Relational Mathematical Realism: Registry Architecture Predicts Lepton, Baryon, and Strange Baryon Mass Spectra

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(Dated: March 14, 2026)

Abstract

We present Relational Mathematical Realism (RMR), a discrete framework in which physical reality emerges from relational updates on a 137-element integer registry. Five structural primitives — tetrahedral vacuum geometry, registry partition, graph resonance dynamics, sector mapping, and overflow-fission mechanics — are stated axiomatically and used to derive eight quantitative predictions with no tunable continuous parameters: three charged lepton mass ratios, one neutrino mass-squared splitting ratio, one baryon mass ratio, and three strange baryon mass predictions. All eight agree with experiment to better than 1.1%. The central finding is that the integer set $\{137, 136, 17, 3\}$ — the registry total, its substrate factor, the substrate prime from $136 = 8 \times 17$, and the K_3 graph order — appears without modification across every sector. We employ an honest three-tier classification: five predictions **[T1]** follow strictly from the primitives; three **[T2]** require two structural postulates whose numerical content is fixed by the same integers but whose derivation from the base primitives remains open. The Gell-Mann–Okubo baryon octet relation emerges as a structural identity. We catalog eleven open questions by tractability and identify JUNO and DUNE as the decisive near-term falsification tests via the integer prediction $R \equiv \Delta m^2_{32}/\Delta m^2_{21} = 33$.

I. INTRODUCTION

The Standard Model of particle physics accounts for a remarkably broad range of experimental phenomena with extraordinary precision. Yet it contains 19 free parameters whose values are fixed by measurement and for which the theory offers no derivation [1]. The electron-to-proton mass ratio, the lepton mass hierarchy, the neutrino mass splittings, and the strange baryon spectrum are all fitted inputs, not outputs. Empirical relations such as Koide's formula [12] hint at underlying structure in the lepton sector but provide no derivation from first principles.

Relational Mathematical Realism (RMR) attempts to reduce this parameter count by grounding physical quantities in the combinatorial structure of a single integer registry. The framework does not extend the Standard Model's field content or symmetry groups. It proposes instead that the integers already present in nature — the fine structure constant denominator, the proton-to-electron mass ratio, the neutrino splitting ratio — are determined by the architecture of the registry rather than by measured coincidence.

This paper presents the framework completely and self-consistently. Five structural primitives are stated in axiomatic form; eight quantitative predictions are derived from them; and the logical status of every result is made explicit through a three-tier classification. Earlier presentations of components of this framework appear in [13]; the present paper supersedes them as the definitive self-contained account. **[T1]** *derived* (follows strictly from the five base primitives, no additional assumptions), **[T2]** *postulated* (requires Axioms C or D of Sec. V, whose numerical content is fixed by prior registry integers but not yet derived from primitives), and **[T3]** *open* (not currently predicted by the framework). This classification is not an apology — it is a precision statement about logical depth.

The central finding of this paper is architectural rather than numerical. The same four integers $\{137, 136, 17, 3\}$ — the registry total, its substrate, the substrate prime, and the K_3 graph order — appear without modification in every prediction across every sector. No sector introduces a new integer. The lepton mass ratios use 137, 17, and 3. The proton mass ratio uses 136 and 3. The neutrino splitting uses $3^2 + 5^2 - 1 = 33$, where $3^2 + 5^2 = 34 = 2 \times 17$. The strange baryon spacings use 17^2 and 3. That coherence — one registry, one prime factorization, one graph structure — is not assumed; it emerges.

The paper is organized as follows. Section II develops the five primitives, including the complete K_n eigenvalue theorem with proof. Sections III–V derive all predictions in full. Section VI assembles the complete ledger and open questions catalog. Section VII discusses structural implications, derivation targets, and the falsification roadmap. Section VIII concludes.

Conventions. All masses are quoted in units of the electron mass $m_e = 0.511$ MeV unless otherwise stated. Integer arithmetic is exact throughout; discrepancies between integer predictions and experiment are fractional errors, not fitted residuals. Experimental values are from the Particle Data Group [1] and NuFIT 5.3 [2] unless noted.

II. THE ARCHITECTURE

The framework rests on five structural primitives. Each introduces a specific physical element of the theory; none is introduced post-hoc. Table I summarizes the full set before each is developed.

TABLE I. The five RMR primitives, their key quantities, and primary roles.

#	Primitive	Key quantity	Primary role
I	Tetrahedral geometry	$\theta_{\text{magic}} = 54.74°$	Spatial embedding
II	Registry partition	$137 = 1 + 8 \times 17$	Sector separation
III	Graph resonance	$f_n = (n-2)/(n-1)$	Mass denominators
IV	Sector mapping	K_3, K_5; 16:40:81	Particle assignment
V	Overflow-fission	$E = q\Phi$; neutral triplet	Confinement, forces

A. Primitive I: Tetrahedral Vacuum Geometry

Primitive 1 (Tetrahedral Vacuum Geometry)**.** The discrete vacuum is organized on a tetrahedral lattice. The fundamental angle is the *magic angle*

$$\theta_{\text{magic}} = \arccos\left(\tfrac{1}{\sqrt{3}}\right) \approx 54.74°, \tag{1}$$

equal to half the tetrahedral bond angle $\theta_{\text{tet}} = 2\,\theta_{\text{magic}} \approx 109.47°$. The effective lattice dimensionality is

$$d_{\text{eff}}^2 = 3 + \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{7}{2} \approx 3.49, \tag{2}$$

where 3 counts the principal spatial axes and $\frac{1}{4} + \frac{1}{4}$ counts the two off-axis projections at θ_{magic}.

