Fundamental Gravitational Retarded Spacetime Shells: Quantum Scale Force Unification

A Geometric Derivation of SU(3) × SU(2) × U(1) from Retarded Discrete Shell Arrival Statistics

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Abstract

We show that the complete Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, together with gravity, emerges from the multipole decomposition of retarded discrete shell arrival statistics in three-dimensional space. The framework rests on three assumptions shared with a companion paper on cosmological applications [1]: (A1) every massive particle periodically appears in 3D space, emitting a discrete spacetime shell propagating at c; (A2) only retarded propagation is physical; (A3) observables are determined by the RMS statistics of shell arrivals.

The retarded Green's function is identified as the shell itself; the wave equation is derived as a consequence, not an assumption. The Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ is derived from the RMS average of Doppler-shifted shell arrival rates. Time dilation, relativistic velocity addition, $E = mc^2$, and $E^2 = (pc)^2 + (mc^2)^2$ all follow from shell geometry without assuming Lorentz invariance.

The monopole ($l = 0$) yields gravity from the mean arrival rate. The dipole ($l = 1$) yields electromagnetism from arrival rate covariance, with $SU(2)_L$ from dipole rotation symmetry and $U(1)_Y$ from shell emission phase. The quadrupole ($l = 2$) provides five symmetric traceless generators; retardation across three spatial rotation planes adds three antisymmetric generators, completing $\mathfrak{su}(3)$. Exactly four forces arise because $l \geq 3$ multipoles produce only contact interactions in 3D, and SU(3) is consistent only with three spatial dimensions.

Quantitative results include: the QCD string tension $\sigma = m^2_{\text{const}}\,\pi/2 = 0.180$ GeV^2 (0.3% agreement); the 125 GeV resonance mass from $m^2_H = m^2_W + m^2_Z + 2v^2\alpha = 125.14$ GeV (0.03% agreement); the electron g-factor $g = 2$ from energy conservation in the pop cycle with the Schwinger correction α/π from environmental jitter; and the

Bohr radius $a_0 = \bar{\lambda}_e/\alpha$ from proton shell spacing. The natural UV cutoff at the electron mass eliminates the need for renormalization. Particle masses and coupling constant values remain empirical inputs.

Contents

1 Introduction

The Standard Model of particle physics is built on the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, with gravity described separately by general relativity. Why this particular group? The standard answer is that it is determined experimentally. This paper demonstrates that it can also be derived geometrically: the full gauge group, including gravity, emerges from the multipole decomposition of a single physical process—the retarded propagation of discrete spacetime shells from massive particles, with observables determined by RMS arrival statistics.

The framework rests on three assumptions shared with a companion paper on cosmological applications [1].

Assumption A1 (Discrete shells at c). Every massive particle periodically appears in three-dimensional space at a characteristic frequency. Each appearance produces a discrete spherical shell of spacetime disturbance that propagates outward at c. The frequency is related to the particle's observed mass by the Compton relation, but the mass itself is an empirical input. Intervals between appearances are intervals of absence from 3D space, during which the particle's energy is conserved in local field configurations.

Assumption A2 (Retarded propagation only). Only causally forward-in-time (retarded) shell propagation is physical. Advanced solutions are excluded. This imposes a fundamental arrow of time on the shell structure.

Assumption A3 (RMS arrival statistics). At any point in space, all physical observables are determined by the root-mean-square statistics of the local shell arrival rate, computed as the superposition of all retarded contributions from the past light cone. The RMS prescription is the unique norm that preserves total power (Parseval's theorem) for independent additive contributions. A *jitter-stable* configuration is one at a local minimum of the RMS arrival rate fluctuation; stable bound states (atoms, nuclei, orbitals) correspond to jitter-stable nodes in the shell interference pattern of the constituent masses.

These assumptions introduce no new fields, extra dimensions, or additional particles. The shells are discrete geometric events. The physics emerges from counting statistics—arrival rates and their correlations—rather than from amplitude interference.

2 The Shell as Retarded Green's Function

A single shell emitted at the origin at $t = 0$ creates a disturbance that, at time t, exists only on the sphere $r = ct$. Its amplitude falls as $1/r$ from geometric dilution over area $4\pi r^2$:

$$G_{\mathrm{ret}}(r,t) = \frac{\delta(t - r/c)}{4\pi r}\,, \quad t > 0\,. \tag{1}$$

This is the retarded Green's function of the three-dimensional wave equation. In the present framework it is not derived from a field equation; it *is* the mathematical description of the physical shell.

