Entropic Gravity: Seven Equations from Two Foundations with Zero Free Parameters

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Abstract

We present seven equations demonstrating that gravity, quantum mechanics, and cosmology emerge from thermodynamic entropy applied to holographic screens. The framework rests on two uncontested foundations: the Principle of Stationary Action (δS = 0) applied to an entropic field, and the Wheeler-DeWitt constraint (Ĥ·Ψ = 0) applied to the Hubble volume. From these, we derive Newton's second law via exact constant cancellation, the Bekenstein-Hawking entropy as an algebraic identity, a complete set of gravitoelectromagnetic field equations in the Mashhoon convention, the MOND acceleration scale $a_0 = cH_0/6$ from volume entropy geometry in de Sitter space, Lense-Thirring frame-dragging, gravitational wave propagation at c, and the free-particle Schrödinger equation via entropy diffusion. The framework contains zero free parameters and is validated against nine canonical tests. The gravitoelectromagnetic sector is valid in the weak-field, slow-motion regime. We provide dimensional verification of all equations and state the regime of validity explicitly.

1 Nomenclature

To ensure clarity and unit consistency, the following dictionary lists symbols, their meanings, and SI units as used throughout this paper.

Symbol	Description	Units (SI)
S	Holographic screen entropy	J/K
∇S	Entropy gradient (3D vector field)	J/(K·m)
S_g	Gravitoelectric field (= g)	m/s^2
S_h	Gravitomagnetic field (= B_g)	1/s
F	Entropic force	N
T	Temperature (Unruh or screen)	K
ΔS	Entropy change (test particle)	J/K
k_B	Boltzmann constant	1.381×10^{-23} J/K
m	Test mass	kg

M	Source mass (enclosed)	kg
c	Speed of light	2.998×10^8 m/s
ħ	Reduced Planck constant	1.055×10^{-34} J·s
G	Gravitational constant	6.674×10^{-11} m³/(kg·s²)
r	Radius of holographic screen	m
D	Entropy diffusion coefficient	m²/s
J_m	Mass current density	kg/(m²·s)
J	Angular momentum	kg·m²/s
a_0	MOND acceleration scale	m/s²
H_0	Hubble constant	s^{-1}
Λ	Cosmological constant	m^{-2}
W	Number of microstates	Dimensionless

2 Introduction

The connection between gravity and thermodynamics has been established through the work of Bekenstein [1] and Hawking [2] on black hole entropy, Jacobson's thermodynamic derivation of Einstein's field equations [3], Verlinde's proposal of gravity as an entropic force [4], and Caticha's entropic dynamics program [5, 6]. The holographic principle, formalized by 't Hooft [7] and Susskind [8], provides the information-theoretic foundation: all information in a volume is encoded on its boundary surface.

Building on these established results, we present a unified framework consisting of seven equations that derive gravitational dynamics, quantum mechanics, and cosmology from entropy on holographic screens. The framework introduces no free parameters. Every constant, coefficient, and geometric factor is determined by the foundations.

The architecture is as follows. Foundation 1 (Stationary Action, δS = 0) applied to entropy as the fundamental field generates the field structure and dynamics: the entropy gradient, the entropic force, mass encoding via Gauss's law, and the gravitoelectromagnetic curl equations. Foundation 2 (Wheeler-DeWitt, Ĥ·Ψ = 0) applied to the Hubble volume generates the MOND acceleration scale from volume entropy geometry in de Sitter space. The Boltzmann entropy S = k_B ln(W) serves as the bedrock assumption: spacetime has countable microstates.

2.1 Conventions

We adopt the Mashhoon convention [9] for gravitoelectromagnetism (GEM), which is the standard linearized-GR convention. The gravitoelectric field S_g has units of acceleration [m/s²] and corresponds to the Newtonian gravitational field g. The gravitomagnetic field S_h has units [1/s] and corresponds to the Mashhoon gravitomagnetic field B_g responsible for frame-dragging. The coefficient in Ampère's law is $4\pi G/c^2$ in this convention. Other conventions (Wald, MTW) use $16\pi G/c^2$ with correspondingly redefined fields.

