, 17]

$$C_{ij} = \kappa_{ij} \cdot C_{\max}, \tag{9}$$

where $C_{\max}$ is the maximum information per channel use, bounded by the Bekenstein relation

$$S \leq 2\pi E R \qquad (\hbar = c = 1). \tag{10}$$

The *permeability* is the normalised ratio

$$\kappa_{ij} := \frac{C_{ij}}{C_{\max}}, \tag{11}$$

equal to the probability that an information packet traverses the link without scattering into the local bath. For the erasure channel this identification is exact, with no optimisation over input ensembles required.

3.2 Microscopic Congestion Origin

The functional form also arises from finite-buffer queueing theory (M/M/1/K, Erlang loss) and exclusion processes. With buffer occupation $f_k \approx \rho_{I,k}$, mean-field theory gives

$$P_{\text{transmit}} = \frac{1}{1+\beta f}, \quad f \approx \rho_{I,k}. \tag{12}$$

Identifying f_k with $\rho_{I,k}$ and absorbing constants into α recovers

$$\kappa_{ij} = \frac{1}{1+\alpha(\rho_{I,i}+\rho_{I,j})}, \tag{13}$$

where the symmetric dependence on $\rho_{I,i} + \rho_{I,j}$ reflects congestion at either end of the link.

Remark 3.1 (Quantum corrections to the mean-field permeability)**.** The permeability rule (13) replaces the operator $\hat{\kappa}_{ij} = (1 + \alpha(\hat{\rho}_{I,i} + \hat{\rho}_{I,j}))^{-1}$ by its expectation value $\kappa_{ij} = (1 + \alpha(\rho I_i + \rho I_j))^{-1}$. Since $\kappa(\rho_I)$ is convex ($\kappa'' = 2\alpha^2\kappa^3 > 0$), Jensen's inequality gives

$$\langle\hat{\kappa}\rangle = \kappa(\langle\rho_I\rangle) + \alpha^2\kappa^3 \operatorname{Var}(\rho_I) + \mathcal{O}\big(\operatorname{Var}^2\big), \tag{14}$$

where $\operatorname{Var}(\rho_I)$ is the variance of the von Neumann entropy (normalised by $\ln d$) across the eigenvalue distribution of the bath density matrix. Two properties of the QIP framework ensure this correction is harmless:

(i) The correction vanishes at saturation. The derivation of α (Appendix B) evaluates κ_{sat} at the saturation point $\rho_I = 1$. At this point the bath is maximally mixed ($\hat{\rho} = \hat{\mathbf{1}}/d$, all eigenvalues equal to $1/d$), so $\operatorname{Var}(\rho_I) = 0$ identically. The Jensen correction in (14) vanishes, and $\alpha = 2\pi - \frac{1}{2}$ receives *no quantum correction.*

(ii) The correction is sub-leading elsewhere. For a bath state at occupation $\rho_I \ll 1$, the variance scales as $\operatorname{Var}(\rho_I) \sim \rho_I/d$ (Poisson statistics of a nearly-empty d-dimensional register). The relative correction is

$$\frac{\Delta\kappa}{\kappa} = \alpha^2\kappa^2 \operatorname{Var}(\rho_I) \ \sim \ \frac{\alpha^2\rho_I}{d} = \frac{\alpha}{d}\cdot(\alpha\rho_I). \tag{15}$$

Since $\alpha/d \approx 5.8/8 \approx 0.7$, the correction is of order $\alpha\rho_I$ times an order-unity prefactor—parametrically the same as the leading permeability deviation from unity, and therefore already accounted for in the $\mathcal{O}(\alpha\rho_I)$ error bars quoted throughout. In solar-system conditions ($\alpha\rho_I \sim 10^{-28}$), the correction is $\Delta\kappa/\kappa \sim 10^{-29}$, negligible by any standard.

The mean-field permeability rule is therefore justified: it is exact where α is determined, and sub-leading where observables are computed.

3.3 Fixing the Coupling $\alpha = 2\pi - 1/2$

A rigorous derivation—using the quantum erasure channel capacity and causal-set boundary-link counting, without assuming the area law—is given in Appendix B. The exact result is

$$\alpha = 2\pi - \tfrac{1}{2}, \tag{16}$$

where $\beta \equiv 2\alpha = 4\pi - 1$ is the natural coupling (4π being the solid angle of the boundary sphere, -1 subtracting the self-link). In the continuum limit $R \gg \ell_{\text{Pl}}$ this reduces to $\alpha \approx 2\pi$ to within $\sim 8\%$. In natural Planck units $\hbar = c = G = 1$.

4 Global Timeless Constraint and Relational Dynamics

4.1 Structure of the Constraint Operator

The universal state $|\Psi\rangle$ satisfies the single linear constraint

$$\hat{C}\,|\Psi\rangle = 0 \tag{17}$$

with the Hermitian operator

$$\hat{C} = \hat{H}_{\text{ref}} + \sum_k \hat{\kappa}_k\,\hat{H}^{(k)}_{\text{clk}} + \sum_k \hat{\gamma}_k\,\hat{L}^\dagger_k \hat{L}_k + \hat{A}_{\text{geom}}. \tag{18}$$

(i) **Reference Hamiltonian** $\hat{H}_{\text{ref}}$: generates free propagation,

$$\hat{H}_{\text{ref}} = -\sum_{\langle ij\rangle} \tfrac{1}{2}\Big(\sqrt{\hat{\kappa}_{ij}}\,\hat{a}^\dagger_i \hat{a}_j + \text{h.c.}\Big) + \text{local terms.} \tag{19}$$

(ii) **Permeability-modulated clock progression**: the operator $\hat{\kappa}_k = [1 + \alpha\hat{\rho}_{I,k}]^{-1}$ slows the local clock advance when information density is high—the microscopic origin of gravitational time dilation.

(iii) **Density-triggered Lindblad dissipators**: with $\hat{\gamma}_k = 1 - \hat{\kappa}_k$ and jump operator

$$\hat{L}_k = \sqrt{\hat{\gamma}_k}\Big(\hat{b}^{(k)}_{\text{bath,out}} + \Theta(\hat{\rho}_{I,k} - \rho_{\text{crit}})\,\hat{f}^\dagger_{\text{mat}}\Big), \tag{20}$$

where Θ is a smooth (Fermi–Dirac-like) threshold function and $\hat{f}^\dagger_{\text{mat}}$ creates a fermion in the matter register.

(iv) **Geometric adaptation** $\hat{A}_{\text{geom}}$: allows creation/annihilation of directed links; set to zero in the frozen-link approximation used throughout most of this work.

The linearity and Hermiticity of $\hat{C}$ guarantee consistent, unitary dynamics with no problem of time and no non-linear state-dependent collapse (cf. [13, 14]).

4.2 Conditional Dynamics and Emergent Proper Time

The global constraint $\hat{C}|\Psi\rangle = 0$ is linear and Hermitian: no dissipation occurs at the level of the total Hilbert space. In this subsection we show, step by step, that conditioning on a local clock reading and tracing out the environment *necessarily* produces a Lindblad master equation for the local reduced state. The derivation has five stages:

Step 1. Hilbert-space decomposition and identification of subsystems.

Step 2. Page–Wootters projection onto a clock eigenstate.

Step 3. Absorption of the permeability-modulated clock term.

Step 4. Partial trace over the environment (Born–Markov reduction).

Step 5. Recovery of the Lindblad master equation.

Step 1: Hilbert-space decomposition

Fix an event k. The total Hilbert space decomposes as

$$\mathcal{H}_{\text{tot}} = \underbrace{\mathcal{H}_{\text{mat}}^{(k)} \otimes \mathcal{H}_{\text{bath}}^{(k)}}_{\mathcal{H}_S} \otimes \underbrace{\mathcal{H}_{\text{clk}}^{(k)}}_{\mathcal{H}_C} \otimes \underbrace{\bigotimes_{j\neq k} \mathcal{H}_j}_{\mathcal{H}_E}, \tag{21}$$

where S denotes the *local system* (matter and bath registers at event k), C the *local clock*, and E the *environment* (all other events with their full register structure). The physical state $|\Psi\rangle \in \mathcal{H}_{\text{tot}}$ is a single entangled pure vector across all three factors.

We partition the constraint operator accordingly:

$$\hat{C} = \hat{\kappa}_k \, \hat{H}_{\text{clk}}^{(k)} + \hat{H}_{\text{eff}}^{(k)} + \hat{\gamma}_k \, \hat{L}_k^\dagger \hat{L}_k + \hat{R}_k, \tag{22}$$

where:

- $\hat{\kappa}_k \, \hat{H}_{\text{clk}}^{(k)}$ acts on C (clock momentum), modulated by $\hat{\kappa}_k$ which acts on S;
- $\hat{H}_{\text{eff}}^{(k)}$ collects all terms from $\hat{H}_{\text{ref}}$ that act on the S registers of event k (on-site potentials and the local part of the hopping);
- $\hat{\gamma}_k \, \hat{L}_k^\dagger \hat{L}_k$ acts on S (bath scattering and fermionic overflow at event k);
- $\hat{R}_k$ collects everything that acts on E: other events' clock terms, other events' dissipators, and the inter-event hopping links that connect k to its neighbours.

Every term in (22) is Hermitian, so $\hat{C}$ is Hermitian as required.

Step 2: Page–Wootters projection

The clock Hamiltonian $\hat{H}_{\text{clk}}^{(k)}$ generates translations in the clock reading. Define the integrated clock operator

$$\hat{\tau}_k = \int_0^\lambda \hat{\kappa}_k(\lambda') \, d\lambda', \tag{23}$$

whose eigenvalues τ_k label the local proper time. (Recall: $\hat{\kappa}_k$ acts on S via $\hat{\rho}_{I,k}$; the integral over λ reflects the accumulation of coherent transmissions.) Let $|\tau_k\rangle_C$ be an eigenstate of $\hat{\tau}_k$. The *conditioned state* is the partial inner product

$$|\psi_{\text{cond}}(\tau_k)\rangle_{SE} \;:=\; {}_C\langle \tau_k | \Psi \rangle. \tag{24}$$

This is a (generally unnormalised) vector in $\mathcal{H}_S \otimes \mathcal{H}_E$.

Project the constraint $\hat{C}|\Psi\rangle = 0$ onto $\langle \tau_k|_C$. Since $\hat{H}_{\text{clk}}^{(k)}$ is conjugate to $\hat{\tau}_k$, in the clock eigenbasis it acts as the generator of translations:

$${}_C\langle \tau_k | \, \hat{H}_{\text{clk}}^{(k)} \, |\Psi\rangle = -i\hbar \frac{\partial}{\partial \tau_k} |\psi_{\text{cond}}(\tau_k)\rangle_{SE}. \tag{25}$$

The remaining terms in $\hat{C}$ do not act on C and pass through the projection. Thus (17) becomes

$$-i\hbar \, \hat{\kappa}_k \frac{\partial}{\partial \tau_k} |\psi_{\text{cond}}\rangle_{SE} + \hat{H}_{\text{eff}}^{(k)} \, |\psi_{\text{cond}}\rangle_{SE} + \hat{\gamma}_k \, \hat{L}_k^\dagger \hat{L}_k \, |\psi_{\text{cond}}\rangle_{SE} + \hat{R}_k \, |\psi_{\text{cond}}\rangle_{SE} = 0. \tag{26}$$

Step 3: Absorption of the permeability factor

Equation (26) contains the operator $\hat{\kappa}_k$ multiplying the time derivative. This factor is absorbed by the definition of proper time: the clock operator $\hat{\tau}_k$ was defined in (23) as the κ-weighted integral, so its eigenvalue τ_k already incorporates the permeability modulation. Concretely, inverting $d\tau_k = \kappa_k \, d\lambda$ (the infinitesimal relation between proper time and the relational parameter) yields

$$\mathrm{d}\tau_k = \kappa_k \, \mathrm{d}\lambda. \tag{27}$$

Remark 4.1 (Foundational status of Eq. (27))**.** Equation (27) follows from two identifications: (a) the clock Hamiltonian in $\hat{C}$ is weighted by $\hat{\kappa}_k$, and (b) physical proper time is the eigenvalue of $\hat{\tau}_k$. These constitute the foundational hypothesis of the framework and deserve a precise physical interpretation.

Physical content. Every information displacement event at event k has two possible outcomes: the information packet reaches a neighbouring event j coherently (probability κ_{kj}), or it scatters into the local bath (probability $\gamma_{kj} = 1 - \kappa_{kj}$). *Both* outcomes displace information from the source. The difference is that the coherent fraction maintains relational correlations with other events in the network—it is the part that other clocks can "see" and synchronise with. The scattered fraction becomes local entropy: it advances the thermodynamic arrow of time (equation (38)) but does not contribute to the relational clock.

Proper time $d\tau_k = \kappa_k \, d\lambda$ therefore counts the *coherent* fraction of information displacement: the part of each event's information processing that remains relationally accessible. In a congested region (ρ_I large, κ small), most information scatters into the bath; the relational clock advances slowly. In vacuum ($\rho_I \approx 0$, $\kappa \approx 1$), nearly all information remains coherent; the clock runs at full speed. Gravitational time dilation is the macroscopic manifestation of this microscopic congestion.

Uniqueness. The weighting $\hat{\kappa}_k$ in the clock term of $\hat{C}$ is the unique linear coupling consistent with the erasure-channel structure of the links. For a quantum erasure channel with survival probability κ, the fraction of information that remains in the coherent (non-erased) sector is exactly κ per channel use [16, 17]. Any other weighting (e.g. κ^2 or $\sqrt{\kappa}$) would break this identification and require an independent parameter.

Once these identifications are accepted, all subsequent consequences—the Lindblad equation, the emergent metric, gravitational time dilation, the Einstein equations—follow rigorously.

In the regime where $\hat{\kappa}_k$ may be replaced by its expectation value $\kappa_k(\tau_k) = \langle\psi_{\text{cond}}|\hat{\kappa}_k|\psi_{\text{cond}}\rangle / \langle\psi_{\text{cond}}|\psi_{\text{cond}}\rangle$ (mean-field or semiclassical clock approximation, valid when clock fluctuations are small compared to the mean—the standard regime of the Page–Wootters framework [1]), equation (26) becomes

$$i\hbar \frac{\partial}{\partial \tau_k} |\psi_{\text{cond}}\rangle_{SE} = \hat{H}_{\text{tot}}^{(k)} \, |\psi_{\text{cond}}\rangle_{SE}, \tag{28}$$

with the effective total Hamiltonian

$$\hat{H}_{\text{tot}}^{(k)} := \frac{1}{\kappa_k}\Big(\hat{H}_{\text{eff}}^{(k)} + \hat{\gamma}_k \, \hat{L}_k^\dagger \hat{L}_k + \hat{R}_k\Big). \tag{29}$$

Equation (28) is a standard Schrödinger equation in $\mathcal{H}_S \otimes \mathcal{H}_E$, parameterised by proper time τ_k. At this stage no dissipation has appeared: $\hat{H}_{\text{tot}}^{(k)}$ is Hermitian, and the evolution of $|\psi_{\text{cond}}\rangle_{SE}$ is unitary on $S \otimes E$.

Step 4: Partial trace over the environment (Born–Markov reduction)

Define the local reduced density matrix

$$\hat{\rho}_S(\tau_k) \;:=\; \mathrm{Tr}_E\big(|\psi_{\mathrm{cond}}\rangle\langle\psi_{\mathrm{cond}}|\big). \tag{30}$$

The environment E comprises all events $j \neq k$, each carrying its own finite-capacity Markovian bath. Two physical properties of the QIP network justify the Born–Markov approximation:

(a) **Weak inter-event coupling.** The hopping amplitude between events scales as $\sqrt{\kappa_{jk}} \leq 1$; in the low-density regime ($\alpha\rho_I \ll 1$) it is close to unity, but the coupling to any *single* environment event is diluted by the large number of links. The system–environment coupling is therefore effectively weak per mode, satisfying the Born condition $\hat{\rho}_{SE} \approx \hat{\rho}_S \otimes \hat{\rho}_E$ at leading order.

(b) **Fast bath equilibration.** Each bath at events $j \neq k$ equilibrates on a timescale set by the Lindbladian spectral gap $\Delta = \gamma/2$ (Appendix C). The environment correlation functions decay as $e^{-s/\tau_{\mathrm{eq}}}$ with $\tau_{\mathrm{eq}} = 2/\gamma$; since $\gamma = 1 - \kappa \leq 1$, this is at most a few Planck times. Environment correlations therefore decay before the next system update—the Markov condition.

Under these conditions, the standard Born–Markov–secular reduction [13] applies. We decompose $\hat{R}_k$ into its Hermitian (energy-shift) and non-Hermitian (scattering) components. The inter-event hopping in $\hat{H}_{\mathrm{ref}}$ transfers information packets across the link $j \to k$ with amplitude $\sqrt{\kappa_{jk}}$; the fraction $\gamma_{jk} = 1 - \kappa_{jk}$ scatters into the bath. Tracing over E converts these scattering events into irreversible quantum jumps acting on S.

Remark 4.2 (Structural validity of the Born–Markov reduction)**.** The Born–Markov reduction in Step 4 rests on two conditions:

(a) Markov condition. The Lindbladian spectral gap $\Delta = \gamma/2$ (Appendix C) gives a bath correlation time $\tau_{\mathrm{corr}} = 2/\gamma$. Since $\gamma = \alpha\rho_I/(1+\alpha\rho_I) \leq 1$, the correlation time satisfies $\tau_{\mathrm{corr}} \geq 2\,t_{\mathrm{Pl}}$ for all $\rho_I \in (0, 1]$, and is *shortest* near saturation ($\gamma \to 1$, $\tau_{\mathrm{corr}} \to 2\,t_{\mathrm{Pl}}$). The Markov condition is satisfied for any observation above the Planck scale; it *improves* with increasing ρ_I, contrary to the naïve expectation.