2.1 The Wave Equation as Consequence

For any function $f(u)$ with $u = t - r/c$, define $\psi(r,t) = f(t - r/c)/r$. The Laplacian in spherical coordinates (assuming angular symmetry) gives:

$$\nabla^2\psi = \frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{d\psi}{dr}\right]. \tag{2}$$

Computing the derivatives with $f = f(t - r/c)$, where $df/dr = -f'/c$:

$$\frac{d\psi}{dr} = -\frac{f'}{cr} - \frac{f}{r^2}, \qquad r^2\frac{d\psi}{dr} = -\frac{f'r}{c} - f, \tag{3}$$

$$\frac{d}{dr}\left[-\frac{f'r}{c} - f\right] = -\frac{f'}{c} + \frac{f''r}{c^2} + \frac{f'}{c} = \frac{f''r}{c^2}. \tag{4}$$

Dividing by r^2:

$$\nabla^2\psi = \frac{f''(t - r/c)}{c^2\, r} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}. \tag{5}$$

Any spherical shell propagating at c with $1/r$ dilution automatically satisfies the wave equation. The wave equation does not generate the physics; the physics generates the wave equation.

2.2 Mass–Energy Equivalence from Shell Periodicity

A massive particle emitting shells at frequency ν produces a periodic disturbance. The fundamental mode gives $E = h\nu = mc^2$. In the shell framework, energy is proportional to shell emission rate and mass is proportional to shell emission rate. The ratio E/m has dimensions of velocity squared; the only velocity is c, giving $E = mc^2$. For a moving particle with RMS-enhanced emission rate $\gamma\nu_0$:

$$E = \gamma\, mc^2. \tag{6}$$

Planck's constant is identified as the energy per shell emission—a conversion factor between frequency and energy, determined by the geometry of one discrete spacetime disturbance.

3 Special Relativity from Shell Geometry

3.1 The Lorentz Factor from RMS Shell Statistics

An observer moving at velocity $v = \beta c$ through an isotropic shell background encounters shells at a Doppler-shifted rate:

$$f(\theta) = \frac{f_0}{1 - \beta\cos\theta}. \tag{7}$$

The RMS average over all solid angles is:

$$\langle f^2\rangle = \frac{f_0^2}{2}\int_{-1}^{1}\frac{dx}{(1-\beta x)^2} = \frac{f_0^2}{1-\beta^2} = \gamma^2 f_0^2, \tag{8}$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-v^2/c^2}}\,. \tag{9}$$

Numerical verification at $\beta = 0,\ 0.1,\ 0.3,\ 0.5,\ 0.8,\ 0.9,\ 0.95,$ and 0.99 confirms agreement with $\gamma^2 = 1/(1-\beta^2)$ to machine precision (15+ decimal places). The Lorentz factor is derived from Galilean kinematics and RMS averaging, without assuming Lorentz invariance.

3.2 Time Dilation

A particle moving at v emits a shell at event 1 and another at event 2 separated by Δt in the lab frame. By event 2, the first shell has expanded to radius $c\Delta t$. The particle is displaced $v\Delta t$ from the center of its own shell. The proper distance from shell center to particle is:

$$ds = \sqrt{(c\Delta t)^2 - (v\Delta t)^2} = c\,\Delta t\,\sqrt{1-\beta^2} = \frac{c\,\Delta t}{\gamma}\,. \tag{10}$$

This is time dilation from the geometry of a particle inside its own expanding shell. The invariant interval $ds^2 = c^2\,dt^2 - dx^2 - dy^2 - dz^2$ is the statement that shells propagate at c.

3.3 Relativistic Velocity Addition

The combined Doppler shift of shell rates gives:

$$w = \frac{u+v}{1+uv/c^2}\,. \tag{11}$$

The speed limit c is automatic: shell Doppler composition can never produce $w \geq c$ because the denominator prevents it algebraically.

3.4 The Energy–Momentum Relation

A moving particle has $E = \gamma mc^2$ and $p = \gamma mv$. Computing $E^2 - (pc)^2$:

$$E^2 = (pc)^2 + (mc^2)^2\,. \tag{12}$$

For massless particles: $E = pc$. For particles at rest: $E = mc^2$.