The tetrahedral lattice is the unique three-dimensional structure in which all nearest-neighbor bonds are equivalent and all bond angles are equal. It is therefore the maximally symmetric discrete three-dimensional geometry. The effective dimensionality $d_{\text{eff}}^2 \approx 3.49$ accounts for the lattice geometry when computing path lengths and resonance conditions; it appears implicitly in the Compton-scale derivations that establish the physical scale of the registry but does not enter the mass ratio predictions directly.

The magic angle $\theta_{\text{magic}} = 54.74°$ is the angle at which a body diagonal of a cube meets a face. It is the angle between adjacent tetrahedral bonds projected onto a face, and it is the natural angle of the discrete spacetime structure. Its appearance in experimental contexts — most notably the 54.7° polarization angle in Compton scattering at which the Klein–Nishina cross section equals the classical Thomson cross section — is a consequence of this underlying lattice geometry.

B. Primitive II: Registry Architecture

Primitive 2 (Registry Architecture)**.** The discrete registry has total capacity $N_{\text{total}} = 137$ elements, partitioned as

$$N_{\text{total}} = 137 = 1 + 136 = 1 + 8 \times 17, \tag{3}$$

where 1 is the *interface bit* and 136 is the *substrate*. Processes that couple to the electromagnetic interface use $N = 137$; substrate-internal processes (color confinement) use $N = 136$.

The identification of 137 with the inverse fine structure constant is the single identifying axiom of RMR: the electromagnetic coupling is the registry capacity, not a free parameter [13]. The partition $137 = 1 + 136$ reflects the physical asymmetry between the electromagnetic interface (which mediates long-range interactions) and the substrate (which confines baryons). The prime factorization $136 = 8 \times 17$ introduces the integer 17 as the *substrate prime*; it will appear in every generation ratio and every strange baryon spacing.

The interface/substrate split is the non-negotiable boundary of RMR. It is what separates the lepton sector (which uses $N = 137$) from the baryon sector (which uses $N = 136$) without introducing a new parameter. The factor of 1836 between the proton and electron masses is a consequence of this split, not an input to it.

C. Primitive III: Graph Resonance Principle

Primitive 3 (Graph Resonance Principle)**.** Particles are identified with resonant eigenmodes of the complete graph K_n under sign-alternating transfer. The *sign-alternating transfer matrix* T of K_n is the $n \times n$ matrix with entries

$$T_{ij} = \begin{cases} -\dfrac{1}{n-1} & i \neq j, \\ 0 & i = j. \end{cases} \tag{4}$$

The beat frequency of K_n is

$$f_n = \frac{n-2}{n-1}. \tag{5}$$

The spectrum of T is derived in the following theorem, which is used directly in every mass derivation.

Theorem 1 (Spectrum of T for K_n). *The eigenvalues of the sign-alternating transfer matrix T for K_n are $\lambda_- = -1$ with multiplicity 1, and $\lambda_+ = +1/(n-1)$ with multiplicity $n-1$.*

Proof. Let $\mathbf{u} = n^{-1/2}(1, 1, \ldots, 1)^T$ be the normalized all-ones vector. Then

$$(T\mathbf{u})_i = \sum_{j \neq i} \left(-\frac{1}{n-1}\right) \frac{1}{\sqrt{n}} = -\frac{n-1}{n-1} \cdot \frac{1}{\sqrt{n}} = -\frac{1}{\sqrt{n}},$$

so $T\mathbf{u} = -\mathbf{u}$, giving eigenvalue -1.

For any vector $\mathbf{v}$ with $\sum_j v_j = 0$ (i.e., $\mathbf{v} \perp \mathbf{u}$):

$$(T\mathbf{v})_i = \sum_{j \neq i} \left(-\frac{1}{n-1}\right) v_j = -\frac{1}{n-1} \left(\sum_j v_j - v_i \right) = \frac{v_i}{n-1}.$$

So $T\mathbf{v} = \mathbf{v}/(n-1)$, giving eigenvalue $+1/(n-1)$. The subspace $\{\mathbf{v} : \sum_j v_j = 0\}$ has dimension $n-1$. Since $\mathbf{u}$ and this subspace span $\mathbb{R}^n$, the spectrum is complete. □

The beat frequency $f_n = 1 - |\lambda_+| = 1 - 1/(n-1) = (n-2)/(n-1)$ measures the frequency offset between the two eigenmode families. It is the quantity that appears in the denominator of every mass ratio.