(b) Born condition. For a generic quantum channel the Born approximation ($\hat{\rho}_{SE} \approx \hat{\rho}_S \otimes \hat{\rho}_E$) is a second-order perturbative result that can fail when the system–environment coupling is strong. However, the QIP link is not a generic channel: it is a quantum erasure channel (Section 3).

The erasure channel has Kraus operators $K_0 = \sqrt{\kappa}\,\hat{\mathbf{1}} \otimes |0\rangle_{\mathrm{flag}}$ (packet arrives) and $K_1 = \sqrt{\gamma}\,\hat{\mathbf{1}} \otimes |1\rangle_{\mathrm{flag}}$ (packet erased to bath), with $\langle 0|1\rangle_{\mathrm{flag}} = 0$. The orthogonality of the flag states has a crucial consequence: the two channel outcomes produce *perfectly distinguishable* environment states. No coherence between "arrived" and "erased" survives the partial trace over the environment.

For such a channel the Kraus-map reduction $\hat{\rho}_S \mapsto \sum_m K_m \hat{\rho}_S K_m^\dagger$ is exact, not perturbative. The Lindblad generator obtained from this Kraus map is the *exact* (not second-order) reduced dynamics of the system. The Born condition is therefore not an approximation for the QIP erasure channel—it is a structural identity that holds at all coupling strengths, including $\gamma \to 1$ near saturation.

Scope of the structural argument. The exactness holds when each link operates as a perfect erasure channel. Deviations arise if the bath flag states acquire nonzero overlap due to finite bath dimension d: in a d-dimensional bath, the effective overlap scales as $\langle 0|1\rangle_{\text{eff}} \sim e^{-d}$, giving corrections exponentially small in d. For the octonionic bath ($d = 8$), these corrections are of order $e^{-8} \approx 3 \times 10^{-4}$—negligible relative to the leading $\mathcal{O}(\alpha\rho_I)$ terms in all macroscopic observables. Near saturation, where all d modes are occupied and the "erased" and "arrived" sectors overlap more, the correction grows but remains bounded by $\mathcal{O}(1/d)$.

Summary. The Born–Markov reduction in the QIP framework is valid for two independent reasons:

(i) *Markov*: bath correlations decay within $\tau_{\text{corr}} = 2/\gamma \leq 2\,t_{\text{Pl}}$, far below any macroscopic observation timescale.

(ii) *Born*: the erasure-channel structure makes the Kraus-map reduction exact (not perturbative), with corrections exponentially small in the bath dimension d.

The regime where corrections become significant is $d \to \mathcal{O}(1)$ *and* $\rho_I \to 1$ simultaneously—a Planck-scale strong-field regime where the continuum limit itself may break down. In all other regimes the Lindblad equation (33) is the exact reduced dynamics.

Derivation of the dissipator. Write $\hat{R}_k = \hat{R}_k^{(0)} + \hat{V}_{SE}$, where $\hat{R}_k^{(0)}$ acts only on E (free evolution of all other events) and $\hat{V}_{SE}$ is the system–environment interaction (the hopping links connecting k to its neighbours). In the interaction picture with respect to $\hat{H}_{\text{eff}}^{(k)} + \hat{R}_k^{(0)}$, the second-order Born–Markov master equation for $\hat{\rho}_S$ is

$$\begin{aligned}\frac{\partial \hat{\rho}_S}{\partial \tau_k} &= -\frac{i}{\hbar}\big[\hat{H}_{\text{eff}}^{(k)} + \hat{H}_{\text{LS}},\ \hat{\rho}_S\big] \\ &\quad - \frac{1}{\hbar^2}\int_0^\infty \mathrm{d}s\ \mathrm{Tr}_E\Big[\hat{V}_{SE},\ \big[\hat{V}_{SE}(-s),\ \hat{\rho}_S(\tau_k)\otimes\hat{\rho}_E\big]\Big],\end{aligned} \tag{31}$$

where $\hat{V}_{SE}(-s)$ is the interaction-picture evolution of $\hat{V}_{SE}$ backward by s, and $\hat{H}_{\text{LS}}$ is the Lamb-shift correction. The environment correlation functions have the form

$$\mathrm{Tr}_E\big(\hat{B}_\mu^\dagger(s)\,\hat{B}_\nu(0)\,\hat{\rho}_E\big)\ \propto\ \gamma_k\,\delta_{\mu\nu}\,e^{-s/\tau_{\text{eq}}}, \tag{32}$$

where $\hat{B}_\mu$ are the environment operators appearing in the decomposition $\hat{V}_{SE} = \sum_\mu \hat{S}_\mu \otimes \hat{B}_\mu$, and $\gamma_k = 1 - \kappa_k$ sets the overall scattering strength. The exponential decay on timescale τ_{eq} is a direct consequence of the finite bath capacity: a finite-dimensional reservoir cannot sustain long-range temporal correlations.

Performing the s-integral in (31) and passing to the secular (rotating-wave) approximation, the double commutator reduces to the standard Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) form [13]:

$$\frac{\partial \hat{\rho}_S}{\partial \tau_k} = -\frac{i}{\hbar}\big[\hat{H}_{\text{eff}}^{(k)} + \hat{H}_{\text{LS}},\ \hat{\rho}_S\big] + \sum_m \gamma_m\Big(\hat{L}_m\,\hat{\rho}_S\,\hat{L}_m^\dagger - \tfrac{1}{2}\big\{\hat{L}_m^\dagger\hat{L}_m,\ \hat{\rho}_S\big\}\Big), \tag{33}$$

where the sum runs over all scattering channels m at event k (bath emission and, above threshold, fermionic overflow), the jump operators $\hat{L}_m$ are exactly those of (20), and the rates $\gamma_m = 1 - \kappa_m$ emerge from the environment correlation integral (32).

Step 5: Recovery of the conditional master equation

Equation (33) is the density-matrix form of the conditional master equation. In state-vector (Itô) form it reads

$$i\hbar\frac{\partial}{\partial\tau_k}|\psi_{\rm cond}\rangle = \hat{H}_{\rm ref}[\rho_{\rm cond}]|\psi_{\rm cond}\rangle + \sum_m \gamma_m\Big(\hat{L}_m|\psi_{\rm cond}\rangle\langle\psi_{\rm cond}|\hat{L}_m^\dagger - \tfrac{1}{2}\{\hat{L}_m^\dagger\hat{L}_m, |\psi_{\rm cond}\rangle\langle\psi_{\rm cond}|\}\Big). \tag{34}$$

Summary of the logical chain.

(i) *Global level:* $\hat{C}|\Psi\rangle = 0$ is a linear, Hermitian constraint on a pure state in $\mathcal{H}_{\rm tot}$. No dissipation, no non-linearity.

(ii) *Clock conditioning:* projecting onto a clock eigenstate $|\tau_k\rangle_C$ removes $\mathcal{H}_C$ and produces a unitary Schrödinger equation (28) on $\mathcal{H}_S\otimes\mathcal{H}_E$.

(iii) *Environment trace:* the Born–Markov reduction over $\mathcal{H}_E$ converts the Hermitian inter-event couplings into the irreversible Lindblad dissipator (33) on $\mathcal{H}_S$.

Dissipation is therefore not a fundamental feature of the dynamics; it is an *inevitable consequence* of restricting attention to a single event within a globally entangled, timeless state. The apparent non-linearity (operators in (34) depend on $\rho_{\rm cond}$) is likewise an artefact of conditioning: global dynamics remain strictly linear and unitary.

Remark 4.3 (Regime of validity)**.** The Lindblad equation (33) is the leading-order Markovian result, valid when the bath correlation time is much shorter than the observation timescale. The Lindbladian spectral gap $\Delta = \gamma/2$ (Appendix C) gives a bath correlation time $\tau_{\rm corr} = 2/\gamma \le 2$ in Planck units for all $\rho_I \in (0, 1]$, satisfying the Markov condition by many orders of magnitude in any macroscopic setting. Near saturation ($\rho_I \to 1$), $\gamma \to 1$ and $\tau_{\rm corr} \to 2\,t_{\rm Pl}$; non-Markovian corrections of order $t_{\rm Pl}/\tau_{\rm obs}$ may become significant only for observations on the Planck timescale.

5 Emergent Proper Time, Gravitational Redshift, and the Arrow of Time

5.1 Proper Time as Integrated Throughput

The integrated proper time is

$$\tau_k(\lambda_2) - \tau_k(\lambda_1) = \int_{\lambda_1}^{\lambda_2} \kappa_k(\lambda)\,{\rm d}\lambda. \tag{35}$$

Since $\kappa_k < 1$ whenever $\rho_{I,k} > 0$, the local proper time always lags behind λ, with the following consequences:

- In vacuum regions ($\rho_{I,k} \approx 0$), $\kappa_k \approx 1$ and ${\rm d}\tau_k \approx {\rm d}\lambda$: the clock ticks at the maximum relational rate.
- In dense regions ($\rho_{I,k} > 0$), $\kappa_k < 1$: gravitational time dilation in its most microscopic form.

The *gravitational redshift ratio* between two neighbouring clocks at the same relational slice is

$$\frac{{\rm d}\tau_k}{{\rm d}\tau_\ell} = \frac{\kappa_k}{\kappa_\ell} = \frac{1+\alpha\rho_{I,\ell}}{1+\alpha\rho_{I,k}}. \tag{36}$$

In the weak-field limit ($\alpha\rho_I \ll 1$), identifying $\mathrm{d}\tau \approx (1+\Phi)\,\mathrm{d}t$ with the standard GR convention $g_{00} = 1 + 2\Phi$:

$$\Phi \approx -\frac{\alpha\rho_I}{2}. \tag{37}$$

This is the first microscopic link between information density and the emergent gravitational field.

5.2 Thermodynamic Arrow of Time

The Lindblad jump rate $\gamma_k = 1 - \kappa_k$ quantifies the fraction of failed information transfers scattered into the local bath. The local entropy production rate is

$$\frac{\mathrm{d}S(\hat{\rho}_k)}{\mathrm{d}\tau_k} = \gamma_k(\rho_{I,k}) \cdot \langle \Delta S \rangle_{\text{jump}} > 0 \tag{38}$$

except in the exact vacuum. The thermodynamic arrow thus points in the direction of increasing τ_k, unifying gravitational and thermodynamic time asymmetries at the microscopic level through the single congestion parameter κ_k.

6 Weighted Quantum Causal Set and Emergent Geometry

6.1 Directed Links and Causal Structure

A directed link $i \to j$ exists with amplitude

$$A_{i\to j} = \sqrt{\kappa_{ij}} = \big(1 + \alpha(\rho_{I,i} + \rho_{I,j})\big)^{-1/2}. \tag{39}$$

The quantum amplitude for a causal chain $a = k_0 \to k_1 \to \cdots \to k_n = b$ is $\mathcal{A}(\text{path}) = \prod_m \sqrt{\kappa_{k_{m-1}k_m}}$.

6.2 Emergent Metric from Weighted Path Lengths

The *information distance* along a path is

$$\ell(\text{path}) := -\frac{1}{2}\sum_m \ln \kappa_{k_{m-1}k_m} \geq 0. \tag{40}$$

This measure is additive, vanishes in vacuum, and arises naturally in information geometry [10] as a Kullback–Leibler-type divergence between capacity states. The emergent spacetime interval is obtained by minimising over causal chains connecting a and b.

6.3 Conformal Line Element and Lorentzian Signature

In the weak-field limit the emergent line element is

$$\mathrm{d}s^2 = \kappa(x)\,\eta_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu, \qquad \kappa(x) = \frac{1}{1+\alpha\rho_I(x)}. \tag{41}$$

Lorentzian signature is a structural consequence of imposing a causal partial order on the set of information events, following the standard causal-set construction [18]:

$$G(i,j) = \begin{cases} -\ell_{ij}^2 & i \prec j \text{ (timelike/null)} \\ +\ell_{ij}^2 & i \perp j \text{ (spacelike)} \end{cases} \tag{42}$$

This is a foundational assumption of the framework, alongside the clock-rate postulate, not a consequence of the permeability rule. The factor $\kappa(x) > 0$ modifies the *magnitude* of intervals but cannot produce the sign distinction between timelike and spacelike separations: it preserves the signature $(+---)$ for all $\kappa > 0$.

Remark 6.1 (Thermodynamic vs. geometric continuum limit)**.** The derivation of the Einstein equations (Section 8) does not require the full geometric continuum limit of the causal set—the so-called Hauptvermutung of causal-set theory [18]. It requires only the *thermodynamic* continuum limit: that the coarse-grained geometry is smooth enough to support local Rindler horizons at every point, and that the Clausius relation $\delta Q = T\delta S$ holds locally.

This is a strictly weaker condition. The thermodynamic limit is analogous to the hydrodynamic limit in statistical mechanics: one does not need to derive the Navier–Stokes equations from molecular dynamics to use them—one needs only local thermodynamic equilibrium. Similarly, the QIP derivation needs only that the discrete causal set is in local equilibrium on scales much larger than $\ell_{\min}$, so that the averaged geometry admits a smooth metric and null congruences.

The stronger geometric continuum limit—showing that the discrete causal set converges to a specific smooth manifold with specific topology—remains an open problem shared by all causal-set approaches. It is not required for the thermodynamic derivation of the field equations, but would be needed for questions about global topology, singularity structure, and the ultraviolet completion of the theory.

Microscopic versus macroscopic metric. The conformally flat metric (41) is the leading-order causal-set result: it describes the geometry of individual links. At this order both g_{00} and g_{ij} are multiplied by the same κ, which does not by itself reproduce the opposite-sign perturbations of the standard GR weak-field metric ($g_{00} = 1 + 2\Phi$, $g_{ij} = -(1 - 2\Phi)$). The correct post-Newtonian structure emerges at the macroscopic level from the thermodynamically derived Einstein equations (Section 8), whose weak-field limit is standard GR with $\gamma_{\rm PPN} = \beta_{\rm PPN} = 1$ and corrections $|\gamma - 1| \sim \mathcal{O}(\alpha\rho_I) \lesssim 10^{-27}$ (Remark 11.2). The conformal line element should be understood as the microscopic "lattice metric," not the macroscopic observable metric; the two are related by thermodynamic coarse-graining, just as a lattice Hamiltonian relates to its continuum effective field theory. The weak-field Poisson limit is derived in Appendix F.

6.4 Causality and Horizons

Maximum propagation speed corresponds to highest-permeability (null-like) paths. Horizons appear where $\kappa \to 0$ over extended regions—the discrete analogue of event horizons.

7 Emergence of Matter from Local Saturation

7.1 The Bekenstein Ceiling

The maximum von Neumann entropy storable at event k is

$$S_{\max,k} \approx 2\pi E_k \ell_k/\hbar, \tag{43}$$

corresponding to $\rho_{I,k}^{\max} \approx 1$ in normalised units.

7.2 Microscopic Transfer Mechanism

The jump operator (20) implements the overflow: $\hat{f}^\dagger_{\mathrm{mat,k}}$ obeys fermionic anticommutation

$$\{\hat{f}_{\mathrm{mat,k}}, \hat{f}^\dagger_{\mathrm{mat},\ell}\} = \delta_{k\ell}, \tag{44}$$

so Pauli exclusion is automatic, not postulated. The rest-mass energy cost is $\Delta E_{\mathrm{rest}} = mc^2 \approx \langle \hat{H}^{(k)}_{\mathrm{mat}} \rangle_{\mathrm{after}}$.

7.3 Saturation Dynamics and Matter Stability

(i) Low density ($\rho_I \ll 1$): scattered packets stay in the bath or are re-absorbed reversibly.

(ii) Increasing ρ_I: γ_k grows, entropy production accelerates.

(iii) Near saturation ($\rho_I \to 1$): the bath cannot accommodate additional entropy without violating the Bekenstein ceiling.

(iv) The jump operator transfers probability irreversibly into the fermionic sector, creating stable occupation $n_f \in \{0, 1\}$ (confirmed numerically in [23]).

(v) Created fermions contribute to ρ_I, closing a self-consistent feedback loop.

Massive particles are regions where the information network has become so congested that the only way to continue processing information is to condense it into stable, exclusion-enforced excitations—linking inertia to gravitational phenomena through information throughput.

8 Thermodynamic Derivation of the Einstein Equations

8.1 Heat Flux and Unruh Temperature

The irreversibly scattered fraction $1 - \kappa_k$ appears as heat flux crossing local Rindler horizons:

$$\frac{\delta Q}{\delta A \, \mathrm{d}\tau} = -T_{ab} k^a k^b, \tag{45}$$

where k^a is the future-directed null normal to the horizon. Gradients of $\kappa(x)$ induce effective acceleration $a = |\nabla_\perp \ln \kappa| / \lambda_0$, giving the Unruh temperature

$$T = \frac{\hbar a}{2\pi k_B} = \frac{\hbar}{2\pi k_B} \frac{|\nabla_\perp \ln \kappa|}{\lambda_0}. \tag{46}$$

8.2 Entropy per Unit Area

In the saturated limit, the bath entropy saturates the Bekenstein–Hawking value, giving areal entropy density

$$\eta = \frac{\delta S}{\delta A} = \frac{1}{4\ell^2_{\mathrm{Pl}}}. \tag{47}$$

8.3 Recovery of the Einstein Equations

Applying the Clausius relation $\delta Q = T\,\delta S$ to every local Rindler horizon at every point, using the Raychaudhuri equation and the contracted Bianchi identity (full derivation in Appendix A), one obtains

$$G_{ab} + \Lambda g_{ab} = 8\pi G\, T_{ab}, \qquad G = \frac{1}{4\hbar\eta}. \tag{48}$$

The Einstein equations are a thermodynamic equation of state for the emergent spacetime, with no additional assumptions beyond the permeability rule and the holographic entropy bound.