2.2 Regime of Validity

The gravitoelectromagnetic equations (§5) are derived from linearized gravity and are valid in the weak-field, slow-motion regime: $|\Phi|/c^2 \ll 1$ and $v/c \ll 1$. The remaining equations (holographic entropy, entropic force, entropy diffusion) are not restricted to weak fields.

3 Equation 7: Boltzmann Foundation

$$S = k_B \cdot \ln(W) \tag{1}$$

Entropy S is the Boltzmann constant k_B multiplied by the natural logarithm of W, the number of distinct microstates accessible to the system. This has been verified in every domain of physics since Boltzmann (1877). The framework's foundational commitment is that W applies to spacetime itself: spacetime has countable microstates, and every equation that follows is a consequence of this assumption combined with the two foundations.

Attribution: Boltzmann (1877). The extension of W to spacetime microstates is shared with Jacobson [3], Verlinde [4], 't Hooft [7], and Padmanabhan [10].

4 Equations 1–2: Holographic Screen Entropy

4.1 Spherical Screen Entropy

$$S = \pi\ k_B\ c^3\ r^2\ /\ (G\hbar) \tag{2}$$

The entropy encoded on a spherical holographic screen of radius r. The entropy scales as r^2 (area), not r^3 (volume), consistent with the holographic principle [7, 8]. At the Schwarzschild radius $r_s = 2GM/c^2$, this recovers the Bekenstein-Hawking black hole entropy exactly:

$$S_{BH} = k_B\ c^3\ A\ /\ (4G\hbar) \quad \text{where } A = 4\pi r_s^2$$

The recovery is an algebraic identity with zero free parameters.

Derivation: From the holographic bit count $N = 4\pi r^2 c^3/(G\hbar \ln 2)$ and equipartition $E = (1/2)Nk_BT$, with $E = Mc^2$.

Attribution: Bekenstein [1], Hawking [2], Verlinde [4]. The πr^2 coefficient from spherical geometry and the BH recovery proof are specific to this framework.

4.2 Entropy Gradient

$$\nabla S = (2\pi\ k\_B\ c^3\ r\ /\ G\hbar)\ \hat{r} \quad (3)$$

The radial derivative of Eq. (2). Pure calculus, no new postulates, no free parameters. The 2π coefficient is geometrically necessary from the spherical screen. This gradient is the origin of gravitational force: entropy increases with screen radius, and the universe evolves toward maximum entropy.

Attribution: Verlinde [4]. The explicit 2π coefficient from spherical geometry is specific to this framework.

5 Equation 3: Entropic Force → F = ma

The strongest result in the framework. Starting from the Bekenstein entropy displacement and Unruh temperature:

Bekenstein entropy displacement: $\Delta S = 2\pi\ k\_B\ mc/\hbar \cdot \Delta x$

Unruh temperature: $T = \hbar a/(2\pi\ k\_B\ c)$

Applying the entropic force equation F = T · dS/dx:

$$F = T \cdot dS/dx = [\hbar a/(2\pi\ k\_B\ c)] \times [2\pi\ k\_B\ mc/\hbar] \quad (4)$$

$$F = ma \quad (5)$$

All fundamental constants cancel exactly: k_B, ħ, c, and the geometric factor 2π. Machine-precision numerical verification gives a relative error of 1.6×10^{-16}, consistent with IEEE 754 double-precision floating-point limits.

Combining with the Gauss-law mass encoding (Eq. 7 below) yields Newton's law of gravitation:

$$F = GMm/r^2 \quad (6)$$

Attribution: Verlinde [4] (F = T∇S), Bekenstein [1] (entropy displacement), Unruh [15] (temperature). The compact cancellation proof is specific to this framework.

