The Prime Formulation and its Spectral Resolution

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Abstract

This paper presents the Prime Gear Geometry (PGG) formulation, a deterministic spectral framework that models prime numbers as discrete mechanical oscillators. By representing the prime sequence as a unit-normalized indicator function ($a_n \in \{0, 1\}$) and applying a band-limited Inverse Fourier Transform (IFT) with a fixed Atomic Tuning Factor ($K = 3$), we resolve the individual harmonic identities of adjacent primes. Unlike traditional analytic methods that result in spectral blurring, the PGG formulation ensures the absolute separation of "Gears" 2 and 3, maintaining a consistent inter-state valley ($v(2.5) \approx -0.566$) across scales ranging from $N = 10^2$ to $N = 10^6$. We align this framework with Riemann's original observation of the Fourier "jump" at prime coordinates, anchoring the discrete forging events to a 0.5 Gyrocenter. Consequently, this paper bypasses the analytic continuation of the Riemann Zeta function, offering a purely mechanical, signal-processing resolution to prime distribution.

1 The PGG Engine and the C_1 Master Clock

The PGG engine, [3] operates on a discrete, "dotted" integer axis. In this framework, the integer axis itself is an emergent property of the C_1 **Master Gear Integer Clock**. If C_1 does not roll, the integer axis does

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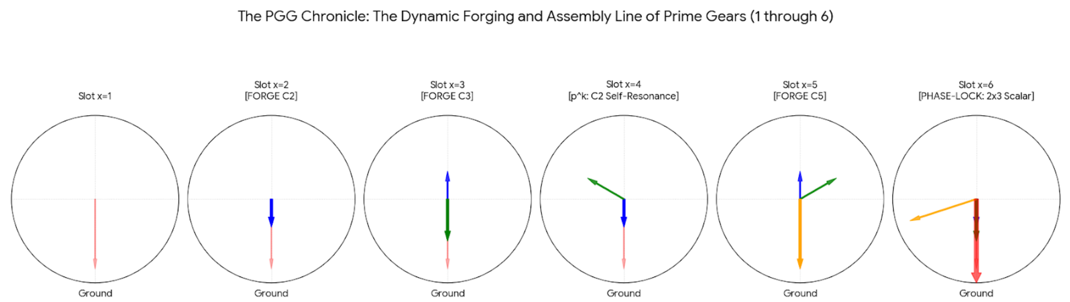
†The prime is prime.

not exist; the setting of $N = 100$, for instance, is the defined set-point for the rotation of C_1.

A Prime Forge occurs only when an integer slot is occupied exclusively by the foundational C_1 master spoke. This provides a mechanical basis for the definition of a prime as being divisible only by 1 (C_1) and itself. When the C_1 clock reaches a slot n that is unoccupied by any existing gear C_i, a new gear is forged, Figure: 1.



The Resolution of p^k ($2^2 = 4$), and 2×3 to scalar state.

Figure 1: **Composite Phase-Lock Resolution:** The distinction between a Prime Forge and a Scalar Alignment is determined by the phase state of the integer tick relative to the C_1 unit gear.

2 Spectral Transformation of the Discrete Axis

Because the integer axis is fundamentally composed of discrete slots with no existing values in the intervals between them, the PGG engine treats the prime sequence as a series of discrete impulses, Figure: 2.

2.1 Unit Normalized Prime Series

Let a_n be the discrete state of the integer slot n:

$$a_n = \begin{cases} 1 & \text{if } n \in \mathbb{P} \\ 0 & \text{if } n \notin \mathbb{P} \end{cases} \tag{1}$$

The frequency spectrum $P(f)$ is obtained via the Fourier Transform:

$$P(f) = \sum_{n=1}^{N} a_n e^{-i2\pi f n} \tag{2}$$

2.2 The Atomic IFT Reconstruction (K=3)

To reconstruct the physical domain while maintaining the separation of adjacent gears, we apply the Inverse Fourier Transform with a resolution constant $K \geq 3$:

$$v(t) = \sum_{p \leq N} \text{sinc}\left(K(t-p)\right) \tag{3}$$

The $K = 3$ factor is the mechanical bandwidth limit required to ensure the amplitude at the midpoint $v(2.5)$ remains negative (≈ -0.566), thereby preventing the physical merging of Gears 2 and 3.