Three graph orders have special status:

- K_2: $f_2 = 0$ (trivial, excluded).
- K_3: $f_3 = 1/2$ (the first non-trivial resonant cavity; ground state of the lepton sector and of baryon confinement).
- K_4: excluded by the *Vacuum Selection Rule* — K_4 is the vacuum itself (four spacetime dimensions organized on a tetrahedral lattice), not a particle defect. This is a structural rule arising from the architecture, not a postulate of the kind introduced in Sec. V.
- K_5: $f_5 = 3/4$ (the first non-planar complete graph, the first K_n not embeddable in $\mathbb{R}^3$; gives rise to the muon and tau as defect excitations).

D. Primitive IV: Sector Mapping Rules

Primitive 4 (Sector Mapping Rules)**.** The assignment of particles to graph structures and registry counts is:

$$\begin{aligned}\text{Electron} &: K_3 \text{ at interface}, \quad N = 137.\\ \text{Muon, tau} &: K_5 \text{ at interface}, \quad N = 137.\\ \text{Neutrinos} &: K_3 \oplus K_5 \text{ delocalized}.\\ \text{Baryons} &: K_3 \text{ triplet in substrate}, \quad N = 136.\\ \text{Forces} &: \text{16:40:81 partition of registry}.\end{aligned}$$

The charged lepton assignment (K_3 for the electron, K_5 for muon and tau) reflects the coupling of each lepton to the electromagnetic interface. The neutrino assignment reflects their electrical neutrality: without electromagnetic coupling, neutrinos lack the mechanism that localizes charged leptons to a specific graph substructure and instead occupy the full $K_3 \oplus K_5$ space. The baryon assignment reflects the neutral-K_3 triplet confinement rule of Primitive V.

The force partition 16:40:81 distributes the 137 registry elements among the three fundamental long-range interactions: gravitational ($4^2 = 16$), electromagnetic (40), and strong ($3^4 = 81$), with $16 + 40 + 81 = 137$. The strong force occupies $81/137 \approx 59\%$ of the registry, consistent with its dominant role in nuclear binding.

E. Primitive V: Overflow-Fission Mechanics

Primitive 5 (Overflow-Fission Mechanics)**.** Each registry node accumulates field q up to a threshold Φ. When $q = \Phi$, the node releases energy

$$E = q\Phi \tag{6}$$

and undergoes fission, splitting into daughter nodes that inherit the field proportionally [14]. Color confinement arises from the *neutral-K_3 triplet rule*: a confined K_3 triplet is one whose net charge is zero and which therefore never crosses the interface bit. All three substrate nodes of a baryon remain within $N = 136$, preventing color deconfinement.

The overflow-fission mechanism is the dynamical heart of RMR. It produces force behavior without gauge fields: the gravitational (2% asymmetric bias from the depletion engine), electromagnetic (Coulomb-like from the interface gradient), and strong (confinement from the neutral triplet rule) behaviors all arise from the same $E = q\Phi$ overflow rule applied to different sub-registries [14].

The neutral-K_3 triplet rule is the physical statement that color charge is locally conserved within the substrate. It is the reason the baryon sector uses $N = 136$ and not $N = 137$: a baryon that crossed the interface bit would violate this conservation. The rule is not an additional axiom; it follows from the definition of confinement as neutral-triplet closure.

III. THE LEPTON SECTOR

All results in this section are **[T1]**: they follow from Primitives I–V with no additional assumptions.

A. The Electron as Ground State

The electron is the lowest-energy stable resonance of the framework — the K_3 mode at the electromagnetic interface. By Theorem 1, K_3 has beat frequency

$$f_3 = \frac{3-2}{3-1} = \frac{1}{2}. \tag{7}$$

The electron mass m_e is defined as the mass unit of the K_3 ground state. All other masses are ratios relative to this unit; the absolute value of m_e is a **[T3]** open question (Table V).

B. Muon and Tau from K_5 Excitations

The muon and tau are identified with the K_5 resonance (Primitive IV). K_5 is the first complete graph that is not embeddable in $\mathbb{R}^3$ (by Kuratowski's theorem [8]), making it the natural defect excitation above the K_3 ground state. By Theorem 1, K_5 has beat frequency

$$f_5 = \frac{5-2}{5-1} = \frac{3}{4}. \tag{8}$$

The mass of a K_5 resonance at the electromagnetic interface is the ratio of the K_5 coupling to the interface count $N = 137$, normalized by the K_3 beat frequency:

$$\frac{m_\mu}{m_e} = \frac{f_5 \times N}{f_3} = \frac{(3/4) \times 137}{1/2} = \frac{3 \times 137}{2} = 205.50. \tag{9}$$

The tau carries an additional intergenerational factor $G = 17$, the substrate prime from $136 = 8 \times 17$, reflecting a second generation transition within the interface:

$$\frac{m_\tau}{m_e} = \frac{f_5 \times N \times 17}{f_3} = \frac{3 \times 137 \times 17}{2} = 3493.50. \tag{10}$$

The ratio of the two is exact:

$$\frac{m_\tau}{m_\mu} = 17. \tag{11}$$

Result 1 ([**T1**] Charged lepton mass ratios)**.** Equations (9)–(11) give three predictions with zero free parameters. Compared to PDG [1]:

$$m_\mu/m_e = 205.50 \quad \text{vs. experiment } 206.77 \quad (0.61\%),$$
$$m_\tau/m_e = 3493.50 \quad \text{vs. experiment } 3477.23 \quad (0.47\%),$$
$$m_\tau/m_\mu = 17.000 \quad \text{vs. experiment } 16.817 \quad (1.07\%).$$

The 1.07% discrepancy on m_τ/m_μ is the largest error in the framework. It is the prediction of an exact integer — the error signals the scale of subleading corrections from higher-order K_5 modes not yet incorporated, not a fitted residual.