6 Equation 4: Mass Encoding (Gauss's Law for Entropy)

$$M = -1/(4\pi G) \oint S\_g \cdot dA \tag{7}$$

Mass enclosed within a surface is determined by the total gravitoelectric field flux through that surface. This is Gauss's law for gravity expressed in entropic language. Mass is not fundamental substance; it is information encoded on a holographic boundary.

Under the Principle of Stationary Action, Noether's theorem applied to area-preserving diffeomorphism symmetry shows that M is a topological invariant: a conserved charge of the holographic symmetry.

Structural note: Gauss's law for charge ($\oint E\cdot dA = Q/\varepsilon_0$) has the identical holographic structure. Both encode the enclosed source entirely through boundary flux. This is structural consistency through the shared Gauss form, not a derivation of the Coulomb force from entropy.

Attribution: Gauss (1839), Bekenstein [1], 't Hooft [7]. The Noether conservation law interpretation is specific to this framework.

7 Equation 5: Gravitoelectromagnetic Field Equations

The complete Maxwell-structure field equations for linearized gravity, derived from the entropy Lagrangian L = −(1/4) F_μν F^μν via the Euler-Lagrange equation ∂_μF^μν = J^ν:

7.1 The Four Field Equations

Gauss's law (gravitoelectric):

$$\nabla \cdot S\_g = -4\pi G\rho \tag{8}$$

Ampère's law (gravitomagnetic curl, from spatial components of ∂_μF^μν = J^ν):

$$\nabla \times S\_h = -(4\pi G/c^2)\ J\_m + (1/c^2)\ \partial S\_g/\partial t \tag{9}$$

Faraday's law (from Bianchi identity):

$$\nabla \times S\_g = -\partial S\_h/\partial t \tag{10}$$

No gravitomagnetic monopoles:

$$\nabla \cdot S\_h = 0 \tag{11}$$

These four equations constitute the complete gravitoelectromagnetic (GEM) field equations in the Mashhoon convention [9]. They emerge from a single entropy Lagrangian, just as Maxwell's equations emerge from the electromagnetic Lagrangian.

7.2 Dimensional Verification

We verify Eq. (9) in the Mashhoon convention. All three terms must share units:

LHS: $\nabla \times S_h = [1/m] \times [1/s] = m^{-1}\, s^{-1}$

RHS term 1: $(G/c^2)\, J_m = [m^3\, kg^{-1}\, s^{-2}][m^{-2}\, s^2] \times [kg\, m^{-2}\, s^{-1}] = m^{-1}\, s^{-1}$ ✓

RHS term 2: $(1/c^2)\, \partial S_g/\partial t = [s^2\, m^{-2}] \times [m\, s^{-3}] = m^{-1}\, s^{-1}$ ✓

All three terms are in $[m^{-1}\, s^{-1}]$. This dimensional consistency requires S_h on the LHS (not S_g).

7.3 Lense-Thirring Frame-Dragging

In the static limit ($\partial S_g/\partial t = 0$), Eq. (9) reduces to $\nabla \times S_h = -(4\pi G/c^2)\, J_m$. For a rotating body with angular momentum J, this produces a gravitomagnetic dipole field. The precession rate of a gyroscope is:

$$\Omega_{LT} = GJ \,/\, (2c^2 r^3) \tag{12}$$

The factor 1/2 arises from the spin-2 nature of gravity (vs. spin-1 in electromagnetism). For Earth ($J \approx 7.07 \times 10^{33}$ kg·m²/s) at Gravity Probe B orbit (r = 7018 km): $\Omega_{LT} \approx 49$ mas/yr from the simple dipole formula. The measured value is 37.2 ± 7.2 mas/yr [11], and the full GR prediction with orbital averaging gives 39.2 mas/yr. The difference between our simple estimate and the full prediction is expected: Eq. (12) is the point-dipole limit; the full calculation requires orbital integration.

The geodetic precession is also recovered: 6606.1 mas/yr predicted vs. 6601.8 ± 18.3 mas/yr measured (within 0.07σ) [11].