3.5 Why c Is the Speed Limit

Spacetime is constituted by accumulated shells. As a particle's velocity approaches c, it approaches the leading edge of its own shells. At $v = c$, the particle would be on its own shell surface—the boundary of the spacetime it has created. Beyond c, there are no shells and therefore no spacetime to travel through. The speed limit is a geometric property of spacetime itself.

4 Mass Threshold Hierarchy

The shell framework contains a natural hierarchy of energy scales determined by the relationship between electromagnetic radiation and mass creation.

Below ∼1 MeV: Pure radiation. Electromagnetic energy propagates through spacetime as $l = 1$ correlations between shell arrivals (Section 5.2). At these energies, radiation cannot create mass and propagates freely.

∼1 MeV (electron mass): Pair production threshold. Above $2m_ec^2 \approx 1.022$ MeV, electromagnetic energy has sufficient density to create electron–positron pairs. This is the natural UV cutoff of the framework: radiation above this energy converts to mass rather than propagating as radiation. No renormalization is required because the mode spectrum has a physical upper bound.

∼140 MeV (pion mass) to ∼938 MeV (proton mass): Strong confinement regime. Energy in this range is confined in closed shell loops—the strong force. Quarks pop at high frequencies but their shells cannot escape; the energy is stored in three-dimensional space between pops as gluon-like loop structures.

Above ∼938 MeV (proton/neutron mass): Spacetime shell creation. The proton (938.3 MeV) and neutron (939.6 MeV) are the lightest stable composite objects above the threshold for creating propagating spacetime shells. They write geometry at a distance: the proton shell spacing is $c/\nu_p = c\,h/(m_pc^2) \approx 1.32$ fm, and the neutron spacing is comparable. Below this scale, energy is confined locally (strong loops, electromagnetic self-energy). Above it, shells propagate and gravity emerges as a macroscopic force. Protons serve as the fundamental clocks, synchronizing and carrying all bound subatomic mass through time via their shell emission cycle. Free neutrons, while above the spacetime threshold, are not independently stable—they decay with a half-life of approximately 10 minutes. Inside a nucleus, neutrons are stabilized by synchronization with the proton's pop cycle.

5 Shell Arrival Statistics and Force Laws

A massive particle at the origin emitting shells at rate ν produces an arrival rate at distance r:

$$\Gamma(r) = \frac{\nu}{4\pi r^2}\,. \tag{13}$$

This is pure geometry: each shell spreads over $4\pi r^2$. The inverse-square law is a consequence of three-dimensional space, not a dynamical equation.

For two sources separated by distance d, the total arrival rate at a field point decomposes into angular multipoles via the Legendre expansion. Each multipole l contributes a distinct angular pattern with characteristic radial scaling $(d/r)^l$.

5.1 $l = 0$: Gravity from the Mean Arrival Rate

The monopole is the angular average. For a source of mass m (emission rate $\nu \propto m$), the mean arrival rate at distance r gives a coincidence rate with a test mass m_2 that scales as $m_1 m_2/r^2$—Newton's law. The mean is always positive and additive, so the monopole interaction is universally attractive and couples to all mass. G_N enters as an empirical parameter.

5.2 $l = 1$: Electromagnetism from Arrival Rate Covariance

The dipole encodes angular asymmetry. While gravity arises from the mean (first moment), electromagnetism arises from the covariance (second moment) of shell arrivals. The key mechanism is the phase relationship between emitters: in-phase emission produces bunched statistics (repulsion from increased jitter), while π-phase-offset emission produces smoothed statistics (attraction from reduced jitter). This phase offset is identified with electric charge. The coupling constant α enters as the phase correlation efficiency.

6 Charge Quantization and Antiparticles

If charge is a phase offset, quantization is constrained by consistency around closed loops. For three particles with pairwise offsets:

$$\phi_{12} + \phi_{23} + \phi_{31} = 0 \pmod{2\pi}\,. \tag{14}$$

The simplest nontrivial solution with equal offsets gives $\phi = 2\pi/3$: three charges separated by 120°, summing to zero—the structure of color charge. For two-particle systems, maximal interaction at $\phi = \pi$ gives particle–antiparticle pairs.

7 The Electroweak Sector: $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ from $l = 1$

The $l = 1$ dipole has three components ($m = -1, 0, +1$) transforming as a vector under rotations. The spin-1 angular momentum matrices J_1, J_2, J_3 acting on this space satisfy:

$$[J_i, J_j] = i\,\epsilon_{ijk}\,J_k\,. \tag{15}$$

In the $\{|+1\rangle, |0\rangle, |-1\rangle\}$ basis:

$$J_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad J_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{16}$$

Direct computation verifies $[J_1, J_2] = iJ_3$ and cyclic permutations. The $\mathrm{SU}(2)_L$ of the weak interaction is the rotation symmetry of the $l = 1$ dipole sector.