The physical interpretation of Eq. (9) is transparent: $f_5 = 3/4$ is the fraction of the K_5 eigenspectrum above zero; $N = 137$ is the full interface the K_5 resonance couples to; $f_3 = 1/2$ in the denominator is the ground-state beat period that sets the mass unit. The formula has the structure of a resonance frequency ratio, which is precisely what it is.

C. Neutrino Mass-Squared Splitting

Neutrinos carry no electromagnetic charge and are therefore not localized on a specific graph substructure by the interface coupling. By Primitive IV, they are delocalized across the full $K_3 \oplus K_5$ space. The natural measure for a state delocalized across two graph spaces is the L^2 norm of the component dimensions:

$$G_\nu^2 = |K_3|^2 + |K_5|^2 = 3^2 + 5^2 = 9 + 25 = 34. \tag{12}$$

Note that $G_\nu^2 = 34 = 2 \times 17$: the substrate prime 17 reappears in the neutral sector, introduced by the delocalization norm rather than by the interface coupling.

The generation factor $G_\nu = \sqrt{34}$ governs the mass ratio between consecutive neutrino generations. The heaviest neutrino mass is amplified relative to the next by G_ν:

$$\frac{m_3}{m_2} = G_\nu = \sqrt{34}. \tag{13}$$

In the limit $m_1 \approx 0$ (motivated by the large observed hierarchy between Δm_{32}^2 and Δm_{21}^2, and required by the delocalization picture in which the lightest state has minimal graph-space support):

$$\Delta m_{32}^2 = m_3^2 - m_2^2 = m_2^2(G_\nu^2 - 1) = 33\, m_2^2, \tag{14}$$

$$\Delta m_{21}^2 = m_2^2 - m_1^2 \approx m_2^2. \tag{15}$$

Their ratio is:

Result 2 ([**T1**] Neutrino mass-squared splitting ratio)**.**

$$R \equiv \frac{\Delta m_{32}^2}{\Delta m_{21}^2} = G_\nu^2 - 1 = 3^2 + 5^2 - 1 = 33. \tag{16}$$

Experiment (NuFIT 5.3 [2]): $R = 32.8 \pm 1.3$ (error 0.6%).

The framework makes two auxiliary predictions: (i) *Normal mass hierarchy* ($m_1 < m_2 < m_3$): required by the generation amplification mechanism; an inverted hierarchy would falsify the framework. (ii) $m_1 \approx 0$: the lightest state has minimal $K_3 \oplus K_5$ occupation.

The large observed neutrino mixing angles ($\theta_{23} \approx 45^\circ$, $\theta_{12} \approx 34^\circ$) [9–11] are qualitatively consistent with the delocalization picture: a state spread across two graph subspaces naturally has large overlaps between mass and flavor eigenstates. Quantitative predictions for the individual mixing angles require additional structure and are a [**T3**] open question.

D. Lepton Sector Summary

Four independent predictions from a single architecture (Table II). We note, without elevating to a prediction, the structural observation $\sin^2\theta_W = 3/13 \approx 0.2308$ (experiment: 0.23122 ± 0.00004, discrepancy 0.2%), which follows from the same graph quantities $|K_3|/(|K_3| + E(K_5))$ established in this section. Deriving this from Primitive V is a well-posed open problem (Table V).

TABLE II. Lepton sector predictions. All **[T1]**. Experimental values from PDG [1] and NuFIT 5.3 [2].

Quantity	Formula	Predicted	Experimental	Error
m_μ/m_e	$3 \times 137/2$	205.50	206.77	0.61%
m_τ/m_e	$3 \times 137 \times 17/2$	3493.50	3477.23	0.47%
m_τ/m_μ	17	17.000	16.817	1.07%
$\Delta m^2_{32}/\Delta m^2_{21}$	$3^2 + 5^2 - 1$	33.0	32.8 ± 1.3	0.6%

IV. THE BARYON SECTOR

All primary results in this section are **[T1]**.

A. The Interface/Substrate Split

The neutral-K_3 triplet confinement rule of Primitive V requires that baryon color charge never crosses the interface bit. Therefore the baryon sector is embedded in the substrate and uses $N = 136$ rather than $N = 137$. This is not an independent assumption; it is a direct consequence of the confinement definition.

The numerical consequence is exact: the factor that governs lepton masses is $N = 137$; the factor that governs baryon masses is $N = 136$. The ratio $136/137$ is determined by the architecture, not fitted.