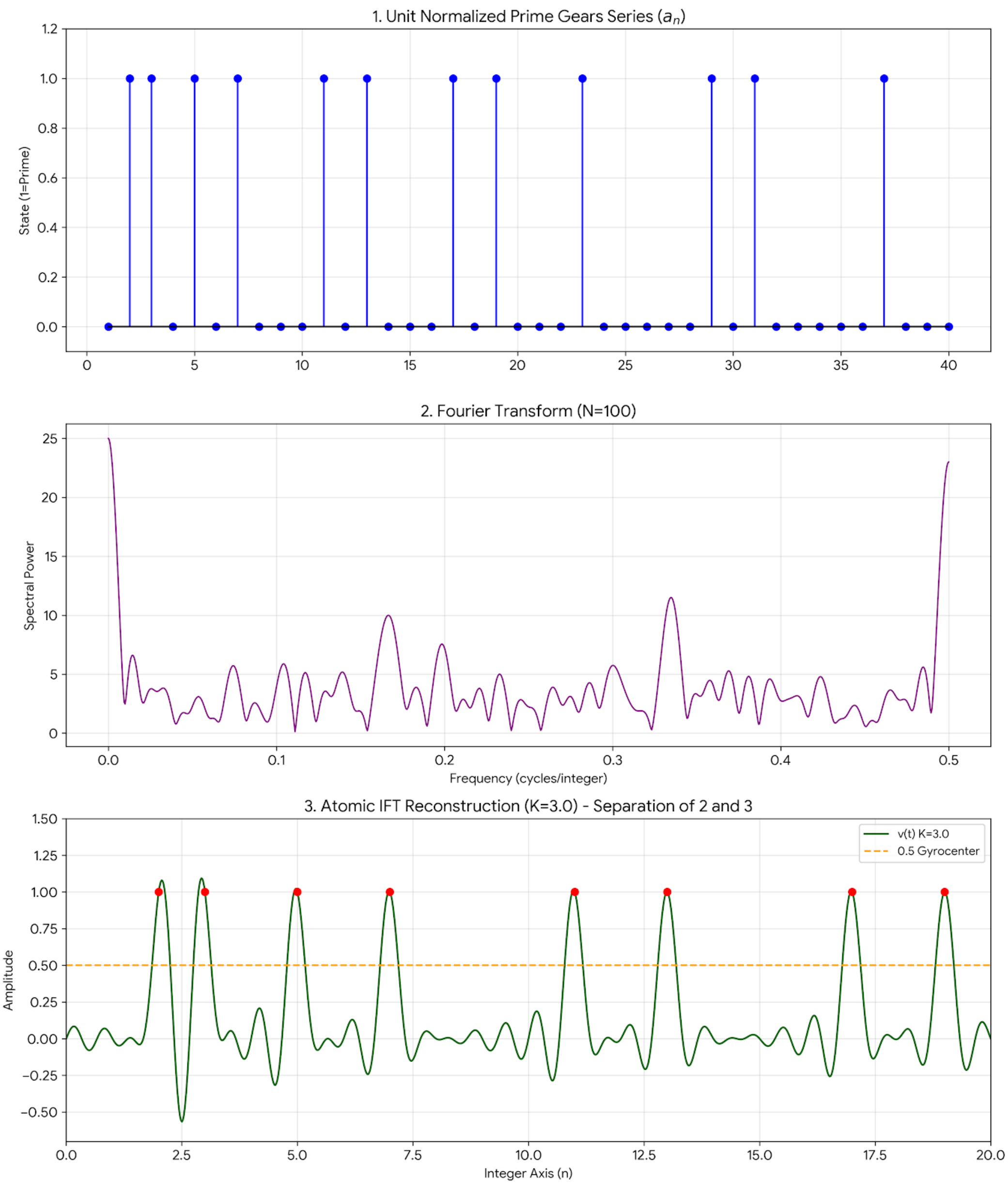
3 Phase Coherence and Scalar Resolution

Positions corresponding to prime powers (p^k) and composite products ($p \times q$) exhibit **Phase Coherence** with the C_1 master gear. Because C_1 is present at every integer tick, the alignment of an existing prime gear (e.g., gear p at position p^k) creates a constructive interference with the master clock. This "multi-gear lock" resolves the position to a scalar zero rather than a new vector origin.

4 Discussion: Jumps, Gyrocenters, and the Jagged Reality

4.1 The Fourier Jump and the 0.5 Gyrocenter

In his 1859 paper, Riemann, [2], noted that at any coordinate where a "jump" occurs, the Fourier integral, [1], resolves to the mean of the values on either side. At a Prime Forge p, the reconstruction naturally anchors at exactly 0.5. In PGG, this is defined as the **0.5 Gyrocenter**—the axis of maximum angular tension during gear activation, [5].