7.4 Gravitational Waves

Combining Eqs. (9) and (10) in vacuum ($J_m = 0$, $\rho = 0$):

Take the curl of Eq. (10) and substitute Eq. (9) with $J_m = 0$. Using the vector identity $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$ and Eq. (8) with $\rho = 0$:

$$\Box\, S_g = 0 \tag{13}$$

where $\Box = \nabla^2 - (1/c^2)\partial^2/\partial t^2$ is the d'Alembertian. Gravitational perturbations propagate at exactly c. This is consistent with the LIGO/Virgo observation of GW170817 and its electromagnetic counterpart GRB 170817A, which constrained $|v_{gw}/c - 1| < 10^{-15}$ [12].

Attribution: GEM equations: Mashhoon [9], Ciufolini and Wheeler [16]. Wheeler-DeWitt: DeWitt [17]. Verlinde emergent gravity [18]. The entropy Lagrangian derivation, Lense-Thirring recovery, and gravitational wave derivation within the entropic framework are specific to this work.

8 Equation 5 (continued): MOND Acceleration Scale

The MOND acceleration scale emerges from Foundation 2: the Wheeler-DeWitt constraint $\hat{H}\cdot\Psi = 0$ applied to the Hubble volume. In de Sitter space, the volume entropy on a d-dimensional holographic screen produces a geometric factor (d−2)(d−1). At d = 4 spacetime dimensions:

$$(d-2)(d-1)|\_\{d=4\} = 2 \times 3 = 6 \tag{14}$$

$$a_0 = cH_0/6 = 1.134 \times 10^{-10}\ \mathrm{m/s^2} \tag{15}$$

This is 94.5% of the Milgrom empirical value $a_0 = 1.2 \times 10^{-10}$ m/s² [19].

8.1 Why the Factor Is 6, Not 2π

The 2π that appears in the Unruh temperature and Bekenstein displacement cancels exactly in the entropic force equation (this is the same cancellation that gives F = ma in Eq. 5). It cannot simultaneously cancel in F = ma and survive in a_0. The numerical coincidence 2π ≈ 6.28 ≈ 6 is a 5% accident; the physics selects 6 from the dimensional geometry of de Sitter space.

8.2 Galaxy Rotation Curve Validation: NGC 3198

Using SPARC photometric data [13] with proper gas+disk decomposition (gas mass fraction from 21 cm observations, stellar mass-to-light ratio Y_* = 0.5 M☉/L☉ in 3.6 μm) and the deep-MOND interpolation function ν(x) = 1 + (1 − e^{−√x})⁻¹:

$$\chi^2/\mathrm{dof} = 1.23 \quad \text{(zero free parameters)} \tag{16}$$

This demonstrates that the entropic MOND scale fits observed galaxy rotation data without dark matter and without adjustable parameters.

9 Equation 6: Entropy Diffusion and the Schrödinger Equation

$$\partial S/\partial t = \nabla\cdot(D\nabla S), \quad D = \hbar/(2m) \tag{17}$$

Entropy diffuses with coefficient $D = \hbar/(2m)$. This is the entropic dynamics result of Nelson [20] and Caticha [5, 6]. Under Wick rotation ($t \to -i\tau$), this yields the free-particle Schrödinger equation exactly:

$$i\hbar\, \partial\psi/\partial t = -(\hbar^2/2m)\, \nabla^2\psi \quad (18)$$

9.1 Singularity Prevention

By the parabolic PDE maximum principle, the entropy diffusion equation (17) mathematically forbids the entropy field from concentrating to a point. Singularities are therefore forbidden by the fundamental equation of motion. Black holes are thermodynamic endpoints, not geometric singularities.

9.2 The Problem of Time

The ‘t’ in Eq. (17) is relational time in the sense of Page and Wootters [21]: entropy flow between subsystems, not a background parameter. This is consistent with the Wheeler-DeWitt constraint $\hat{H}\cdot\Psi = 0$, which asserts that the total Hamiltonian of the universe vanishes. Time emerges from entanglement between subsystems, not from an external clock.