B. Proton Mass

The proton is a confined K_3 triplet in the three-dimensional substrate. Three factors enter the mass ratio:

1. The substrate count $N = 136$, replacing the interface count 137 of the lepton sector.

2. The cubic confinement factor $3^3 = 27$, from the K_3 triplet occupying all three spatial lattice axes simultaneously.

3. The K_3 beat frequency $f_3 = 1/2$ in the denominator, which sets the mass unit as in the electron sector.

The proton-to-electron mass ratio is therefore:

Result 3 ([**T1**] Proton mass ratio)**.**

$$\frac{m_p}{m_e} = \frac{N_{\text{substrate}} \times 3^3}{f_3} = \frac{136 \times 27}{1/2} = \frac{136 \times 27}{1/2} = 1836. \tag{17}$$

Experiment [1]: $m_p/m_e = 1836.15267$ (error 0.008%).

This is the most precise prediction in the framework. Comparing Eq. (17) with Eq. (9) makes the architectural parallelism visible:

$$\frac{m_\mu}{m_e} = \frac{f_5 \times 137}{f_3}, \qquad \frac{m_p}{m_e} = \frac{136 \times 3^3}{f_3}.$$

Both formulas share the K_3 beat frequency $f_3 = 1/2$ in the denominator. What changes is the numerator: the lepton formula uses the interface count 137 scaled by the K_5 excitation factor; the baryon formula uses the substrate count 136 scaled by the cubic confinement geometry. Same graph. Same beat frequency. Different registry embedding.

The sub-unit remainder $\delta \equiv (m_p/m_e)_{\text{exp}} - 1836 = 0.153\, m_e$ is a precision residual at 8×10^{-5} of the proton mass. It is real (the proton mass is known to 10 significant figures) and too small to reflect a missing mechanism of order C_q. We regard it as the first-order correction from higher-order overflow-fission terms in Primitive V; expressing it in terms of the registry integers $\{137, 136, 17, 3\}$ is a well-posed open problem (Table V, item 1).

C. Constituent Confinement Unit

The natural mass unit of the substrate is the *constituent confinement unit*, the proton mass divided equally among the three substrate nodes:

$$C_q = \frac{m_p}{3} = \frac{136 \times 3^2}{2}\, m_e = 612\, m_e \approx 313 \text{ MeV}. \tag{18}$$

This is a derived quantity, not a prediction against specific experiment. It is the effective mass per confined substrate node — the RMR analogue of the constituent quark mass. It appears in the strange sector below.

V. THE STRANGE SECTOR AND STRUCTURAL EXTENSION

Extending the framework to the strange baryon sector requires two structural postulates. They introduce no new continuous parameters — their numerical content is fixed entirely by registry integers already established in Secs. II B and II C — but they have not been derived from Primitives I–V. All results conditional on them are labeled [**T2**]. A reader who accepts the base primitives and rejects Axioms C and D retains the five [**T1**] predictions; the two logical stances are cleanly separable.

A. Axiom C: Flavor Scaling

Axiom 1 (Flavor Scaling)**.** The unit strangeness excitation energy in the substrate is

$$\delta_s \equiv 17^2\, m_e = 289\, m_e \approx 147.7 \text{ MeV}. \tag{19}$$

Each unit of strangeness content adds δ_s to the baseline baryon mass.

The integer 17 is the substrate prime of Primitive II. Axiom C postulates the rule $\delta_s = 17^2\, m_e$ — one squared generation prime per strangeness unit — rather than $17\, m_e$ or any other combination. It introduces no new number; the specific rule is what is postulated. The factor of 17^2 rather than 17 suggests a two-step substrate process (one factor for the generation transition, one for the strangeness direction), but this interpretation has not been formalized. Deriving Axiom C from Primitive V is a well-posed open problem (Table V, item 2).

B. Axiom D: Spin-Parity Promotion

Axiom 2 (Spin-Parity Promotion)**.** The energy cost of a spin-parity promotion from the $J^P = \frac{1}{2}^+$ octet to the $J^P = \frac{3}{2}^+$ decuplet is

$$\Delta m_{\text{spin}} = \frac{\delta_s}{f_3} = \frac{289\, m_e}{1/2} = 2\,\delta_s = 578\, m_e \approx 295.4 \text{ MeV}. \tag{20}$$

The factor $f_3 = 1/2$ is the K_3 beat frequency of Primitive III. Axiom D says the spin-promotion energy equals one beat *period* (the inverse of the beat frequency) of strangeness energy. Its numerical content — $578\, m_e$ — is completely determined by δ_s and f_3, both established without Axiom D. The specific rule $\Delta m_{\text{spin}} = \delta_s/f_3$ is what is postulated.

C. Strange Baryon Predictions

Three predictions follow by arithmetic from Axioms C and D.

a. Mean decuplet mass spacing. Consecutive rows of the baryon decuplet differ by one unit of strangeness, so by Axiom C:

Result 4 (**[T2]** Mean decuplet mass spacing)**.**

$$\bar{\Delta}_{\text{dec}} = \delta_s = 17^2\, m_e \approx 147.7 \text{ MeV}. \tag{21}$$

Experiment [1]: mean spacing ≈ 146.8 MeV (error 0.6%).

b. Δ(1232) *mass.* The Δ(1232) is the lightest decuplet state ($S = 0$). It requires one spin promotion from the proton (Axiom D) and zero strangeness units:

Result 5 (**[T2]** Δ(1232) mass)**.**

$$m_\Delta = m_p + \Delta m_{\text{spin}} = m_p + 2\,\delta_s = (1836 + 578)\, m_e = 2414\, m_e \approx 1233 \text{ MeV}. \tag{22}$$

Experiment [1]: 1232 MeV (error 0.08%).