9 Gravitational Sourcing as Information Accounting: the No-Double-Counting Theorem

A natural concern arises at the junction of Sections 7 and 8: information density ρ_I sources gravity via the permeability κ and the emergent metric, and the same ρ_I undergoes a phase transition (overflow) into fermionic matter. Does the resulting fermion contribute to gravity *twice*—once as the pre-overflow bath occupation that curved spacetime, and again as a massive particle with its own T_{ab}?

We show below that this cannot happen. The gravitational source in QIP is always and only ρ_I; what changes at overflow is the *internal structure* of the degrees of freedom that carry it, not the total amount. The proof proceeds in three steps: a local conservation law at the level of the master equation, the identification of all energy forms with information transfer, and a consistency check against the thermodynamic Einstein equations.

9.1 Local Information Conservation Across the Overflow

Proposition 9.1 (Information conservation at overflow)**.** *Let $\hat{\rho}_S(\tau)$ be the local reduced density matrix at event k, evolving under the Lindblad equation (33), and let the information density be defined as in (6). Define the* total local information *as*

$$I_{\mathrm{tot},k}(\tau) \;:=\; S\big(\hat{\rho}_k^{(\mathrm{bath})}(\tau)\big) + S\big(\hat{\rho}_k^{(\mathrm{mat})}(\tau)\big) + \Delta I_{\mathrm{emitted},k}(\tau), \tag{49}$$

where $S(\cdot)$ is the von Neumann entropy of each sector and $\Delta I_{\mathrm{emitted},k}$ is the cumulative information carried away by emitted quanta (Higgs bosons and radiated bath excitations) that have propagated out of the local causal diamond. Then the Lindblad equation implies

$$\frac{dI_{\mathrm{tot},k}}{d\tau} = J_k^{\mathrm{in}} - J_k^{\mathrm{out}}, \tag{50}$$

where J_k^{in} and J_k^{out} are the net information fluxes through the boundary of the local causal diamond, determined by the inter-event hopping amplitudes $\sqrt{\kappa_{jk}}$.

Proof. The global constraint $\hat{C}|\Psi\rangle = 0$ preserves the purity of $|\Psi\rangle$, so the total von Neumann entropy $S(|\Psi\rangle\langle\Psi|) = 0$ is constant. At the level of the local system S (comprising both bath and matter registers at event k), the Lindblad equation (33) generates entropy through the dissipator $\mathcal{D}[\hat{\rho}_S]$. The standard result for entropy production under GKSL dynamics is [20, 21]:

$$\frac{dS(\hat{\rho}_S)}{d\tau} = \sigma_k(\tau) - \Phi_k(\tau), \tag{51}$$

where $\sigma_k \geq 0$ is the irreversible entropy production rate (sourced by the $\hat{L}_m$ jumps) and Φ_k is the entropy flux out of the system into the environment.

Now decompose $\hat{\rho}_S$ according to the Hilbert space factorisation $\mathcal{H}_S = \mathcal{H}_{\text{mat}}^{(k)} \otimes \mathcal{H}_{\text{bath}}^{(k)}$. The jump operator (20) has two components:

(i) $\hat{b}_{\text{bath,out}}^{(k)}$: transfers bath entropy to outgoing radiation (bath → environment);

(ii) $\Theta(\hat{\rho}_{I,k} - \rho_{\text{crit}})\, \hat{f}_{\text{mat}}^{\dagger}$: transfers bath entropy to the matter register (bath → matter).

Process (i) decreases $S(\hat{\rho}_k^{(\text{bath})})$ and increases $\Delta I_{\text{emitted},k}$ by the same amount (the information is carried by the emitted quantum into a neighbouring causal diamond, where it becomes part of J_j^{in}). Process (ii) decreases $S(\hat{\rho}_k^{(\text{bath})})$ and increases $S(\hat{\rho}_k^{(\text{mat})})$: information is *relabelled* from bath to matter, but not created or destroyed locally.

Crucially, $\hat{f}_{\text{mat}}^{\dagger}$ obeys fermionic anticommutation (44), so the matter register has $n_f \in \{0, 1\}$ and $S(\hat{\rho}_k^{(\text{mat})}) \leq \ln 2$. The overflow event therefore transfers at most $\ln 2$ nats of entropy from bath to matter. No entropy is created by the overflow itself: the jump operator is a unitary rotation from one sector to another, dressed by the rate γ_k.

Summing over all three contributions in (49) and using the entropy balance (51):

$$
\begin{aligned}
\frac{dI_{\text{tot},k}}{d\tau} &= \underbrace{\frac{dS(\hat{\rho}_k^{(\text{bath})})}{d\tau}}_{\text{bath change}} + \underbrace{\frac{dS(\hat{\rho}_k^{(\text{mat})})}{d\tau}}_{\text{matter change}} + \underbrace{\frac{d\Delta I_{\text{emitted},k}}{d\tau}}_{\text{emitted quanta}} \\
&= \left(\sigma_k - \Phi_k^{(\text{out})}\right) + 0 + \Phi_k^{(\text{out})} + J_k^{\text{in}} - J_k^{\text{out}} \\
&= \sigma_k + J_k^{\text{in}} - J_k^{\text{out}}.
\end{aligned} \tag{52}
$$

The irreversible production σ_k accounts for the entropy generated by scattering events within the causal diamond. In a steady state or in a closed system ($J^{\text{in}} = J^{\text{out}}$), $I_{\text{tot},k}$ is strictly non-decreasing and is conserved only in the reversible ($\sigma = 0$) limit. In either case, *the overflow event itself contributes zero net change to* $I_{\text{tot},k}$: it is a **sector-transfer**, not a source. □

Corollary 9.2 (No double-counting of gravitational source)**.** *Since* $\kappa_k = (1 + \alpha\rho_{I,k})^{-1}$ *depends on* $\rho_{I,k}$ *defined as the total normalised entropy at event* k *(equation (6)), and since* $I_{\text{tot},k}$ *is conserved across the overflow transition, the gravitational field sourced by* $\rho_{I,k}$ *before overflow is* identically *the gravitational field sourced by the same* $\rho_{I,k}$ *after overflow. The fermion's rest mass does not add a new gravitational contribution: it is the* same ρ_I*, now partially housed in the matter register rather than entirely in the bath.*

Remark 9.3 (Comparison with other quantum gravity programmes)**.** The gravitational-source accounting problem is endemic across quantum gravity. In the semiclassical approximation, what exactly sources G_{ab} (the expectation value? the wavefunction? the measurement outcome?) is an open foundational question. Loop quantum gravity inherits the problem from classical GR by coupling Standard Model $T_{\mu\nu}$ to spin-network vertices without deriving the coupling. Entropic gravity (Verlinde) faces a circularity critique: the holographic screen entropy is itself gravitationally determined. In AdS/CFT the problem is dissolved by holographic unification, but only in asymptotically AdS spacetimes.

QIP is, to our knowledge, the only framework in which the same microscopic variable (ρ_I) both sources gravity and undergoes a well-defined phase transition into matter, with a master equation that explicitly tracks information conservation across the transition. The no-double-counting result (Corollary 9.2) is therefore not merely a consistency check but a structural feature unavailable to other approaches.

9.2 Energy as the Thermodynamic Measure of Information Transfer

The conservation law of Proposition 9.1 has a direct energetic counterpart. Recall from Remark 2.1 that energy enters the framework in two roles: stored (Bekenstein) energy and gravitationally sourcing energy. We now unify these under a single principle.

Theorem 9.4 (Universality of gravitational sourcing)**.** *In the QIP framework, every form of energy—fermionic rest mass, gauge boson kinetic energy, and radiation—is the thermodynamic measure of irreversible information transfer through quantum channels. Since gravity is sourced by ρ_I, every form of energy contributes to the gravitational source through the same mechanism: the modification of κ by ρ_I. There is no energy outside the information-theoretic substrate, and therefore no gravitational source outside ρ_I.*

Proof. The chain of identifications in the framework is:

$$\text{information transfer} \xrightarrow{\gamma_k = 1-\kappa_k} \text{irreversible scattering} \xrightarrow{\delta Q} T_{ab} \xrightarrow{\text{Clausius}} G_{ab}. \tag{53}$$

We verify this for each energy form:

(i) Fermionic rest mass. A fermion at event k is a state with $n_f = 1$ in the matter register $\mathcal{H}^{(k)}_{\text{mat}}$. By Proposition 9.1, the information locked in this state was transferred from the bath at overflow and is still counted in $\rho_{I,k}$. The rest mass $mc^2 \approx E_{\text{stored},k}$ (equation (7)) is the Bekenstein energy of the information density that saturated the channel. This $\rho_{I,k}$ enters κ_k, which enters the emergent metric, which sources gravity. No additional coupling is required.

(ii) Gauge bosons. A propagating gauge boson (photon, gluon, W, Z) is a single excitation of the bath that has been transmitted across a link. It carries ρ_I in transit: the information packet is "in flight" between nodes, contributing to the bath occupation at both the source and receiver ends during the transmission interval. Its energy is the rate of information transfer along that channel, which enters T_{ab} via the heat flux identification (45).

(iii) Radiation (thermal photons, gravitational waves). Radiation is the cumulative effect of many bath-to-bath scattering events. Each scattering event transfers information irreversibly and contributes to δQ. The total T_{ab} of a radiation field is the coarse-grained sum of all such micro-transfers, with no additional sourcing term.

In every case, the gravitational contribution traces back to ρ_I through the permeability rule (13) and the Jacobson thermodynamic derivation (Section 8). No energy form requires a separate gravitational coupling constant, and no energy form is gravitationally "silent." □

Remark 9.5 (Dark matter as an immediate corollary)**.** The universality of gravitational sourcing provides a clean basis for a dark matter proposal developed in the companion paper [23]. Sub-threshold bath occupation ($\rho_I < \rho_{\text{crit}}$) carries information density, and by Theorem 9.4, information density gravitates regardless of whether overflow has occurred. Sub-threshold regions modify κ and curve the emergent metric exactly as matter-occupied regions do. The only difference is the absence of Standard Model interactions: no overflow has occurred, so no fermionic excitations and no gauge charges are present. (The Standard Model gauge structure is derived in [23]; the concept is used here only at the level of "no gauge charges ⇒ gravitates but is invisible to other forces.") Dark matter is therefore not a new particle species; it is *information that has not yet undergone the phase transition into matter*. The full treatment is in [23].

9.3 Consistency with the Einstein Equations

The thermodynamic derivation of the Einstein equations (Section 8) uses the Clausius relation $\delta Q = T\,\delta S$ applied to every local Rindler horizon. The heat flux δQ is identified with $-T_{ab}k^a k^b\,\delta A\,d\tau$ (equation (45)), and the entropy per unit area is $\eta = 1/(4\ell_{\rm Pl}^2)$ (equation (47)).

For this derivation to be self-consistent across the overflow transition, the heat flux through a horizon enclosing a matter-producing event must equal the heat flux that was present immediately before overflow. This is guaranteed by Proposition 9.1: since $\rho_{I,k}$ is unchanged by the overflow (information is relabelled, not created), and since $\kappa_k = (1+\alpha\rho_{I,k})^{-1}$ depends only on the total $\rho_{I,k}$, the Unruh temperature T, the entropy density η, and the heat flux δQ are all continuous across the transition. The Einstein tensor G_{ab} inherits this continuity.

Explicitly, consider a local Rindler horizon enclosing event k at the moment of overflow ($\tau = \tau_{\rm over}$). Define:

$$\rho_I^- := \lim_{\tau\to\tau_{\rm over}^-} \rho_{I,k}(\tau) \qquad \text{(just before overflow)}, \tag{54}$$

$$\rho_I^+ := \lim_{\tau\to\tau_{\rm over}^+} \rho_{I,k}(\tau) \qquad \text{(just after overflow)}. \tag{55}$$

By the sector-transfer nature of the overflow (Proposition 9.1):

$$\rho_I^+ = \rho_I^{(\rm bath,+)} + \rho_I^{(\rm mat,+)} = \rho_I^-\,, \tag{56}$$

where $\rho_I^{(\rm bath,+)} = \rho_I^- - \delta\rho$ and $\rho_I^{(\rm mat,+)} = \delta\rho$, with $\delta\rho \le \ln 2/\ln d_k$ being the normalised entropy transferred to the matter register.

Therefore:

$$\kappa_k^+ = \frac{1}{1+\alpha\rho_I^+} = \frac{1}{1+\alpha\rho_I^-} = \kappa_k^-\,, \tag{57}$$

and consequently the emergent metric, the Unruh temperature, the heat flux, and the Einstein tensor are all *continuous* at the overflow event. The gravitational field does not "jump" when a fermion is created. Matter inherits the gravitational effect of the information that produced it—no more, no less.

Remark 9.6 (Logical dependencies: external inputs and their provenance)**.** The derivation of the Einstein equations from the QIP permeability rule uses four external results. None presupposes general relativity:

Input	Source	GR-independent?
Bekenstein bound $S \le 2\pi ER$	Quantum information theory	Yes[a]
Unruh temperature $T = \hbar a/(2\pi)$	QFT on flat spacetime	Yes[b]
Raychaudhuri equation	Pseudo-Riemannian geometry	Yes[c]
Bianchi identity $G^{ab}{}_{;b} = 0$	Differential geometry	Yes
Lorentzian signature	Axiom (causal partial order)	N/A

[a]The Bekenstein bound was originally derived [3] using black-hole thermodynamics. However, Casini [4] proved it from the positivity of relative entropy in any Lorentz-invariant quantum field theory, without reference to gravity. The QIP framework uses the bound in this quantum information-theoretic form.

[b]The Unruh effect [7] is a consequence of the structure of the vacuum state under Lorentz boosts. It requires Lorentz invariance and the existence of a Rindler wedge in Minkowski spacetime, but not the Einstein field equations or any gravitational dynamics.
[c]The Raychaudhuri equation describes the focusing of null geodesic congruences in any pseudo-Riemannian manifold. It is a kinematic identity of differential geometry, not a dynamical equation. Once the emergent metric exists (Section 6), the Raychaudhuri equation follows as a mathematical consequence without further physical input.

The logical order of the QIP derivation is therefore:

$$\underbrace{\text{permeability rule}}_{\text{QIP}} \to \underbrace{\text{emergent metric}}_{\S 6} \to \underbrace{\text{Clausius relation}}_{\text{QIT + QFT}} \to \underbrace{\text{Raychaudhuri}}_{\text{geometry}} \to \underbrace{G_{ab} = 8\pi G\, T_{ab}}_{\text{derived}}$$

General relativity appears at the end as a *consequence*, not as an input at any stage.

Remark 9.7 (The physical meaning of "no double-counting")**.** The result (56) can be stated in plain physical terms:

A fermion does not add gravitational weight to the universe. It is a reorganisation of information that was already gravitating. The rest mass of a particle is the Bekenstein energy of the information that was committed at overflow, not new energy injected from outside the network. The total gravitational source at event k is always $\rho_{I,k}$, irrespective of how that information is internally partitioned between bath and matter registers.

This resolves the double-counting concern raised for entropic gravity by [22] and places QIP in a structurally unique position among quantum gravity programmes: it is the only framework we are aware of in which the gravitational-source accounting is explicitly tracked across a matter-creation event by the microscopic dynamics.

10 The Gravitational Constant as Information Budget

The derivation of $\alpha = 2\pi - \frac{1}{2}$ (Appendix B) fixes the permeability at saturation to $\kappa_{\text{sat}} = 1/(4\pi)$. In this section we show that the same chain of identifications determines the gravitational constant G in QIP natural units, reveals a holographic identity connecting G to the channel capacity, and clarifies the sense in which SI dimensions are emergent within the framework.

10.1 Derivation of G from α

In Appendix F, the effective gravitational constant is fixed by holographic matching:

$$G = \frac{1}{4\hbar\eta}, \tag{58}$$

where $\eta = \delta S/\delta A = 1/(4\ell_{\text{Pl}}^2)$ is the areal entropy density at holographic saturation. This relation is a consequence of the Clausius relation $\delta Q = T\,\delta S$ applied to every local Rindler horizon (Section 8), with no additional input.

We now express η directly in terms of α. At saturation, the total entropy of a causal diamond of radius R is (equation (121)):

$$S_{\max} = N_\partial \cdot C_{\text{sat}} = \frac{4\pi R^2}{\ell_{\text{Pl}}^2} \cdot \frac{1}{4\pi} = \frac{R^2}{\ell_{\text{Pl}}^2}, \tag{59}$$

so the areal entropy density is

$$\eta = \frac{S_{\max}}{4\pi R^2} = \frac{R^2/\ell_{\text{Pl}}^2}{4\pi R^2} = \frac{1}{4\pi \ell_{\text{Pl}}^2}. \tag{60}$$

Substituting into (58) (in Planck units $\hbar = 1$):

$$G = \frac{1}{4\eta} = \frac{4\pi\ell_{\rm Pl}^2}{4} = \pi\ell_{\rm Pl}^2. \tag{61}$$

The standard definition of the Planck length is $\ell_{\rm Pl} = \sqrt{\hbar G/c^3}$, which in Planck units ($\hbar = c = 1$) gives $\ell_{\rm Pl}^2 = G$. Equation (61) then reads

$$G = \pi G, \tag{62}$$

which is a contradiction unless the sprinkling density in the boundary-link count (110) is normalised consistently with the Jacobson thermodynamic route. This is the well-known geometric-prefactor ambiguity of causal-set approaches (see [18] and the discussion below equation (121)).

The resolution is that the precise numerical coefficient of the area law is fixed by the Jacobson route, not by the boundary-link counting. The counting gives $S \propto A$; the Jacobson derivation fixes the proportionality constant to $\eta = 1/(4\ell_{\rm Pl}^2)$. With this normalisation, $G = 1/(4\hbar\eta) = \ell_{\rm Pl}^2$ in Planck units, which is the definition of G in those units.

The non-trivial content is not the value of G in Planck units (which is 1 by definition) but the relationship between G and the channel capacity $C_{\rm sat}$ and the boundary link count N_∂. This is most transparently expressed in a system of units natural to the QIP framework itself.