The $\mathrm{U}(1)_Y$ hypercharge arises from the phase symmetry of shell emission: the RMS arrival rate is invariant under global phase rotations $\psi \to e^{i\alpha}\psi$.

The retarded propagator adds a time-orientation to the dipole, creating an axial component that violates parity. The full $l = 1$ sector contains both vector (electromagnetic) and axial-vector (weak) components. Their mixing corresponds to the Weinberg angle. The $l = 1$ dipole plus retardation provides: $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with $3 + 1 = 4$ generators.

8 The Color Sector: $\mathrm{SU}(3)_C$ from $l = 2$ Plus Retardation

8.1 Three-Axis Quadrupole and Color Neutrality

The quadrupole ($l = 2$) is computed over three orthogonal spatial axes. Each axis defines a pattern $P_2(\cos\theta_i)$ satisfying an exact identity:

$$P_2(\cos\theta_x) + P_2(\cos\theta_y) + P_2(\cos\theta_z) = 0\,. \tag{17}$$

This follows from $P_2(n_i) = (3n_i^2 - 1)/2$ and $n_x^2 + n_y^2 + n_z^2 = 1$. The three axial patterns span a two-dimensional subspace and form vectors summing to zero—the geometric analog of color neutrality.

8.2 Completing the Algebra: $5 + 3 = 8$

The $l = 2$ symmetric traceless tensor in three dimensions has five independent components. The full $\mathfrak{su}(3)$ algebra is eight-dimensional. The remaining three generators arise from the antisymmetric phase structure introduced by retardation: the arrow of time (A2) creates a phase asymmetry in each of the three independent rotation planes of three-dimensional space (xy, xz, yz). Each plane contributes one antisymmetric generator. Three spatial dimensions, three rotation planes, three antisymmetric generators.

This decomposition is verified against the Gell-Mann matrices [4]: λ_1, λ_3, λ_4, λ_6, λ_8 are real symmetric (five); λ_2, λ_5, λ_7 are imaginary antisymmetric (three). The $5 + 3 = 8$ split is basis-independent: the adjoint representation of SU(3) decomposes under transposition into a five-dimensional symmetric and three-dimensional antisymmetric subspace regardless of basis choice.

8.3 Short-Range Force and Flux Tube Geometry

The $l = 2$ contribution falls as d^2/r^4 in the far field, producing a force scaling as $1/r^5$—naturally short-range. The nodal structure of $P_2(\cos\theta)$ channels the field into axial concentrations between sources, qualitatively resembling QCD flux tubes.

9 The Complete Standard Model Gauge Group

Table 1: Origin of the Standard Model gauge group from retarded shell multipoles.

Multipole	Symmetric (geometry)	Antisymmetric (retardation)	Total	Gauge group
$l = 0$	1	0	1	Gravity
$l = 1$	3	1 (phase)	$3 + 1$	$\mathrm{SU}(2) \times \mathrm{U}(1)$
$l = 2$	5	3 (rotation planes)	8	SU(3)
Total	9	4	$12 + 1$	SM + Gravity

All 12 generators of $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ plus gravity are accounted for with none missing and none extra.

10 Why Exactly Four Fundamental Forces

The interaction strength of multipole l scales as $(d/r)^l/r$. For $l \geq 3$, this falls as d^3/r^4 or faster—negligible beyond the source size. Higher multipoles are contact interactions, not fundamental forces. Exactly four forces arise as a geometric consequence of three-dimensional space.

Furthermore, SU(3) requires exactly three orthogonal axes. In $d = 2$, only SU(2) is possible. In $d = 4$, SU(4) would appear. The observed SU(3) is consistent only with $d = 3$.

11 Confinement and the QCD String Tension

11.1 Quarks as Captured Shell Loops

Quarks have Compton frequencies substantially higher than the inverse confinement scale (~ 1 fm). Their shells cannot propagate to infinity: they are captured by neighboring quarks and curve back, forming closed loops. The strong force is gravity in the regime where shell loops close.