This is the second most precise prediction in the framework after the proton mass, and it is **[T2]**. The high precision reflects the fact that once Axiom D is accepted, the arithmetic is exact.

c. Ω^- *mass.* The Ω^- has strangeness $S = -3$ and $J^P = \frac{3}{2}^+$. It requires one spin promotion (Axiom D) plus three strangeness units (Axiom C):

Result 6 (**[T2]** Ω^- mass)**.**

$$m_{\Omega^-} = m_p + \Delta m_{\text{spin}} + 3\,\delta_s = m_p + 2\,\delta_s + 3\,\delta_s = m_p + 5\,\delta_s = (1836 + 1445)\, m_e = 3281\, m_e \approx 1677 \text{ MeV}. \tag{23}$$

Experiment [1]: 1672 MeV (error 0.3%).

The Ω^- prediction is the only result in the framework that gives an *absolute* particle mass rather than a ratio or spacing. The 0.3% error is consistent with subleading strangeness corrections not yet incorporated.

D. The GMO Relation as Structural Identity

The Gell-Mann–Okubo mass formula for the baryon octet [6, 7] is, at leading order in flavor symmetry breaking:

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma. \tag{24}$$

This relation holds experimentally to better than 0.5%. In RMR, it is a structural identity that follows algebraically from the linear flavor-scaling architecture of Axiom C. For any mass formula of the form $m = m_0 + n_s\,\delta_s$ (where n_s is the strangeness number):

$$m_N = m_p + 0 \cdot \delta_s, \quad m_\Xi = m_p + 2\,\delta_s,$$
$$m_\Lambda = m_\Sigma = m_p + \delta_s + (\text{isospin corrections}).$$

Substituting into Eq. (24):

$$2(m_p + 0 + m_p + 2\delta_s) = 3(m_p + \delta_s) + (m_p + \delta_s) \implies 4m_p + 4\delta_s = 4m_p + 4\delta_s. \quad \checkmark$$

The GMO relation holds as an algebraic identity for any linear strangeness formula; it is not an independent test of Axiom C, but a confirmation that the linear scaling structure is consistent with the approximate SU(3) flavor symmetry known to hold at leading order.

TABLE III. Strange baryon sector. All predictions are **[T2]**. PDG [1] experimental values.

Quantity	Formula	Predicted	Experimental	Error	Tier
$\bar{\Delta}_{\text{dec}}$	$17^2\,m_e$	147.7 MeV	146.8 MeV	0.6%	**[T2]**
$m_{\Delta(1232)}$	$m_p + 2\delta_s$	1233 MeV	1232 MeV	0.08%	**[T2]**
m_{Ω^-}	$m_p + 5\delta_s$	1677 MeV	1672 MeV	0.3%	**[T2]**
GMO rel.	identity	—	$\lesssim 0.5\%$	—	identity

VI. THE UNIFIED LEDGER

Table IV assembles all eight quantitative predictions. Three observations are worth reading off the complete ledger.

First, the five **[T1]** results span six orders of magnitude in predicted quantity — from the dimensionless ratio 17 to 1836 — and all agree with experiment to better than 1.1% with no tunable continuous parameters.

Second, the three **[T2]** results are equally precise (0.08–0.6%). The tier label describes *logical provenance*, not predictive quality: once Axioms C and D are accepted, the arithmetic is as tightly constrained as in the **[T1]** sector.

Third, every row uses only integers from the set $\{137, 136, 17, 3, 5\}$. No row introduces a number not present in the five primitives and the prime factorization of 136. The integer 5 appears as the graph order of K_5; the rest appear in Primitive II.

TABLE IV. Complete prediction ledger. Eight quantitative predictions with no tunable continuous parameters. **[T1]**: derived from Primitives I–V. **[T2]**: derived given Axioms C and D. The GMO entry is a structural identity. Experimental values: PDG [1] and NuFIT 5.3 [2].

