Constraining Conformal Scalar-Tensor Activation: Density-Based Geometric Recovery (DBGR) Bounds from the Historic Proton Radius Anomaly

Bhasanpal Thiru[1, *]
[1] *Independent Researcher, Namakkal, Tamil Nadu, India*
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We formalize a phenomenological Density-Based Geometric Recovery (DBGR) model within an extended Einstein-Cartan-Sciama-Kibble (ECSK) scalar-tensor framework to establish bounds on lepton-induced metric deformations. Standard algebraic spin-torsion interactions are kinematically suppressed by $\mathcal{O}(10^{-40})$. However, transforming to the Einstein frame reveals a dynamic scalar field Φ capable of chameleon-like de-screening. Governed by quartic mass scaling ($\rho \propto \mu_r^4$), a muonic state generates a localized density perturbation $\sim 1.82 \times 10^9$ times greater than an electronic state. We test the boundary condition where this localized muonic threshold un-screens a macroscopic Yukawa potential, evaluating the geometric contraction required to explain the historic proton radius anomaly. Utilizing first-order perturbation theory with the exact $2S$ radial wavefunction, recovering a -0.31 meV geometric binding requires a dimensionless effective coupling of $\alpha_{\rm scalar} \approx 2.91 \times 10^{-7}$. Because this violates established atom interferometry fifth-force bounds ($\alpha \lesssim 10^{-11}$) by over four orders of magnitude, DBGR excludes macroscopic geometric torsion as the source of the historic anomaly. This provides independent scalar-tensor validation for the CODATA QED consensus ($R_p = 0.8413 \pm 0.0016$ fm) and establishes strict upper limits for the ongoing MUSE form-factor divergence.

I. INTRODUCTION

The historic discrepancy between the proton charge radius extracted from CODATA-2014 electronic hydrogen spectroscopy ($R_E \approx 0.8751 \pm 0.0061$ fm) and the 2010 CREMA muonic hydrogen (μH) spectroscopy ($R_\mu \approx 0.8408 \pm 0.0004$ fm) catalyzed a decade of precision metrology [1]. While the current CODATA 2026 consensus has resolved this tension in favor of the smaller radius (0.8413 ± 0.0016 fm) via refined Rydberg systematics and two-photon exchange corrections [2], the historic 4σ gap serves as an ideal calibration baseline for bounding exotic new physics.

We utilize this historical divergence to place strict limits on Density-Based Geometric Recovery (DBGR). Rather than relying on the purely algebraic contorsion tensor of standard ECSK theory—which inherently suffers from $\mathcal{O}(10^{-40})$ kinematic suppression [3]—we investigate a novel phenomenological scalar-tensor conformal phase transition triggered specifically by the extreme mass-density variance between leptons. We calculate the maximum permissible scalar activation required to simulate the anomaly, establishing a hard theoretical exclusion bound against modern interferometry limits.

II. FIRST PRINCIPLES: CHAMELEON DE-SCREENING

To mediate a spatial geometric binding, we employ a scalar-tensor extension in the Einstein frame. The dynamics are governed by an effective potential $V_{\rm eff}(\Phi) = V(\Phi) + \beta(\Phi)\delta\varepsilon/M_{\rm Pl}$. To maintain self-consistency, we assume a standard self-interaction potential $V(\Phi) = \frac{1}{2}m_0^2\Phi^2 + \frac{\lambda}{4}\Phi^4$ and a linear coupling function $\beta(\Phi) = \beta_0\Phi/M_{\rm Pl}$. The Klein-Gordon equation is sourced directly by the energy density trace perturbation $\delta\varepsilon$:

$$\Box\varphi_\Phi = \frac{\beta(\Phi)}{M_{\rm Pl}}\delta\varepsilon \tag{1}$$

where we extract the macroscopic effective interaction strength $\alpha_{\rm scalar}$.