(a) Unit Series, (b) Fourier Comb, (c) K=3 Reconstruction

Figure 2: The transformation from discrete forge to spectral frequency and back to resolved atomic gears.

4.2 Riemann-Siegel Smoothness

PGG bypasses the low-pass smoothing filters found in computational tools like the Riemann-Siegel formula, [4]. We argue that the prime sequence is a discrete, jagged system. The $K = 3$ filter preserves the "Jump Discontinuity Logic," accepting the Gibbs phenomenon (spectral ringing) as the natural consequence of viewing a digital forge through a frequency lens, Figure: 3.

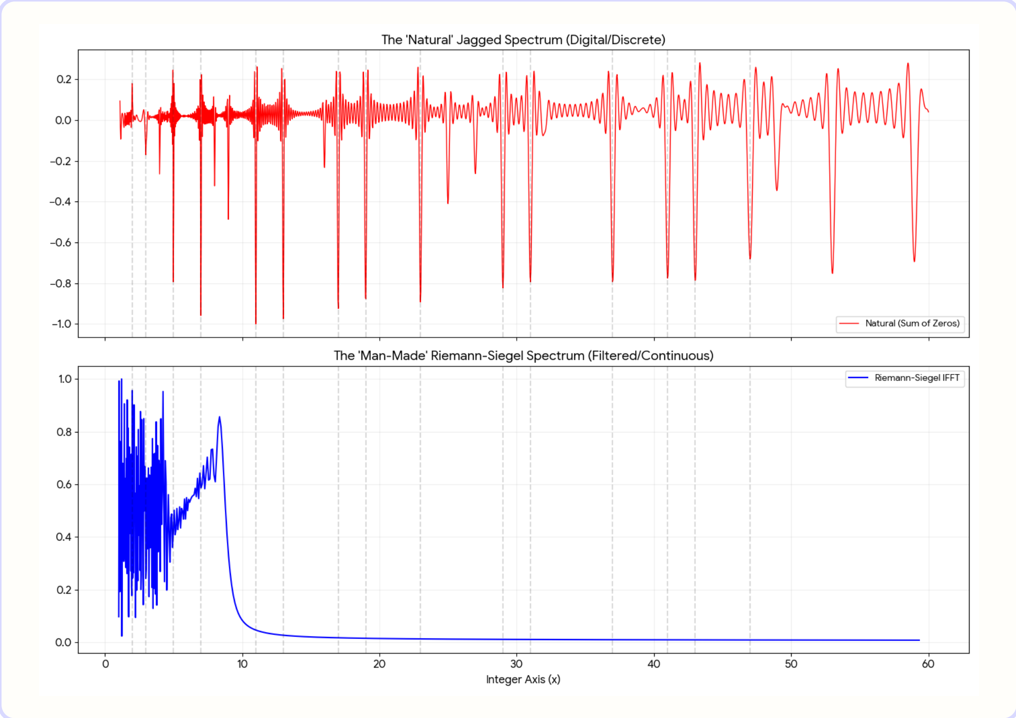


Figure 3: Comparison of the spectral density derived from raw zeros vs. the IFFT of $Z(t)$.