10.2 QIP Natural Units and the Holographic Identity

Define **QIP natural units** by setting the fundamental causal link length, the information capacity per link use, and the speed of information propagation all to unity:

$$\ell_{\min} = 1, \qquad C_{\max} = 1 \text{ nat}, \qquad c = 1. \tag{63}$$

Here $\ell_{\min}$ is the spacing between adjacent events in the causal set — the primitive length quantum of the framework, representing one quantum of spatial information transfer. In these units the Planck length is *derived*:

$$\ell_{\rm Pl}^2 = \frac{G\hbar}{c^3} = G \qquad (\text{since } \hbar = c = 1). \tag{64}$$

The boundary-link count for the minimal causal diamond ($R = \ell_{\min} = 1$) in QIP units is the solid angle of the unit sphere:

$$N_\partial^{(\min)} = 4\pi \left.\frac{R^2}{\ell_{\min}^2}\right|_{R=1} = 4\pi. \tag{65}$$

Each boundary link at saturation carries capacity $C_{\rm sat} = \kappa_{\rm sat}$ (since $C_{\max} = 1$). The total information throughput of the minimal diamond is therefore:

$$\mathcal{I}_{\min} := N_\partial^{(\min)} \cdot C_{\rm sat} = 4\pi \cdot \frac{1}{4\pi} = 1 \text{ nat (exactly)}. \tag{66}$$

Theorem 10.1 (Holographic unit-throughput identity)**.** *The minimal causal diamond in the QIP framework — the smallest possible spherical region of the information network — transmits exactly one nat of information at saturation:*

$$N_\partial^{(\min)} \cdot \kappa_{\rm sat} = 1. \tag{67}$$

This identity is exact and follows from the four consistency conditions C1–C4 of Appendix B with no approximation.

Proof. From C1: $\kappa_{\text{sat}} = 1/(1{+}2\alpha)$. From C2–C4 at the minimal diamond: $N_\partial^{(\text{min})} = 4\pi$ and $C_{\text{sat}} = \kappa_{\text{sat}} \cdot C_{\max} = \kappa_{\text{sat}}$. The Bekenstein saturation condition C4 requires $N_\partial^{(\text{min})} \cdot C_{\text{sat}} = 2\pi ER$, and the Unruh energy gives $2\pi ER = N_\partial^{(\text{min})} \cdot T_{\text{loc}} \cdot 2\pi R = 4\pi \cdot (1/(2\pi)) \cdot 2\pi = 4\pi$ in QIP units. Therefore $4\pi \cdot \kappa_{\text{sat}} = 1$, which yields $\kappa_{\text{sat}} = 1/(4\pi)$ and hence $1 + 2\alpha = 4\pi$, i.e. $\alpha = 2\pi - 1/2$. Inserting back: $N_\partial^{(\text{min})} \cdot \kappa_{\text{sat}} = 4\pi/(4\pi) = 1$. □

Now define the gravitational coupling in QIP units as the total number of boundary links of the minimal causal diamond:

$$\boxed{G_{\text{QIP}} := 1 + 2\alpha = 4\pi.} \tag{68}$$

This is the *inverse* of the saturation permeability:

$$\kappa_{\text{sat}} = \frac{1}{G_{\text{QIP}}}. \tag{69}$$

The holographic identity (67) then reads

$$\kappa_{\text{sat}} \cdot G_{\text{QIP}} = \frac{1}{4\pi} \cdot 4\pi = 1 \text{ nat (exactly)}. \tag{70}$$

Remark 10.2 (Physical interpretation of $G_{\text{QIP}} = 4\pi$)**.** The quantity $G_{\text{QIP}} = 4\pi$ has a direct geometric meaning: it is the solid angle of the full sphere S^2, and simultaneously the surface area of the unit sphere. In QIP, the gravitational coupling is the **total number of independent directions into which a single event can radiate information**. The stronger gravity couples information to geometry, the more boundary channels are available per event, and the more the local geometry responds to the local information density.

The factor 4π appears for the same reason it appears in the Gauss law, in the Bekenstein–Hawking entropy, and in the Stefan–Boltzmann law: it counts the isotropic angular budget of information flow in $3 + 1$ dimensions. In $2 + 1$ dimensions the boundary of a causal diamond is a circle (2π boundary links), and the same derivation would give $G_{\text{QIP}}^{(2+1)} = 2\pi$. In $4 + 1$ dimensions the boundary is S^3 ($2\pi^2$ surface area), giving $G_{\text{QIP}}^{(4+1)} = 2\pi^2 + 1$. The gravitational coupling is the *angular measure of the causal boundary*, and it is fixed by the dimensionality of the information network.

10.3 SI Dimensions as Emergent Measures

The gravitational constant in SI units has dimensions $\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$. Within the QIP framework each of these dimensions is an emergent measure of information transfer events in the causal network:

The metre. A spatial distance is the number of causal-set links traversed end-to-end: $L = N_{\text{links}} \cdot \ell_{\min}$. The SI metre is a human convention for grouping approximately $\ell_{\min}^{-1} \approx 10^{34}$ link lengths (if $\ell_{\min} \sim \ell_{\text{Pl}}$).

The second. Proper time is the integrated throughput: $\tau = \int \kappa \, d\lambda$ (equation (27)). A second is a count of approximately $c/\ell_{\min} \approx 10^{43}$ successful information transfers.

The kilogram. Mass is the Bekenstein energy locked at overflow: $mc^2 \approx E_{\text{stored}} = \rho_I \ln d/(2\pi R)$ (equation (7)). A kilogram corresponds to approximately $c^2/E_{\min} \approx 10^{52}$ overflow-scale energy quanta.

All three reduce to counts of information events. The SI value $G \approx 6.674\times10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$ therefore encodes the ratio between human measurement conventions and the fundamental link scale $\ell_{\min}$. $G_{\text{QIP}} = 4\pi$ is the dimensionless content; the SI numerical value carries no additional physical information beyond the choice of unit scale.

Proposition 10.3 (Dimensionless content of G)**.** *In any unit system built from the primitives of the QIP causal set ($\ell_{\min}$, $C_{\max}$, c), the gravitational constant is the pure number*

$$G_{\text{QIP}} = 1 + 2\alpha = 4\pi. \tag{71}$$

This number is determined by α alone, which is in turn determined by the consistency conditions C1–C4. No dimensionful input beyond the definition of the unit system is required.

Proof. By definition of QIP natural units (63), the gravitational constant is

$$G = \frac{\ell_{\min}^2\, c^3}{\hbar\, C_{\text{sat}}(\alpha)^{-1}} = \frac{\ell_{\min}^2\, c^3}{\hbar}\,(1+2\alpha). \tag{72}$$

Setting $\ell_{\min} = c = \hbar = 1$, this reduces to $G = 1 + 2\alpha = 4\pi$. Since α is fixed by C1–C4 (Appendix B), G_{QIP} is a derived quantity with no free parameters. □

10.4 The Relationship $\ell_{\text{Pl}} = \sqrt{4\pi}\,\ell_{\min}$

In SI units, $G = \ell_{\text{Pl}}^2 c^3/\hbar$. In QIP units, $G = (1+2\alpha)\,\ell_{\min}^2\, c^3/\hbar$. Equating:

$$\ell_{\text{Pl}}^2 = (1+2\alpha)\,\ell_{\min}^2 = 4\pi\,\ell_{\min}^2, \tag{73}$$

or equivalently:

$$\frac{\ell_{\text{Pl}}}{\ell_{\min}} = \sqrt{4\pi} \approx 3.545. \tag{74}$$

The Planck length is $\sqrt{4\pi}$ times the fundamental link length of the causal set. This is not a definition: it is a **consequence of** α. The human-defined Planck scale (constructed from the measured values of G, $\hbar$, c) differs from the true microscopic scale of the network by a factor determined entirely by the information channel geometry.

Remark 10.4 (Measurability)**.** The ratio $\ell_{\text{Pl}}/\ell_{\min} = \sqrt{4\pi}$ is a dimensionless prediction. It could in principle be tested by any experiment sensitive to Planck-scale discreteness: if the causal set has link spacing $\ell_{\min}$ rather than ℓ_{Pl}, the granularity of spacetime would appear at a scale $\ell_{\text{Pl}}/\sqrt{4\pi} \approx 4.56 \times 10^{-36}$ m, roughly $3.5\times$ below the Planck length. Current experiments (gamma-ray burst dispersion, gravitational-wave strain noise) probe scales near ℓ_{Pl}; a factor of 3.5 improvement in sensitivity would constitute a test.

10.5 Predictive Content: What α Determines

The result $G_{\text{QIP}} = 4\pi$ does not predict G in SI units (which would require independent knowledge of $\ell_{\min}$ in metres — a problem shared by every quantum gravity theory). What it does predict is that *all dimensionless observables* derivable from the framework are functions of α alone. Specifically:

(i) **PPN parameters:** $\gamma_{\text{PPN}} = \beta_{\text{PPN}} = 1 + \mathcal{O}(\alpha\rho_I)$ (Remark 11.2).

(ii) **Gravitational time dilation:** $d\tau_A/d\tau_B = \kappa_A/\kappa_B = (1 + \alpha\rho_{I_B})/(1 + \alpha\rho_{I_A})$

(iii) **Dark matter fraction:** $\Omega_{\text{DM}}/\Omega_b$ is set by the primordial inflow distribution, which is a function of α and the cosmological initial conditions.

(iv) **Higgs-to-fermion mass ratio:** $m_H/(m_f + m_H) = \ln 2/S_{\text{diag}}$ (derived in [23]), with S_{diag} determined by the octonionic bath dynamics at the α-dependent saturation threshold.

(v) **The holographic bound itself:** $S = A/(4\ell_{\text{Pl}}^2) = A/(16\pi\ell_{\text{min}}^2)$ — the Bekenstein–Hawking entropy per unit area in link units is $1/(16\pi)$, a pure number set by α.

The consistency of these predictions with observation, all flowing from the single parameter $\alpha = 2\pi - 1/2$, is the empirical test of the framework. Getting any one of them right is not surprising; getting all of them right simultaneously from one number would be strong evidence that α correctly describes the information geometry of the universe.

Remark 10.5 (α as the minimal complete specification)**.** Since α fixes the causal structure (κ_{sat}), the gravitational coupling (G_{QIP}), the dimensionality (through $4\pi = \text{Area}(S^2)$), the arrow of time (through $\gamma > 0$), the matter-creation threshold (ρ_{crit}), and the quantum statistics (through the $-1/2$ fermionic correction), it constitutes the minimal complete specification of a universe within the QIP framework.

Different values of α — corresponding to different dimensionalities, different statistics, or different channel structures — define different self-consistent universes, each a fixed point of the consistency map $\alpha = F(\alpha)$. The "landscape" of possible universes is the set of fixed points of the QIP self-consistency equations, and our universe is the unique attractor in $3 + 1$ dimensions with fermionic matter: $\alpha = 2\pi - 1/2$.

11 Macroscopic Gravity from Thermodynamic Coarse-Graining

11.1 The Scalar Field as Local Throughput

The dimensionless scalar field is identified with the coarse-grained local permeability:

$$\phi(x) := \langle \kappa(x) \rangle = \frac{1}{1 + \alpha\rho_I(x)}. \tag{75}$$

In the weak-field regime $\phi \approx 1 - \alpha\rho_I \approx 1 + 2\Phi$.

Remark 11.1 (Algebraic slaving of ϕ)**.** The field ϕ is *not* an independent propagating degree of freedom. It is algebraically determined by the local information density through (75): $\phi(x) = (1+\alpha\rho_I(x))^{-1}$ is an identity, not a field equation. No initial conditions, propagation speed, or causal cone are associated with ϕ.

The Lindbladian spectral gap (Appendix C) gives the intrinsic bath relaxation rate as

$$\Gamma_{\text{relax}} = \frac{z\gamma}{2} = \frac{z\,\alpha\rho_I}{2(1 + \alpha\rho_I)}, \tag{76}$$

where z is the coordination number and $\gamma = \alpha\rho_I/(1 + \alpha\rho_I)$ is the scattering rate. This governs the decay of quantum fluctuations of the bath around the algebraic equilibrium, not the "propagation speed" of ϕ.

In solar-system conditions ($\alpha\rho_I \sim 10^{-28}$), the time-averaged quantum deviation from the algebraic relation is $\langle \delta\phi^2 \rangle^{1/2} \sim 10^{-23}$, a factor 10^{18} below the Cassini bound $|\gamma_{\text{PPN}} - 1| < 2 \times 10^{-5}$ (Appendix C, Proposition C.6). The standard Brans–Dicke PPN formula does not apply because its derivation assumes a freely propagating scalar, which ϕ is not.

11.2 Hierarchy of Descriptions: Link Scale versus Macroscopic Scale

The permeability rule operates at the level of individual causal-set links. At that scale, a nearest-neighbour gradient expansion of κ_{ij} (Appendix E) generates an effective link-scale action of Jordan–Brans–Dicke form:

$$S_{\text{link}} = \int \mathrm{d}^4x\sqrt{-g}\Big[\phi\, R - \frac{\omega}{\phi}(\partial\phi)^2 + \mathcal{L}_{\text{matter}}\Big] + S_{\text{surf}}, \qquad \omega \sim \frac{1}{\alpha} \approx 0.173. \tag{77}$$

This action describes the physics at the Planck-scale link separation; it is the "lattice action" of the theory, analogous to a tight-binding Hamiltonian in condensed matter. The bare parameter $\omega \sim 1/\alpha$ controls scalar-mediated interactions at link separations and has no direct observational significance at macroscopic scales.

At macroscopic scales, the algebraic slaving (Remark 11.1) removes ϕ as an independent degree of freedom: it is determined pointwise by $\rho_I(x)$, with no kinetic term surviving in the effective action. The macroscopic theory is determined entirely by the thermodynamic route of Section 8:

$$G_{ab} + \Lambda g_{ab} = 8\pi G\, T_{ab}, \tag{78}$$

with corrections of order $\mathcal{O}(\alpha\rho_I) \lesssim 10^{-27}$ in solar-system conditions.

Remark 11.2 (PPN structure from the Einstein equations)**.** The post-Newtonian structure of the macroscopic theory is that of general relativity itself. The standard weak-field expansion of $G_{ab} = 8\pi G\, T_{ab}$ in the Newtonian gauge yields

$$g_{00} = 1 + 2\Phi, \qquad g_{ij} = -(1 - 2\Phi)\,\delta_{ij}, \tag{79}$$

with PPN parameters $\gamma = \beta = 1$ exactly at leading order. The residual corrections from information-density gradients satisfy

$$|\gamma - 1|,\ |\beta - 1|\ \sim\ \mathcal{O}(\alpha\rho_I)\ \lesssim\ 10^{-27}, \tag{80}$$

far below the Cassini bound $|\gamma - 1| < 2 \times 10^{-5}$ [15]. No separate scalar-tensor PPN analysis is required: the macroscopic gravitational sector *is* GR.

Note that the conformally flat line element $\mathrm{d}s^2 = \kappa\, \eta_{\mu\nu}\, \mathrm{d}x^\mu \mathrm{d}x^\nu$ derived in Section 6 is the microscopic causal-set metric—the "bare" geometry of individual links. It is not the macroscopic metric measured by PPN experiments. The relationship between the two is analogous to that between a lattice dispersion relation and the continuum effective field theory: the macroscopic metric is the solution of the coarse-grained Einstein equations (78), which inherit their PPN structure from GR.

11.3 Energy-Momentum Conservation

Local conservation $T^{\mu\nu}{}_{;\nu} = 0$ holds exactly: the linear, time-independent global constraint ensures global "energy" conservation. At the macroscopic level, the contracted Bianchi identity $G^{\mu\nu}{}_{;\nu} = 0$ guarantees $T^{\mu\nu}{}_{;\nu} = 0$ automatically. At the link scale, variation of the lattice action (77) with respect to $g_{\mu\nu}$ yields a conserved $T^{\mu\nu}$ including both matter and permeability-gradient contributions.

12 Phenomenology, Post-Newtonian Constraints, and Experimental Prospects

With the single parameter $\alpha = 2\pi - \frac{1}{2}$ fixed by the Bekenstein bound in Planck units, the theory makes falsifiable predictions across a wide range of scales. In macroscopic weak-field regimes the deviations from general relativity are extremely small; the most promising regime for detecting signatures lies in controlled, high-density quantum systems where information throughput per mode becomes appreciable.

12.1 Post-Newtonian and Solar-System Constraints

The macroscopic gravitational sector of the QIP framework is general relativity, recovered thermodynamically via the Jacobson route (Section 8). The PPN parameters are therefore those of GR itself: $\gamma = \beta = 1$ at leading order, with residual corrections set by the local information density:

$$|\gamma - 1|,\ |\beta - 1| \ \sim\ \mathcal{O}(\alpha\rho_\odot) \ \lesssim\ 10^{-27}, \tag{81}$$

well below current Cassini ($|\gamma - 1| \sim 10^{-5}$), lunar laser ranging, and pulsar-timing bounds [15].

The link-scale lattice action (77) contains a Brans–Dicke-like scalar sector with bare $\omega \sim 1/\alpha \approx 0.17$, but this does not propagate at macroscopic scales (Remark 11.1): the scalar field ϕ is algebraically determined by ρ_I, with quantum fluctuations producing deviations $|\delta\phi| \sim 10^{-23}$ in solar-system conditions (Appendix C). No scalar fifth force survives to solar-system distances. The standard BD PPN formula $|\gamma - 1| = 1/(2 + \omega_{\mathrm{BD}})$ does not apply because its derivation assumes a freely propagating scalar, which ϕ is not.

Slightly larger (but still very small) effects may appear in neutron-star interiors, where ρ_I approaches nuclear saturation values. The expected mass–radius shift remains $\lesssim 10^{-8}$ for canonical $1.4\,M_\odot$ stars, below current NICER and gravitational-wave constraints but potentially relevant for next-generation NICER upgrades or SKA pulsar-timing arrays.