This explains several QCD features simultaneously. The force is short-range (loops are closed). It is confining (separating quarks stretches the loops, costing energy proportional to distance). And 99% of proton mass is binding energy (dominated by loop energy, not bare quark masses: $m_u + m_u + m_d \approx 9$ MeV vs. $m_p = 938$ MeV). Gluons correspond to the loops themselves—they have no independent shell emission and therefore cannot propagate freely. The loops are not static arcs but rotate at the constituent quark frequency. The time-averaged energy distribution of a rotating semicircular arc, integrated over one full rotation period, produces a cylindrically symmetric energy density concentrated along the axis between the two quarks—the tube-like geometry observed in lattice QCD simulations of the chromoelectric flux tube.

11.2 String Tension from Semicircular Loop Geometry

The shell loop follows a semicircular arc of radius $d/2$ between quarks at separation d. The intrinsic energy density along the loop is $\sigma_0 = m_{\mathrm{const}}^2$ in natural units, where m_{const} is the constituent quark mass. The arc length is $\pi d/2$, so the total loop energy is $E = m_{\mathrm{const}}^2 \cdot \pi d/2$. The string tension is:

$$\sigma = \frac{dE}{dd} = m_{\mathrm{const}}^2 \cdot \frac{\pi}{2}\,. \tag{18}$$

For $m_{\mathrm{const}} = 338$ MeV:

$$\sigma = (0.338\ \mathrm{GeV})^2 \times \frac{\pi}{2} = 0.1795\ \mathrm{GeV}^2\,. \tag{19}$$

The observed value is $\sigma \approx 0.18$ GeV^2, giving agreement to 0.3%. Exact agreement requires $m_{\mathrm{const}} = 338.5$ MeV. Sensitivity: $m_{\mathrm{const}} = 330$ MeV gives $\sigma = 0.171$ GeV^2 (5% off); $m_{\mathrm{const}} =$

345 MeV gives $\sigma = 0.187$ GeV^2 (3.9% off). The constituent quark mass is an effective quantity within the literature range 330–350 MeV; the semicircular geometry is the simplest closed path but has not been derived from shell dynamics.

11.3 Linear Confinement and Pair Creation

The loop geometry produces linear confinement: $E \propto d$, constant force at large separation. When d is large enough that the loop energy exceeds the pair-creation threshold, the loop snaps and each end forms a new closed loop—string breaking.

12 The 125 GeV Resonance as Electroweak Sector Magnitude

12.1 Degree-of-Freedom Counting

The $l = 1$ sector has four degrees of freedom: three from SU(2) rotation and one from U(1) phase. Through jitter stabilization (the process by which the RMS fluctuation pattern settles into a stable configuration), three modes acquire mass (W^+, W^-, Z^0) and one remains massless (γ). Four in, four out. No fifth degree of freedom exists.

12.2 The Sector Magnitude

The total energy of the $l = 1$ sector, computed as the Pythagorean magnitude including escaped photon energy, is:

$$m_H^2 = m_W^2 + m_Z^2 + 2v^2\alpha\,, \tag{20}$$

where $m_W^2 = 6460.8$ GeV^2 (charged jitter modes), $m_Z^2 = 8315.2$ GeV^2 (neutral jitter mode), and $2v^2\alpha = 884.8$ GeV^2 (photon energy escaped during stabilization: $v = 246.22$ GeV is the electroweak VEV, α the fine-structure constant, factor of 2 for both polarizations).

$$m_H = \sqrt{6460.8 + 8315.2 + 884.8}\ \text{GeV} = 125.14\ \text{GeV}\,. \tag{21}$$

The observed resonance is 125.10 ± 0.14 GeV [2, 3]: agreement to 0.03%.

If correct, the "Higgs boson" is not a fundamental particle but the total energy magnitude of the electroweak jitter structure. The observed decay channels are the ways this energy manifests through W and Z couplings. Branching ratios are identical to Standard Model predictions because fermions couple to the $l = 1$ sector in proportion to their mass, and therefore cannot distinguish between interpretations.

12.3 Distinguishing Tests

Higgs self-coupling. If the 125 GeV resonance is not a particle, di-Higgs production should be absent or suppressed. The High-Luminosity LHC is expected to measure this.
Off-shell behavior. A sector magnitude does not have a propagator extending above its mass. High-energy off-shell measurements could probe this.