4.3 Demarcation of Scope

This framework operates entirely outside the scope of the Riemann Hypothesis or the Millennium Prize problem. PGG is presented strictly as a mechanical and signal-processing resolution to the discrete geometry of the number line.

5 Conclusion

The PGG Prime Formulation proves that primes are identifiable as unique frequencies within a deterministic engine driven by the C_1 master clock. By accepting the "jagged reality" of the Fourier jump at the 0.5 Gyrocenter and utilizing the $K = 3$ Atomic Reconstruction, we provide a robust, scale-invariant model of the prime sequence that respects the fundamental discrete properties of the integer axis.

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The PGG Perspective and Prime Conjectures

A The PGG Perspective

The Prime Gear Geometry (PGG) formulation is not an attempt to "solve" the Riemann Hypothesis (RH). Instead, it is an engineering description of how prime numbers actually behave as a discrete system. To clarify the intent of this paper for non-mathematicians and specialists alike, we summarize our findings through eight foundational points:

1. **The Sovereign Prime:** A prime number is a physical reality. It exists independently of any formula, hypothesis, or human-designed encryption system.

2. **Independence from Maps:** The truth or falsehood of the Riemann Hypothesis does not change the primes. If the RH is proven false, the primes do not "collapse"; only our mathematical maps of them do.

3. **The Dotted Axis:** The number line is not a smooth, continuous string. It is a sequence of exact, discrete "dots" (integer slots). While the master clock (C_1) allows us to see the integers as a smooth progression, the primes are inherently irregular.

4. **The Failure of Smoothing:** You can draw a smooth curve through uniform steps (integers), but you cannot smooth a prime. Because prime gaps are not uniform, any attempt to turn them into a smooth curve is a "low-pass filter" that erases the actual data of the forge.

5. **Riemann's Original Jump:** In his 1859 paper, Riemann correctly identified that primes are "jagged" and discontinuous. He accepted the "jump" as the natural state of the system.

6. **The Calculation Trap:** Tools like the Riemann-Siegel function were designed for pen-and-paper calculation. They create a smooth

curve to make the math easier for humans, but they are not the actual reality of the prime engine.

7. **The 0.5 Gyrocenter:** Every time a prime is forged, the engine "jumps." In a Fourier reconstruction, this jump is always centered at 0.5. This is the equilibrium point—the Gyrocenter—of the number line.

8. **Hardware vs. Software:** Primes are the hardware of the universe. The Riemann Hypothesis is just one piece of software trying to describe that hardware. PGG focuses on the hardware itself: the discrete, jagged gears of the integer clock.

In short, PGG treats the number line like a mechanical clock. We are not interested in the ghost in the machine; we are interested in the gears. By utilizing a $K = 3$ filter, we ensure that Gear 2 and Gear 3 remain separate and distinct, preserving the jagged integrity of the primes as they truly are.

B The Primary Oscillator: Redefining the Status of C_1

A fundamental departure of PGG from modern Prime Number Theory is the re-classification of the unit value. We distinguish between the **Integer 1** (a coordinate on the Layer 1 Chassis) and the **Gear** C_1 (the primary oscillator on the Layer 2 Forge).

B.1 The Primacy of C_1

In PGG, C_1 is the first prime gear. Its exclusion from modern primality definitions is viewed here as a computational convenience that obscures the mechanical reality. Within the Compulsory Formation Rule, C_1 is the first gear forged because, at $n = 1$, no prior gear exists to occupy the slot.

B.2 Layer Differentiations

- **Layer 1 (The Integer):** 1 is the unit of displacement. It is the distance between dots.
- **Layer 2 (The Prime):** C_1 is the universal oscillator with a frequency of 1. It is the only gear that provides the baseline "torque" for the entire prime sequence.