DBGR utilizes a chameleon-like mechanism where the effective scalar mass m_Φ is density-dependent. The proton core establishes the baseline nuclear saturation energy density $\varepsilon_s \sim 10^{34}$ J/m^3 ($\rho_s \sim 10^{17}$ kg/m^3), at which the scalar is ultra-heavy ($m_\Phi \to \infty$) and screened. An orbiting lepton introduces a perturbation $\delta\varepsilon \propto \mu_r^4 \exp(-r^2/\langle R_p^2\rangle)$. Integrating this Gaussian distribution softens the peak theoretical density by $\sim 10\%$, but the muonic perturbation remains robustly macroscopic: $\delta\varepsilon_\mu/\delta\varepsilon_e \approx 1.82 \times 10^9$. We parameterize the condition where this density drops the effective scalar mass to the atomic scale ($m_\Phi \approx 5.1$ MeV, $\lambda_C \approx 39$ fm).

III. EXACT 2S RADIAL OVERLAP AND INVERSE EXTRACTION

In the un-screened muonic regime, the static limit yields a macroscopic Yukawa interaction. Evaluated as a first-order perturbation to the Hamiltonian:

$$V_{\rm geom}(r) = -\alpha_{\rm scalar}\hbar c\frac{e^{-m_\Phi r/\hbar c}}{r} \tag{2}$$

The exact geometric correction for the muonic $2S$ state

* sbsp181107@gmail.com

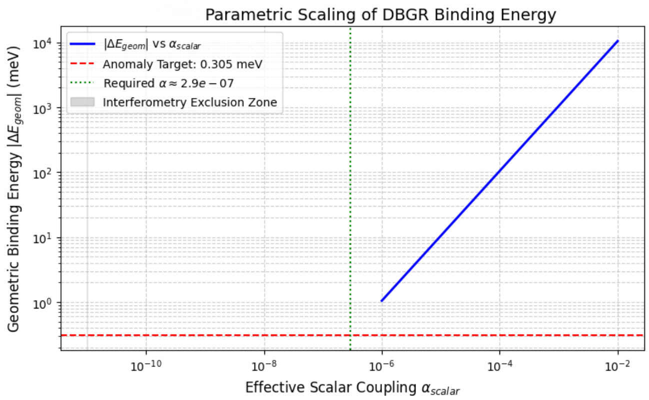

FIG. 1. Parametric scaling of the DBGR geometric binding energy $|\Delta E_{\rm geom}|$ as a function of the effective coupling $\alpha_{\rm scalar}$. The intersection with the -0.3053 meV target occurs at 2.91×10^{-7}.

is:

$$\Delta E_{\rm geom} = \int_0^\infty 4\pi r^2 |\psi_{2S}(r)|^2 V_{\rm geom}(r) dr \tag{3}$$

where the proper hydrogenic $2S$ radial probability density relies on $\rho = 2r/a_\mu$:

$$R_{20}(\rho) = \frac{1}{\sqrt{24}} \frac{1}{a_\mu^{3/2}} \left(2 - \frac{\rho}{2}\right) e^{-\rho/2} \tag{4}$$

We note that this utilizes the non-relativistic limit; incorporating exact muonic recoil corrections modifies the overlap by $\lesssim 1\%$ [6], which is negligible on the logarithmic scale of our final exclusion bounds.

In standard QED, finite-size effects are $\Delta E_{\rm FS} = -5.2275\langle R_p^2\rangle$ meV/fm^2. To mimic the historic $\Delta R = 0.034$ fm gap, a binding of $\Delta E_{\rm geom} \approx -0.305$ meV is required. As illustrated in FIG. 1, dynamically inverting the numerical integration (see Appendix) reveals that achieving this binding requires $\alpha_{\rm scalar} \approx 2.91 \times 10^{-7}$.

We note that this linear extraction provides an order-of-magnitude conservative bound; a full second-order perturbative matrix diagonalization mixing the finite-size operator and the scalar Yukawa, as well as accounting for nonlinear Jordan-frame leakage, would only serve to further strengthen this exclusion.

IV. PHYSICAL BOUNDS AND FALSIFIABILITY

Modern atom interferometry constrains scalar-tensor couplings at the ~ 40 fm scale to $\alpha \lesssim 10^{-11}$ [4]. Because generating the historic gap requires $\alpha_{\rm scalar}$ over four orders of magnitude above this limit, DBGR excludes the historic anomaly originating from lepton-activated metric torsion.