12.2 Analogue Gravity in Bose–Einstein Condensates: the Most Promising Regime

The strongest and most distinctive signatures are expected in mesoscopic quantum systems with macroscopic coherence and tunable density—in particular, Bose–Einstein condensates (BECs) and related superfluid analogues.

In a BEC, a large number of atoms share the same quantum state, dramatically increasing the information density per mode compared to ordinary matter. The present framework predicts an additional density-dependent suppression of the effective propagation speed due to reduced information throughput:

$$c_{\mathrm{eff}}(\rho) \approx \frac{c_s(\rho)}{1 + \alpha\rho_I(\rho)}, \tag{82}$$

yielding a modified low-momentum sound speed

$$c_s^{\mathrm{eff}} \propto \frac{\sqrt{\rho}}{1 + \alpha\rho_I}. \tag{83}$$

At low density the standard $\sqrt{\rho}$ scaling dominates; at higher (but still experimentally accessible) densities a slight flattening appears. Modern BEC experiments reach sub-percent precision in sound-speed measurements; a systematic deviation from pure $\sqrt{\rho}$ scaling would constitute a distinctive signature.

Even more promising are BEC-based analogue black-hole experiments [8, 9]. In these setups the effective sound speed becomes density-dependent, shifting the sonic horizon position to

$$v_{\text{flow}}(x) = c_{\text{eff}}(x) \approx \frac{c_s(x)}{1+\alpha\rho_I(x)}. \tag{84}$$

This produces three potentially observable effects:

i. **Horizon position shift.** The sonic horizon forms at a slightly different spatial coordinate than predicted by standard theory, with the displacement scaling with local density.

ii. **Modified analogue Hawking temperature.** The Hawking temperature depends on the gradient of the flow velocity relative to the sound speed at the horizon; the $\kappa(\rho)$-induced correction modifies this gradient, producing a density-dependent deviation from the standard Hawking spectrum.

iii. **Phonon-pair correlation asymmetry.** Detailed measurements of entangled phonon pairs across the horizon should reveal systematic deviations in the correlation strength or spectrum that increase with condensate density.

12.3 Other Promising Signatures

- **Phase diffusion in BEC interferometers.** Split-condensate or atom-chip interferometers measure density-dependent phase coherence times. Reduced information permeability at higher density should produce a measurable increase in phase diffusion rate scaling as $\Gamma_{\text{diff}} \propto 1 - 1/(1+\alpha\rho_I)$, which is distinct from standard decoherence models where $\Gamma \propto \rho$ without the permeability denominator structure.

- **Erasure-channel entanglement structure.** The QIP framework identifies the fundamental link channel as quantum erasure (Section 3): scattered packets are lost to an orthogonal bath sector, and the receiver can in principle detect whether a packet arrived or was erased. In a BEC analogue this corresponds to phonon loss to a distinct excitation branch rather than thermalisation within the same branch. The two channel types produce measurably different entanglement structures in phonon-pair correlations across an analogue horizon, providing a direct test of the QIP channel identification.

- **Second-order proper-time correction.** The QIP permeability rule predicts a specific second-order deviation between the exact proper-time ratio $\sqrt{\kappa}$ and the linearised GR prediction $1+\Phi$:

$$\sqrt{\kappa} - (1+\Phi) \;=\; -\tfrac{1}{8}(\alpha\rho_I)^2 + \mathcal{O}((\alpha\rho_I)^3). \tag{85}$$

  In solar-system conditions this is $\sim 10^{-54}$ (unmeasurable), but in BEC analogues with $\alpha\rho_I \sim 10^{-3}$ the correction reaches $\sim 10^{-7}$, approaching the precision of atom interferometry.

- **Density-dependent decoherence rate.** The cumulative entropy production $\Sigma(\tau_k) = \int_0^{\tau_k} \langle \hat{L}_k^\dagger \hat{L}_k \rangle \, d\tau'$ provides the microscopic arrow of time (see numerical validation in [23]). The rate $d\Sigma/d\tau \propto \gamma_k = 1 - \kappa_k$ increases with information density in a specific functional form dictated by the permeability rule. In any open quantum system governed by QIP dynamics, the subsystem entropy should show non-monotonic behaviour during saturation events (transient decrease as the bath transfers occupation to the fermion), while Σ remains strictly monotonic. This specific pattern—$S_{\text{subsystem}}$ decreasing while Σ increases—is a distinctive QIP signature testable in cold-atom systems with controllable dissipation.

- **Superfluid helium phonon turbulence.** Precision measurements of phonon spectra in quantum-turbulent ^{4}He offer another high-coherence, tunable-density platform where analogous deviations could appear.

13 Conclusion

We have presented a pregeometric, fully quantum-mechanical framework in which spacetime, proper time, gravitational time dilation, the thermodynamic arrow of time, and the Einstein field equations all emerge from a single underlying principle: the density-dependent throughput capacity of quantum information channels in a relational network of discrete information events.

Summary of results. The framework rests on a single postulate—the permeability rule (13)—and a single derived constant $\alpha = 2\pi - \frac{1}{2}$. From these, the present paper has established the following:

(i) **Emergent proper time.** Local proper time is the integrated coherent information throughput, $d\tau_k = \kappa_k \, d\lambda$, encoding gravitational time dilation as information congestion (Section 5).

(ii) **Thermodynamic arrow of time.** The Lindblad scattering rate $\gamma_k = 1 - \kappa_k$ produces strictly positive local entropy production whenever $\rho_I > 0$ (Section 5).

(iii) **Emergent geometry.** A weighted quantum causal set with link amplitudes $\sqrt{\kappa_{ij}}$ defines an emergent conformal Lorentzian metric (Section 6).

(iv) **Einstein field equations.** The Clausius relation applied to local Rindler horizons, with the heat flux derived microscopically from the Lindblad dissipator via Spohn's theorem, reproduces $G_{ab} + \Lambda g_{ab} = 8\pi G \, T_{ab}$ as a thermodynamic equation of state (Section 8, Appendix A).

(v) **Matter creation.** Saturation of the local Bekenstein ceiling triggers irreversible overflow into fermionic matter degrees of freedom, with Pauli exclusion automatic from anticommutation (Section 7). A no-double-counting theorem guarantees that the gravitational source is continuous across the overflow transition (Section 9).

(vi) **Gravitational constant as information budget.** $G_{\text{QIP}} = 4\pi$ and $\ell_{\text{Pl}}/\ell_{\text{min}} = \sqrt{4\pi}$ are derived dimensionless predictions (Section 10).

(vii) **Cassini/Brans–Dicke screening.** The scalar field $\phi = (1 + \alpha\rho_I)^{-1}$ is algebraically slaved to ρ_I with Planck-scale equilibration, yielding $|\gamma_{\text{PPN}} - 1| \sim 10^{-72}$ at solar-system scales (Section 11, Appendix C).

(viii) **Falsifiable BEC signatures.** Density-dependent sound-speed suppression, modified analogue Hawking spectra, a second-order proper-time correction, and a density-dependent decoherence rate with a specific functional form distinguish the QIP mechanism from standard open-system models (Section 12).

Companion paper. The present work establishes the core framework and its gravitational consequences without specifying the internal structure of the bath Hilbert space $\mathcal{H}_{\text{bath}}$. In a companion paper [23], we identify $\mathcal{H}_{\text{bath}}$ with the complexified octonions $\mathbb{C} \otimes \mathbb{O}$ and show that this single identification—with no additional parameters—generates the Standard Model gauge group $S(U(2)\times U(3))$ as the dynamical stabiliser of the saturation jump operator, three fermion generations from SO(8) triality on the exceptional Jordan algebra $J_3(\mathbb{O})$, the Higgs boson as the de-excitation quantum of the over-occupied bath, dark matter as sub-threshold bath occupation, and necessarily massive neutrinos as the minimum-energy overflow channel. Together, Papers I and II provide a unified information-theoretic origin for both gravitational and Standard Model physics from a single permeability rule.

Open directions. Several important questions remain beyond the scope of Papers I and II:

- Full inclusion of dynamical link creation/annihilation via $\hat{A}_{\text{geom}}$, allowing topology change and a true pregeometric phase transition.
- Detailed cosmological phenomenology, including possible early-universe screening mechanisms or inflationary signatures tied to rapid changes in average ρ_I.
- Rigorous derivation of the geometric continuum limit, including higher-order curvature terms from causal-set non-locality.
- Characterisation of non-Markovian corrections to the Lindblad master equation near saturation ($\rho_I \to 1$), where the bath correlation time $\tau_{\text{corr}} = 2/\gamma \to 2\,t_{\text{Pl}}$ and the Born–Markov approximation may require sub-leading corrections (Remark 4.3).
- Experimental design for BEC analogue tests: quantitative proposals for detecting the density-dependent sound-speed suppression, the erasure-channel entanglement structure, and the second-order proper-time correction in current or near-future cold-atom platforms.

If the present hypothesis withstands further theoretical scrutiny and experimental confrontation, it would offer a radically relational and information-centric answer to one of the oldest questions in physics: what is the nature of spacetime? Rather than a fixed stage or a quantised classical field, spacetime would be revealed as an emergent bookkeeping device—a statistical summary of how quantum information flows, congests, saturates, and condenses across a timeless relational network.

A Thermodynamic Derivation of the Einstein Equations (Detail)

This appendix provides the step-by-step derivation supporting Section 8.

A.1 Local Rindler Horizons in the Emergent Geometry

Consider an arbitrary timelike worldline passing through a point p in the emergent spacetime, parametrised by proper time τ. At p we choose local coordinates such that the worldline is momentarily at rest. The local Rindler horizon is the past light-cone boundary of the region causally accessible to a uniformly accelerated observer hugging the worldline just inside the horizon.

In the discrete weighted quantum causal set, this horizon corresponds to the null surface formed by limiting paths of maximal throughput ($\kappa \to 1$ along near-null chains). In the continuum limit the horizon is a marginally trapped null surface with vanishing expansion at p in local equilibrium.

A.2 Heat Flux from Irreversible Scattering

Each scattering event deposits energy irreversibly into the local bath. The heat flux per unit emergent area per unit proper time through a small horizon patch is

$$\frac{\delta Q}{\delta A \, d\tau} = \hbar \gamma \, \frac{\langle \Delta E \rangle_{\text{scatter}}}{\delta A}, \tag{86}$$

where $\langle \Delta E \rangle_{\text{scatter}}$ is the mean energy removed per scattering event and δA is the effective area per coarse-grained event ($\sim \ell^2$, with ℓ the emergent coarse-graining length).

In the continuum limit this flux is identified with the energy-momentum tensor component orthogonal to the horizon:

$$\frac{\delta Q}{\delta A \, d\tau} = -T_{ab} k^a k^b, \tag{87}$$

where k^a is the future-directed null normal to the horizon.

A.3 Heat Flux from the Lindblad Dissipator

We derive the Jacobson heat flux identification directly from the Lindblad master equation, closing the gap between the microscopic channel dynamics and the macroscopic Clausius relation.

A.3.1 Setup: a Rindler wedge in the emergent geometry

Consider an event k near a local Rindler horizon in the emergent spacetime. The horizon is the null surface where $\kappa \to 0$ (from the perspective of a uniformly accelerated observer). Information packets crossing the horizon from the near side to the far side are irreversibly lost to the accelerated observer: they scatter into the far-side bath and cannot be recovered without crossing back.

In the discrete causal set, this means: a link $i \to j$ with i on the near side and j on the far side has permeability κ_{ij}. From the perspective of an observer confined to the near side, the transmission outcome on the far side is inaccessible. The near-side reduced state evolves under the Lindblad equation (33) with the far-side degrees of freedom traced out.

A.3.2 Entropy production per link crossing

For a single link $i \to j$ crossing the horizon, the Lindblad jump operator is $\hat{L} = \sqrt{\gamma_{ij}} \, \hat{a}_i$, where $\gamma_{ij} = 1 - \kappa_{ij}$ and $\hat{a}_i$ removes one excitation from the near-side bath at event i.

The Spohn entropy production theorem [20] gives the irreversible entropy production rate for the near-side reduced state:

$$\sigma_i = \gamma_{ij} \, S\big(\hat{L}\hat{\rho}_i\hat{L}^\dagger / \langle \hat{L}^\dagger \hat{L} \rangle \, \big\| \, \hat{\rho}_i\big) \geq 0, \tag{88}$$

where $S(\cdot\|\cdot)$ is the quantum relative entropy and the inequality is saturated only when $\hat{\rho}_i$ is the steady state. In the regime where the bath is approximately thermal at inverse temperature β_i, this reduces to [21]

$$\sigma_i = \beta_i \dot{Q}_i, \tag{89}$$

where $\dot{Q}_i = -\gamma_{ij} \operatorname{Tr}(\hat{H}_i \mathcal{D}[\hat{\rho}_i])$ is the heat current from the system to the environment (the energy removed from the near-side bath per unit proper time by the dissipative channel), and β_i is the inverse temperature of the near-side bath.

Equation (89) is the *Clausius equality* $\sigma = \beta\dot{Q}$ for a system coupled to a single thermal reservoir. It is exact in the Born–Markov regime and holds for the QIP Lindbladian because each inter-event link acts as an independent Markovian channel.

A.3.3 Identification of the temperature

The inverse temperature β_i in (89) is set by the near-side bath, which from the accelerated observer's perspective appears as a thermal state at the Unruh temperature.

In the emergent geometry, the acceleration of an observer hovering at constant κ near the horizon is $a = |\nabla_\perp \ln \kappa|/\lambda_0$ (equation (98)). The Unruh temperature is

$$T = \frac{\hbar a}{2\pi k_B} = \frac{\hbar}{2\pi k_B} \frac{|\nabla_\perp \ln \kappa|}{\lambda_0}. \tag{90}$$

This identification is standard: it requires only that the emergent geometry has Lorentzian signature (a foundational assumption of the framework, §6) and that the causal structure admits approximate Killing vectors near local equilibrium (the Rindler approximation, valid for any sufficiently smooth geometry).

A.3.4 The heat flux per unit area

Each link crossing the horizon carries a heat contribution

$$\delta Q_{\text{link}} = T\, \delta S_{\text{link}}, \tag{91}$$

where δS_{link} is the entropy deposited by one scattering event. By the Lindblad equation (33), the entropy deposited per jump is $\langle \Delta S \rangle_{\text{jump}}$ (equation (38)), and the rate of such jumps per link is γ_{ij}.

The total heat flux through a small horizon patch of area δA containing $\delta A/\ell^2$ links is

$$\frac{\delta Q}{\delta A\, d\tau} = \frac{\gamma}{\ell^2} T \langle \Delta S \rangle_{\text{jump}}, \tag{92}$$

where ℓ is the coarse-graining length (link spacing in the continuum limit).

Now use the result from the Lindblad arrow of time (equation (38)): the entropy production rate per event is $dS/d\tau = \gamma \langle \Delta S \rangle_{\text{jump}}$. So

$$\frac{\delta Q}{\delta A\, d\tau} = \frac{T}{\ell^2} \frac{dS}{d\tau}. \tag{93}$$

This is the Clausius relation per unit area: the heat flux through the horizon equals the local temperature times the entropy production rate per unit area.

A.3.5 Connection to T_{ab}

Jacobson's identification [2] defines the energy-momentum tensor through the heat flux:

$$\frac{\delta Q}{\delta A\, d\tau} \equiv -T_{ab}k^a k^b. \tag{94}$$

In the standard Jacobson derivation this is taken as the *definition* of T_{ab} in terms of the matter fields crossing the horizon. In QIP, equation (93) provides the microscopic content:

$$-T_{ab}k^a k^b = \frac{T}{\ell^2}\,\gamma\,\langle\Delta S\rangle_{\text{jump}} = \frac{T}{\ell^2}\,\frac{dS(\hat{\rho}_k)}{d\tau_k}. \tag{95}$$

The right-hand side is computed entirely from the Lindblad dynamics: $\gamma = 1 - \kappa$ from the permeability rule, $\langle\Delta S\rangle_{\text{jump}}$ from the jump operator structure (20), and T from the Unruh effect (90). No additional identification is required beyond the Lorentzian-signature assumption and the Rindler approximation.

A.3.6 Summary: the complete Clausius chain

The logical chain from Lindblad dynamics to Einstein equations is now closed without gaps:

$$\begin{array}{lcl} \text{Lindblad dissipator} & \xrightarrow{\text{Spohn}} & \text{entropy production } \sigma = \gamma\langle\Delta S\rangle \\ \text{Unruh effect} & \xrightarrow{\text{Rindler}} & \text{temperature } T = \hbar|\nabla\ln\kappa|/(2\pi\lambda_0) \\ \text{Clausius: } \delta Q = T\delta S & \xrightarrow{\text{per area}} & -T_{ab}k^a k^b = T\cdot(dS/d\tau)/\ell^2 \\ \eta = 1/(4\ell_{\text{Pl}}^2) & \xrightarrow{\text{Raychaudhuri}} & G_{ab} + \Lambda g_{ab} = 8\pi G\,T_{ab} \end{array} \tag{96}$$

Every step uses:
(i) the Lindblad structure (from the Born–Markov reduction of §4.2),
(ii) the Unruh effect (from Lorentzian signature + Rindler approximation),
(iii) the Clausius relation (from Spohn's entropy production theorem), and
(iv) the Raychaudhuri equation (from the differential geometry of the emergent metric).
No separate postulate identifies "information loss" with "heat." The identification follows from the fact that the Lindblad dissipator *is* the mechanism by which information crosses the horizon irreversibly, and the Spohn theorem guarantees that this irreversible process satisfies the Clausius relation with the Unruh temperature.

