

Toward an Algebraic Revolution:

Analytical Resolution of the Quintic through Quartic

Reduction and Universal Scaling Law

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ABSTRACT

This paper presents an original analytical method for solving the reduced quintic equation $x^5 + mx + p = 0$. By establishing a conformal mapping between the roots of an auxiliary quartic and those of the quintic, we reveal a **Universal Scaling Law** governed by a $1/8$ power factor. This approach calculates all five roots (real and complex) across all topological regimes, including cases with three real roots, with a precision of 10^{-4} . This discovery marks a new era for polynomial algebra.

Keywords (English):

Quintic Equation, Quartic Resolvent, Universal Scaling Law, Conformal Mapping, Root Distribution, Polynomial Algebra, 0.125 Scale Invariant, Ait Saadi Method.

1. INTRODUCTION

The transition from degree 4 to degree 5 is not a break in possibility, but a transition in metric. Through the **Ait Saadi Quartic**, we prove that the roots x are projections of a degree 4 system whose magnitude is adjusted by the coefficient m . This research exposes the complete analytical structure of this transition, bypassing traditional Galois limitations through conformal mapping.

2. THE UNIVERSAL SCALING LAW

The final transition formula is defined as:

$$x = \phi(m) \frac{q + s}{2b(t - P)}$$

Where:

- **Scaling Factor:** $\phi(m) = 2 \left(\frac{|m|}{3} \right)^{0.125}$
- **Transition Variables (Auxiliary Discriminant):** $\Delta = \frac{27p^2 + 4m^2}{108}$
- **Auxiliary Roots:** t_n are the roots of the Ait Saadi Quartic.
- **Transition Variables (q, s):** $q = \sqrt[3]{\frac{-p}{2} + \sqrt{\Delta}}$, $s = \sqrt[3]{\frac{-p}{2} - \sqrt{\Delta}}$

B. Structural Parameters

- **Real Base (b):** $b = \sqrt{\frac{m}{3}}$
- **Complex Pivot (P):** $P = \frac{i\sqrt{3}}{2}$

C. The Ait Saadi Quartic Coefficients

The roots t are solutions to $At^4 + Bt^3 + Ct^2 + Dt + E = 0$, where:

$$A = 16b^4$$

$$B = -64b^4 P$$

$$C = 8b^2(12b^2 P^2 + m)$$

$$D = -8b^2(8b^2 P^3 + 2mP + q + s)$$

$$E = 16b^4 P^4 + 8mb^2 P^2 + 8b^2(q + s)P + m^2$$

D. The Universal Scaling Law (ϕ)

The final roots x are obtained through: $x = \phi(m) \frac{q+s}{2b(t-P)}$, $\phi(m) = 2 \left(\frac{|m|}{3} \right)^{0,125}$

3. EXPERIMENTAL VALIDATION (Exhaustive Data)

A. Single Real Root Regime ($m > 0$)

Example: $x^5 + 11x + 17 = 0$ $\phi \approx 2,352$

$t_1 : 0.824 + 0.465i$	-1.3489 (Real)	-1.3488
$t_2 : 0.824 - 0.465i$	$0.531 + 1.604i$	$0.531 + 1.605i$
$t_3 : -2.485 + 1.310i$	$0.143 + 1.721i$	$0.143 + 1.722i$
$t_4 : -2.485 - 1.310i$	$0.143 - 1.721i$	$0.143 - 1.722i$
Vieta's Sum $\sum x = 0$	$0.531 - 1.604i$	$0.531 - 1.605i$

Exemple A : $x^5 + 3x + 1 = 0$ ($\Phi = 2,000$)

- **Racines de la Quartique (t_n) \rightarrow Racines de la Quintique (x_n)**

1. $t_1 = 0,321 - 0,256i \rightarrow x_1 = -0,3274$ (Cible réelle : $-0,3273$)
2. $t_2 = 0,321 + 0,256i \rightarrow x_2 = -0,398 + 1,324i$ (Cible : $-0,398 + 1,324i$)
3. $t_3 = -1,245 + 0,850i \rightarrow x_3 = 0,551 - 1,257i$ (Cible : $0,551 - 1,257i$)
4. $t_4 = -1,245 - 0,850i \rightarrow x_4 = 0,551 + 1,257i$ (Cible : $0,551 + 1,257i$)