13 Electron Structure and the g-Factor

13.1 The Electron Pop Cycle

The electron (mass 0.511 MeV) is below the threshold for creating propagating spacetime shells. It pops at its Compton frequency ($\nu_e = m_e c^2/h \approx 1.236 \times 10^{20}$ Hz), but its energy between pops is stored in the local magnetic field configuration, not in propagating spacetime disturbances.

Each pop reinforces the magnetic field. When the electron pops out (absent from 3D space), the field it created begins to decay but does not fully collapse before the next pop. The field is the energy storage mechanism that conserves the electron's energy between appearances.

13.2 $g = 2$ from Energy Conservation

The magnetic moment of a charged particle with angular momentum S has a baseline value $\mu = (e/2m)S$, corresponding to $g = 1$. In the shell framework, the electron's total energy is split between two phases of the pop cycle: present in 3D (creating the field) and absent (field holding the energy). Each phase contributes equally to the measured magnetic moment because they carry the same energy in two states of the same cycle. Energy conservation requires the stored field to return exactly one unit of magnetic moment when the electron pops back in:

$$g = 1\ (\text{pop}) + 1\ (\text{stored field}) = 2\,. \tag{22}$$

This is the Dirac value, arising from energy conservation in the pop cycle rather than from the Dirac equation.

13.3 The Schwinger Correction from Jitter

In a universe containing other matter, the electron is not stationary between pops. It jitters through the spacetime written by the proton (or nucleus) it is bound to, driven by the local thermal and quantum environment. When it pops back in, it is slightly displaced from where it popped out. The stored magnetic field at the new position differs slightly from where it was created, producing a small correction to the handoff.

This correction scales as α (the electromagnetic self-interaction coupling) divided by π (geometric averaging over jitter directions):

$$\frac{g-2}{2} = \frac{\alpha}{\pi} + \mathcal{O}(\alpha^2) \approx 0.00116\,. \tag{23}$$

The measured value is $(g-2)/2 = 0.00115965218$ (CODATA 2022). The leading term matches to 0.15%.

The correction is positive ($g > 2$) because the field misalignment from jitter adds a small extra torque—the displaced electron pops into a field configuration that gives it a slight additional kick. Energy is conserved; the jitter energy comes from the thermal and gravitational environment.

In a hypothetical universe containing only one proton and one electron with no thermal background, $g = 2$ exactly: no jitter, no displacement, perfect handoff.

13.4 The Bohr Radius from Shell Spacing

The proton pops at its Compton frequency $\nu_p = m_p c^2/h \approx 2.27 \times 10^{23}$ Hz, creating spacetime shells spaced by $c/\nu_p \approx 1.32$ fm (the proton scale). The electron, bound to the proton, must sit in a jitter-stable node of the proton's shell pattern—a region of low RMS fluctuation where it can resonate stably with the nuclear pop cycle.

The electron's reduced Compton wavelength is $\bar{\lambda}_e = \hbar/(m_e c) \approx 3.86 \times 10^{-13}$ m. The electromagnetic coupling α determines how strongly the electron's pop cycle couples to the proton's shell pattern. The stable resonance condition gives:

$$a_0 = \frac{\bar{\lambda}_e}{\alpha} = \frac{\hbar}{m_e c \alpha} = 5.292 \times 10^{-11} \text{ m} . \tag{24}$$

This is the Bohr radius exactly. It is not set by abstract quantum numbers but by the physical distance at which the electron's intrinsic scale and its electromagnetic coupling to the proton's shell geometry balance.

Adding neutrons to the nucleus (isotopes) changes the shell pattern: more baryons popping means a wider, more complex interference structure. More protons and more neutrons create more jitter-stable nodes at different radii and angles—the electron orbitals of heavier elements. The shapes of these orbitals (s, p, d, f) reflect the geometry of the nuclear shell interference pattern.

14 Weinberg Angle at the Unification Scale

At energies where all multipole sectors contribute equally, the electromagnetic fraction of the electroweak–strong generator space is determined by multipole dimensions. The trace normalization condition for the hypercharge generator Y relative to the SU(2) and SU(3) generators, evaluated over one fermion generation whose content is geometrically constrained (three colors from $d = 3$, electroweak doublets from $l = 1$), gives:

$$\sin^2 \theta_W = \frac{3}{3+5} = \frac{3}{8} . \tag{25}$$

This equals the SU(5) grand unified theory prediction [5]. In the Standard Model, this runs to ≈ 0.231 at the Z-pole. The framework constrains the gauge groups and color multiplicity geometrically, which constrains the generator traces. The specific hypercharge assignments ($Y = 1/3$ for quarks, $Y = -1$ for leptons) are consistent with the phase-loop quantization of Section 6 but have not been uniquely derived from A1–A3.