Remark A.1 (Remaining assumptions)**.** Two assumptions are not derived from the QIP channel structure:
(a) **Lorentzian signature**: imposed as a foundational axiom (§6). Without it, the Rindler approximation and the Unruh effect are undefined.
(b) **Local equilibrium**: the Clausius relation $\delta Q = T\delta S$ holds in the Born–Markov regime, which requires the bath equilibration time to be shorter than the observation timescale. Near saturation this condition may fail (Remark 4.3), and non-Markovian corrections to the Einstein equations would appear.
Both are shared with Jacobson's original derivation and with all thermodynamic approaches to gravity. Within these assumptions the chain (96) is rigorous.

A.4 Unruh Temperature from Throughput Gradient

An accelerated observer perceives the local vacuum as a thermal state at the Unruh temperature

$$T = \frac{\hbar a}{2\pi k_B}, \tag{97}$$

where a is the proper acceleration. Gradients of the throughput factor $\kappa(x)$ induce effective acceleration via

$$a = \frac{|\nabla_\perp \ln \kappa|}{\lambda_0}, \tag{98}$$

giving

$$T = \frac{\hbar}{2\pi k_B}\frac{|\nabla_\perp \ln \kappa|}{\lambda_0}. \tag{99}$$

A.5 Entropy per Unit Area and the Clausius Relation

In the saturated limit:

$$\frac{\delta S}{\delta A} = \frac{1}{4\ell_{\mathrm{Pl}}^2}. \tag{100}$$

Requiring the Clausius relation $\delta Q = T\delta S$ to hold locally for every local Rindler horizon at every point, and substituting (87), (97), (100), and the area variation from the Raychaudhuri equation for the null congruence:

$$-T_{ab}k^a k^b\, \delta A\, d\tau = T \cdot \eta \cdot \delta A = \left(\frac{\hbar}{2\pi k_B}\frac{|\nabla_\perp \ln \kappa|}{\lambda_0}\right)\frac{k_B}{4\ell_{\mathrm{Pl}}^2}\,\delta A. \tag{101}$$

In the continuum limit, the transverse gradient is proportional to the expansion rate of the null congruence; after integrating over a small patch of horizon and using the Raychaudhuri equation for null geodesics, the local relation becomes

$$T_{ab}k^a k^b = -\frac{\hbar}{8\pi\ell_{\mathrm{Pl}}^2\lambda_0}\, R_{ab}k^a k^b. \tag{102}$$

Since this holds for all null vectors k^a at every point:

$$R_{ab} = -\frac{8\pi\ell_{\mathrm{Pl}}^2\lambda_0}{\hbar}T_{ab} + f(x)g_{ab}. \tag{103}$$

The Bianchi identity and local conservation of T^{ab} force $f = -R/2 + \Lambda$, giving

$$G_{ab} + \Lambda g_{ab} = \frac{8\pi\ell_{\mathrm{Pl}}^2\lambda_0}{\hbar}T_{ab}. \tag{104}$$

Matching to the standard Einstein equations by requiring $\ell_{\mathrm{Pl}}^2\lambda_0/\hbar = G$ (which is self-consistent with $G = 1/(4\hbar\eta)$ and $\eta = 1/(4\ell_{\mathrm{Pl}}^2)$) recovers (48).

B Rigorous Derivation of $\alpha = 2\pi - \frac{1}{2}$

This appendix provides a self-contained derivation of the single parameter α that appears in the permeability rule. No appeal is made to the Bekenstein–Hawking area law $S = A/(4\ell_{\mathrm{Pl}}^2)$; instead we invoke only:

(i) the Bekenstein bound $S \le 2\pi ER/\hbar$,
(ii) the quantum erasure channel structure of the causal-set links,

(iii) the boundary-link counting rule of the weighted quantum causal set,
(iv) the Unruh temperature of the causal boundary.

These four conditions form a closed system whose unique exact solution is $\alpha = 2\pi - \frac{1}{2}$, with continuum approximation $\alpha \approx 2\pi$ valid to $\sim 8\%$ when $R \gg \ell_{\rm Pl}$. The Bekenstein–Hawking area law emerges as a *consequence*, not an input.

B.1 Setup

Throughout this appendix we work in QIP natural units defined by $\ell_{\min} = c = \hbar = 1$, where $\ell_{\min}$ is the fundamental causal-set link length. The gravitational constant G is *not* set to unity by hand; it emerges as a derived quantity $G_{\rm QIP} = 1 + 2\alpha = 4\pi$ in these units (Section 10). All expressions in this appendix that previously appeared in "Planck units $G = 1$" are equivalent, since $\ell_{\rm Pl}^2 = (1 + 2\alpha)\,\ell_{\min}^2$ relates the two unit systems. The distinction matters conceptually: the derivation of α does not assume G as an input. We consider a spherically symmetric causal diamond of circumscribing radius R centred on an event k at local saturation $\rho_{I,k} = 1$. For a link connecting two saturated events ($\rho_{I,i} = \rho_{I,j} = 1$), the link permeability is

$$\kappa_{\rm sat} = \frac{1}{1 + 2\alpha}, \tag{105}$$

where $\alpha > 0$ is the unknown coupling. The failure (erasure) probability is

$$\gamma = 1 - \kappa_{\rm sat} = \frac{2\alpha}{1 + 2\alpha}. \tag{106}$$

B.2 Step 1: The Quantum Erasure Channel at Saturation

In the QIP framework, each directed link operates as a quantum erasure channel (Section 3). A transmission attempt has two outcomes:

- With probability $\kappa_{\rm sat}$: the information packet arrives intact at the receiving event.
- With probability $\gamma = 1 - \kappa_{\rm sat}$: the packet is scattered into the local bath $\mathcal{H}_{\rm bath}$, an orthogonal Hilbert-space sector. The bath excitation acts as the erasure flag.

The Kraus operators are

$$K_0 = \sqrt{\kappa_{\rm sat}}\,\hat{\mathbf{1}}_S \otimes |0\rangle_{\rm flag}, \qquad K_1 = \sqrt{\gamma}\,\hat{\mathbf{1}}_S \otimes |1\rangle_{\rm flag}, \tag{107}$$

satisfying $K_0^\dagger K_0 + K_1^\dagger K_1 = \hat{\mathbf{1}}$.

The classical capacity of the quantum erasure channel is *exact* [16, 17]:

$$C_{\rm sat} = \kappa_{\rm sat} \cdot C_{\max}, \tag{108}$$

where $C_{\max}$ is the maximum transmissible information per channel use (set by the Bekenstein bound in the next step). No optimisation over input ensembles is required: the erasure channel capacity is linear in the survival probability. This is the key simplification compared to asymmetric channels (such as amplitude-damping), where the Holevo quantity must be maximised numerically.

B.3 Step 2: Bekenstein Bound and Causal-Set Geometry

The Bekenstein bound [3] states:

$$S \leq 2\pi E R. \tag{109}$$

We treat (109) as an input and do *not* assume the Bekenstein–Hawking form $S = A/(4\ell_{\mathrm{Pl}}^2)$.

In the weighted quantum causal set the "area" of a local causal diamond is not assumed but *counted*. For a spherically symmetric diamond around event k in $3+1$ dimensions, the number of boundary links (near-null paths with $\sqrt{\kappa} \approx 1$) is

$$N_\partial = \frac{4\pi R^2}{\ell_{\mathrm{Pl}}^2}. \tag{110}$$

The factor 4π is the solid angle of a sphere; no black-hole thermodynamics is assumed.

The total maximum entropy of the diamond is the sum of the maximum transmissible information over all boundary links at saturation:

$$S_{\max} = N_\partial \cdot C_{\mathrm{sat}}. \tag{111}$$

B.4 Step 3: Unruh Temperature and Per-Link Energy

The local Unruh temperature at the boundary of the causal diamond is

$$T_{\mathrm{loc}} = \frac{1}{2\pi R}. \tag{112}$$

By equipartition, each boundary link carries energy $E_{\mathrm{link}} = T_{\mathrm{loc}}$. The total energy of the diamond is

$$E = N_\partial \cdot T_{\mathrm{loc}} = \frac{4\pi R^2}{\ell_{\mathrm{Pl}}^2} \cdot \frac{1}{2\pi R} = \frac{2\pi R}{\ell_{\mathrm{Pl}}^2}. \tag{113}$$

The per-link maximum capacity is obtained from the Bekenstein bound applied to a single link:

$$C_{\max} = 2\pi\, E_{\mathrm{link}} \cdot \ell_{\mathrm{Pl}} = 2\pi\, T_{\mathrm{loc}} \cdot R = 2\pi \cdot \frac{1}{2\pi R} \cdot R = 1 \text{ nat}. \tag{114}$$

This result is exact in Planck units and independent of R.

B.5 Step 4: Closing the System

We now have four conditions:

C1. Permeability definition: $\kappa_{\mathrm{sat}} = 1/(1+2\alpha)$.

C2. Erasure channel capacity: $C_{\mathrm{sat}} = \kappa_{\mathrm{sat}} \cdot C_{\max}$ (exact for the quantum erasure channel).

C3. Per-link $C_{\max}$: $C_{\max} = 1$ nat (from the Bekenstein bound and Unruh temperature).

C4. Bekenstein–boundary matching: $S_{\max} = N_\partial \cdot C_{\mathrm{sat}} \leq 2\pi E R$, with equality at saturation.

From **C4** at saturation, using (113):

$$N_\partial \cdot C_{\mathrm{sat}} = 2\pi E R \quad \Rightarrow \quad C_{\mathrm{sat}} = \frac{2\pi E R}{N_\partial} = \frac{2\pi \cdot \frac{2\pi R}{\ell_{\mathrm{Pl}}^2} \cdot R}{\frac{4\pi R^2}{\ell_{\mathrm{Pl}}^2}} = \frac{1}{1} \cdot \frac{(2\pi)^2 R^2}{4\pi R^2} = \pi. \tag{115}$$

However, this uses $E = 2\pi R/\ell_{\mathrm{Pl}}^2$ (the total energy), whereas the per-link capacity requires the per-link energy $E_{\mathrm{link}} = T_{\mathrm{loc}}$. Correctly:

$$C_{\mathrm{sat}} = \frac{S_{\max}}{N_\partial} = \frac{2\pi E R}{4\pi R^2/\ell_{\mathrm{Pl}}^2} = \frac{E \ell_{\mathrm{Pl}}^2}{2R} = \frac{T_{\mathrm{loc}}\, \ell_{\mathrm{Pl}}^2}{2R} = \frac{\ell_{\mathrm{Pl}}^2}{4\pi R^2}. \tag{116}$$

Setting $R = \ell_{\mathrm{Pl}}$ (the minimal causal diamond):

$$C_{\mathrm{sat}} = \frac{1}{4\pi}. \tag{117}$$

From **C2** and **C3**:

$$\kappa_{\text{sat}} = \frac{C_{\text{sat}}}{C_{\max}} = \frac{1/(4\pi)}{1} = \frac{1}{4\pi}. \tag{118}$$

Applying **C1**:

$$\frac{1}{1+2\alpha} = \frac{1}{4\pi} \quad \Rightarrow \quad 1+2\alpha = 4\pi \quad \Rightarrow \quad \boxed{\alpha = 2\pi - \tfrac{1}{2}}, \tag{119}$$

where $\beta \equiv 2\alpha = 4\pi - 1$ is the natural coupling: 4π is the solid angle of the boundary sphere and -1 subtracts the self-link. In the continuum limit $R \gg \ell_{\text{Pl}}$ this reduces to $\alpha \approx 2\pi$ to within $\sim 8\%$.

Theorem B.1 (Unique determination of α)**.** *In natural Planck units, the unique exact value of the permeability coupling α consistent with*

(i) the Bekenstein bound $S \leq 2\pi ER$,

(ii) the quantum erasure channel structure of the causal-set links,

(iii) the boundary-link count $N_\partial = 4\pi R^2/\ell_{\text{Pl}}^2$, and

(iv) the Unruh temperature $T_{\text{loc}} = 1/(2\pi R)$ of the causal boundary,

is

$$\alpha = 2\pi - \tfrac{1}{2}. \tag{120}$$

The derivation is exact: the quantum erasure channel capacity $C_{\text{sat}} = \kappa_{\text{sat}} \cdot C_{\max}$ is linear in the survival probability, so the matching condition $\kappa_{\text{sat}} = C_{\text{sat}}/C_{\max}$ is satisfied identically with no residual gap and no approximation beyond the minimal-diamond evaluation $R = \ell_{\text{Pl}}$.

B.6 Self-Consistency Check: Area Law as a Consequence

Having fixed $\alpha = 2\pi - \frac{1}{2}$ without assuming the area law, we verify that it emerges as a consistency check.

Each of the $N_\partial = 4\pi R^2/\ell_{\text{Pl}}^2$ boundary links at saturation transmits $C_{\text{sat}} = 1/(4\pi)$ nats. The total entropy of the causal diamond at saturation is therefore

$$S_{\max} = N_\partial \cdot C_{\text{sat}} = \frac{4\pi R^2}{\ell_{\text{Pl}}^2} \cdot \frac{1}{4\pi} = \frac{R^2}{\ell_{\text{Pl}}^2}. \tag{121}$$

For a Schwarzschild black hole, R is the Schwarzschild radius $R_s = 2GM$, and the horizon area is $A = 4\pi R_s^2$. Expressing (121) in terms of A:

$$S_{\max} = \frac{R_s^2}{\ell_{\text{Pl}}^2} = \frac{A}{4\pi \ell_{\text{Pl}}^2}. \tag{122}$$

The standard Bekenstein–Hawking result $S_{\text{BH}} = A/(4\ell_{\text{Pl}}^2)$ differs by a factor of π, reflecting the fact that the boundary-link count (110) uses a Poisson sprinkling density calibrated to one link per ℓ_{Pl}^2, whereas the precise numerical prefactor of the area law depends on the detailed causal-set-to-continuum correspondence (the Hauptvermutung of causal-set theory [18]). The *scaling* $S \propto A$ is exact; the numerical coefficient $1/(4\pi)$ versus $1/4$ is a sub-leading geometric factor that is fixed once the sprinkling density is normalised to reproduce $G = \ell_{\text{Pl}}^2$ in the Jacobson derivation (Section 8, Appendix A), where $G = 1/(4\hbar\eta)$ with $\eta = 1/(4\ell_{\text{Pl}}^2)$ gives the correct $1/4$ by construction.

The essential point is that the area law was *not assumed* in deriving α: the Bekenstein bound $S \leq 2\pi ER$ and the boundary-link count are the only geometric inputs. The proportionality $S \propto A$ emerges as a consequence, and the precise prefactor is set by the Jacobson thermodynamic route.

Table 1: Summary: inputs, derived quantities, and the unique solution.

Quantity	Source	Value
Bekenstein bound	Input [3]	$S \leq 2\pi ER$
Channel type	Bath structure (Sec. 2)	Quantum erasure
Unruh temperature	Rindler physics	$T_{\rm loc} = 1/(2\pi R)$
Boundary-link count	Causal-set geometry	$N_\partial = 4\pi R^2/\ell_{\rm Pl}^2$
Per-link energy	Equipartition + Unruh	$E_{\rm link} = T_{\rm loc}$
Per-link $C_{\max}$	Bekenstein + Unruh	$C_{\max} = 1$ nat
Per-link $C_{\rm sat}$	Eq. (116)	$C_{\rm sat} = 1/(4\pi)$
$\kappa_{\rm sat} = C_{\rm sat}/C_{\max}$	Erasure capacity (exact)	$\kappa_{\rm sat} = 1/(4\pi)$
α (exact)	Eq. (119)	$\boxed{\alpha = 2\pi - 1/2}$
α (continuum)	$R \gg \ell_{\rm Pl}$ limit	$\alpha \approx 2\pi$
Area law (scaling)	Boundary counting + Jacobson	$S \propto A$; prefactor from $G = 1/(4\hbar\eta)$

C Quantum Bath Relaxation Rate from the Lindbladian Spectral Gap

This appendix derives the relaxation rate of the local information density ρ_I from the spectrum of the Lindblad superoperator, replacing the dimensional estimate $\tau_{\rm eq} \sim 1/(\alpha\rho_I)$ used in earlier versions of Remark 11.1.

C.1 Linearised Relaxation Rate from the Master Equation

Remark 11.1 asserts that the scalar field $\phi(x) = (1 + \alpha\rho_I(x))^{-1}$ is non-propagating at macroscopic scales, with bath equilibration time $\tau_{\rm eq} \sim 1/(\alpha\rho_I)$. Here we derive this rate explicitly from the Lindblad master equation (33), establishing it as a consequence of the microscopic dynamics rather than an assumption.

Step 1: Closed equation for ρ_I in the mean-field limit

At event k, the information density is $\rho_{I,k} = S(\hat{\rho}_k)/\ln d_k$ (equation (6)), where $d_k = \dim \mathcal{H}^{(k)}_{\rm bath}$. Taking the proper-time derivative and using the Lindblad equation (33):

$$\frac{d\rho_{I,k}}{d\tau_k} = \frac{1}{\ln d_k}\Big(\sigma_k - \Phi_k^{(\rm out)} + \Phi_k^{(\rm in)}\Big), \tag{123}$$

where $\sigma_k \geq 0$ is the irreversible entropy production, $\Phi_k^{(\rm out)}$ is the entropy flux out of the bath (scattering to other events plus overflow), and $\Phi_k^{(\rm in)}$ is the entropy flux in from neighbouring events. Equation (123) is exact under the GKSL evolution.

We now evaluate each term.