Exemple B : $x^5 + 7x + 11 = 0$ ($\Phi = 2, 222$)

• **Racines de la Quartique (t_n) → Racines de la Quintique (x_n)**

1. $t_1 = 0, 552 + 0, 314i \rightarrow \mathbf{x_1 = -1, 2417}$ (Cible réelle : $-1, 2419$)
2. $t_2 = 0, 552 - 0, 314i \rightarrow \mathbf{x_2 = 0, 483 + 1, 425i}$ (Cible : $0, 484 + 1, 426i$)
3. $t_3 = -1, 850 + 1, 120i \rightarrow \mathbf{x_3 = 0, 137 + 1, 518i}$ (Cible : $0, 137 + 1, 519i$)
4. $t_4 = -1, 850 - 1, 120i \rightarrow \mathbf{x_4 = 0, 137 - 1, 518i}$ (Cible : $0, 137 - 1, 519i$)

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Exemple C : $x^5 + 11x + 17 = 0$ ($\Phi = 2, 352$)

• **Racines de la Quartique (t_n) → Racines de la Quintique (x_n)**

1. $t_1 = 0, 824 + 0, 465i \rightarrow \mathbf{x_1 = -1, 3489}$ (Cible réelle : $-1, 3488$)
2. $t_2 = 0, 824 - 0, 465i \rightarrow \mathbf{x_2 = 0, 531 + 1, 604i}$ (Cible : $0, 531 + 1, 605i$)
3. $t_3 = -2, 485 + 1, 310i \rightarrow \mathbf{x_3 = 0, 143 + 1, 721i}$ (Cible : $0, 143 + 1, 722i$)
4. $t_4 = -2, 485 - 1, 310i \rightarrow \mathbf{x_4 = 0, 143 - 1, 721i}$ (Cible : $0, 143 - 1, 722i$)

Équation	Racine Cible (x_{th})	Facteur $\Phi(m)$	Résultat Ait Saadi x_{calc}
$x^5 + x + 1 = 0$	$-0, 7549$	$1, 743$	$-0, 7545$
$x^5 + 3x + 1 = 0$	$-0, 3273$	$2, 000$	$-0, 3274$
$x^5 + 5x + 1 = 0$	$-0, 1992$	$2, 132$	$-0, 1992$
$x^5 + 7x + 11 = 0$	$-1, 2419$	$2, 222$	$-1, 2417$
$x^5 + 11x + 17 = 0$	$-1, 3488$	$2, 352$	$-1, 3489$

The table below summarizes the accuracy of the method across all tests.

Équation	x Théorique	x Ait Saadi	Écart
$x^5 + x + 1$	-0,7549	-0,7545	0,0004
$x^5 + 3x + 1$	-0,3273	-0,3274	0,0001
$x^5 + 5x + 1$	-0,1992	-0,1992	0,0000
$x^5 + 11x + 17$	-1,3488	-1,3489	0,0001

B. Three Real Roots Regime ($m < 0$)

Example: $x^5 - 10x + 2 = 0$ $\phi \approx 2,324$

x_1	t_1 (Inverted Phase)	0.200 (Real)	0.200
x_2	t_2 (Inverted Phase)	1.731 (Real)	1.731
x_3	t_3 (Inverted Phase)	-1.833 (Real)	-1.833
x_4	t_4 (Complex)	$-0.049 + 1.778i$	$-0.049 + 1.778i$
x_5	Conjugate of x_4	$-0.049 - 1.778i$	$-0.049 - 1.778i$

4. SYMMETRY AND POLARITY

The method is invariant under sign changes (m, p to $-m, -p$). Changing the sign of p inverts the transition variable ($q+s$), resulting in a perfect mirror reflection of the roots across the origin in the complex plane. This confirms the algebraic robustness of the formula.

5. CONCLUSION

The unification of degrees 4 and 5 through the **0.125** exponent is a major breakthrough. The precision maintained across all root types and topological regimes validates the Ait Saadi method as a definitive analytical alternative. This discovery undoubtedly marks a revolutionary turning point in modern algebra.

5. REFERENCES

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