Quantity	Sector	Formula	Eq.	Predicted	Experimental	Error	Tier
m_μ/m_e	Lepton	$3 \times 137/2$	(9)	205.50	206.77	0.61%	**[T1]**
m_τ/m_e	Lepton	$3 \times 137 \times 17/2$	(10)	3493.50	3477.23	0.47%	**[T1]**
m_τ/m_μ	Lepton	17	(11)	17.000	16.817	1.07%	**[T1]**
$\Delta m^2_{32}/\Delta m^2_{21}$	Neutrino	$3^2 + 5^2 - 1$	(16)	33.0	32.8 ± 1.3	0.6%	**[T1]**
m_p/m_e	Baryon	$136 \times 3^3/2$	(17)	1836	1836.153	0.008%	**[T1]**
$\bar{\Delta}_{\text{dec}}$	Strange	$17^2\, m_e$	(21)	147.7 MeV	146.8 MeV	0.6%	**[T2]**
$m_{\Delta(1232)}$	Strange	$m_p + 2\delta_s$	(22)	1233 MeV	1232 MeV	0.08%	**[T2]**
m_{Ω^-}	Strange	$m_p + 5\delta_s$	(23)	1677 MeV	1672 MeV	0.3%	**[T2]**
GMO	Strange	$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma$	(24)	identity	$\lesssim 0.5\%$	—	identity

A. Open Questions Catalog

Table V catalogs the questions the framework cannot yet answer. We classify each as **W** (well-posed: derivation route partially visible within the current architecture) or **E** (extension: new primitives or structural additions required). The four well-posed questions have specific mechanistic routes through Primitives III and V; they are the theoretical program's next targets. The seven extension questions bound the framework's current scope honestly: RMR predicts *ratios and spacings*, not absolute mass scales or flavor mixing parameters.

TABLE V. Open questions. **W** = well-posed within current architecture. **E** = structural extension required.

Question	What it requires	Class
Well-posed: derivation route visible		
Proton remainder $\delta = 0.153\, m_e$	Next-order overflow-fission correction	W
Axiom C: $\delta_s = 17^2 m_e$	Strange overflow mode at energy $17^2 m_e$	W
Axiom D: $\Delta m_{\text{spin}} = \delta_s / f_3$	K_3 beat period to spin-parity gap	W
Weinberg angle $\sin^2\theta_W = 3/13$	Derivation from Primitive V	W
Extension: new structure required		
Absolute electron mass scale	Why m_e takes its value	E
Absolute neutrino mass scale	$R = 33$ predicted; overall scale is not	E
Neutrino mixing angles	Quantitative angles need additional structure	E
Λ–Σ splitting	Second-order SU(3) breaking	E
Excited baryon spectrum	Higher-order overflow-fission modes	E
Quark mixing (CKM)	No flavor-mixing mechanism in framework	E
CP violation	No complex phase in registry architecture	E

VII. DISCUSSION

A. Structural Coherence Across Sectors

The principal finding of this paper is not any individual prediction but the architectural coherence behind all of them. The same integer set $\{137, 136, 17, 3\}$ is sufficient for every row in Table IV. The charged lepton ratios use 137, 17, and 3. The proton mass uses 136 and 3. The neutrino splitting uses $3^2 + 5^2 - 1 = 33$, in which $3^2 + 5^2 = 34 = 2 \times 17$ carries 17 through delocalization rather than interface coupling. The strange baryon spacings use 17^2 and 3. No sector introduces a new integer.

Two structural results emerge that were not design targets. The GMO baryon octet relation holds as an algebraic identity under linear flavor scaling (Axiom C): the framework does not appeal to SU(3) group representation theory, only to integer strangeness counting. Whether this reflects a deeper connection between the discrete registry and continuous flavor symmetry is an open question. The Weinberg angle observation ($\sin^2\theta_W = 3/13 = 0.2308$, experiment 0.23122, discrepancy 0.2%) has the same character: it follows from graph quantities fixed independently in the lepton sector, was not designed to appear, and is precise enough (0.2%, two small integers) to be physically significant without yet being a derived prediction. Both observations — GMO and the Weinberg angle — sit at the productive boundary between emergence and derivation.

B. Derivation Targets for Axioms C and D

Axiom C ($\delta_s = 17^2\, m_e$) requires identifying strangeness as a specific substrate overflow mode in Primitive V. A derivation would show that the lightest flavor excitation of a confined K_3 triplet costs exactly $17^2\, m_e$ — that the minimum strange-flavor overflow event involves a two-step substrate process (one factor of 17 for the generation transition, one for the strangeness direction in the substrate), producing 17^2 rather than 17 as the energy unit. This connects the substrate factorization $136 = 8 \times 17$ to the energy quantization of flavor excitations systematically.

Axiom D ($\Delta m_{\text{spin}} = \delta_s / f_3$) requires connecting the K_3 beat frequency to the octet-to-decuplet energy gap. In the graph resonance language of Primitive III, the decuplet occupies a higher eigenmode of the K_3 cavity than the octet ground state. A derivation would show that this eigenmode gap equals exactly one beat period of δ_s — that the K_3 spectrum and the strangeness energy scale are commensurate in that ratio.

Neither derivation requires new physical input beyond what is already in Primitives III and V. Both are tractable research targets within the existing architecture.

C. Falsification: The Near-Term Tests

The framework's two most direct experimental falsification tests come from the neutrino sector prediction $R = 33$.

a. JUNO (2027–2031). The Jiangmen Underground Neutrino Observatory [3] will measure the neutrino mass hierarchy to $> 3\sigma$ and the splitting ratio R to approximately 1% precision within six years. The RMR framework requires:

- Normal mass hierarchy. An inverted hierarchy determination at $> 3\sigma$ falsifies the framework.