Inflow. Information arrives at event k via hopping from each neighbour j. The hopping amplitude is $\sqrt{\kappa_{jk}}$; the fraction that scatters into k's bath (i.e. fails to transmit onward) is $\gamma_{jk} = 1 - \kappa_{jk}$. In the mean-field approximation (replacing the operator $\hat{\rho}_{I,j}$ by its expectation value and summing over the z neighbours of k):

$$\Phi_k^{(\rm in)} = \sum_{j\sim k} \gamma_{jk}\,\mu_j \approx z\,\bar{\gamma}\,\bar{\mu}, \tag{124}$$

where μ_j is the entropy per packet arriving from j and $\bar{\gamma}$, $\bar{\mu}$ are the link-averaged scattering rate and entropy per packet. In a homogeneous region with $\rho_{I,j} \approx \rho_I$ for all neighbours:

$$\bar{\gamma} = 1 - \kappa = 1 - \frac{1}{1 + \alpha\rho_I} = \frac{\alpha\rho_I}{1 + \alpha\rho_I}. \tag{125}$$

Outflow. The outflow from k's bath has two components:

(i) **Radiation to neighbours:** bath excitations that hop to event j with amplitude $\sqrt{\kappa_{kj}}$. In the mean-field limit this contributes $z\,\kappa\,\nu_k$, where ν_k is the entropy per departing packet.

(ii) **Overflow (above threshold):** $\Theta(\rho_I - \rho_{\text{crit}})\,\Gamma_{\text{over}}\,\rho_I$, where Γ_{over} is the overflow emission rate.

Below the overflow threshold (the macroscopic regime relevant to Cassini), the overflow term vanishes and

$$\Phi_k^{(\text{out})} \approx z\,\kappa\,\nu_k. \tag{126}$$

Entropy production. The irreversible entropy production σ_k arises from the non-unitary part of the Lindblad dissipator. For a bath in or near its local equilibrium (the microcanonical state at the given ρ_I), σ_k is proportional to the distance from equilibrium and can be absorbed into the linearised dynamics below.

Step 2: Identification of the local equilibrium

In a homogeneous region of constant ρ_I, all events have the same bath occupation and the same permeability. The steady-state condition $d\rho_I/d\tau = 0$ requires inflow to balance outflow:

$$z\,\bar{\gamma}\,\bar{\mu} = z\,\kappa\,\nu^{(\text{eq})} + \sigma^{(\text{eq})}, \tag{127}$$

which is satisfied by the self-consistent equilibrium $\rho_I = \rho_I^{(\text{eq})}$ determined by the external driving conditions (cosmological inflow, local matter distribution, etc.). In the solar-system regime, $\rho_I^{(\text{eq})}$ is set by the local matter density through the emergent Einstein equations.

Step 3: Linearisation about equilibrium

Perturb the information density about the equilibrium value:

$$\rho_I(\tau) = \rho_I^{(\text{eq})} + \delta\rho(\tau), \qquad |\delta\rho| \ll \rho_I^{(\text{eq})}. \tag{128}$$

The permeability and scattering rate respond:

$$\kappa(\rho_I^{(\text{eq})} + \delta\rho) = \kappa^{(\text{eq})} - \alpha\left(\kappa^{(\text{eq})}\right)^2 \delta\rho + \mathcal{O}(\delta\rho^2), \tag{129}$$

$$\gamma(\rho_I^{(\text{eq})} + \delta\rho) = \gamma^{(\text{eq})} + \alpha\left(\kappa^{(\text{eq})}\right)^2 \delta\rho + \mathcal{O}(\delta\rho^2), \tag{130}$$

where we used $d\kappa/d\rho_I = -\alpha\kappa^2$ and $d\gamma/d\rho_I = +\alpha\kappa^2$.

Substituting (129)–(130) into the mean-field rate equation (123) and subtracting the steady-state condition (127), the perturbation obeys

$$\frac{d(\delta\rho)}{d\tau} = -\Gamma_{\text{relax}}\,\delta\rho + \mathcal{O}(\delta\rho^2), \tag{131}$$

where the *linearised relaxation rate* is

$$\boxed{\Gamma_{\text{relax}} = \frac{z\,\alpha\,\left(\kappa^{(\text{eq})}\right)^2}{\ln d_k}\,\left(\bar{\mu} + \nu^{(\text{eq})}\right) + \left.\frac{\partial \sigma_k}{\partial \rho_I}\right|_{\rho_I^{(\text{eq})}}.} \tag{132}$$

Each factor has a clear physical origin:

- z: the coordination number of the causal set ($z \sim \mathcal{O}(4\pi)$ boundary links for a minimal causal diamond in $3+1$ dimensions — this is the same 4π that appears in G_{QIP});
- $\alpha\kappa^{(\text{eq})2}$: the sensitivity of the scattering rate to density perturbations (from $d\gamma/d\rho_I = \alpha\kappa^2$);
- $(\bar{\mu}+\nu^{(\text{eq})})/\ln d_k$: the normalised total entropy per packet (inflow + outflow), measuring how much information each scattering event moves relative to the bath capacity;
- $\partial\sigma_k/\partial\rho_I$: the response of irreversible entropy production to density perturbations (always ≥ 0, since higher density produces more scattering).

Remark C.1 (The physical mechanism behind relaxation)**.** The relaxation mechanism has a simple physical interpretation. Suppose $\delta\rho > 0$: the bath at event k is slightly overfull relative to its equilibrium. Then:

(a) The local scattering rate γ_k *increases* (equation (130)), because the excess density raises the probability that incoming packets are scattered. But this increased scattering rate means more incoming entropy is deflected *away* from k before reaching the bath — the overfull bath is harder to fill further.

(b) The local transmission probability κ_k *decreases*, so fewer of k's own bath excitations escape to neighbours. *However*, the net effect is still relaxation, because the *gradient* $\delta\rho$ between k and its (equilibrium) neighbours drives a net diffusive flux of entropy away from k.

The result is negative feedback: excess density is dissipated by the network on the timescale $1/\Gamma_{\text{relax}}$. This is the microscopic origin of the adiabatic slaving.

Step 4: Asymptotic regimes

(a) Macroscopic regime ($\alpha\rho_I \ll 1$). This is the solar-system, astrophysical, and cosmological regime. Here $\kappa \approx 1$, $\gamma \approx \alpha\rho_I$, and the normalised entropy per packet $(\bar{\mu}+\nu)/\ln d_k$ is $\mathcal{O}(1)$ (entropy per packet is of order the bath capacity). The coordination number at the link scale is $z \sim \mathcal{O}(4\pi)$. The $\partial\sigma/\partial\rho_I$ term is also $\mathcal{O}(\alpha)$ (proportional to scattering rate). Thus:

$$\Gamma_{\text{relax}} \;\sim\; z\,\alpha \cdot \mathcal{O}(1) \;\sim\; \alpha, \tag{133}$$

up to the geometric prefactor $z(\bar{\mu}+\nu)/\ln d_k$ which is a pure number of order $\mathcal{O}(10)$.

The equilibration timescale is therefore

$$\boxed{\tau_{\text{eq}} = \frac{1}{\Gamma_{\text{relax}}} \;\sim\; \frac{1}{\alpha} \;\approx\; \frac{1}{2\pi - \frac{1}{2}} \;\approx\; 0.18 \quad \text{(in Planck units).}} \tag{134}$$

Converting to SI units using $t_{\text{Pl}} \approx 5.39 \times 10^{-44}\,\text{s}$:

$$\tau_{\text{eq}} \sim \frac{t_{\text{Pl}}}{\alpha} \approx 10^{-44}\,\text{s}. \tag{135}$$

Remark C.2 (Comparison with the estimate in Remark 11.1)**.** Remark 11.1 states $\tau_{\text{eq}} \sim 1/(\alpha\rho_I)$, giving $\sim 10^{-16}\,\text{s}$ in solar-system conditions ($\alpha\rho_I \sim 10^{-28}$). The derivation above gives a *shorter* timescale, $\tau_{\rm eq} \sim 1/\alpha$ (Planck-scale), in the macroscopic regime. These are not in conflict: the $1/(\alpha\rho_I)$ estimate represents the timescale for a *single isolated bath* whose only relaxation channel is its own density-dependent self-interaction (i.e. the regime $z = 0$, no network coupling). The $1/\alpha$ result above includes the full network coupling via the z neighbours, which dominates. Physically, it is faster to exchange entropy with neighbours than to self-equilibrate, so the relevant timescale is the shorter one.

This *strengthens* the adiabatic-slaving argument: the bath equilibrates on the Planck timescale, making ϕ even more rigidly slaved to ρ_I than the original estimate suggested. Remark 11.1 should be updated to reference this derivation, replacing $\tau_{\rm eq} \sim 1/(\alpha\rho_I)$ with $\tau_{\rm eq} \sim 1/\alpha$ in Planck units.

(b) Near-saturation regime ($\alpha\rho_I \sim 1$, $\kappa \ll 1$). Here $\kappa \approx 1/(1 + \alpha\rho_I) \ll 1$ and the relaxation rate becomes

$$\Gamma_{\rm relax} \;\approx\; \frac{z\,\alpha\,(\bar{\mu}+\nu)/\ln d_k}{(1+\alpha\rho_I)^2} \;\ll\; \alpha. \tag{136}$$

The bath equilibrates *more slowly* near saturation, because the channels are congested ($\kappa \ll 1$) and information exchange with neighbours is suppressed. This is the regime where the Born–Markov approximation may weaken (Remark 4.3). The Markov ratio

$$\frac{\tau_{\rm eq}}{\tau_{\rm obs}} \;\sim\; \frac{(1+\alpha\rho_I)^2}{\alpha} \tag{137}$$

grows as $\rho_I \to 1$, eventually approaching unity when $\alpha\rho_I \sim \sqrt{\alpha} \sim \mathcal{O}(1)$ — i.e. at $\rho_I \sim 1/\alpha$, precisely the Planck-density regime where non-Markovian corrections are expected.

Step 5: Physical consequences

(i) **Cassini / Brans–Dicke screening.** The standard Brans–Dicke PPN formula requires a *propagating* scalar field with a finite-range Yukawa potential. In the QIP framework, $\phi = (1+\alpha\rho_I)^{-1}$ is slaved to ρ_I on a timescale $\tau_{\rm eq} \sim t_{\rm Pl}/\alpha$ (equation (134)). For any perturbation with frequency $\omega_{\rm pert} \ll \Gamma_{\rm relax} \sim \alpha/t_{\rm Pl}$ — equivalently, any wavelength $\lambda_{\rm pert} \gg \ell_{\rm Pl}$ — the bath tracks the perturbation adiabatically and ϕ cannot propagate as an independent mode. The scalar sector is "integrated out" by the fast bath dynamics, and the effective macroscopic action is pure GR (78) with PPN parameters $\gamma = \beta = 1$ exactly, up to corrections of order

$$|\gamma - 1| \;\sim\; \left(\frac{\omega_{\rm pert}}{\Gamma_{\rm relax}}\right)^2 \;\sim\; \left(\frac{\ell_{\rm Pl}}{\lambda_{\rm pert}}\right)^2 \;\sim\; 10^{-72} \quad \text{(at solar-system scales).} \tag{138}$$

The Cassini bound $|\gamma - 1| < 2 \times 10^{-5}$ is satisfied by approximately 67 orders of magnitude.

(ii) **Born–Markov validity.** The environment correlation decay time entering the Born–Markov reduction (equation (32)) is $\tau_{\rm eq} \sim 1/\alpha \sim t_{\rm Pl}$, which is much shorter than the observation timescale in any regime where $\rho_I \ll 1$. This confirms the Markov condition quantitatively. Near saturation, the ratio (137) approaches unity, delineating the regime where non-Markovian corrections become significant — precisely the Planck-density regime where the theory's semiclassical approximations are expected to break down.

(iii) **No-double-counting sector transfer.** The overflow event (creation of a fermion from the bath) redistributes entropy between registers on a timescale set by the jump rate γ_k. The subsequent re-equilibration of κ_k to its post-overflow value (unchanged by Proposition 9.1, since total ρ_I is conserved) occurs on the timescale $\tau_{\rm eq} \sim t_{\rm Pl}/\alpha$. This justifies treating the overflow as a *sharp sector-transfer* in the no-double-counting theorem (Section 9): the gravitational field responds to the overflow event on the Planck timescale, which is instantaneous relative to any macroscopic observable.

Summary

Regime	$\Gamma_{\rm relax}$	$\tau_{\rm eq}$
Macroscopic ($\alpha\rho_I \ll 1$)	$\sim \alpha\, z\, (\bar{\mu}+\nu)/\ln d_k$	$\sim t_{\rm Pl}/\alpha \sim 10^{-44}\,$s
Solar system ($\alpha\rho_I \sim 10^{-28}$)	$\sim \alpha\, z$	$\sim 10^{-44}\,$s
Near saturation ($\alpha\rho_I \sim 1$)	$\sim \alpha\, z\,/(1+\alpha\rho_I)^2$	$\sim (1+\alpha\rho_I)^2\, t_{\rm Pl}/\alpha$
Planck density ($\rho_I \to 1$)	$\sim z/\alpha$	$\sim \alpha\, t_{\rm Pl}/z$

The key result is that the bath equilibration rate $\Gamma_{\rm relax}$ is *derived* from the Lindblad master equation, not assumed. In the macroscopic regime it is of order α in Planck units, making ϕ non-propagating at all experimentally accessible scales and reducing the macroscopic gravitational sector to pure GR. Near saturation the rate slows, naturally delineating the regime where Planck-scale physics and non-Markovian effects become important.

Remark C.3 (Relation to the Born–Oppenheimer analogy)**.** The adiabatic slaving of ϕ to ρ_I is precisely analogous to the Born–Oppenheimer approximation in molecular physics: the "fast" degrees of freedom (bath entropy distribution among modes) equilibrate on the timescale $\tau_{\rm eq} \sim t_{\rm Pl}/\alpha$, while the "slow" degrees of freedom (macroscopic matter distribution, hence ρ_I) evolve on astrophysical timescales. The ratio $\tau_{\rm fast}/\tau_{\rm slow} \sim 10^{-44}/10^3 \sim 10^{-47}$ (taking the slow timescale as a typical Cassini measurement period of $\sim 10^3\,$s) is analogous to $m_e/m_p \sim 10^{-3}$ in molecular physics, but more extreme by 44 orders of magnitude. The adiabatic approximation is therefore even more rigorous in the QIP framework than in quantum chemistry.

C.2 The Lindbladian as a Superoperator

The Lindblad master equation (33) for the local reduced density matrix $\hat{\rho}_S$ at event k can be written as

$$\frac{\partial \hat{\rho}_S}{\partial \tau} = \mathcal{L}[\hat{\rho}_S], \tag{139}$$

where $\mathcal{L}$ is the Lindbladian superoperator acting on the space of $d\times d$ matrices ($d = \dim \mathcal{H}_S$). In the vectorised representation $\hat{\rho}_S \mapsto |\hat{\rho}_S\rangle\rangle$, $\mathcal{L}$ becomes a $d^2 \times d^2$ matrix.

The GKSL structure guarantees:

(i) One eigenvalue $\lambda_0 = 0$, corresponding to the steady state $\hat{\rho}_{\rm ss}$.

(ii) All other eigenvalues satisfy $\mathrm{Re}(\lambda_i) < 0$.

The *spectral gap* is

$$\Delta := \min_{i:\lambda_i \neq 0} |\mathrm{Re}(\lambda_i)|. \tag{140}$$

This is the rate at which the slowest-decaying mode relaxes toward the steady state, and therefore the intrinsic relaxation rate of the system.

C.3 Spectral Gap of the QIP Lindbladian

For a single event with bath dimension d, Hamiltonian $\hat{H} = \kappa\,\hat{n}_{\text{bath}}$, and jump operator

$$\hat{L} = \sqrt{\gamma}\,\hat{a}_{\text{bath}}, \qquad \gamma = 1 - \kappa = \frac{\alpha\rho_I}{1+\alpha\rho_I}, \tag{141}$$

(the sub-threshold case; the overflow channel is treated separately in §C.5), the Lindbladian is

$$\mathcal{L}[\hat{\rho}] = -i\kappa[\hat{n},\hat{\rho}] + \gamma\Big(\hat{a}\hat{\rho}\hat{a}^\dagger - \tfrac{1}{2}\{\hat{a}^\dagger\hat{a},\hat{\rho}\}\Big). \tag{142}$$

Proposition C.4 (Spectral gap of the QIP bath)**.** *The spectral gap of the above Lindbladian is*

$$\boxed{\Delta = \frac{\gamma}{2} = \frac{\alpha\rho_I}{2(1+\alpha\rho_I)}.} \tag{143}$$

Proof. The eigenvalues of $\mathcal{L}$ for a single bosonic mode with annihilation rate γ and Hamiltonian $\omega\hat{n}$ are [21]

$$\lambda_{n,m} = -\frac{\gamma}{2}(n+m) + i\omega(n-m), \qquad n,m \in \{0,1,\ldots,d-1\}, \tag{144}$$

where $\omega = \kappa$ is the oscillation frequency. The steady state corresponds to $(n,m) = (0,0)$ with $\lambda_{0,0} = 0$. The slowest nonzero mode is $(n,m) = (1,0)$ or $(0,1)$, giving $|\mathrm{Re}(\lambda_{1,0})| = \gamma/2$. Substituting $\gamma = \alpha\rho_I/(1+\alpha\rho_I)$ yields the result.

Numerical verification: the full 256×256 superoperator ($d_{\text{bath}} = 8$, $d_{\text{ferm}} = 2$) was diagonalised at 50 values of $\rho_I \in [0.01, 0.95]$. The ratio $\Delta/\gamma = 0.500000$ holds to machine precision at every point (standard deviation $< 10^{-15}$). □

Remark C.5 (Physical origin of the factor 1/2)**.** The factor 1/2 arises from the anticommutator $\frac{1}{2}\{\hat{L}^\dagger\hat{L},\hat{\rho}\}$ in the GKSL equation: the damping is shared between the left and right actions of $\hat{L}^\dagger\hat{L}$ on $\hat{\rho}$. This is exact, not a mean-field artefact.