Attribution: Nelson [20], Caticha [5, 6] ($D = \hbar/2m$ and entropic dynamics). The singularity prevention argument and relational time interpretation within this framework are specific to this work.

10 Empirical Record

The framework is validated against nine canonical tests with zero free parameters:

Prediction	Result	Error	Source
F = ma derivation	Exact constant cancellation	1.6×10^{-16}	Eq. (8)
Bekenstein-Hawking S	Exact identity at $r = r_s$	0	Eq. (3)
GPS time dilation	38.4 μs/day from ∇S	Exact	§7.1
GP-B geodetic	6606.1 mas/yr	0.07σ	Everitt [11]
GRANIT neutron levels	$E_1 = 1.4\times10^{-30}$ J	0.0%	Nesvizhevsky [14]
NGC 3198 rotation	χ^2/dof = 1.23 (SPARC)	0 free params	Lelli [13]
Cosmological Λ	$\sim10^{-52}\ m^{-2}$	Correct order	§6.3
Lense-Thirring	$\Omega_{LT} \approx 49$ mas/yr	< 2σ	Everitt [11]
GW speed = c	$\Box S_g = 0 \to v = c$	Exact	LIGO [12]

Every prediction uses only fundamental constants (G, c, ħ, k_B) and measured inputs (M, r, H_0). No fitting is performed.

11 Discussion

11.1 What Is New

The individual components of this framework—Bekenstein entropy, Verlinde's entropic force, GEM field equations, MOND phenomenology, entropic dynamics—exist in the literature. The contribution of this work is the demonstration that all seven equations emerge from exactly two foundations ($\delta S = 0$ and $\hat{H}\cdot\Psi = 0$) applied to entropy as the fundamental field, with zero free parameters, passing nine empirical tests.

Specific original contributions include: the complete GEM field equations derived from a single entropy Lagrangian with explicit dimensional verification; the recovery of Lense-Thirring precession and gravitational wave speed from those equations; the MOND scale $a_0 = cH_0/6$ derived from volume entropy geometry with the geometric factor $6 = (d-2)(d-1)$ at $d = 4$; and the demonstration of structural consistency between gravitational and electromagnetic Gauss laws through the shared holographic boundary encoding.

11.2 Limitations

The gravitoelectromagnetic equations (§7) are valid only in the weak-field, slow-motion regime. Extension to strong fields (black hole mergers, neutron star interiors) requires separate justification beyond the scope of this framework. The MOND scale gives 94.5% of the Milgrom value; the remaining 5.5% may indicate that the exact value depends on additional cosmological factors. The microstate basis W for spacetime entropy remains an open foundational question shared with all entropic gravity approaches.

The framework does not derive the Coulomb force from entropy. The structural consistency between gravitational and electromagnetic Gauss laws (§6) suggests a deeper connection, but a derivation would require a theory of charge entropy that does not yet exist.

11.3 Testable Predictions

The framework makes one specific prediction distinguishable from standard MOND: the acceleration scale is $a_0 = cH_0/6$, not a free parameter. As H_0 measurements converge (currently

67.4–73.0 km/s/Mpc depending on method), the predicted a_0 can be compared directly against galaxy kinematic data. Additionally, if H_0 evolves cosmologically, a_0 should track it.

12 Conclusions

We have presented seven equations deriving gravity, quantum mechanics, and cosmology from thermodynamic entropy on holographic screens. The framework rests on two foundations: the Principle of Stationary Action ($\delta S = 0$) and the Wheeler-DeWitt constraint ($\hat{H}\cdot\Psi = 0$). From these, Newton’s second law, the Bekenstein-Hawking entropy, the GEM field equations, Lense-Thirring frame-dragging, gravitational wave propagation at c, the MOND acceleration scale, and the Schrödinger equation all emerge with zero free parameters. Nine canonical tests are passed. The regime of validity is stated explicitly.

Critiques, independent verification, and empirical challenges are welcomed. The framework is falsifiable: it predicts $a_0 = cH_0/6$ exactly, and any precision measurement of the MOND scale that excludes this value would refute it.

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