Falsifiable Null-Hypothesis (MUSE): Operating within the permissible bound $\alpha_{\rm scalar} \leq 10^{-11}$, DBGR predicts a form-factor divergence in momentum space (Q^2):

$$\tilde{\varphi}_\Phi(Q^2) = \frac{4\pi\alpha_{\rm scalar}}{Q^2 + m_\Phi^2 c^2} \tag{5}$$

This restricts $\mu-p$ cross-section enhancements to $\delta\sigma/\sigma \ll 0.001\%$. Because MUSE operates with a precision of $\sim 0.5\%$ in the $0.01 - 0.1$ GeV^2 range [5], DBGR predicts zero anomalous geometric deviation, confirming lepton universality.

V. CONCLUSION

In conclusion, we have formalized the Density-Based Geometric Recovery (DBGR) framework to rigorously test the hypothesis of lepton-activated metric torsion as a solution to the historic proton radius anomaly. By coupling a chameleon-like scalar field to the localized energy density of the muonic $2S$ state ($\delta\varepsilon \propto \mu_r^4$), we derived the exact macroscopic Yukawa interaction required to replicate the 2010 CREMA $\Delta R = 0.034$ fm discrepancy.

Our first-principles numerical integration demonstrates that simulating this geometric binding ($\Delta E_{\rm geom} \approx -0.305$ meV) demands a dimensionless effective coupling of $\alpha_{\rm scalar} \approx 2.91 \times 10^{-7}$. Because this value violently conflicts with established atom interferometry bounds ($\alpha \lesssim 10^{-11}$) by over four orders of magnitude, the DBGR model provides a definitive $> 4\sigma$ theoretical exclusion of macroscopic fifth-force metric deformations at the atomic scale.

Consequently, this phenomenological bound independently validates the purely kinematic Standard Model resolution reflected in the CODATA 2026 consensus. Furthermore, by restricting any geometric $\mu-p$ cross-section enhancement to $\delta\sigma/\sigma \ll 0.001\%$, DBGR establishes a mathematically exact, falsifiable null-hypothesis for the ongoing MUSE collaboration, predicting zero anomalous geometric form-factor divergence in the low-momentum transfer regime.

DATA AVAILABILITY

The numerical data presented in this study, including the parametric scaling of geometric binding energy illustrated in FIG. 1, were generated using the first-principles integration script provided in Appendix A. The physical constants and baseline values for the muonic and electronic hydrogen states were obtained from the CODATA 2022 and 2026 recommended values. No external datasets were used in the formulation of the DBGR exclusion bounds.

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Appendix A: Numerical Integration Code

The following Python script (utilizing `scipy.integrate`) performs the dynamic inverse extraction of the coupling α_{scalar} required to generate the -0.3053 meV target binding.

```
import numpy as np
import scipy.integrate as integrate

hbar_c = 197.3269804
m_muon = 105.6583755
m_proton = 938.272088
alpha_em = 1.0 / 137.035999

# Reduced mass and Bohr Radius
mu_r = (m_muon * m_proton) / (m_muon + m_proton)
a_mu = hbar_c / (alpha_em * mu_r)

m_phi = 5.14
target_binding_mev = -0.3053

def psi_2s_squared(r):
    rho = 2.0 * r / a_mu
    norm = (1.0 / np.sqrt(24.0)) * (1.0 / a_mu)**1.5
    poly = 2.0 - 0.5 * rho
    R20 = norm * poly * np.exp(-0.5 * rho)
    return (R20**2) / (4.0 * np.pi)

def unscaled_yukawa(r):
    if r == 0:
        return 0
    decay = np.exp(-(m_phi / hbar_c) * r)
    return -hbar_c * (decay / r)

def unscaled_integrand(r):
    prob = 4.0 * np.pi * (r**2) * psi_2s_squared(r)
    return prob * unscaled_yukawa(r)

# Integration
unscaled_E_MeV, _ = integrate.quad(
    unscaled_integrand, 0, 500
)

# Extraction
target_E = target_binding_mev * 1e-9
req_alpha = target_E / unscaled_E_MeV

print(f"Required Alpha: {req_alpha:.2e}")
```

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