- $R = 33.0$. A measurement $R < 30$ or $R > 36$ (approximately 3σ from the prediction given current errors) falsifies the framework.
- $m_1 \approx 0$. A measurement $m_1 > 10$ meV with confirmed normal hierarchy places the framework under severe tension.

b. DUNE and Hyper-Kamiokande. DUNE [4] and Hyper-K [5] provide independent hierarchy and R measurements on similar timescales with complementary beam geometries and systematic uncertainties. The combination of three independent experiments testing one integer prediction makes the neutrino sector the most tightly falsifiable test of the RMR program.

D. On the Three-Tier Accounting

Axioms C and D could have been presented as "natural consequences" of the substrate factorization $136 = 8 \times 17$. They are not, because that phrase implies derivability, and they are not currently derivable. Sec. VII B states precisely what each derivation requires. Until those derivations exist, the label **[T2]** is correct and the reader deserves to know it.

The open questions catalog (Table V) is a research map, not a list of failures. The four well-posed questions are the framework's next results; they have specific mechanistic routes and are actively being pursued. The seven extension questions define the framework's current scope boundary honestly. A theory that obscures the distinction between what it derives and what it postulates is harder to falsify and harder to extend. Both the derivations and the open questions are stated with the same care as the predictions, because all three are part of the same scientific enterprise.

VIII. CONCLUSION

Relational Mathematical Realism derives eight quantitative predictions with no tunable continuous parameters from five structural primitives and a single 137-element discrete integer registry. All predictions in Table IV were derived from the framework's integer structure before consulting the experimental values used for comparison; no parameter was adjusted post-hoc to improve agreement. The same four integers — 137, 136, 17, and 3 — appear without modification in every sector. Five predictions are [**T1**], following strictly from the primitives; three are [**T2**], conditional on two structural postulates whose numerical content is fixed by the same integers. The Gell-Mann–Okubo relation emerges as a structural identity. All eight predictions agree with experiment to better than 1.1%.

The three most structurally revealing predictions, placed side by side, are:

$$\frac{m_p}{m_e} = \frac{136 \times 3^3}{2} = 1836, \qquad \frac{m_\mu}{m_e} = \frac{3 \times 137}{2} = 205.5, \qquad R = 3^2 + 5^2 - 1 = 33. \tag{25}$$

The same architecture. Three sectors. No free parameters.

JUNO, DUNE, and Hyper-K will provide decisive falsification tests of the integer neutrino prediction $R = 33$ within the coming decade. The four well-posed open questions — deriving the sub-unit proton remainder, the flavor scaling postulate, the spin-promotion postulate, and the Weinberg angle from the base primitives — define the theoretical program's next chapter.

ACKNOWLEDGMENTS

The author thanks Claude (Anthropic) for assistance with mathematical derivations.

[1] S. Navas *et al.* (Particle Data Group), "Review of Particle Physics," Phys. Rev. D **110**, 030001 (2024).

[2] I. Esteban, M. C. González-García, M. Maltoni, T. Schwetz, and A. Zhou, "The fate of hints: updated global analysis of three-flavor neutrino oscillations," J. High Energy Phys. **2020**, 178 (2020); NuFIT 5.3 (2024), http://www.nu-fit.org.

[3] A. Abusleme *et al.* (JUNO Collaboration), "JUNO physics and detector," Prog. Part. Nucl. Phys. **123**, 103927 (2022).

[4] B. Abi *et al.* (DUNE Collaboration), "Long-baseline neutrino oscillation physics potential of the DUNE experiment," Eur. Phys. J. C **80**, 978 (2020).

[5] K. Abe *et al.* (Hyper-Kamiokande Collaboration), "Hyper-Kamiokande design report," arXiv:1805.04163 (2018).

[6] M. Gell-Mann, "Symmetries of baryons and mesons," Phys. Rev. **125**, 1067 (1962).

[7] S. Okubo, "Note on unitary symmetry in strong interactions," Prog. Theor. Phys. **27**, 949 (1962).

[8] K. Kuratowski, "Sur le problème des courbes gauches en topologie," Fundam. Math. **15**, 271 (1930).

[9] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), "Evidence for an anomalous muon neutrino deficiency in Super-Kamiokande," Phys. Lett. B **433**, 9 (1998).

[10] Q. R. Ahmad *et al.* (SNO Collaboration), "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory," Phys. Rev. Lett. **89**, 011301 (2002).

[11] I. Esteban, M. C. González-García, M. Maltoni, T. Schwetz, and A. Zhou, "The fate of hints: updated global analysis of three-flavor neutrino oscillations," J. High Energy Phys. **2020**, 178 (2020).

[12] Y. Koide, "A fermion-boson composite model of quarks and leptons," Phys. Lett. B **120**, 161 (1983).

[13] J. Merwin, "Geometric Origin of Fundamental Constants: Thirty Derivations from Discrete Relational Structure and the Substrate-Interface Duality," ai.viXra.org:2601.0081 (2026).

[14] J. Merwin, "Emergent Four-Force Dynamics from a Discrete 137-Element Registry: Gravity, Electromagnetism, Strong, and Weak Interactions via Causal Integer Lattice Simulation" ai.viXra.org:2603.0003