15 CP Violation and the Arrow of Time

Assumption A2 imposes a fundamental arrow of time. The strong CP problem—why QCD does not violate CP despite the equations allowing it—dissolves: the strong force consists of closed shell loops propagating forward in time. There is no theta parameter because the time-ordering is physical. Matter–antimatter asymmetry is built into the framework at the level of assumptions: matter is the configuration that pops forward in time, and A2 excludes the time-reversed alternative. The asymmetry is not generated by a small perturbation but is a structural feature of retarded propagation.

16 Additional Structural Results

16.1 Pauli Exclusion from Shell Statistics

Two identical particles at the same location emit identical shells, producing maximum variance (jitter). Since stable configurations minimize jitter, identical particles in the same state are maximally unstable and are driven apart—the Pauli exclusion principle without an additional postulate.

16.2 Spin-$1/2$ from Retardation

The electron's pop cycle (A1) creates a two-state system: present in 3D and absent. Retardation (A2) makes the two halves physically inequivalent—popping in (creating a field in a pre-existing environment) is not the time-reverse of popping out (leaving a field to propagate). The combined PT transformation satisfies $(PT)^2 = 1$ with $PT \neq 1$—the Kramers doublet structure characteristic of half-integer spin. The full identity requires traversing the pop cycle twice (4π), the defining property of spin-1/2.

A complete construction of the spinor representation from shell geometry, including the specific transformation matrices, remains an open problem.

16.3 Three Particle Generations

The $l = 2$ quadrupole is defined over three orthogonal spatial axes. Each axis supports a distinct quadrupole pattern, providing a geometric basis for exactly three fermion generations. The CKM mixing matrix corresponds to misalignment between $l = 2$ (mass/color) and $l = 1$ (weak) axes. The mass hierarchy between generations has not been derived.

16.4 The Higgs Field as Jitter Background

The Higgs field of the Standard Model corresponds to the baseline RMS fluctuation of the $l = 1$ sector. The jitter pattern settles into a stable configuration where SU(2) modes acquire mass (W, Z) and one remains massless (photon). The 125 GeV resonance is a perturbation of this background (Section 12).

16.5 Virtual Particles as Energy in Transit

Virtual particles correspond to electromagnetic waves ($l = 1$ correlations) carrying energy between stable masses during intervals when those masses are absent from 3D space. Energy conservation is maintained because energy alternates between mass (present) and radiation (absent). Vacuum energy is finite because the number of masses is finite.

17 Qualitative Implications

The following are consistent with the framework at a qualitative level. Quantitative derivations have not been completed.

17.1 Black Holes

Mass inside a black hole continues to pop and emit shells. These shells propagate at c through the interior geometry but cannot escape classically. However, the interior shell density accumulates with each pop—shells stack but cannot stack infinitely tight due to discrete spacing. Over time, the accumulated interior shell pressure may exceed the exterior trapping condition, allowing energy to leak out. Smaller black holes have less interior volume, reach the stacking threshold sooner, and would radiate faster—consistent with the inverse-mass dependence of Hawking radiation. A precise calculation requires knowledge of the interior mass distribution.

17.2 Quantum Entanglement

The framework contains no nonlocal interactions. Correlated particles share phase relationships encoded in their shell structures from the moment of creation. Measurement reads information already present in the local shell configuration. Bell inequality violations would arise if shell phase correlations exceed classical hidden-variable bounds, but a rigorous derivation has not been performed.

17.3 Atomic Structure

Nuclear shell emissions create interference patterns with regions of low jitter. Electrons exist stably in these low-jitter zones—the orbitals. Discrete energy levels correspond to specific radii where jitter-stable nodes occur. Chemical bonds arise from shared jitter-stable zones between adjacent nuclei. Molecular geometry is determined by the shell interference pattern of constituent nuclei.

18 Empirical Inputs

The framework treats the following as measured inputs, not derived quantities:

- **Particle masses.** Shell emission frequencies are set by observed masses. The mass hierarchy across generations is not derived.
- **The fine-structure constant α.** The phase correlation efficiency of the $l = 1$ sector. Whether derivable from A1–A3 remains open.
- **Newton's constant G_N.** The effective coupling of the complete local shell environment, including the full retarded history of all baryonic matter and primordial plasma in the past light cone [1]. An environmental quantity measured at $\eta = 1$, not a fundamental constant.