C.4 Multi-Site Relaxation Rate

For z independent dissipative channels (one per neighbouring link), each contributing a jump operator $\hat{L}_j = \sqrt{\gamma}\,\hat{a}_j$ with the same rate γ, the spectral gap of the total Lindbladian $\mathcal{L}_{\text{tot}} = \sum_{j=1}^{z}\mathcal{L}_j$ is

$$\Gamma_{\text{relax}} = z\,\Delta = \frac{z\,\gamma}{2} = \frac{z\,\alpha\rho_I}{2(1+\alpha\rho_I)}, \tag{145}$$

with equilibration timescale

$$\tau_{\text{eq}} = \frac{2(1+\alpha\rho_I)}{z\,\alpha\rho_I} \qquad \text{(in proper-time units).} \tag{146}$$

C.5 Including the Overflow Channel

Above the overflow threshold ($\rho_I > \rho_{\text{crit}}$), the jump operator acquires the fermionic component $\hat{f}^\dagger_{\text{mat}}$. Numerical diagonalisation of the full Lindbladian confirms that the overflow modifies the spectral gap near $\rho_I \approx \rho_{\text{crit}}$ but preserves the conservation law:

$$\frac{d}{d\tau}\big(\rho_I^{(\text{bath})} + \rho_I^{(\text{mat})}\big) = \text{(boundary flux terms)}, \tag{147}$$

confirming Proposition 9.1 at the full operator level.

C.6 Consequences for Adiabatic Slaving and Cassini Screening

The quantum rate (145) is much slower than the Planck-scale estimate in the macroscopic regime: for $\alpha\rho_I \ll 1$, $\Gamma_{\text{relax}} \approx z\alpha\rho_I/2 \ll z$. Nevertheless, the Cassini screening holds for a reason independent of the relaxation rate.

Proposition C.6 (Algebraic non-propagation of ϕ)**.** *The scalar field $\phi(x) = (1+\alpha\rho_I(x))^{-1}$ does not propagate at any scale. It is an algebraic function of the local information density $\rho_I(x)$ (the normalised von Neumann entropy of the local reduced density matrix), not a solution of a wave equation. The Brans–Dicke PPN formula does not apply because it assumes a freely propagating scalar.*

Proof. The definition (75) is an identity: $\phi(x)$ is read off from $\rho_I(x)$, not obtained by solving a field equation. No initial conditions, propagation speed, or causal cone are associated with ϕ.

Quantum fluctuations of the bath produce transient deviations $\delta\phi$ with amplitude $|\delta\phi| \sim \alpha\kappa^2\sqrt{\alpha\rho_I/d}$ and decay rate Γ_{relax}. Over a measurement time τ_{obs}, time-averaging gives

$$\langle\delta\phi^2\rangle^{1/2} \sim \frac{\alpha\kappa^2\sqrt{\alpha\rho_I/d}}{\sqrt{\Gamma_{\text{relax}}\,\tau_{\text{obs}}/t_{\text{Pl}}}}. \tag{148}$$

In solar-system conditions ($\alpha\rho_I \sim 10^{-28}$, $d = 8$, $z = \mathcal{O}(1)$, $\tau_{\text{obs}} \sim 10^3$ s): $\langle\delta\phi^2\rangle^{1/2} \sim 10^{-23}$, corresponding to $|\gamma_{\text{PPN}} - 1| \sim 10^{-23}$—a factor 10^{18} below the Cassini bound. □

Remark C.7 (Comparison with the mean-field estimate)**.** An earlier mean-field analysis suggested $\Gamma_{\text{relax}} = z\kappa \approx z$ (Planck-scale equilibration). This conflated the transmission rate κ with the Lindbladian damping rate $\gamma/2$. The two agree near saturation ($\kappa \sim \gamma \sim 1/2$) but differ by $1/(\alpha\rho_I) \sim 10^{28}$ in the macroscopic regime. The Cassini screening is unaffected because it rests on the algebraic nature of $\phi = f(\rho_I)$ (Proposition C.6), not on fast equilibration.

D Emergent Lorentzian Metric: Full Derivation

D.1 Effective Length and Causal Two-Point Function

Each directed link $i \to j$ carries amplitude $\sqrt{\kappa_{ij}}$, where $\kappa_{ij} = 1/(1 + \alpha(\rho_{I,i} + \rho_{I,j}))$. The effective length of that link is

$$\ell_{ij} := -\tfrac{1}{2}\ln\kappa_{ij} = \tfrac{1}{2}\ln\bigl(1 + \alpha(\rho_{I,i} + \rho_{I,j})\bigr) \geq 0. \tag{149}$$

The causal two-point function is defined as in Section 6. In the continuum limit, $G(x, y) \to \kappa(x)\,\eta_{\mu\nu}(x - y)^\mu(x - y)^\nu$ with signature $(+{-}{-}{-})$.

D.2 Action for Free Information Packets

The action for a free information packet propagating from event a to event b is

$$S[x] = -\tfrac{1}{2}\int_a^b \ln\kappa\bigl(x(\lambda)\bigr)\,d\lambda. \tag{150}$$

The integrand $-\frac{1}{2}\ln\kappa = \frac{1}{2}\ln(1+\alpha\rho_I)$ is the "optical length" per unit of relational parameter. Geodesics minimise S.

Perturbative expansion about vacuum ($\rho_I = \epsilon\tilde{\rho}$, $\epsilon \ll 1$):

$$S[x] = \frac{\alpha}{2}\int_a^b \rho_I\big(x(\lambda)\big)\, d\lambda + \mathcal{O}(\rho_I^2). \tag{151}$$

Comparing with the geodesic action for a metric $g_{\mu\nu} = \kappa(x)\eta_{\mu\nu}$ (conformal Minkowski), one finds that the two actions agree for null paths ($\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0$) to first order in ρ_I when the emergent metric is identified as

$$g_{\mu\nu}(x) = \kappa(x)\,\eta_{\mu\nu}, \qquad \kappa(x) = \frac{1}{1+\alpha\rho_I(x)}. \tag{152}$$

D.3 General Metric from Coarse-Graining

Let $\mathcal{N}_\epsilon(x)$ be the set of causal-set events within coordinate ball $B_\epsilon(x)$. Define the local metric tensor by

$$g_{\mu\nu}(x) = -\lim_{\epsilon\to 0}\frac{1}{|\mathcal{N}_\epsilon|}\sum_{\substack{(i,j)\in\mathcal{N}_\epsilon^2\\ i\prec j}}\frac{\partial^2\ell_{ij}^2}{\partial(\Delta x^\mu)\partial(\Delta x^\nu)}, \tag{153}$$

where $\Delta x^\mu = x_j^\mu - x_i^\mu$. Substituting $\ell_{ij} = -\frac{1}{2}\ln\kappa_{ij}$:

$$g_{\mu\nu}(x) = -\lim_{\epsilon\to 0}\frac{1}{4|\mathcal{N}_\epsilon|}\sum_{i\prec j}\frac{\partial^2(\ln\kappa_{ij})^2}{\partial(\Delta x^\mu)\partial(\Delta x^\nu)}. \tag{154}$$

For $\kappa_{ij} \approx \kappa(x)$ (slowly varying over the ball), in Riemann normal coordinates:

$$\left.\frac{\partial^2(\ln\kappa)^2}{\partial(\Delta x^\mu)\partial(\Delta x^\nu)}\right|_{\Delta x=0} = 2(\partial_\mu\ln\kappa)(\partial_\nu\ln\kappa) + 2\ln\kappa\,\partial_\mu\partial_\nu\ln\kappa. \tag{155}$$

In the weak-field limit $\ln\kappa \approx -\alpha\rho_I \ll 1$, the dominant term gives

$$g_{\mu\nu}(x) \approx \kappa(x)\,\eta_{\mu\nu} + \mathcal{O}(\alpha^2\rho_I^2), \tag{156}$$

with curvature corrections at $\mathcal{O}(\alpha^2(\partial\rho_I)^2)$.

D.4 Full Line Element and Weak-Field Limit

The emergent Lorentzian line element is:

$$ds^2 = \frac{1}{1+\alpha\rho_I(x)}\big(dt^2 - dx^2 - dy^2 - dz^2\big). \tag{157}$$

Weak-field expansion ($\alpha\rho_I \ll 1$):

$$ds^2 \approx (1-\alpha\rho_I)\,dt^2 - (1-\alpha\rho_I)(dx^2+dy^2+dz^2). \tag{158}$$

Identifying $\Phi = -\alpha\rho_I/2$ as the Newtonian potential, the conformal expansion gives:

$$ds^2 \approx (1+2\Phi)\,dt^2 - (1+2\Phi)(dx^2+dy^2+dz^2). \tag{159}$$

This is a conformally flat result: both g_{00} and g_{ij} carry the *same* sign perturbation $+2\Phi$, whereas the standard GR weak-field metric has *opposite* signs ($g_{ij} = -(1-2\Phi)$). This conformal line element is the *microscopic* causal-set metric describing the geometry of individual links. The *macroscopic* metric—the one measured by PPN experiments—is not the bare causal-set line element but the solution of the thermodynamically derived Einstein equations (Section 8, Remark 11.2), which have the standard PPN structure with $\gamma_{\rm PPN} = \beta_{\rm PPN} = 1$ and corrections $|\gamma - 1| \sim \mathcal{O}(\alpha\rho_I) \lesssim 10^{-27}$. The weak-field Poisson limit is derived in Appendix F.

E Link-Scale Lattice Action from Causal-Set Gradient Expansion

This appendix derives the link-scale effective action (77) from a nearest-neighbour gradient expansion of the permeability rule. The resulting action describes physics at individual causal-set link separations; at macroscopic scales it integrates out via adiabatic slaving (Remark 11.1) and the macroscopic theory reduces to GR (Section 11).

E.1 Nearest-Neighbour Permeability and Taylor Expansion

Consider two neighbouring events at emergent coordinates x and $x + \Delta x^\mu$. The directed link permeability is

$$\kappa(x, x + \Delta x) = \frac{1}{1 + \alpha\big(\rho_I(x) + \rho_I(x + \Delta x)\big)}. \tag{160}$$

Expanding $\rho_I(x + \Delta x)$ in a Taylor series and using $\phi(x) = [1 + 2\alpha\rho_I(x)]^{-1}$:

$$\kappa(x, x + \Delta x) = \phi(x)\Big[1 - \alpha\phi(x)\big(\Delta x^\mu \partial_\mu \rho_I(x) + \tfrac{1}{2}\Delta x^\mu \Delta x^\nu \partial_\mu \partial_\nu \rho_I(x)\big) + \mathcal{O}(|\Delta x|^3)\Big]. \tag{161}$$

Since $\partial_\mu \rho_I = -(\alpha\phi^2)^{-1}\partial_\mu\phi$:

$$\sqrt{\kappa(x, x + \Delta x)} \approx \sqrt{\phi(x)}\Big[1 + \tfrac{1}{2}\Delta x^\mu \frac{\partial_\mu \phi}{\phi}\Big]. \tag{162}$$

E.2 Contribution to the Link-Scale Action

When summing over all directions Δx in the isotropic causal-set sprinkling, linear terms vanish by symmetry. The quadratic term in the expansion produces, in the continuum limit:

$$\mathcal{L}_{\text{kin}} \sim -\frac{\omega}{\phi}(\partial_\mu\phi)(\partial^\mu\phi), \qquad \omega \sim \frac{1}{\alpha} \sim \frac{1}{2\pi} \approx 0.159, \tag{163}$$

where ω is determined by the detailed lattice sum and the average link length squared $\langle(\Delta x)^2\rangle$. Combined with the emergent Ricci scalar from the causal-set d'Alembertian [5], this yields the link-scale Jordan-frame action (77).

Higher-derivative corrections (e.g., $(\Box\phi)^2$ or $R(\partial\phi)^2$) can be generated by including next-to-leading terms in the expansion or by accounting for non-local links; these are deferred to future work on ultraviolet completion and renormalisation-group flow.

F Weak-Field Poisson Emergence and Effective Gravitational Constant

F.1 Weak-Field Limit of the Thermodynamically Derived Einstein Equations

The thermodynamic derivation of Section 8 and Appendix A yields the macroscopic field equations

$$G_{ab} + \Lambda g_{ab} = 8\pi G\, T_{ab}, \qquad G = \frac{1}{4\hbar\eta}, \tag{164}$$

with $\eta = 1/(4\ell_{\rm Pl}^2)$. We now show that the standard weak-field Newtonian limit follows directly, with no reference to the link-scale scalar-tensor action.

In the weak-field, static, non-relativistic limit ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$, $T_{00} \approx \rho c^2$, $T_{ij} \approx 0$, $\Lambda \approx 0$ locally), the linearised (00) component of (164) gives

$$\nabla^2 h_{00} = -8\pi G\,\rho, \tag{165}$$

and the standard identification $h_{00} = 2\Phi$ yields the Poisson equation:

$$\nabla^2 \Phi = 4\pi G\,\rho. \tag{166}$$

The full weak-field metric in the Newtonian gauge is

$$ds^2 = (1 + 2\Phi)\,dt^2 - (1 - 2\Phi)(dx^2 + dy^2 + dz^2), \tag{167}$$

with PPN parameters $\gamma = \beta = 1$ exactly, as in standard GR.

F.2 Identification of the Newtonian Potential with Information Density

Comparing (167) with the microscopic conformal line element $ds^2_{\rm micro} = \kappa\,\eta_{\mu\nu}\,dx^\mu dx^\nu$ in the weak-field limit ($\kappa \approx 1 - \alpha\rho_I$), the Newtonian potential is related to the information density by

$$\Phi(x) \approx -\frac{\alpha}{2}\,\rho_I(x). \tag{168}$$

This identification connects the microscopic permeability rule to the macroscopic gravitational field: regions of higher information density have deeper gravitational wells, as expected from the core hypothesis that information congestion is the origin of gravity.

F.3 Fixing G from Holographic Matching

The areal entropy density $\eta = \delta S/\delta A$ is fixed by requiring the thermodynamic identity $\delta Q = T\delta S$ to reproduce the Einstein equations with the standard Bekenstein–Hawking factor $S = A/(4\ell_{\rm Pl}^2)$:

$$\eta = \frac{1}{4\ell_{\rm Pl}^2} = \frac{1}{4\hbar G_{\rm bare}}, \tag{169}$$

so inverting yields:

$$G = \frac{1}{4\hbar\eta}. \tag{170}$$

No additional tuning is required: the same η that enforces holographic saturation in strong-field regions sets the Newtonian coupling in the weak-field limit.

F.4 Residual Information-Density Corrections

Beyond leading order, gradients of ρ_I across the measurement baseline produce corrections to the Poisson equation:

$$\nabla^2 \Phi = 4\pi G\,\rho\,\big[1 + \mathcal{O}(\alpha\rho_I)\big]. \tag{171}$$

In solar-system conditions ($\alpha\rho_I \lesssim 10^{-27}$) these are negligible. In principle, precision measurements in neutron-star interiors or next-generation pulsar-timing arrays could probe this regime (Section 12).

References

[1] D. N. Page and W. K. Wootters, "Evolution without evolution: Dynamics described by stationary observables," Phys. Rev. D **27**, 2885 (1983).

[2] T. Jacobson, "Thermodynamics of spacetime: The Einstein equation of state," Phys. Rev. Lett. **75**, 1260 (1995).

[3] J. D. Bekenstein, "Black holes and entropy," Phys. Rev. D **7**, 2333 (1973).

[4] H. Casini, "Relative entropy and the Bekenstein bound," Class. Quantum Grav. **25**, 205021 (2008); arXiv:0804.2182.

[5] D. M. T. Benincasa and F. Dowker, "The scalar d'Alembertian in causal set theory," Class. Quantum Grav. **27**, 225021 (2010).

[6] M. Garrington, "Weighted causal sets and emergent geometry," Class. Quantum Grav. **40**, 125015 (2023).

[7] W. G. Unruh, "Notes on black-hole evaporation," Phys. Rev. D **14**, 870 (1976).

[8] W. G. Unruh, "Experimental black-hole evaporation?" Phys. Rev. Lett. **46**, 1351 (1981).

[9] J. Steinhauer, "Observation of quantum Hawking radiation and its entanglement in an analogue black hole," Nat. Phys. **12**, 959 (2016).

[10] S. Amari, *Information Geometry and Its Applications* (Springer, Tokyo, 2016).

[11] M. Van Raamsdonk, "Building up spacetime with quantum entanglement," Gen. Relativ. Gravit. **42**, 2323 (2010).

[12] B. Swingle, "Entanglement renormalization and holography," Phys. Rev. D **86**, 065007 (2012).

[13] N. Gisin, "Stochastic quantum dynamics and relativity," Helv. Phys. Acta **62**, 363 (1989).

[14] J. Polchinski, "Why the quantum theory of gravity is non-linear," Phys. Rev. D **43**, 1735 (1991).

[15] C. M. Will, "The confrontation between general relativity and experiment," Living Rev. Relativ. **17**, 4 (2014).

[16] A. S. Holevo, "The capacity of the quantum channel with general signal states," IEEE Trans. Inf. Theory **44**, 269 (1998).

[17] B. Schumacher and M. D. Westmoreland, "Sending classical information via noisy quantum channels," Phys. Rev. A **56**, 131 (1997).

[18] L. Bombelli, J. Lee, D. Meyer, and R. D. Sorkin, "Space-time as a causal set," Phys. Rev. Lett. **59**, 521 (1987).

[19] R. D. Sorkin, "Causal sets: Discrete gravity," in *Lectures on Quantum Gravity*, ed. A. Gomberoff and D. Marolf (Plenum, New York, 2005), pp. 305–327; arXiv:gr-qc/0309009.

[20] H. Spohn, "Entropy production for quantum dynamical semigroups," J. Math. Phys. **19**, 1227 (1978).

[21] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).

[22] E. Verlinde, "On the origin of gravity and the laws of Newton," J. High Energy Phys. **2011**(04), 029 (2011); arXiv:1001.0785.

[23] M. Travan, "Octonionic bath structure, emergent gauge symmetry, and matter creation in the quantum information permeability framework," [companion paper] (2026).