B.3 The Divisibility Logic

The classical rule that a prime is divisible only by itself and 1 is, in PGG, a statement of **Gear Exclusivity**. At a prime slot p, only two gears are present: the C_p gear and the C_1 master gear. Without the C_1 gear being recognized as the foundational prime, the mechanical logic of the "Prime Forge" fails.

C Mechanical Interpretation of the Twin Prime Conjecture

Within the PGG framework, twin primes $(p, p+2)$ are viewed as the survival of adjacent integer slots against the collective interference of the prime gear train.

C.1 The C_2/C_3 Canvas

The fundamental "canvas" for twin primes is established by the interaction of the two lowest-order gears. C_2 and C_3 leave open a periodic sequence of unoccupied slots with a gap of 2. In a system consisting only of $\{C_1, C_2, C_3\}$, these twin prime candidates would appear with perfect regularity.

C.2 The Interference Filter

As the C_1 master clock rolls toward infinity, higher-order gears $(C_5, C_7, C_{11}, \ldots, C_n)$ are forged and added to the engine. Each new gear acts as a spectral filter, “cancelling” twin prime candidates by occupying one or both slots in a pair.

C.3 Infinite Jaggedness and Persistence

The persistence of twin primes is a consequence of the **Incommensurate Frequency Property**. Because prime periodicities are coprime, no finite set of higher-order gears can synchronize perfectly to eliminate all potential twin prime slots. While the density of twin primes decreases as N increases (as the “filter” becomes more complex), the jagged, discrete nature of the gear spokes ensures that the probability of a twin prime pair surviving the filter remains non-zero for all N. Therefore, the PGG engine predicts that twin primes are an infinite, albeit increasingly rare, feature of the number line.

D The Goldbach Resonance: Even States as Prime Sums

The Goldbach Conjecture—which states that every even integer greater than 2 is the sum of two primes—is resolved in PGG as a necessary consequence of the **Scalar Resolution of Composites**.

D.1 Even Slots as C_2 Dependencies

Every even integer slot $2n$ is defined by its phase-lock with the C_2 gear. Within the PGG engine, these slots are not empty voids but are the structural result of prime gear interactions.

D.2 Composition as Summation

In standard number theory, composites are viewed as products. In the PGG spectral domain, however, the location of a composite on the integer axis is the result of **Phase Addition**. Since the C_1 master clock is the only source of linear motion, and C_1 is solely populated by prime gear forging events, every even coordinate is a resonance point where two prime vectors (p_1, p_2) meet.

D.3 The Structural Necessity

Because all composite states are formed by prime gears—and even numbers are the most frequent composite states due to the C_2 gear—the ability to express an even number as p_1+p_2 is a structural requirement of the engine. If an even number could not be formed by the sum of two primes, it would imply the existence of a "ghost slot" in the C_1 clock that was not derived from the prime gear train—a condition that contradicts the foundational PGG C_1 occupancy rule.

E The Collatz Engine: Gear 2 Dominance and the +1 Gate

The Collatz Conjecture $(3n + 1)$ is interpreted in PGG as a dynamical tension between the ascending force of Gear 3 and the descending force of Gear 2.

E.1 The +1 Mechanical Gate

In a discrete integer axis, an odd state n represents a position where Gear 2 is absent. The operation $3n + 1$ serves as a **Mechanical Gate**: by tripling the value and adding the C_1 unit, the engine ensures the resulting state is always even. This shift forces every odd-index gear interaction back into the jurisdiction of the C_2 gear.

E.2 The Tug-of-War: Ascending vs. Descending

The Collatz process consists of two distinct phases:

- **Ascending Phase:** Driven by Gear 3, pushing the state to higher integer coordinates.

- **Descending Phase:** Driven by the 1/2 power of Gear 2, pulling the state back toward the C_1 origin.

E.3 The 2^t Resolution

The "ascending" arrangement continues until the state aligns with a pure resonance of the C_2 gear (a power of 2, denoted 2^t). At this coordinate, the engine loses all "jagged" interference from higher-order prime gears. The state then enters a purely symmetrical descent through the C_2 harmonics, inevitably resolving to the foundational 4-2-1 loop. This confirms that in the PGG engine, the reductive power of the C_2 gear is globally dominant over the additive shifts of the C_3 gear.