- **The electroweak VEV v.** The total energy scale of the $l = 1$ sector.

19 Structural Constraints

The following are structural consequences of the framework:

1. **No fifth force.** $l \geq 3$ multipoles are contact interactions in 3D. Discovery of a fifth force with octupole symmetry would falsify the framework.
2. **No superpartners.** The framework generates exactly the Standard Model gauge group. Continued null results at colliders are consistent.
3. **Gravitational wave polarizations.** Only tensor $(+, \times)$ modes at c. Scalar or vector gravitational wave polarizations would contradict the monopole origin of gravity.
4. **Color group is** $\mathrm{SU}(3)$**.** Three-axis quadrupole constrains color to $\mathrm{SU}(3)$.
5. **Three spatial dimensions.** $\mathrm{SU}(3)$ is consistent only with $d = 3$.
6. **Three generations only.** Three spatial axes, three quadrupole patterns, three fermion generations.
7. **No fundamental scalars.** The framework generates no spin-0 particles. The 125 GeV resonance is a sector magnitude, not a particle.
8. **No Higgs self-coupling.** Di-Higgs production should be absent or suppressed.
9. **Weinberg angle.** $\sin^2\theta_W = 3/8$ at the unification scale, consistent with $\mathrm{SU}(5)$ boundary condition.

20 Open Problems and Limitations

Coupling constants. The values of G_N, α, G_F, and α_s are not determined by A1–A3.

Particle masses and generations. The mass hierarchy ($m_e : m_\mu : m_\tau = 1 : 207 : 3477$) is not derived. Extensive numerical exploration did not produce mass ratios from pure geometry.

Spinor representation. The spin-1/2 argument (Section 16.2) provides the correct structure but a complete mathematical construction of spinors from shell geometry is needed.

Confinement. The semicircular loop geometry produces the correct string tension but has not been derived from shell dynamics. A first-principles derivation of loop geometry would strengthen the result.

Higgs mass formula. The $2v^2\alpha$ photon escape term is physically motivated but not rigorously derived from A1–A3. The 0.03% agreement is striking but the derivation would benefit from a more formal treatment.

Hypercharge assignments. The specific values $Y = 1/3$ (quarks) and $Y = -1$ (leptons) are consistent with phase-loop quantization but not uniquely derived.

Radiative corrections. The framework does not address renormalization or loop-level effects beyond the leading Schwinger term.

21 Conclusion

The Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ plus gravity is the unique gauge structure that emerges from the multipole decomposition of retarded discrete shell arrival statistics in three-dimensional space. Each piece has a geometric origin: SU(3) from three-axis quadrupole plus retardation across three rotation planes, SU(2) from dipole rotation, U(1) from emission phase, gravity from the mean arrival rate. Exactly four forces arise because only $l = 0, 1, 2$ produce non-contact interactions in 3D.

The framework reproduces quantitative results—string tension (0.3%), 125 GeV resonance (0.03%), $g = 2$ from energy conservation, Schwinger correction α/π, Bohr radius $\bar{\lambda}_e/\alpha$—using zero free parameters beyond measured masses and couplings. The natural UV cutoff at the electron mass eliminates the vacuum energy problem without renormalization.

The decisive tests remain the CMB power-spectrum re-fit described in [1], the Higgs self-coupling measurement at the High-Luminosity LHC, and continued precision tests of the structural constraints.

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References

[1] A. T. Hawkins, "Fundamental Gravitational Retarded Spacetime Shells: Cosmic Scale," preprint (2026).

[2] ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012).

[3] CMS Collaboration, Phys. Lett. B **716**, 30 (2012).

[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[5] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[6] Particle Data Group, R. L. Workman *et al.*, Phys. Rev. D **110**, 030001 (2024).

[7] J. Schwinger, Phys. Rev. **73**, 416 (1948).

[8] R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. B **28**, 128 (1968).

[9] A. Einstein, Annalen der Physik **17**, 891 (1905).

[10] ACME Collaboration, V. Andreev *et al.*, Nature **562**, 355 (2018).

[11] LIGO/Virgo/KAGRA Collaboration, Phys. Rev. D **103**, 122002 (2021).

[12] LIGO/Virgo Collaboration, Phys. Rev. Lett. **119**, 161101 (2017).

[13] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).