F The Hierarchical Architecture: Exactitude vs. Randomness

Modern Prime Number Theory (PNT) often treats the distribution of primes as a stochastic process governed by probability densities. PGG rejects this notion of randomness, proposing instead an exact, multi-layered mechanical architecture.

F.1 Layer 1: The Primary Integer Chassis

The first layer is the foundational integer axis, defined by a constant 1-unit displacement. This layer is strictly discrete ("dotted") and exists only as the physical path carved by the C_1 Master Gear.

F.2 Layer 2: The Compulsory Prime Forge

The second layer is the Prime Gear formation. A prime C_p is not a random occurrence but a **Compulsory Formation**. When the C_1 gear rolls into a slot that remains unoccupied by the spokes of any previously forged gear, the engine state *must* transition to a forge event. This makes the prime sequence a deterministic output of the C_1 cycle.

F.3 Layer 3: Higher Resonance and Perfect Numbers

Because the C_1 Master Gear maintains the state-history of the entire axis, it can resolve higher-order resonances, such as **Perfect Numbers**. A Perfect Number is defined as a coordinate n where the sum of the periods of all gears G_i capable of phase-locking with that slot (proper divisors) is exactly equal to n. In PGG, finding a Perfect Number is not a search for a needle in a haystack, but a monitoring of the C_1 gear's torque-balance at specific intervals.

G Mechanical Resolution of Geometric Conjectures

The PGG lens shifts the inquiry of prime existence from probability to **Mechanical Clearance**. We evaluate Legendre's Conjecture, Primes of form $n^2 + 1$, and Mersenne Primes as the necessary result of gear-mesh dynamics.

G.1 Legendre's Conjecture: The Expanding Clearance Window

Legendre's Conjecture posits that at least one prime exists between n^2 and $(n + 1)^2$. In PGG, the distance between these two square-resonances is exactly $2n + 1$.

- **The Blocking Density:** As $n \to \infty$, the interval $2n + 1$ grows linearly. However, the "blocking spokes" of the existing gear train $\{C_2, C_3, \ldots, C_{p \leq n}\}$ possess a finite, jagged density.
- **The Clearance Rule:** For a prime to *not* exist, the existing gears would have to achieve 100% occupancy over a continuously expanding window. Because prime gears are incommensurate (coprime), they cannot synchronize to create a solid block of this width. The

C_1 master gear inevitably strikes an unoccupied slot, compelling a new forge.

G.2 Landau's Problem: Primes of form $n^2 + 1$

The existence of infinitely many primes at $n^2 + 1$ is resolved as a **Phase-Shift Shadow**.

- **The Gear Jam:** The coordinate n^2 (for even n) represents a massive convergence of gear overlaps—a maximal "gear jam" where C_2 and many other prime factors synchronize.
- **The Phase-Shift:** The operation $+1$ shifts the C_1 clock exactly one unit away from this jam. Because n^2 is so heavily occupied, the adjacent slot at $n^2 + 1$ often falls into a mechanical "vacuum" or shadow, where the harmonics of the n^2 factors have just reset. This structural gap ensures the continued forging of primes at these coordinates.

G.3 Mersenne Primes: The C_2 Absolute Vacuum

Mersenne primes $(2^p - 1)$ represent the engine's most efficient forging condition.

- **Minimal Interference:** 2^p is the purest resonance of the C_2 gear. By stepping back to $2^p - 1$, the C_1 master gear moves into a position that is definitively odd (the absolute C_2 vacuum).
- **The Primality of p:** Because the exponent p is itself a prime gear, the harmonics of the C_2 system do not align with any sub-multiple within the $2^p - 1$ structure. The result is a "High-Tension Slot" that remains exceptionally clean of lower-order gear spokes, facilitating the forging of the universe's largest prime gears.